Closed string tachyons and semiclassical instabilities

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We conjecture that the end point of bulk closed string tachyon decay at any nonzero coupling is the annihilation of space-time by Witten's bubble of nothing, resulting in a topological phase of the theory. In support of this we present a variety of situations in which there is a correspondence between the existence of perturbative tachyons in one regime and the semiclassical annihilation of space-time. Our discussion will include many recently investigated scenarios in string theory including Scherk-Schwarz compactifications, Melvin magnetic backgrounds, and noncompact orbifolds. We use this conjecture to investigate a possible web of dualities relating the eleven-dimensional Fabinger-Horava background with nonsupersymmetric string theories. Along the way we point out where our conjecture resolves some of the puzzles associated with bulk closed string tachyon condensation.

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I. INTRODUCTION AND THE CONJECTURE

One of the most satisfying recent results in string theory involves the fate of theories with open string tachyons. Sen conjectured that the condensation of these tachyons corresponds to the decay of unstable *D*-brane configurations leaving a supersymmetric vacuum (with or without stable *D*-branes) [1–3]. Support for this conjecture has come from techniques using conformal field theory, open string field theory, and noncommutative geometry.

There is in addition a natural picture that emerges wherein the open string tachyonic instability is the perturbative manifestation (for certain values of the background moduli) of an instability which also admits a semiclassical description (for other values of the background moduli). The simplest example of this type is the D-$\bar{D}$ system where for small enough separation there exists an open string tachyonic mode which mediates annihilation of the pair in the manner of Sen, while for large separation the tachyonic mode becomes massive and the same annihilation instability may be described by a sphaleron solution of the Born-Infeld action [4,5].

The case for closed string tachyons is much less understood. Recent works on systems with closed string tachyons [6–8] have all pointed towards the conclusion that the end point of the instability is a supersymmetric closed string vacuum much like the case for open string tachyons [9–11]. However not all tachyons are equal. We believe that if the coupling is nonzero, closed string tachyons will have a more drastic effect on the theory than the open string tachyons for the following reason. For open string tachyons to arise one must have *D*-branes in some closed string background (space-time). According to Sen the height of the tachyon potential is given by the tension of the relevant *D*-brane(s). At the location of the decaying *D*-brane(s) one has (before the decay happens and for nonzero coupling) positive curvature. (In the case of say $D9$-$\overline{D9}$ in a type IIB background one would have dS space to begin with.) After the decay one would have a flat closed string background. On the other hand the bulk closed string tachyon of type 0B exists already in a flat background. This means that the flat space background corresponds to the unstable point (a maximum or a saddle point) of the tachyon potential. Even if there is a minimum to the tachyon potential the end point of the decay will not be one of the known stable flat space backgrounds of string theory. This is illustrated in Fig. 1.

In [6] it has been conjectured that the end point of the decay of type 0A or 0B due to the bulk tachyon in these theories is the supersymmetric type IIA or IIB theory. These arguments however are dependent on the equivalance of certain M-theory backgrounds with type 0 and type II in certain Melvin magnetic backgrounds. (We will review these arguments later.) Precisely at the point where one has the flat type 0A background however the region in which this equivalence holds shrinks to zero. This renders the picture of magnetic flux shielding considerably less trivial. On the other hand in a very recent paper by David *et al.* [8], sigma model renormalization group (RG) arguments have been used to show that the end point of the decay of type 0A in flat space is type IIA in flat space. How then can we reconcile the argument of the previous paragraph with this claim?

The point is that sigma model arguments are made in a particular background and give a setup in which perturbation

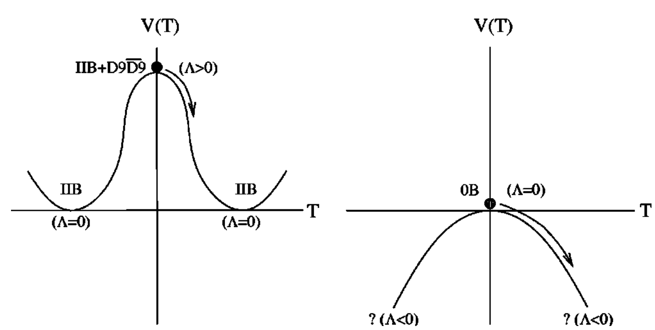

FIG. 1. The tachyon potentials for an unstable *D*-brane in dS space on the left versus the flat space tachyon of type 0A on the right.

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theory around that background can be done. One can calculate *S*-matrix elements for arbitrary numbers of particles on the assumption that the coupling is so weak that the back reaction on the background can be ignored. Of course if the coupling were exactly zero there would be no back reaction and as the tachyon slides down the potential there will be no change in the background and it is consistent to argue that the end point is indeed type IIA. However in this paper we are interested in the question of what happens to nonsupersymmetric theories at finite (nonzero) coupling. In this case one really needs to take into account the discussion of the previous paragraph. In fact even if the tachyon potential bottomed out, at any nonzero coupling the best that one could hope for is to end up with a supersymmetric (SUSY) string theory (say type IIA for type 0A) in AdS space.

What then might be the end point of this decay (for any theory with bulk tachyons—not just type 0A or 0B) at nonzero coupling? For now we note that another end point seems plausible, that of space-time annihilation. The motivation for this conjecture comes from a semi-classical argument first presented by Witten [12] and directly parallels the case for D-$\overline{D}$ annihilation. This instability dissappears in the zero coupling limit and so may only be associated with a tachyonic instability at nonzero coupling.

A precise formulation of the conjecture is as follows. Suppose we have a theory **A** on a background **X** which admits a semiclassical instability (determined by an analysis of the low-energy effective field theory). Now also assume that this semiclassical instability varies smoothly as the moduli determining **X** are varied. If by adjusting the moduli of **X** we reach a region in which the semiclassical analysis is invalid (but otherwise would lead to an instability), and we are instead afforded a perturbative description of the quantum theory **Y**, then the semiclassical instability should be reflected by a tachyonic instability in the perturbative description **Y**. In addition, the end point of both instabilities are to be identified.

We may consider a strong and weak form of the conjecture above distinguished as follows:

Strong: A semiclassical instability as described above predicts the existence of a tachyonic perturbative description of the resulting theory and we should identify the end point of condensation of the tachyon with that of the semiclassical instability.

