B production asymmetries in perturbative QCD

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This paper explores a new mechanism for *B* production in which a $\bar{b}$ quark combines with a light parton from the hard-scattering process before hadronizing into the *B* hadron. This heavy-quark recombination mechanism can be calculated within perturbative QCD up to a few nonperturbative constants. Though suppressed at large transverse momentum by a factor $\Lambda_{\text{QCD}} m_b / p_\perp^2$ relative to $\bar{b}$ fragmentation production, it can be important at large rapidities. A signature for this heavy-quark recombination mechanism in $p\bar{p}$ colliders is the presence of rapidity asymmetries in *B* cross sections. Given reasonable assumptions about the size of nonperturbative parameters entering the calculation, we find that the asymmetries are only significant for rapidities larger than those currently probed by collider experiments.

I. HEAVY QUARK PRODUCTION

Perturbative QCD calculations of heavy hadron production are based on the factorization theorems of QCD [1]. These theorems imply that, for sufficiently large transverse momentum, production of a heavy-light hadron occurs in two stages: first, a heavy quark is produced in a short-distance partonic process; second, the heavy quark fragments into the hadron. They allow cross sections to be factorized into partonic cross sections and fragmentation functions. For example, the formula for the *B* production cross section in a gluon-gluon collision is

$$d\sigma[gg \to B + X] = d\hat{\sigma}[gg \to b\bar{b}] \otimes D_{\bar{b} \to B}. \tag{1}$$

(We denote hadrons with *b* quarks as $\bar{B}$ and hadrons with $\bar{b}$ antiquarks as *B*.) The symbol $\otimes$ represents convolution over the final-state momentum fraction, *z*, of the *B* hadron in the $\bar{b}$ quark jet. The fragmentation function $D_{\bar{b} \to B}(z)$ is a universal function that involves long-distance physics and hence cannot be computed perturbatively. On the other hand, the partonic cross section $d\hat{\sigma}[gg \to b\bar{b}]$ depends only on short-distance physics and can be calculated in perturbation theory. To obtain cross sections for hadronic collisions, partonic cross sections such as Eq. (1) are convolved with parton distribution functions for the initial-state partons.

The QCD factorization theorems are exact only in the limit of large transverse momentum, $p_\perp \to \infty$. For finite $p_\perp$, one expects corrections that are suppressed by powers of $\Lambda_{\text{QCD}}/p_\perp$. In this paper we will argue that there are $O(\Lambda_{\text{QCD}} m_b / p_\perp^2)$ corrections that are calculable within perturbative QCD up to a few multiplicative constants. These corrections come from processes in which a light quark *q* participating in the hard-scattering process combines with

the $\bar{b}$ and they subsequently hadronize into a state including the *B* hadron. We will refer to this as a $\bar{b}q$ recombination process. In $p\bar{p}$ colliders, this process can be distinguished qualitatively from $\bar{b}$ fragmentation because it gives interesting rapidity asymmetries in *B* hadron production. Though not important in the central rapidity region, $\bar{b}q$ recombination can actually give a larger contribution than $\bar{b}$ fragmentation at large rapidities.

Before describing this mechanism in detail, it is worthwhile to briefly summarize the current status of *B* production. Theoretical calculations of heavy quark production have been performed to next leading order (NLO) in perturbative QCD [2–4]. A comparison of these perturbative QCD calculations with collider data is reviewed in [5,6]. At the Fermilab Tevatron [7–11], the shape of the $p_\perp$ distribution is in agreement with experiments, while its normalization appears to be somewhat underpredicted by the theory. However, even at NLO the theoretical prediction suffers from considerable uncertainty due to dependence on the renormalization scale. Setting the renormalization scale, μ_R, equal to $(p_\perp^2 + m_b^2)^{1/2}$ gives a prediction that is roughly a factor of 2 below experiment, while for $\mu_R \approx (p_\perp^2 + m_b^2)^{1/2}/2$ the theoretical prediction can accomodate the data. The strong sensitivity to the renormalization scale indicates that higher order perturbative corrections are important. Recent studies that attempt to resum logarithmically enhanced higher order corrections can be found in [12–15]. The resummation of higher order terms of the form $\alpha_s^n \ln^n(p_\perp/m_b)$ and $\alpha_s^n \ln^{n+1}(p_\perp/m_b)$ gives results with reduced scale dependence and the prediction of the resummed calculation is similar to that of the fixed NLO calculation with the lower values of μ_R. Thus it seems at least possible that higher order perturbative QCD calculations would account for the shape and normalization of the experimentally observed $p_\perp$ distribution.

Most measurements of heavy quark production focus on the central rapidity region. Less is known about the cross section in the forward regions. The DØ collaboration [16] has measured the production of muons coming from *b* decay in the rapidity range $2.4 < |y^\mu| < 3.2$. They found that the excess over the theoretical prediction in the forward region is

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greater than in the central region, indicating perhaps that large rapidity b production is not as well understood. If one chooses μ_R so that the central region is described correctly, the NLO calculation of the cross section at large rapidity is too small by a factor of 2. It has been suggested that modification of the heavy quark fragmentation function [6] or a reorganization of the perturbative calculation [13] could modify the forward-to-central rapidity ratio. However, Collider Detector at Fermilab (CDF) [10] measurements of $b\bar{b}$ rapidity correlations agree well with NLO QCD calculations. CDF measured the ratio of central ($|y|<0.6$) to forward ($2.0<|y|<2.6$) b production subject to the constraint that the other b was produced with $|y|<1.5$ and that both b's had $p_\perp>25$ GeV. It is important to point out that one cannot directly compare the DØ and CDF experiments because (a) they involve slightly different rapidity regions, (b) DØ makes no cuts on the second b produced in the event and (c) most importantly, the DØ experiment uses a much smaller $p_\perp$ cut of 5 GeV.

II. HEAVY-QUARK RECOMBINATION MECHANISM

Our investigation of novel mechanisms for B production is motivated in part by the possibility that there are important contributions to B production at large rapidity that are not accounted for in Eq. (1). This paper will consider contributions to B production in which a light quark involved in the short-distance process combines with the $\bar{b}$ and they subsequently hadronize into the final-state B meson. This $\bar{b}q$ recombination contribution is distinct from what is included in standard fragmentation calculations, where the parton process $qg\to b+\bar{b}+q$ is taken into account at next-to-leading order. Over most of the phase space, the q in the final state produces a jet that is distinct from those produced by the b and $\bar{b}$. There is a small contribution from the corner of phase space where the q has small momentum in the rest frame of the $\bar{b}$. However, in this region, the q and $\bar{b}$ can bind and large nonperturbative effects can enhance the cross section. Our mechanism takes these nonperturbative effects into account.

The word "recombination" in high energy physics dates back to the *parton recombination model* invented by Das and Hwa in 1977 to describe the production of mesons at low $p_\perp$ in hadronic collisions [17]. In this model, most of the momentum of the meson comes from the momentum of a valence parton in one of the colliding hadrons. The meson is formed by that valence parton recombining with a sea parton from that same hadron. Our recombination mechanism for heavy meson production is analogous in that it also allows a valence parton to deliver most of its momentum to the heavy meson, although in this case the valence parton combines with a heavy quark created in the hard scattering. Our recombination mechanism is also analogous to a "higher twist" mechanism for the inclusive production of light mesons at large $p_\perp$ studied by Berger, Gottschalk, and Sivers [18]. This mechanism also involves a parton from the colliding hadron that is involved in the hard scattering and ends up as a constituent of the large-$p_\perp$ meson. The cross section is suppressed by $\Lambda_{QCD}^2/p_\perp^2$ relative to the fragmentation mecha-

nism, and the nonperturbative factor involves a function that describes the distribution of the light-cone momentum in the meson. In our mechanism for heavy meson production, the cross section is suppressed only by $\Lambda_{QCD}m_b/p_\perp^2$, and the nonperturbative factor can be reduced to a few numbers. For $p_\perp\sim m_b$, the contribution from our mechanism is suppressed only by Λ_{QCD}/m_b. In this region of $p_\perp$, there are other contributions suppressed by Λ_{QCD}/m_b in which the parton that combines with the heavy quark is a spectator not involved in the hard scattering [19]. String fragmentation models for hadronization provide an explicit model for these contributions [20]. Another mechanism that gives contributions at $p_\perp\gg m_b$ that are suppressed by Λ_{QCD}^2/m_b^2 is intrinsic heavy flavor [21,22]. These other mechanisms may give contributions to the integrated cross section that are comparable to that of our heavy-quark recombination mechanism, but they should be smaller for $p_\perp\gg m_b$. Even for $p_\perp\sim m_b$, our mechanism may be numerically important in certain kinematic regions, such as large rapidity, because it allows most of the momentum of a valence parton to be delivered to the heavy meson.

To motivate our recombination mechanism for heavy mesons, it is helpful to make an analogy with quarkonium production [24–27]. Quarkonium production cross sections also obey a factorization formula:

$$d\sigma[ij\to H+X]=\sum_n d\hat{\sigma}[ij\to(Q\bar{Q})^n+X]\langle 0|\mathcal{O}_n^H|0\rangle,$$
(2)

where i,j represent initial-state partons and $Q\bar{Q}$ represents the heavy quark-antiquark pair. The $Q\bar{Q}$ are produced in the short-distance process with small relative momentum in a state with definite angular momentum and color quantum numbers, which are collectively denoted by n. The $Q\bar{Q}$ pair in the state n subsequently evolves into a final hadronic state that includes the quarkonium, H. The probability for this is given by a vacuum matrix element of a local operator in nonrelativistic QCD (NRQCD), denoted by $\mathcal{O}_n^H$. If the $Q\bar{Q}$ produced in the short-distance process is a color-singlet and has the same angular momentum quantum numbers as the quarkonium being produced, the matrix element has a physical interpretation in terms of the nonrelativistic wave function of the $Q\bar{Q}$ pair in the bound state.

The quarkonium factorization formula suggests that there are analogous contributions to B production:

$$d\sigma[qg\to B+X]=\sum_n d\hat{\sigma}[qg\to(\bar{b}q)^n+b]\,\rho[(\bar{b}q)^n\to B].$$
(3)

(A similar mechanism for $\bar{B}$ production is obtained by replacing $b\leftrightarrow\bar{b}$ and $q\to\bar{q}$.) The notation $(\bar{b}q)^n$ means that the q has small momentum in the $\bar{b}$ rest frame and that the $\bar{b}$ and the q are in a state with definite color and angular momentum quantum numbers specified by n. The factor $\rho[(\bar{b}q)^n\to B]$

represents the probability for $(\bar{b}q)^n$ to evolve into a final state that includes the *B* hadron. We refer to the process in Eq. (3) as $\bar{b}q$ recombination.

Note that the color and spin quantum numbers of the $(\bar{b}q)^n$ need not coincide with those in the quark model for the *B* hadron. The $(\bar{b}q)^n$ can evolve into a state that contains the *B* hadron and an arbitrary number of partons that have small momentum in the *B* hadron rest frame. In this nonperturbative transition, soft gluons can radiate color and flip the spin of the light quark so the $(\bar{b}q)^n$ produced in the short-distance process need not be a color-singlet and its angular momentum quantum numbers can differ from those of the final-state *B* hadron. In quarkonia production, these nonperturbative transitions are governed by the multipole expansion and the nonperturbative matrix elements appearing in Eq. (2) scale with definite powers of the typical velocity, v, of the heavy quarks in the bound state. Those transitions for which the color and angular momentum quantum numbers of the $(Q\bar{Q})^n$ do not match those of *H* are suppressed by powers of v, and are therefore controlled by the heavy quark mass. We expect no such suppression for the parameters $\rho[(\bar{b}q)^n \rightarrow B]$ in Eq. (3).

Furthermore, it is important to observe that the *B* meson need not have the same light quark flavor quantum numbers as the $(\bar{b}q)^n$ pair that is produced in the short-distance process. The hadronization process can involve the creation of a light quark-antiquark pair, $q'\bar{q}'$, and the $\bar{b}$ can bind with the q' to form a *B* meson with a different light quark flavor from the $(\bar{b}q)^n$ that is produced in the hard scattering. This is an important difference from quarkonium production, where the total heavy quark and heavy antiquark numbers are separately conserved because the hadronization process is governed by long-distance physics. Light quark-antiquark pair production could of course have a nonperturbative suppression factor. For instance, there is such a suppression factor in the large N_c limit of QCD. Thus a $(\bar{b}u)^n$ could prefer to hadronize into a B^+ rather than a B^0. If such a suppression factor exists, it is not controlled by the heavy quark mass and should therefore be roughly the same for *b* and *c* hadrons. The flavor dependence of the nonperturbative factor $\rho[(\bar{b}q)^n \rightarrow B]$ is important for predictions of flavor-specific *B* hadron production cross sections. In this paper, we will for simplicity consider only cross sections that are summed over all *B* hadrons. For this inclusive cross section, the nonperturbative factor in Eq. (3) can therefore be denoted simply by ρ_n.