Weak: A semiclassical instability as described above can be related to the tachyonic mode whenever it exists in a perturbative description of the resulting theory by identification of the end points of the instabilities.

The weak form allows for theories which do not reduce to tachyonic perturbative descriptions. We will see examples of each case below.

The conjecture above will in many cases involve extrapolations from strong coupling regions. As the systems are nonsupersymmetric, such extrapolations are unprotected and hence we do not know how to prove them. Support for this conjecture comes from three directions. (1) The close analogy to open string tachyons in unstable *D*-brane systems for which there is much support. (2) The existence of closed string tachyons in perturbative limits of systems exhibiting semiclassical instabilities. (3) The fact that flat space decay end points do not seem plausible for finite coupling as argued above.

The outline of this paper is as follows. In the next section (Sec. II A) we will first apply the conjecture in systems with an eleven-dimensional starting point. These are important because their perturbative limits involve the well known ten-dimensional string theories. Starting with semiclassically unstable circle and interval compactifications of M theory we identify the tachyonic perturbative limits involving type 0A or 0B and nonsupersymmetric heterotic strings on flat backgrounds, Melvin magnetic backgrounds, and noncompact orbifolds. We then move on in Sec. II B to similar considerations for ten-dimensional starting points which admit a greater degree of control and in many instances may be related to the eleven-dimensional cases by a ‘‘9-11’’ flip duality. Along the way we will encounter several situations for which recent analyses have led to conflicting conclusions and we will discuss these issues. We end with some conclusions (in particular for using the Scherk-Schwarz) mechanism for SUSY breaking and directions for future work.

II. APPLICATIONS OF THE CONJECTURE

A. 11D→10D

In this section we will discuss circle and interval compactifications of M theory. The semiclassical instabilities arise in the eleven-dimensional low-energy gravity theory as a result of the Kaluza-Klein structure of the vacuum. We adapt several results from [13] to the case of ten noncompact dimensions and discuss our own ideas on the relevance of the semiclassical decay evolution. Identification of perturbative string limits requires an extrapolation from strong to weak coupling and in the absence of supersymmetry is unprotected.

1. Twisted circle $M^{10}\times S^1_{R,B}$

Consider eleven-dimensional M theory on a background which locally resembles $M^{10}\times S^1$

$$ds_{11}^2=-dt^2+d\rho^2+\rho^2 d\phi^2+dy_i dy^i+dx_{11}^2,\quad i=3,\ldots,9 \tag{1}$$

but differs globally by the nontrivial identifications:

$$x_{11}\sim x_{11}+2\pi n_1 R \tag{2}$$

$$\phi\sim\phi+2\pi n_1 BR+2\pi n_2 .$$

We designate such a ‘‘twisted’’ circle by $S^1_{R,B}$. Let us choose a periodic spin structure for the S^1_R. The twist parameter B takes values $0\leqslant|B|<2/R$. For $B=0$ this is a supersymmetric compactification while for $B\neq 0$ the spacetime supersymmetry is completely broken. The effective theory governing the low-energy dynamics will generically incorporate Einstein-Hilbert gravity. It has been known for some time that gravity on a Kaluza-Klein background of this form exhibits a semiclassical instability towards the annihilation of spacetime (first discussed for five dimensions in [14,15] and later extended to eleven dimensions in [10]). This instability is mediated by a bounce solution that takes the form of an eleven-dimensional Euclidean Kerr black hole solution:

$$ds_{11}^2=\left(1-\frac{\mu}{r^6\Sigma}\right)dx_0^2-\frac{2\mu\alpha\sin^2\theta}{r^6\Sigma}dx_0 d\phi$$
$$+\frac{\Sigma}{r^2-\alpha^2-\mu r^{-6}}dr^2+\Sigma d\theta^2$$
$$+\frac{\sin^2\theta}{\Sigma}\left[(r^2-\alpha^2)\Sigma-\frac{\mu}{r^6}\alpha^2\sin^2\theta\right]d\phi^2$$
$$+r^2\cos^2\theta(d\chi^2+\sin^2\chi d\Omega_6^2) \quad (3)$$

where $\Sigma\equiv r^2-\alpha^2\cos^2\theta$, μ is the black hole mass parameter, α is a single complexified angular momentum parameter, and we have written $d\Omega_7$ as $d\chi^2+\sin^2\chi d\Omega_6$ for later convenience. The identifications (2) are most easily expressed in eleven-dimensional $SO(2)$-coordinates on S_1:

$$t,\rho,\phi^{(2\pi)},x_3,x_4,x_5,x_6,x_7,x_8,x_9,x_{11}$$

but the instanton is more easily expressed in $SO(9)$-coordinates on S_8:

$$x_0,r,\theta^{(\pi/2)},\phi^{(2\pi)},\chi^{(2\pi)},\theta_1^{(\pi)},\theta_2^{(\pi)},\theta_3^{(\pi)},\theta_4^{(\pi)},\theta_5^{(\pi)},\theta_6^{(\pi)}.$$

To match the bounce solution (3) to the unstable background (1),(2) the background parameters must satisfy

$$R=\frac{\mu}{4r_H^7-3\alpha^2 r_H^5} \quad (4)$$
$$B=\frac{\alpha r_H^6}{\mu}-\frac{\alpha}{|\alpha|R}$$

where r_H is the location of the Euclidean black hole horizon satisfying

$$r_H^2=\alpha^2+\frac{\mu}{r_H^6}. \quad (5)$$

The coordinate singularity sets a lower bound on the range of the radial coordinate

$$r\geqslant r_H. \quad (6)$$

To estimate the decay rate we evaluate the Euclidean action I for the bounce solution (3) and calculate $\Gamma\sim e^{-I}$. This evaluates to

$$\Gamma\sim e^{-\pi^5\mu R/96G_{11}} \quad (7)$$

where G_{11} is the eleven-dimensional Newton's constant. Evaluating the decay rate in terms of the background parameters R,B involves untangling expressions (4), (5) which can be quite difficult. The task simplifies for two important parameter regions [15]:

For $|B|\sim 0$ the expression for μ reduces to $\mu=(\frac{7}{2})^7(R/|B|^7)$, which clearly diverges for $|B|\to 0$. The decay rate vanishes rendering the theory stable against the semiclassical instability.

For $|B|=1/R$ (corresponding to $\alpha=0$) the expression simplifies to $\mu=(4R)^8$. One can demonstrate that this is actually a minimum of $\mu(B)$ and hence represents the most unstable background.