It is only necessary to consider $(\bar{b}q)^n$ with relative orbital angular momentum $L=0$. We will show below that the recombination constants $\rho[(\bar{b}q)^n \rightarrow B]$ for $L=0$ are proportional to Λ_{QCD}/m_b, so the recombination contributions to the cross section are suppressed by $\Lambda_{QCD}m_b/p_\perp^2$. Producing $(\bar{b}q)^n$ in a higher orbital angular momentum state requires keeping additional factors of the light quark momentum in the hard-scattering matrix element. The light quark momentum in the bound state is $O(\Lambda_{QCD})$ and these additional factors of Λ_{QCD} must be compensated by powers of m_b or $p_\perp$.

So up to corrections suppressed by additional powers of Λ_{QCD}/m_b or $\Lambda_{QCD}/p_\perp$, it is sufficient to compute production of $(\bar{b}q)^n$ in the spectroscopic states $^1S_0^{(1)}$, $^3S_1^{(1)}$, $^1S_0^{(8)}$ and $^3S_1^{(8)}$, where the superscript in parentheses indicates either color-singlet or color-octet. Furthermore, heavy quark spin symmetry [28,29] relates the transition amplitudes of $^1S_0$ and $^3S_1$ states. Thus the calculation involves only two nonperturbative parameters, ρ_1 for color-singlet and ρ_8 for color-octet production. Heavy quark flavor symmetry can also be used to relate the production parameters for *b* hadrons to those for *c* hadrons.

In most regions of phase space, $\bar{b}$ fragmentation dominates over the $\bar{b}q$ recombination mechanism. However, the $\bar{b}q$ recombination cross section is peaked in the forward region where the $(\bar{b}q)$ pair emerges in approximately the same direction as the initial-state light quark. As shown below, it is possible for $\bar{b}q$ recombination to be comparable to $\bar{b}$ fragmentation in the forward region. In addition to enhancing the cross section, recombination leads to an interesting rapidity asymmetry in *B* hadron production at $p\bar{p}$ colliders. Let the proton direction define positive rapidity. If the light quark in the initial state is from the proton, the $(\bar{b}q)$, and therefore the *B* hadron, will emerge preferentially with large positive rapidity, while if the initial-state light quark comes from the antiproton, the *B* hadron will prefer to have negative rapidity. Since the proton has more u,d than $\bar{u},\bar{d}$, there should be an excess of $(\bar{b}u)$ and $(\bar{b}d)$ at positive rapidity, while the $\bar{u},\bar{d}$ in the antiproton should lead to an excess of $(b\bar{u})$ and $(b\bar{d})$ at negative rapidity. This rapidity asymmetry is a striking prediction of this mechanism.

Rapidity asymmetries have also been predicted on the basis of string fragmentation models for hadronization [20,23]. In these models, the asymmetry arises from a process called the "beam drag effect" in which the $\bar{b}$ produced by the hard scattering combines with a spectator quark from an initial-state hadron. Since the beam drag effect necessarily involves soft momentum scales, this nonperturbative contribution is part of the corrections to Eq. (1) suppressed by at least $\Lambda_{QCD}^2/p_\perp^2$ at large $p_\perp$. We emphasize that our recombination mechanism generates the asymmetry by a perturbative mechanism in which a $\bar{b}$ combines with a light quark participating in the hard scattering process. Thus, it is suppressed at large $p_\perp$ only by $\Lambda_{QCD}m_b/p_\perp^2$. Our process is therefore more important than the beam drag effect for $p_\perp \gg m_b$. Even for $p_\perp \sim m_b$, our process may be numerically more important at large rapidity, because it allows most of the momentum of a valence parton to be delivered to the heavy meson.

III. HEAVY-QUARK RECOMBINATION CROSS SECTION

This paper presents calculations of the $\bar{b}q$ recombination contribution to the inclusive cross section for *B* hadrons at the Tevatron. We will focus on the rapidity dependence of *B* hadron production. The rapidity dependence of the inclusive cross section for $\bar{B}$ hadrons can be obtained by replacing $y \rightarrow -y$. The leading order diagrams contributing to qg
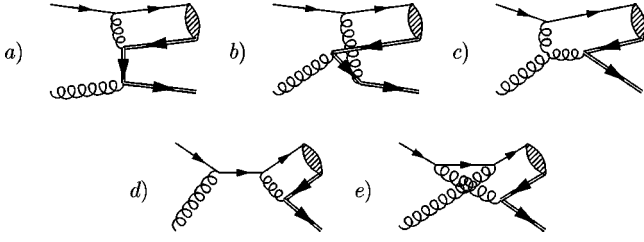
FIG. 1. Diagrams for $\bar{b}q$ recombination into a B meson. Light quarks are single lines, heavy quarks are double lines and the shaded blob is the B meson.

$\rightarrow(\bar{b}q)+b$ are shown in Figs. 1(a)–(e). The single lines represent light quarks and double lines are heavy quarks. The shaded blob represents the transition amplitude for the $(\bar{b}q)$ to hadronize into a state including a B hadron.

Let us denote the momentum of the initial-state light quark as p, the initial-state gluon as l, the final-state light quark as p_q, the momentum of the $\bar{b}$ (which ends up in the B) as p_b, and the momentum of the b quark as k. In the rest frame of the B hadron, p_b has a large component of $O(m_b)$ while all components of p_q are $O(\Lambda_{\mathrm{QCD}})$. Therefore, in the perturbative amplitude we can take $p_q \rightarrow 0$ up to corrections of $O(\Lambda_{\mathrm{QCD}}/m_b)$. The $O(p_q^{-1})$ piece of the amplitude is

$$\mathcal{M}_{(\bar{b}q)} = g_s^3 \bar{u}(p_q)\left(\frac{A_1}{2p \cdot p_q} + \frac{A_2}{2l \cdot p_q}\right) v(p_b) \qquad (4)$$

$$= g_s^3 \mathrm{Tr}\left[v(p_b)\bar{u}(p_q)\left(\frac{A_1}{2p \cdot p_q} + \frac{A_2}{2l \cdot p_q}\right)\right],$$

where A_1 and A_2 are given by

$$A_1 = T^b \gamma^\mu u(p) \cdot \bar{u}(k)\left(T^a T^b \not{\epsilon} \frac{1}{-\not{l}+\not{k}-m_b}\gamma_\mu\right.$$

$$+ T^b T^a \gamma_\mu \frac{1}{\not{l}-\not{p}_b-m_b}\not{\epsilon}$$

$$\left.+ if^{abc}T^c \frac{(\not{l}-\not{p})\epsilon_\mu + 2p\cdot\epsilon\,\gamma_\mu - 2\,l_\mu\not{\epsilon}}{(k+p_b)^2}\right), \qquad (5)$$

$$A_2 = -T^a T^b \frac{\not{k}\not{l}\,\gamma^\mu}{(k+p_b)^2} u(p) \cdot \bar{u}(k) T^b \gamma_\mu .$$

The trace in Eq. (4) is over both Dirac and color indices. The polarization vector of the gluon is ϵ_μ. Contributions to A_1 come from diagrams in Figs. 1(a)–(c), while A_2 comes from Fig. 1(e). Note that Fig. 1(d) does not have any terms that are singular as $p_q \rightarrow 0$, so we neglect its contribution relative to the terms kept in Eq. (5).

At this point we need to project the partonic amplitude onto a matrix element that gives the transition amplitude to a state with a B hadron. Let us assume for the moment that the $(\bar{b}q)$ evolves into the exclusive state consisting of a single B meson, which is possible only if the $(\bar{b}q)$ are in a color-

singlet state. [Later, we will consider the contribution of $(\bar{b}q)$ evolving into final states containing additional soft hadrons.] Then we should replace the spinors in Eq. (4) with the valence light-cone wave function of the B meson. The formalism we use is similar to that used for exclusive B decays and form factors [30–35]. We will employ the conventions of [32,33]. The light-cone wave function of the B meson is

$$\langle B(v)|\bar{q}_{\beta,j}(z)b_{\alpha,i}(0)|0\rangle$$

$$= \frac{\delta_{ij}}{N_c}\frac{if_B m_B}{4}\left[\frac{1-\not{v}}{2}\left\{2\bar{\phi}_+(t) - \not{z}\frac{\bar{\phi}_-(t)-\bar{\phi}_+(t)}{t}\right\}\gamma_5\right]_{\alpha\beta}.$$

$$\qquad (6)$$

In Eq. (6), z is a light-like vector, $p_b = m_b v$ and $t = v\cdot z$. The α, β are Dirac indices, i, j are color indices. For $(\bar{b}q)$ in a $^3S_1^{(1)}$ state, we should replace the B meson with a B^* meson and the γ_5 with $\not{\epsilon}$, where ϵ^μ is the polarization vector of the B^*. The right-hand side is the most general Lorentz decomposition of the non-local matrix element consistent with heavy quark symmetry. The functions $\bar{\phi}_\pm(t)$ are defined so that $\bar{\phi}_\pm(0) = 1$ and f_B is the B meson decay constant. The amplitude for producing the B meson is obtained by making the following substitution in the matrix element for the partonic cross section in Eq. (4) [33]:

$$v_i(p_b)\bar{u}_j(p_q)$$

$$\rightarrow -\frac{\delta_{ij}}{N_c}\frac{if_B}{4}\frac{\not{p}_b-m_b}{2}\int_0^\infty d\omega\left(2\phi_+(\omega) + \int_0^\omega d\eta(\phi_-(\eta)\right.$$

$$\left. - \phi_+(\eta))\gamma^\mu\frac{\partial}{\partial p_q^\mu}\right)\gamma_5\Big|_{p_q=\omega v}. \qquad (7)$$

The functions $\phi(\omega)$ are the Fourier transforms of $\bar{\phi}(t)$. When making this substitution, p_q should be replaced with ωv only after taking derivatives on the amplitude. It is straightforward to take derivatives and integrate by parts to obtain

$$\mathcal{M}_B = \frac{g_s^3}{N_c}\mathrm{Tr}\left[(\not{p}_b-m_b)\gamma_5\left\{\left(f_+ - \frac{\not{p}}{2p\cdot v}f_-\right)\frac{A_1}{2p\cdot v}\right.\right.$$

$$\left.\left.+ \left(f_+ - \frac{\not{l}}{2l\cdot v}f_-\right)\frac{A_2}{2l\cdot v}\right\}\right], \qquad (8)$$

where the $f_\pm$ are defined by

$$f_+ = \frac{f_B}{4}\int_0^\infty d\omega\frac{\phi_+(\omega)}{\omega}, \qquad (9)$$

$$f_- = \frac{f_B}{4}\left(\int_0^\infty d\omega\frac{\phi_-(\omega)-\phi_+(\omega)}{\omega} + \phi_-(0)\right).$$

In the last equation, we have used $\phi_+(0) = 0$. Since $\phi_-(0) \neq 0$, the integral in the definition of f_- is divergent [32,33]. This divergence is cut off by the transverse momentum of the

light quark in the *B* meson. If the cross section depended on f_- it would indicate that the recombination mechanism is sensitive to soft physics. Fortunately, this is not the case. It is possible to show that $\not{p}A_1 = \not{l}A_2 = 0$. For example,

$$\not{l}A_2 \sim \not{l}\not{k}\not{l} = 2\epsilon \cdot l \;\; \not{l} - \not{k}\not{l}^2 = 0. \tag{10}$$

Thus all terms proportional to f_- give a vanishing contribution to the amplitude. The final result for the amplitude for the exclusive recombination production of a *B* meson is

$$\mathcal{M}_B = \frac{g_s^3 m_b f_+}{N_c} \mathrm{Tr}\left[(\not{p}_b - m_b)\gamma_5\left(\frac{A_1}{2p\cdot p_b} + \frac{A_2}{2l\cdot p_b}\right)\right]. \tag{11}$$

This amplitude can be obtained from the partonic amplitude of Eq. (4) by making a few simple substitutions. For the Dirac spinors, we make the substitution

$$v_i(p_b)\bar{u}_j(p_q) \to \frac{\delta_{ij}}{N_c} m_b f_+ (\not{p} - m_b)\gamma_5, \tag{12}$$

and then we replace all other occurrences of p_q in Eq. (4) with p_b. Since f_B scales like $\Lambda_{\mathrm{QCD}}^{3/2}/m_b^{1/2}$ and $\phi_+(\omega)$ scales like $\Lambda_{\mathrm{QCD}}^{-1}$, we conclude that f_+ scales like $(\Lambda_{\mathrm{QCD}}/m_b)^{1/2}$.