In fact we have found with some numerical work that the decay rate is a monotonically decreasing function of R for fixed $B\neq 0$ indicating the expected stability of the decompactified theory and in turn the maximum instability of the theory as $R\to 0$. For fixed R one can also demonstrate that the decay rate is a monotonically increasing function for B increasing from $0\to 1/R$ beyond which it monotonically decreases as B approaches $2/R$.

The evolution of the background (1),(2) after the decay is determined by finding a zero-momentum surface in the bounce solution and using this as inital data for an analytic continuation back to Lorentzian signature. Such a zero-momentum surface is given by $\chi=\pi/2$, so we may continue Eq. (3) by sending $\chi\to\pi/2+i\tau$ to obtain

$$ds_{11}^2=\left(1-\frac{\mu}{r^6\Sigma}\right)dx_0^2-\frac{2\mu\alpha\sin^2\theta}{r^6\Sigma}dx_0 d\phi$$
$$+\frac{\Sigma}{r^2-\alpha^2-\mu r^{-6}}dr^2+\Sigma d\theta^2$$
$$+\frac{\sin^2\theta}{\Sigma}\left[(r^2-\alpha^2)\Sigma-\frac{\mu}{r^6}\alpha^2\sin^2\theta\right]d\phi^2$$
$$+r^2\cos^2\theta(-d\tau^2+\cosh^2\tau d\Omega_6). \quad (8)$$

To get a feel for what the metric above describes let us first identify the spatial infinity limit with the pre-decay geometry. This is nontrivial owing to the double analytic continuation ($t\to ix_0$, $\chi\to\pi/2+i\tau$) that we have used to get to this expression. Just after the decay the geometry far from the decay nucleus should be in its pre-decay form. Evaluating Eq. (8) for $r\to\infty$ we find

$$ds_{11}^2(r\to\infty)\sim dx_0^2+dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2-r^2\cos^2\theta d\tau^2$$
$$+r^2\cos^2\theta\cosh^2\tau d\Omega_6. \quad (9)$$

In this form it is not obvious that this metric describes asymptotically flat space. To see this we first introduce radial coordinates $(\hat{\rho},\hat{r})$ defined by

$$\hat{\rho}=r\sin\theta \quad (10)$$
$$\hat{r}=r\cos\theta$$

and then introduce "flat" coordinates

$$\tilde{r}=\hat{r}\cosh\tau \quad (11)$$
$$\tilde{\tau}=\hat{r}\sinh\tau.$$

In these coordinates Eq. (9) takes the form

$$ds^2_{11}(r\to\infty)\sim dx_0^2+d\hat{\rho}^2+\hat{\rho}^2 d\phi^2+d\tilde{r}^2+\tilde{r}^2 d\Omega_6-d\tilde{\tau}^2 \tag{12}$$

which clearly describes a flat eleven-dimensional spacetime.

The full post-decay metric expressed in these flat coordinates is extremely complicated, however the most striking feature of the decay scenario is easily seen as a result of Eq. (11). Note that the coordinate redefinitions imply

$$\tilde{r}^2+\hat{\rho}^2-\tilde{\tau}^2=r^2. \tag{13}$$

A trivial algebraic rearrangement of Eq. (13) combined with the coordinate minimum for r in Eq. (6) implies the existence of a totally geodesic submanifold which is growing in time

$$(\tilde{r}^2+\hat{\rho}^2)_{min}=r_H^2+\tilde{\tau}^2. \tag{14}$$

For the coordinate region inside of the expanding bubble $\tilde{r}^2+\hat{\rho}^2<r_H^2+\tilde{\tau}^2$ the metric degrees of freedom cease to exist. This is the "bubble of nothing" annihilation of spacetime first described by Witten [12,16]. It is a difficult picture to consider but is strikingly reminiscent of the idea of a purely topological phase of gravity. One should here consider the corresponding story for unstable open string theories in which the decay (either via condensation of tachyons or sphaleron mediated semiclassical processes) often leads to an annihilation of the open string degrees of freedom. For unstable D-branes one always has the closed string vacuum to leave behind, but for an unstable closed string vacuum the natural result seems, though perfectly analogous, considerably more catastrophic.

A great deal of discussion has been aimed at elucidating the picture of this semiclassical decay in terms of a dimensionally reduced theory [6,13–15]. This has led to a number of seemingly strange equivalences. A very simple example involves two different ten-dimensional descriptions of the same eleven-dimensional process [17]. In one case the decay involves spacetime falling into a pointlike singularity at an ever increasing rate, while the other description resembles the (considerably less catastrophic) shielding of a Kaluza-Klein magnetic field via pair production of magnetic monopoles [18]. While these are certainly very interesting results, we take here the view that exactly when a Kaluza-Klein reduction becomes appropriate we lose the eleven-dimensional classical gravity approximation used in these calculations. At sufficiently small length scales quantum M-theory effects become important. The appropriate quantum description in many cases will be in terms of a perturbative string theory on the reduced background. We should not concern ourselves with the dimensionally reduced picture of the semiclassical instability. Instead we should look for perturbative manifestations of this instability. Why then identify the end points of the semiclassical and perturbative decays? Again a chief motivation is the analogy with unstable open string theories where we see the semiclassical instability described in [4,5] go over to the tachyonic instability elucidated by Sen in [1–3].

If the size of the radius shrinks below the eleven-dimensional Planck length $R<l_P$ then the eleven-dimensional gravity approximation used above breaks down. However we are in most cases afforded a description of the resulting dynamics in terms of weakly coupled string theory. For $B=0$ this of course reduces to supersymmetric type IIA strings in a flat background. The bounce action above diverges for this particular case reflecting that the eleven-dimensional theory is actually supersymmetric and hence stable. We will now move on to the unstable $B\neq 0$ cases and discuss their perturbative limits.

2. Melvin models

For values of $0\leq|B|<1/R$ and $R<l_P$ we may reduce the background (1),(2) along the Killing vector $l=\partial_{x_{11}}-B\partial_\phi$ to obtain a Melvin magnetic flux tube background (a fluxbrane) [13,14] which is described by

$$ds^2_{10}=\Lambda^{1/2}(-dt^2+d\rho^2+dy_i dy^i)+\Lambda^{-1/2}\rho^2 d\tilde{\phi}^2 \tag{15}$$

$$e^{4\Phi/3}=\Lambda=1+\rho^2B^2 \tag{16}$$

$$A_{\tilde{\phi}}=\frac{B\rho^2}{2\Lambda} \quad\Rightarrow\quad \frac{1}{2}F_{\mu\nu}F^{\mu\nu}|_{\rho=0}=B^2 \tag{17}$$

where $\tilde{\phi}\equiv\phi-Bx_{11}$. This curved ten-dimensional background incorporates an axially symmetric RR two-form field strength parametrized by its central $(\rho=0)$ value B, and a nontrivial dilaton which grows as we move away from the $\rho=0$ hyperplane for $B\neq 0$. To determine the perturbative content of the theory we should recall that the eleven-dimensional starting point was M theory on a flat Kaluza-Klein background. For the periodic choice of spin structure on the S^1_R factor and for $B=0$ this reduces to type IIA strings on M^{10} as discussed above. For $B\neq 0$ we should then obtain type IIA strings propagating on the Melvin background [19].

Quantizing strings on the background (15), (16), (17) faces the twin difficulties of incorporating RR flux and a curved geometry and is beyond current understanding. Applying the strong form of the conjecture discussed in the introduction would however imply that the corresponding closed string fluctuations should admit at least one tachyonic mode whose condensation would also lead to the annihilation of the spacetime.

The theory for $|B|\neq 0$ is continuously connected to the supersymmetric type IIA vacuum. It may seem natural that condensation of a closed string tachyon would in this case relax the value of $|B|$ to zero, restoring the supersymmetric vacuum [20]. In this sense the Melvin magnetic flux would represent an excited state in the type IIA theory, decaying by flux dissipation [21]. However the Melvin background does not merely constitute weakly coupled type IIA string theory with some additional unstable flux. As one can see from the nontrivial dilaton profile (16) a description in terms of any

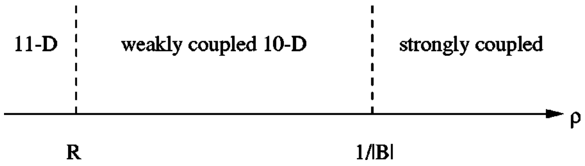

FIG. 2. Radial range of weakly coupled 10D physics for a noncritical Melvin background.

weakly coupled string theory will only be possible in the spatial region $R<\rho<1/|B|$; see Fig. 2.

For $\rho<R$ we invalidate the Kaluza-Klein ansatz, while for $\rho>1/|B|$ the string coupling becomes strong. In either case we must utilize the eleven-dimensional description.

3. Critical Melvin and type 0A: The Scherk-Schwarz circle

For $|B|=1/R$ the effect of the twisted identifications (2) is to accompany a $2\pi R$ translation in x_{11} (generated by $n_1\rightarrow n_1+1$) with a 2π rotation in ϕ. This forces fermions to pick up a -1 when transported around the compact circle [so called Scherk-Schwarz (SS) boundary conditions [22]] and leaves bosons unaffected. Our starting point was a periodic spin structure on the S^1_R so the net effect of $|B|=1/R$ is to exactly reverse this choice of spin structure. Thus one may consider the "critical" case of $|B|=1/R$ in either of two ways:

(a) Periodic spin structure on S^1_R and $|B|=1/R$. This case will again reduce to type IIA strings propagating on a Melvin magnetic background. However in this critical case (Fig. 3) the theory is nowhere described by a weakly coupled ten-dimensional string theory since the relevant region shrinks to zero.

(b) Antiperiodic spin structure on S^1_R and $|B|=0$. In this case the resulting ten-dimensional background is flat M^{10}. Bergman and Gaberdiel have considered M theory compactified on a Scherk-Schwarz circle and conjectured that the appropriate perturbative degrees of freedom are type 0A strings [23,24]. While this conclusion is still unverified it agrees with our conjecture in the sense that the spectrum of type 0A strings on M^{10} admits a closed string tachyon. The end point of condensation of the type 0A closed string tachyon would then be identified with the annihilation of spacetime.

Thus far only the special cases $|B|=0,1/R$ admit realiable perturbative information. The presence of RR flux and the curvature of spacetime for $|B|\neq 0$, $1/R$ renders even a spectrum calculation beyond our reach, however, we will later discuss similar models in nine dimensions for which the full spectrum is trivially obtained.

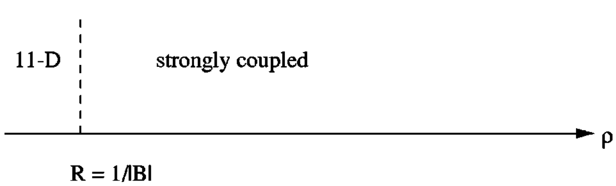

FIG. 3. Radial range of perturbative descriptions for a critical Melvin background.

4. Noncompact orbifolds

When the twist parameter takes special values of the form $|B|=1/NR$ or $|B|=1/R+1/NR$ we can perform $SL(2,Z)$ transformations on the (x_{11},ϕ) compactification torus and reduce to a ten-dimensional background of the form [6]

$$ds^2_{10}=\eta_{\mu\nu}dx^\mu dx^\nu+d\hat{r}^2+\frac{\hat{r}^2}{N^2}d\hat{\phi}^2 \tag{18}$$

$$e^{4\Phi/3}=N^2 \tag{19}$$

$$A_{\hat{\phi}}=\frac{1}{2}R\frac{N-1}{N}. \tag{20}$$

The resulting spacetime is that of a noncompact orbifold with the fundamental region a cone of deficit angle $2\pi/N$. The dilaton in this case is constant throughout the spacetime. If we start with periodic spin structure on the S^1_R then for $|B|=1/NR$ the correct perturbative degrees of freedom involve type 0A strings. For periodic spin structure on the S^1_R and $|B|=1/R+1/NR$ we transform to $B'=B-(B/|B|)(1/R)$ and reverse the spin structure as in Sec. II A 3. This reduces to type IIA strings propagating on the orbifold background (18),(19),(20).

Even though the geometry is locally flat, string quantization on this background is difficult owing to the presence of the Ramond-Ramond (RR) Wilson line. Our conjecture would imply the existence of a tachyonic instability in the perturbative description for either $|B|=1/NR$ or $|B|=1/R+1/NR$ with the end point of its condensation involving the annihilation of spacetime.