Note that the final expression obtained for the production matrix element is very similar to what would be obtained in a quarkonium production calculation. If we replaced the light quark with a charm quark in the above calculation, then the outgoing momentum would be $p_q = r p_B$ and $p_b = (1-r)p_B$, where $r = m_c/(m_c + m_b)$ and p_B is the B_c momentum. The projection operator for the B_c is

$$v(p_b)_i\bar{u}_j(p_q) \to \delta_{ij}\frac{f_{B_c}}{12}(\not{p}_B - m_{B_c})\gamma_5. \tag{13}$$

Thus the matrix element for *B* production via $(\bar{b}q)$ in a $^1S_0^{(1)}$ state can be obtained by taking the $r \to 0$ limit of the matrix element for B_c production and replacing $f_{B_c}/(4r) \to f_+ m_b$. The resulting cross sections are equivalent to modelling the *B* meson as a nonrelativistic bound state with a light constituent quark.

In calculating the inclusive cross section, one must also include processes in which the $(\bar{b}q)$ hadronizes into multiparticle final states, such as $B\pi, B\pi\pi$, etc. After squaring the matrix element, the exclusive cross section is proportional to f_+^2. The field theoretic expression for $f_+^* f_+$ involves a projection operator $|B(v)\rangle\langle B(v)|$. The inclusive cross section will instead involve the inclusive projection operator

$$\sum_X |B(v)+X\rangle\langle B(v)+X|, \tag{14}$$

where the sum is over all possible soft hadrons X in the rest frame of the *B*. After manipulations analogous to the exclusive case, we would ultimately arrive at an expression proportional to a double moment of some density matrix instead of $\phi^*(\omega')\phi(\omega)$. The bottom line is that the inclusive cross

section for color-singlet $\bar{b}q$ recombination can be obtained from the exclusive cross section simply by replacing f_+^2 by a different nonperturbative parameter ρ_1. The parameter ρ_1 is proportional to the probability for the color-singlet $(\bar{b}q)$ to evolve into any state containing the *B* hadron. Like f_+^2, the parameter ρ_1 scales like $\Lambda_{\mathrm{QCD}}/m_b$.

If the vacuum saturation approximation is applied to the sum over soft hadrons in Eq. (14), it would give f_+^2 as an estimate for ρ_1. In quarkonium production [24], the vacuum saturation approximation can be applied to color-singlet matrix elements with the appropriate angular momentum quantum numbers. It is a controlled approximation with corrections that scale as v^4. In the case of *B* hadrons, the vacuum saturation approximation is not a controlled approximation and it is likely to drastically underestimate the value of ρ_1. There is no reason to expect the contributions from $B\pi$, $B\pi\pi$, etc. to be suppressed relative to *B*. The extraction of the parameter f_+ from exclusive *B* decays could be used to place a lower bound on ρ_1, but we emphasize that it should be regarded as a weak lower bound.

Our final result for the color-singlet $\bar{b}q$ recombination contributions to the inclusive cross section for *B* hadron production in qg collisions is given by

$$\frac{d\hat{\sigma}}{dt}[\bar{b}q(^1S_0^{(1)})] = \frac{2\pi^2\alpha_s^3}{243}\frac{m_b^2}{S^3}\left[-\frac{64\,U}{S}\right.$$
$$+\frac{m_b^2}{T}\left(79 - \frac{112\,U}{T} - \frac{64\,U^2}{T^2}\right)$$
$$\left. -\frac{16\,m_b^4\,S}{U\,T^2}\left(1 - \frac{8\,U}{T}\right)\right], \tag{15}$$

$$\frac{d\hat{\sigma}}{dt}[\bar{b}q(^3S_1^{(1)})] = \frac{2\pi^2\alpha_s^3}{243}\frac{m_b^2}{S^3}\left[-\frac{64\,U}{S}\left(1 + \frac{2\,U^2}{T^2}\right)\right.$$
$$-\frac{m_b^2}{T}\left(19 - \frac{28\,T}{U} + \frac{368U}{T} + \frac{4\,T^2}{U^2}\right.$$
$$\left.\left. -\frac{64\,U^2}{T^2}\right) - \frac{48m_b^4\,S}{U\,T^2}\left(1 - \frac{8\,U}{T}\right)\right] \tag{16}$$

where we have defined $S = (p+l)^2$, $T = t - m_b^2 = (p - p_b)^2 - m_b^2 = -2p\cdot p_b$ and $U = (l - p_b)^2 - m_b^2 = -2l\cdot p_b$. To understand the behavior of the cross section, it is useful to express the variables S, T and U in terms of $p_\perp$ and $\Delta y = \bar{y} - y$, where $\bar{y}$ and y are the rapidities of the $\bar{b}$ and b, respectively:

$$S = 2(p_\perp^2 + m_b^2)(1 + \cosh\Delta y),$$

$$T = -(p_\perp^2 + m_b^2)(1 + e^{-\Delta y}),$$

$$U = -(p_\perp^2 + m_b^2)(1 + e^{\Delta y}).$$

At fixed Δy, $d\sigma/dt = \rho_1\,d\hat{\sigma}/dt \sim \Lambda_{\mathrm{QCD}}m_b/(p_\perp^2 + m_b^2)^3$. For large $p_\perp$, the cross sections $d\sigma/dt$ for recombination scale

like $\Lambda_{\text{QCD}} m_b / p_\perp^6$, while the partonic cross sections that appear in the $\bar{b}$ fragmentation formula in Eq. (1) scale like $1/p_\perp^4$. Thus, recombination is suppressed at large $p_\perp$ by $\Lambda_{\text{QCD}} m_b / p_\perp^2$ relative to $\bar{b}$ fragmentation, consistent with the QCD factorization theorems. Although the recombination cross sections are peaked in the forward region, they are always finite as the b quark mass cuts off all infrared divergences.

It is also possible for a color-octet $(\bar{b}q)$ to hadronize into a state with a B plus soft hadrons. To obtain the color-octet production cross section starting from the partonic amplitude in Eq. (4), we first make the substitution

$$v_i(p_b)\bar{u}_j(p_q) \rightarrow \sqrt{\frac{2}{N_c}} T_{ij}^a m_b f_+^8 (\not{p} - m_b) \gamma_5, \qquad (17)$$

and replace all other occurrences of p_q with p_b. After squaring the matrix element and summing over colors in the final state, $(f_+^8)^2$ is replaced with ρ_8, a factor that is proportional to the probability for a $(\bar{b}q)$ in a color-octet state to hadronize into a state including the B hadron. The normalization in Eq. (17) is chosen so that ρ_8/ρ_1 is the ratio of the probabilities for a color-octet $(\bar{b}q)$ and a color-singlet $(\bar{b}q)$ to hadronize into a B hadron. Since both ρ_1 and ρ_8 are $O(\Lambda_{\text{QCD}}/m_b)$, we might expect this ratio to be approximately 1. Our results for the color-octet $\bar{b}q$ recombination contributions to the inclusive cross section for B hadron production in qg collisions are

$$\frac{d\hat{\sigma}}{dt}[\bar{b}q(^1S_0^{(8)})] = \frac{4\pi^2\alpha_s^3}{243} \frac{m_b^2}{S^3} \left[-\frac{U}{S}\left(4 + \frac{9\,T}{U} + \frac{9\,T^2}{U^2}\right) \right.$$
$$- \frac{m_b^2}{2T}\left(79 + \frac{14\,U}{T} + \frac{8\,U^2}{T^2} - \frac{18T^2}{U^2}\right)$$
$$\left. + \frac{m_b^4 S}{UT^2}\left(8 + \frac{9T}{U} + \frac{8\,U}{T}\right)\right], \qquad (18)$$

$$\frac{d\hat{\sigma}}{dt}[\bar{b}q(^3S_1^{(8)})] = \frac{4\pi^2\alpha_s^3}{243} \frac{m_b^2}{S^3} \left[-\frac{U}{S}\left(22 + \frac{9\,T}{U} + \frac{18\,U}{T}\right.\right.$$
$$\left. + \frac{9\,T^2}{U^2} + \frac{8\,U^2}{T^2}\right) - \frac{m_b^2}{2T}\left(233 + \frac{10U}{T}\right.$$
$$\left. + \frac{316\,T}{U} + \frac{266\,T^2}{U^2} - \frac{8\,U^2}{T^2}\right)$$
$$\left. + \frac{3\,m_b^4 S}{U\,T^2}\left(8 + \frac{9\,T}{U} + \frac{8\,U}{T}\right)\right]. \qquad (19)$$

The normalization of the $\bar{b}q$ recombination contributions to the inclusive cross section for B hadron production depend on the parameters ρ_1 and ρ_8. Since their values are not known, we will focus on the qualitative aspects of the rapidity distributions in $p\bar{p}$ collisions. Note that $\bar{b}q$ recombination can actually contribute to B hadron production in two ways:
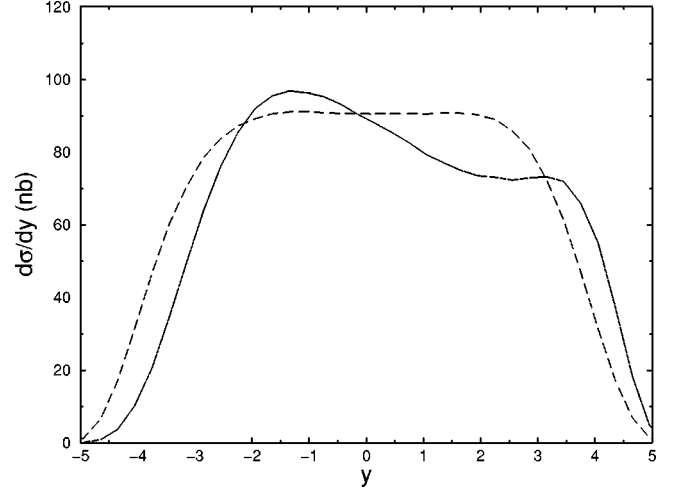

FIG. 2. Color-singlet (solid) and color-octet (dashed) contributions to the rapidity distribution of B hadrons in $p\bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV, with a $p_\perp > 5$ GeV cut. We have chosen $\rho_1 = 0.1$ and $\rho_8 = 0.17$, so color-singlet and color-octet cross sections are roughly equal at central rapidity.

$$(a) \quad \sum_n d\hat{\sigma}[qg \rightarrow (\bar{b}q)^n + b]\,\rho_n[(\bar{b}q)^n \rightarrow B],$$

$$(b) \quad \sum_{\bar{q},n} d\hat{\sigma}[\bar{q}g \rightarrow (b\bar{q})^n + \bar{b}]\,\rho_n \otimes D_{\bar{b}\rightarrow B}. \qquad (20)$$