This result seems to contradict the conclusions reached in [7] where it was found that the effect of tachyon condensation in noncompact orbifolds is to "un-orbifold" the theory restoring the "underlying" supersymmetric closed string vacuum. This deserves some discussion. Among the arguments in [7] was the observation that the orbifold fixed plane represents a curvature singularity in a locally flat spacetime which may be viewed as a localized excitation above the underlying background. In fact for the special cases Z_N with N odd it was pointed out that the closed string tachyons are localized to the orbifold fixed plane and by an analogy with open string tachyons localized to unstable *D*-branes represent an instability towards decay of the localized energy density restoring the supersymmetric vacuum. The elegant analysis in [7] is sound, however when one tries to apply their conclusions to the present scenario we find some obstacles.

First of all the analysis was performed in the zero coupling limit. As we argued in the introduction, for zero coupling the tachyon may condense without affecting the underlying background. We expect that for nonzero coupling the tachyon will have a more dramatic effect on the background. Secondly to connect these orbifolds to the twisted circle compactifications discussed above one must include the RR Wilson line (20). In [7] the spectrum of the theory was computed without incorporating any Wilson line. As we have mentioned earlier including the RR Wilson line is difficult, however one can circumvent this difficulty by looking at the corresponding situation obtained by compactifying a ten-dimensional theory on a twisted circle. In this case the Wilson line will arise in the Neveu-Schwarz–Neveu-Schwarz (NSNS) sector and the spectrum can be evaluated exactly. We will discuss these in more detail in Sec. II B but for now we point out that one important effect of including the Wilson line is the localization of closed string tachyons for any value of $0<|B|<1/R$ (particularly $|B|=1/NR$ with N even or odd). In addition the "curvature singularity as a localized excitation above a flat background" argument needs to be reconsidered. For a Wilson line of the form (20) the ten-dimensional geometry actually lifts to a flat eleven-dimensional geometry which is everywhere regular. Probing distances very close to the orbifold fixed plane invalidates the Kaluza-Klein ansatz and we should replace the reduced theory by its eleven-dimensional interpretation.

5. The Scherk-Schwarz interval $M^{10}\times I_L^{SS}$

Consider now Horava-Witten (HW) theory [25,26], i.e., eleven-dimensional M theory compactified on a line segment of length L. In addition to the bulk degrees of freedom anomaly cancellation requires an $E8$ gauge theory to live on each ten-dimensional wall bounding the bulk spacetime. Fabinger and Horava (FH) considered the scenario that results from reversing the chirality of fermions living on one of the walls [27]. This breaks the spacetime supersymmetry and renders the theory unstable. FH demonstrated the existence of an attractive Casimir force between the walls and then went on to discuss a semiclassical instability towards formation of a wormhole-like tube connecting the two walls, the interior of which has no metric degrees of freedom. This tube grows radially outward eating up both the $E8$ walls and the bulk spacetime. This system is equivalent to a compactification of M theory on a Scherk-Schwarz circle of radius L/π followed by a Z_2 orbifolding. This may be viewed as a Z_2 orbifolding of the critically twisted circle $S^1_{L/\pi,\pi/L}$ discussed in Sec. II A 3. The relevant bounce solution is simply the Z_2 invariant form of Eq. (3) evaluated at $R_{FH}=L/\pi$ and $B=\pi/L$:

$$ds_{11}^2=\left(1-\frac{\mu}{R^8}\right)dx_{11}^2+\left(1-\frac{\mu}{R^8}\right)^{-1}dr^2 + r^2(d\chi^2+\sin^2\chi d\Omega_8). \tag{21}$$

For the critical case we may easily express the mass parameter μ in terms of the background parameters

$$\mu=\left(\frac{4L}{\pi}\right)^8. \tag{22}$$

Borrowing expression (7) for the decay rate we find

$$\Gamma\sim e^{-2^{11}L^9/3\pi^4G_{11}}. \tag{23}$$

Analysis of the post decay evolution proceeds along the lines of Sec. II A 1. The picture is that of a Z_2 projection of Witten's spherically symmetric bubble of nothing expanding in time. We now discuss two possible perturbative limits of the FH scenario.

TABLE I. Consistent nonsupersymmetric heterotic string theories in ten dimensions.

Gauge symmetry	Tachyon representation	Chiral
$H_{SO(16)\times SO(16)}$	tachyon-free	yes
$H_{SO(32)}$	(32_v)	no
$H_{SO(8)\times SO(24)}$	$(8_v,1)$	yes
$H_{SU(16)\times SO(2)}$	$(1,2_v)$	yes
$H_{SO(16)\times E8}$	$(16_v,1)$	yes
$H_{E7\times SU(2)\times E7\times SU(2)}$	(1,2,1,2)	yes
H_{E8}	(1)	no

6. The case of the shrinking interval $M^{10}\times I_{L\to 0}^{SS}$

Consider the situation where the two $E8$ walls come together. For $L\sim l_P$ the eleven-dimensional gravity approximation breaks down. We might anticipate a result similar to the HW case for which the appropriate description as the two $E8$ walls come together is in terms of weakly coupled heterotic $E8\times E8$ string theory ($H^{susy}_{E8\times E8}$) on M^{10}. For the case of FH the resulting perturbative string description must have broken supersymmetry. Furthermore, our conjecture in its strong form would imply that the resulting string theory should have a tachyonic mode which mediates the annihilation of spacetime. There are seven candidate nonsupersymmetric heterotic string theories [28]. Their relevant properties are summarized in Table I.