In (a), the $(b)\bar{q}$ hadronizes into the B hadron. In (b), the $(b\bar{q})$ hadronizes into a $\bar{B}$, and the B comes from the fragmentation of the associated $\bar{b}$. In $p\bar{p}$ collisions, both processes in Eq. (20) lead to asymmetric B hadron rapidity distributions. As discussed earlier, process (a) yields B hadrons strongly peaked in the proton direction. The most important contribution to process (b) comes when the initial-state $\bar{q}$ is a $\bar{u}$ or $\bar{d}$ from the antiproton, and the resulting B hadron cross section is peaked at negative rapidity. Since (a) produces B's with positive rapidity while (b) produces B's with negative rapidity, the asymmetry in the rapidity distribution tends to get washed out when the two effects are combined. The extent to which this happens depends on the color state of the $(\bar{b}q)$ and $(b\bar{q})$. This is illustrated in Fig. 2, which shows the rapidity distribution of B hadrons from $\bar{b}q$ recombination in $p\bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV, with a $p_\perp > 5$ GeV cut. We have set $m_b = 4.75$ GeV and used the CTEQ5L [40] parton distribution functions. We used the one-loop running formula for $\alpha_s(\mu_R)$ with parameter $\Lambda^{(5)} = 0.146$ GeV and set the renormalization and factorization scales to $(m_b^2 + p_\perp^2)^{1/2}$. The color-singlet and color-octet $\bar{b}q$ recombination cross sections are the solid and dashed lines, respectively. The unknown parameters ρ_1 and ρ_8 are $O(\Lambda_{\text{QCD}}/m_b)$ and therefore are expected to be less than 1. We have chosen $\rho_1 = 0.1$ and $\rho_8 = 0.17$. Their relative normalization is such that the two distributions are approximately equal at central rapidity. Color-singlet recombination produces an asymmetric rapidity distribution with a small peak in the forward direction from process (a) and a large peak in the backward direction from
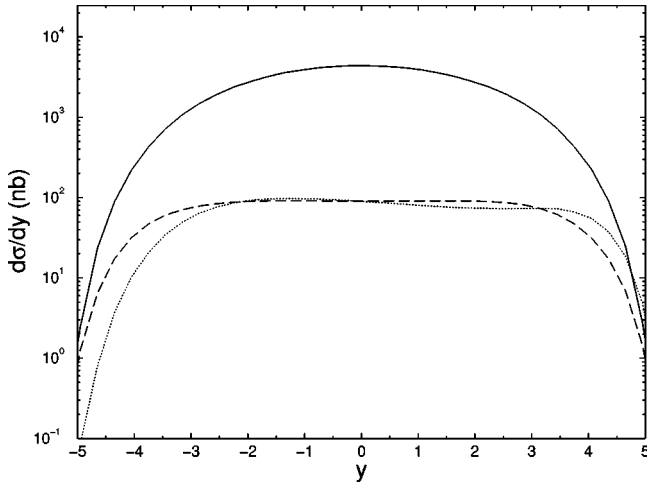
FIG. 3. Rapidity distribution of *B* hadrons in $p\bar{p}$ collisions at $\sqrt{s}=1.8$ TeV, with a $p_\perp>5$ GeV cut. Solid line is NLO *b* fragmentation contribution, dotted and dashed lines are color-singlet and color-octet contributions. The parameters ρ_1 and ρ_8 are the same as in Fig. 2.
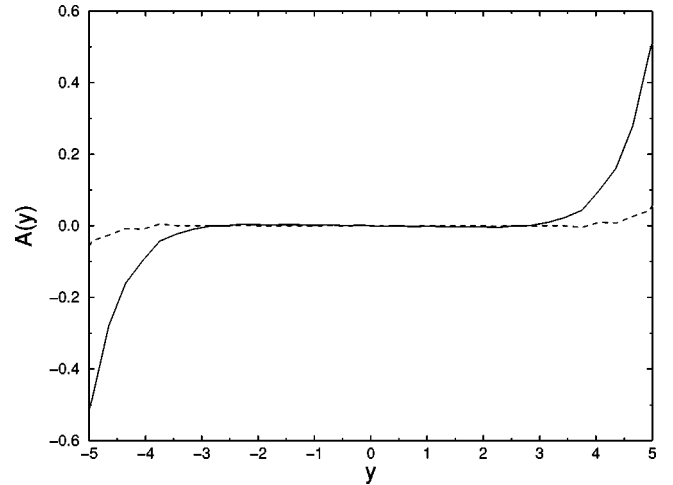

FIG. 4. Rapidity asymmetry for *B* hadrons in $p\bar{p}$ collisions at $\sqrt{s}=1.8$ TeV, with a $p_\perp>5$ GeV cut. The solid line is color-singlet; the dashed line is color-octet. The parameters ρ_1 and ρ_8 are the same as in Fig. 2.

process (b). In color-octet production, the processes (a) and (b) combine to yield a surprisingly symmetric rapidity distribution. Thus the *B* rapidity asymmetries can give important information on the relative importance of color-octet and color-singlet production mechanisms.

IV. COMPARISON WITH FRAGMENTATION MECHANISM

Next we compare the $\bar{b}q$ recombination cross sections with the NLO perturbative QCD calculations of $\bar{b}$ fragmentation production of *B* hadrons in $p\bar{p}$ colliders. The rapidity distributions are compared in Fig. 3. The solid line shows the $\bar{b}$ fragmentation contribution, which is obtained by using the LO cross section for $\bar{b}$ production convolved with CTEQ5L parton distribution functions [40], then multiplied by a *K* factor of 2.0 to give a normalization consistent with the NLO calculation [4]. In [4,10], it is demonstrated that the shapes of the NLO and LO rapidity distributions are nearly identical, so multiplying the LO cross section by a *K* factor gives an adequate representation of the NLO calculation. The partonic cross section should be convolved with a *b* quark fragmentation function. We have checked using the Peterson fragmentation function [41] that the shape of the rapidity distribution is essentially unaffected by this convolution. Also depicted in Fig. 3 are the $\bar{b}q$ recombination cross sections for *B* hadrons, including both processes in Eq. (20). The dotted and dashed lines show the rapidity distributions for color-singlet and color-octet $\bar{b}q$ recombination, respectively. The parameters ρ_1 and ρ_8 are the same as in Fig. 2. We emphasize that the normalization of the recombination contributions in Fig. 2 is uncertain, and that only their shapes are determined. It is clear that the recombination cross section can be as large as the fragmentation contribution for sufficiently large rapidity, however the point at which this occurs depends sensitively on ρ_1 and ρ_8.

It is also interesting to look at the rapidity asymmetry of *B* production, which is defined by

$$A(y)=\frac{\sigma_B(y)-\sigma_B(-y)}{\sigma_B(y)+\sigma_B(-y)}. \qquad (21)$$

This is shown in Fig. 4. Color-singlet recombination, shown by the solid line, only gives a noticeable asymmetry for $|y|>3$, while color-octet recombination, shown by the dashed line, gives a negligible asymmetry for $|y|<5$. For the values of ρ_1 and ρ_8 chosen in this paper, the asymmetry is unobservable at the rapidities currently probed by the DØ experiment. The asymmetries can of course be larger or smaller depending on the values of ρ_1 and ρ_8.

In order to assess how important the $\bar{b}q$ recombination mechanism is in the rapidity regions that are accessible experimentally, one needs to know the normalization of the cross section, which is determined by the parameters ρ_1 and ρ_8. Unfortunately, these parameters are unknown. The vacuum saturation approximation gives the weak lower bound $\rho_1>f_+^2$. If one makes the mild assumption that the light-cone wave function $\phi_+(\omega)$ is positive definite, it is possible to place a lower bound : $f_+\geqslant0.1$. [31].[1] The light-cone wave function models employed in [31] give estimates for f_+ in the range 0.1–0.2. We conclude that $\rho_1>0.01$.

The actual determination of ρ_1 and ρ_8 would require phenomenological analysis of *B* hadron production in other high energy physics processes. We can extract estimates of these parameters from a constituent quark model for heavy-meson production developed by Berezhnoy, Kiselev and Likhoded [36–39]. In this model, the *B* hadron cross section is obtained from a factorization formula analogous to Eq. (2) for $\bar{b}c$ hadrons by replacing m_c with a constituent quark mass, m_q, for the light quark and computing the parton cross sec-

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[1]We thank Dan Pirjol for discussions on this point.

tion to leading order in perturbative QCD. In the simplest form of this model, only the color-singlet S-wave matrix element, $\langle \mathcal{O}_1^B \rangle$, is kept and it is estimated from quark model wave functions [36]. This model was previously used to calculate heavy quark fragmentation functions [42,43]. However, the model can easily be extended by relaxing the vacuum saturation approximation for $\langle \mathcal{O}_1^B \rangle$ and including a color-octet S-wave matrix element $\langle \mathcal{O}_8^B \rangle$ [37–39]. The parton cross sections in this model can include fragmentation, recombination, and other contributions, depending on the high energy process being considered. All the contributions are calculated in terms of the two matrix elements $\langle \mathcal{O}_1^B \rangle$ and $\langle \mathcal{O}_8^B \rangle$. Our recombination cross sections can be obtained from calculations using this model by isolating the recombination contributions and then keeping the leading terms in the limit $r = m_q/(m_q + m_b) \rightarrow 0$. We can therefore use the matrix elements of [37–39] to estimate our parameters ρ_1 and ρ_8.

The simplest way to determine the values of ρ_1 and ρ_8 that correspond to the matrix elements $\langle \mathcal{O}_1^B \rangle$ and $\langle \mathcal{O}_8^B \rangle$ of [37–39] is to compare their calculations of D^* fragmentation functions [37] with what we would get if we applied our formalism. For the parameters $\rho_n[c\bar{q} \rightarrow D^*]$, we find $\rho_{1,8}^{D^*} = 3\langle \mathcal{O}_{1,8} \rangle/(4m_c m_q^2)$, where $m_q = 0.3$ GeV is a constituent quark mass. From studies of photoproduction and electroproduction [38,39], they extract $\langle \mathcal{O}_1 \rangle + \langle \mathcal{O}_8 \rangle/8 = 0.25$ GeV^3 and $\langle \mathcal{O}_8 \rangle = 0.33$–$0.49$ GeV^3. The corresponding values of the parameters in our formalism are $\rho_1^{D^*} + \rho_8^{D^*}/8 = 1.4, \rho_8^{D^*} = 1.8$–$2.8$. We can estimate the corresponding parameters $\rho_{1,8}^{B^*}$ for B^* production by multiplying by a factor of m_c/m_b. This is because $\rho_{1,8}^{B^*}$ scale with the heavy quark mass as $1/m_b$. Note that we are assuming that the moment of the light-cone wave function appearing in the definition of f_+ does not exhibit a strong dependence on the heavy quark mass between the scale m_c and m_b. The resulting estimates are $\rho_1 \approx 0.4$ and $\rho_8 \approx 0.8$, which are larger than the values we used in the figures.

So far we have focused exclusively on the contributions in which the parton that combines with the $\bar{b}$ is the light quark q. There are also processes in which a $\bar{q}$ participates in recombination:

$$d\sigma[\bar{q}g \rightarrow B + X] = \sum_n d\hat{\sigma}[\bar{q}g \rightarrow (\bar{b}\bar{q})^n + b] \, \rho_n[\bar{b}\bar{q} \rightarrow B]. \tag{22}$$

The LO diagrams for the process in Eq. (22) are identical to the diagrams shown in Fig. 1, except the light quark lines are replaced with light antiquark lines. Since the diagrams for $\bar{b}\bar{q}$ recombination are almost identical to those for $\bar{b}q$ recombination, we expect that the cross section will be peaked in the direction of the incoming $\bar{q}$. The B hadron will emerge preferentially in the $\bar{p}$ direction rather than the p direction, so one would expect that the asymmetry would be the reverse of that seen in Fig. 4. In the case of color-octet production, we saw that the asymmetry was almost completely washed out by the process (b) in Eq. (20). Thus, it is difficult to tell how

big the asymmetries from $\bar{b}\bar{q}$ recombination will be without an explicit calculation. The $\bar{b}\bar{q}$ recombination process will introduce additional nonperturbative parameters. To lowest order in Λ_{QCD}/m_b and $\Lambda_{QCD}/p_\perp$, the $\bar{b}$ and $\bar{q}$ are in an $L = 0$ state. Their color state can be either the $\bar{6}$ or the 3 of $SU(3)$. Heavy quark spin symmetry relates the spin states so there will be two parameters, ρ_3 and ρ_6, that are analogous to the ρ_1 and ρ_8 parameters for $\bar{b}q$ recombination. Intuitively, one might expect $(\bar{b}q)$ to prefer to hadronize to B mesons and the diquark $(\bar{b}\bar{q})$ to prefer to hadronize to the Λ_b by picking up another light antiquark. This intuition leads one to expect that $\bar{b}q$ recombination is more important for heavy meson production and that $\bar{b}\bar{q}$ recombination is more important for heavy baryon production. This would probably lead to rapidity asymmetries that have opposite signs for Λ_b baryons and B mesons.

One could also consider processes in which a gluon from the hard scattering combines with the $\bar{b}$. These do not produce any interesting asymmetries because the gluon distributions in the p and $\bar{p}$ are identical. The $(\bar{b}g)$ can be in a $\bar{3}$, 6 or $\overline{15}$ of $SU(3)$, so three nonperturbative parameters are required to describe this recombination mechanism. At large rapidity, the parton cross sections for $(\bar{b}g)$ may be larger than those for $(\bar{b}q)$. For reasonable values of ρ_1 and ρ_8, the $\bar{b}q$ recombination cross section is too small to explain the excess b production at large rapidity observed by the DØ Collaboration [11]. However, it is possible that the discrepancy could be explained by $\bar{b}g$ recombination.

The excess b production at large rapidity observed by the DØ collaboration cannot be explained by the conventional fragmentation mechanism. We have proposed a heavy-quark recombination mechanism that is suppressed by $\Lambda_{QCD}m_b/p_\perp^2$ at central rapidity, but may give an important contribution to the cross section at large rapidity. In $p\bar{p}$ collisions, our mechanism generates asymmetries in the rapidity distributions of B hadrons. The observation of rapidity asymmetries at the Tevatron that are too large to be explained by string fragmentation models of hadronization would provide qualitative evidence for our mechanism. Our mechanism involves several nonperturbative parameters, so quantitative evidence would require comparing its predictions with data on B production in other high energy physics experiments. We have calculated the cross sections for $\bar{b}q$ recombination in qg collisions and studied their effects on B production at the Tevatron. Future work should involve a complete calculation of the other recombination contributions discussed in this paper, and an exploration of the implications of heavy-quark recombination mechanisms for b and c hadron production in ep collisions and fixed target experiments.

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