To identify the best candidate theory we consider the membrane world-volume anomaly analysis of HW [26]. For a topologically stabilized membrane wrapping a large $S^1_{x^9}$ and stretched between the two walls, the right-moving $8''$ fermions [29] induce a three-dimensional gravitational anomaly since the world volume has orbifold singularities, i.e. it is not a smooth manifold. To cancel this anomaly one must add left-moving current algebra modes with $c=16$. Since the anomaly is localized and evenly distributed between the two boundaries of the world volume, the current algebra modes should be evenly distributed between the two ends as well. In the HW case spacetime supersymmetry is preserved, and the only supersymmetric string theory with this world-sheet structure is the $H^{susy}_{E8\times E8}$ theory [30]. If the spacetime supersymmetry is broken, as in the case at hand, then we should look for nonsupersymmetric strings with this world-sheet structure. Only two of the seven cases above are of this type; the $H_{E8\times SO(16)}$ and $H_{SO(16)\times SO(16)}$ theories. There are two additional reasons which lead to the choice of $H_{E8\times SO(16)}$ as the $L\to 0$ limit of FH. Motivated by our conjecture, we choose the only one of these two that is tachyonic. This would follow from the strong form of the conjecture. An indication that this is plausible was worked out in [27] where the mass of a membrane state stretched between the two walls was calculated and shown to become tachyonic when the two walls are sufficiently close, i.e. $L<l_P$. Of course the membrane energy calculation becomes invalid precisely in this regime, but it does seem indicative of a

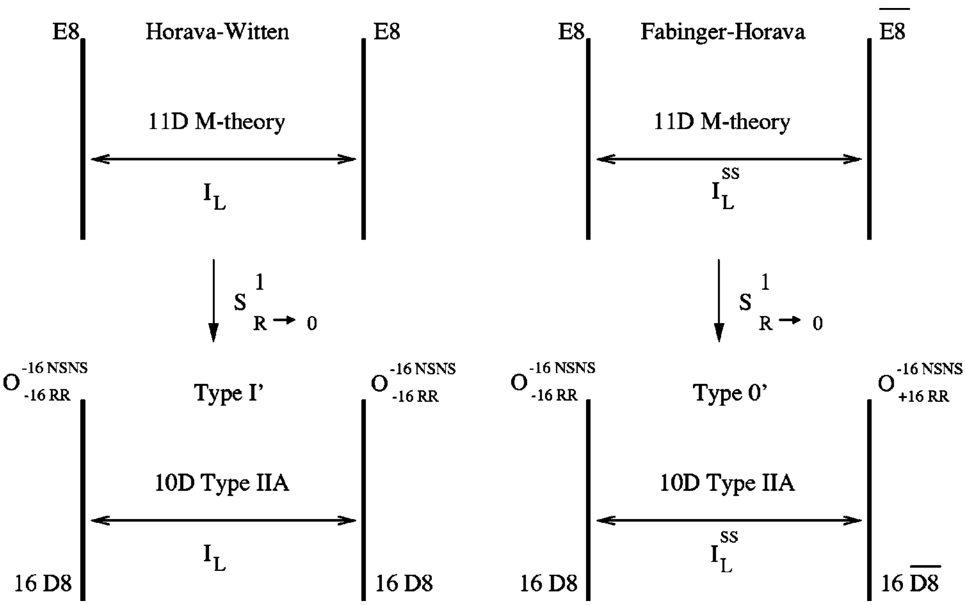

FIG. 4. Compactifications of HW and FH on transverse circles. The charges carried by the orientifold planes are explicitly designated.

continued instability of the theory. Furthermore, the Gliozzi-Scherk-Olive (GSO) constraints which lead to the $H_{E8\times SO(16)}$ theory differ from those that lead to the $H^{susy}_{E8\times E8}$ theory by a twist of $\exp(i\pi F_B)$ which affects only half of the left moving current algebra modes [31]. It may seem odd that the gauge group is broken asymmetrically as the two-walls come together since there seems to be no "preferred" wall. However both $E8$ and $SO(16)$ require 16 current algebra fermions on the corresponding worldsheet so in a sense the walls are on equal footing. The effect of "flipping" one of the $E8$ walls on the M-theory side must translate into a modification of the GSO projection on one of the two sets of 16 current algebra fermions on the heterotic string world sheet. In any case, the two walls coming together ventures through intermediate coupling regions (for which there is no known description) unprotected by supersymmetry, rendering the specific mechanism behind the spacetime gauge symmetry breaking difficult to study.

7. The case of the shrinking transverse circle $M^9\times S^1_{R\to 0}\times I^{SS}_{L=finite}$

Now consider the FH background keeping the Scherk-Schwarz interval length L large and further compactifying the theory on a transverse circle S^1_R with a periodic choice of spin structure. For $R<l_P$ we should be able to describe the system by a nonsupersymmetric variant of the familiar type I′ theory (type 0′) as shown in Fig. 4.

In the familiar supersymmetric case type I′ is obtained from type IIA compactified on a circle by dividing out by the Z_2 symmetry $g=I\Omega$ where Ω is the world sheet orientation reversal and I: $x^9\to -x^9$. In the original M-theory picture of the FH construction the reversal of orientation of one of the E_8 walls may be accomplished by dividing the theory by an additional Z_2 symmetry generated by $g'=S(-1)^{F_s}$ giving

$$M^9\times S^1_R\times S^1_{L/\pi}/(Z_2\times Z_2') \tag{24}$$

where in the M-theory case the first Z_2 is generated by just the reflection. Thus the theory that we get on reduction to ten dimensions $R\to 0$ should be type IIA on $M^9\times S^1_{L/\pi}/(Z_2\times Z_2')$ where the two symmetries are generated by g and g'. This is equivalent to a theory defined in [32,33] and has an orientifold 8-plane at one fixed point and an anti-orientifold at the other fixed point. There is no need to add D-branes to cancel RR tadpoles but getting flat space would require the cancellation of the NSNS tadpoles and thus would entail the presence of 16 $D8$-branes and 16 $\overline{D8}$-branes with the passage to the corresponding M theory case possible when the former are coincident with the orientifold plane and the latter with the anti-orientifold plane. Thus this limit of the FH theory is simply an orientation reversed version of the type I′ theory. We should then really be considering an S^1 compactification of FH with a Wilson line Y which breaks the $E8\times E8$ gauge symmetry to $SO(16)\times SO(16)$. This has a potentially tachyonic mode coming from the twisted sector [23], with mass

$$m^2=\frac{L^2}{4\pi^2\alpha'^2}-\frac{2}{\alpha'} \tag{25}$$

which becomes tachyonic when $L<2\pi\sqrt{2\alpha'}$. This state clearly survives the g and g' projections.

It should be stressed that this is a closed string tachyon coming from the lowest winding mode of the twisted sector of the theory. In addition of course the theory has open string tachyons coming from the open strings stretched between the D-$\overline{D}$ when they get within a distance $\pi\sqrt{2\alpha'}$ of each other. The D-branes are attracted to each other and will annihilate due to this, leaving us with a background that will have a negative cosmological constant (due to the negative tension of the orientifold anti-orientifold system). What would one expect to be the end point of the decay of the closed string tachyon?

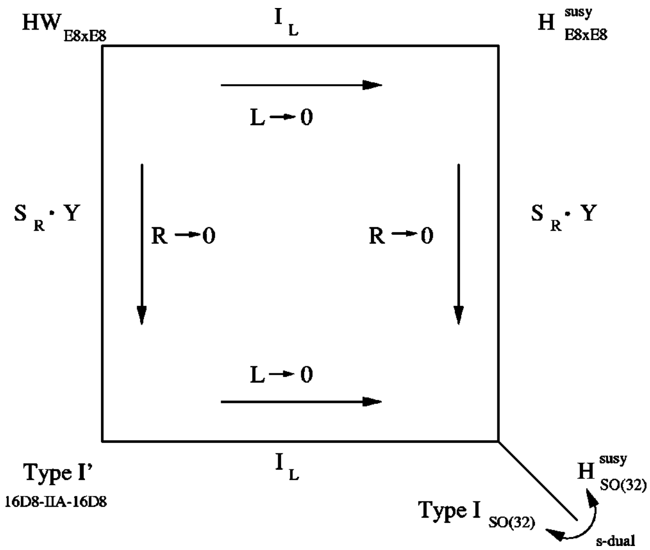

FIG. 5. The standard web of dualities obtained by supersymmetric compactifications of the Horava-Witten theory [25,45].

The answer to this according to our conjecture should be obtained from the picture of semiclassical vacuum bubble decay that one has when $L > 2\pi\sqrt{2\alpha'}$ in analogy with the corresponding M-theory FH case. Thus as in the FH case one expects this theory to be subject to space time annihilation. One might then conjecture that for $L < 2\pi\sqrt{2\alpha'}$ also the end point of the tachyonic decay should also be interpreted as the catastrophic annihilation of the background, even though strictly speaking this is not a region where the geometrical argument of Witten is directly applicable.

In the region for which the lowest mode becomes tachyonic $L < 2\pi\sqrt{2\alpha'}$ the nine-dimensional theory should be replaced by an appropriate *T*-dual description. In the supersymmetric case the appropriate *T*-dual description is the type I theory. This theory can also constructed from type IIB by gauging the world-sheet parity Ω (type IIB or Ω) and adding 32 $D9$-branes to cancel the resulting massless RR and NSNS tadpoles. At strong coupling this theory is described by the weakly coupled $H^{susy}_{SO(32)}$ theory. See Fig. 5.

For the *T* dual of type $0'$ one expects a perturbative description in terms of a nonsupersymmetric analog of type I theory. Before discussing this theory in detail we use the strong form of our conjecture to anticipate some of its features. The semiclassical instability in eleven dimensions annihilates both the gauge degrees of freedom and the spacetime. In the compactification at hand the gauge degrees of freedom and the spacetime are described by two different sectors of the theory (the former by open strings and the latter by closed strings). The single semiclassical annihilation instability of the eleven-dimensional theory should then descend to two tachyonic instabilities, one leading to the annihilation of the gauge degrees of freedom and the other leading to the annihilation of spacetime [34].

To construct the *T*-dual theory we permute the two Z_2 symmetries. In the *strict* $L \to 0$ limit one is then left with type 0B or Ω, which has been called type 0 theory (a nonsupersymmetric analog of type I). This theory has been constructed in [23,35–37] and indeed contains both open string tachyons charged under the gauge symmetry as well as a closed string tachyon (the type 0B tachyon survives the Ω projection). The massless NSNS tadpole contribution for type 0B or Ω is twice that in type IIB or Ω, so to formulate the type 0B or Ω theory in flat space (which we expect for the *T*-dual description of type $0'$ in flat space) requires the addition of 64 $D9$-branes. The absence of massless RR tadpoles implies that 32 of these should be $D9$-branes and the other 32 $\overline{D9}$-branes. However the type 0 theories exhibit two types of Dp-brane for any given p distinguished by their charge under the twisted sector fields [23,37]. If we designate these by Dp' and Dp'', then the massless tadpoles can be cancelled by adding n $D9'-\overline{D9'}$ pairs and $32-n$ $D9''-\overline{D9''}$ pairs. The resulting gauge symmetry is then $SO(n)\times SO(n)\times SO(32-n)\times SO(32-n)$. Of course we originally had the 16 $D8$-branes and $16'\overline{D8}$-branes of type $0'$, but these are standard type II $D8$-branes. For finite radius on the *T*-dual side the situation can be described by the splitting of type II Dp-branes into pairs of type 0 Dp'/Dp''-branes on the dual circle [38], however some subtleties for the strict $L \to 0$ limit are unresolved [39]. This is presently under investigation [40].

This gauge symmetry enhancement is an unusual feature of Scherk-Schwarz compactifications. The point is that for a Scherk-Schwarz compactification there are new twisted sector states in the theory which only become light as $R \to 0$. In the closed string sector these states form an essential part of the conjectured relationship between type 0A and M-theory [23] where they become the additional NSNS and RR fields of type 0A (relative to type IIA). In the case of open strings they lead to the gauge symmetry enhancement discussed above [40].

8. Type 0 at strong coupling

Our analysis thus far has been based on perturbative constructions and nonperturbative relations motivated by the tachyon-semiclassical instability conjecture. At this point we will take an aside from the main line of this paper to complete the picture that seems to emerge. If we are bold enough to push the admittedly speculative results of Sec. II A 7 to strong coupling we may expect that the *S* dual of the type 0 theory discussed above will involve a nonsupersymmetric heterotic string theory with a tachyonic instability which is charged under the gauge symmetry. The task then is to identify which of the seven candidate theories is appropriate. We should point out the similarity between the limits of FH that we have been considering and the standard picture for the HW background 5. Where the type $0'$ theory resulted from compactifying the FH theory on a circle with a Wilson line Y breaking the $E8\times E8$ to $SO(16)\times SO(16)$, we can consider also compactifying the $H_{E8\times SO(16)}$ theory on a circle with the same Wilson line Y. The resulting *T*-dual description is the $H_{SO(32)}$ theory [41]. This leads us to conjecture that the *S*-dual of type 0 is described by the $H_{SO(32)}$ string as in Fig. 6.

An unusual feature of this *S*-duality proposal is the change in rank of the gauge symmetry group. Though *S* du-

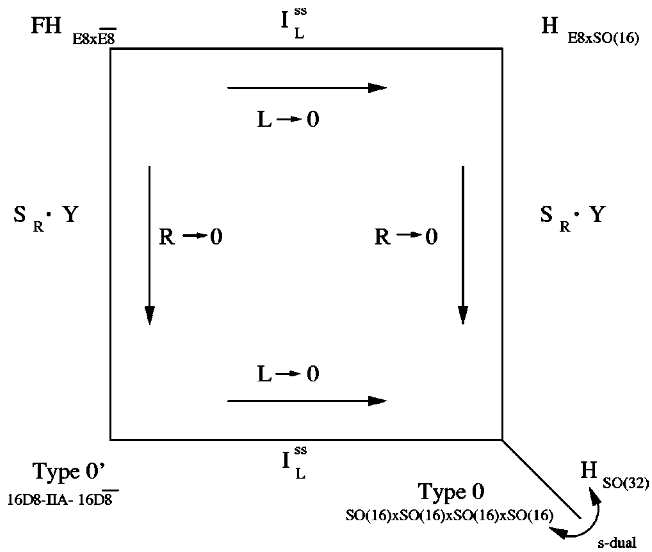

FIG. 6. The conjectured web of dualities obtained from simple compactifications of the Fabinger-Horava background; cf. Secs. II A 6 and II A 7.

alities of this type are know in field theory [42], we know of no such example in string theory. Of course the *S*-duality relationship that we have proposed has its geometric origin in a compactification torus which involves a Scherk-Schwarz cycle. The subtleties involved in a strict zero-radius limit for a Scherk-Schwarz circle should manifest itself when taking the strong coupling limit of type 0. This mismatch in gauge group rank was pointed out in a closely related context in [43,44]. It also motivated the authors of [37] to conjecture that the strong coupling dual of the ten-dimensional type 0 theory is the $D=26$ bosonic string compactified on the $SO(32)$ lattice since this is the only possible closed string theory with a rank 32 gauge group. Our conjecture stems from a larger scheme of dualities (presented in Fig. 6) akin to the familiar web of dualities shown in Fig. 5. We find these similarities very compelling and are presently working to understand the gauge symmetry enhancement issue in more detail [40].

A standard technique for supporting *S*-duality conjectures is to find a stable soliton that becomes light in the strong coupling limit and identify its fluctuation spectra with that of the fundamental degrees of freedom in the dual theory. In type I or heterotic $SO(32)$ duality for instance the massless fluctuations of the type I *D* string (with mass inversely proportional to the string coupling) are identified with world-sheet fields of the *F* string in $H^{susy}_{SO(32)}$ [45]. In particular the *DD* open string modes become the *F*-string fields with spacetime quantum numbers while the *DN* open string modes go over to the current algebra degrees of freedom.

Trying to apply this reasoning to the present case immediately confronts an ambiguity in that there are two types of nontachyonic *D* string present in type 0 [23,37]. Furthermore the fluctuation spectrum on either *D* string does not match up with the world sheet structure of the $H_{SO(32)}$ *F* string. The resolution of this ambiguity has already been suggested in [23] based on observations noted earlier in [46]. The appropriate soliton to consider is a bound state of the two *D* strings present in the theory. In particular the modes of open strings stretched between the two *D* strings give rise to the world sheet fermions carrying a spacetime vector index in the dual theory. These bound states are very interesting on their own in so far as they are very Bogomol'nyi-Prasad-Sommerfield-(BPS)-like despite being nonsupersymmetric. For example when parallel two of these bound states exhibit no force on one another in a manner analogous to BPS *D* strings. In addition the bound state is decoupled from all of the twisted sector fields in the theory including the bulk tachyon [46]. Similar proposals have been discussed for the self-duality of type 0B [23,47]. A thorough understanding of these soliton bound states would provide considerable support for the picture that we have outlined and is presently under investigation.

B. 10D→9D

We now turn to applications of the conjecture for compactifications from ten to nine dimensions. Some advantages over the previous discussion are that any Kaluza-Klein gauge field will now reside in the NSNS sector of the perturbative string descriptions and there is no coupling interpretation for compact dimensions (thus avoiding problems with strong coupling extrapolations).

Perhaps the most important aspect of starting in ten dimensions is that we have at our disposal the full quantum theory (as opposed to its low-energy effective field theory limit in the eleven-dimensional case). Since the $M^9\times S^1_{R,B}$ is flat (though globally nontrivial), string quantization on this background is straightforward. We can always reduce the theory to nine dimensions to obtain the corresponding curved NSNS Melvin backgrounds, but for our purposes the spectrum calculation in ten dimensions will suffice.

There are numerous supersymmetric ten dimensional starting points. We will first briefly present the ten-dimensional version of the twisted circle semiclassical instability which parallels Sec. II A 1. This analysis will apply to any perturbative string theory compactified on a twisted circle [48]. We will then move on to a case by case analysis of the small *R* limit. Type IIA or IIB exhibit similar behavior as do the two heterotic theories. Type I we discuss on its own.

1. Twisted circle $M^9\times S^1_{R,B}$

Our discussion of the semiclassical instability of twisted circle compactifications in 11D carries over to this case with little change. We will quickly highlight the results. The ten-dimensional geometry is flat with the nontrivial identifications

$$x_9\sim x_9+2\pi n_1 R \tag{26}$$

$$\phi\sim\phi+2\pi n_1 BR+2\pi n_2 .$$

The ten-dimensional Euclidean Kerr bounce solution is given by

$$ds_{10}^2=\left(1-\frac{\mu}{r^5\Sigma}\right)d\tau^2-\frac{2\mu\alpha\sin^2\theta}{r^5\Sigma}d\tau d\phi+\frac{\Sigma}{r^2-\alpha^2-\mu r^{-5}}dr^2+\Sigma d\theta^2+\frac{\sin^2\theta}{\Sigma}\left[(r^2-\alpha^2)\Sigma-\frac{\mu}{r^5}\alpha^2\sin^2\theta\right]d\phi^2+r^2\cos^2\theta d\Omega_6. \tag{27}$$

To match this bounce solution to the twisted circle background the black hole mass and angular momentum parameters (μ,α) must satisfy

$$R=\frac{2\mu}{7r_H^6-5\alpha^2 r_H^4} \tag{28}$$

$$B=\frac{\alpha r_H^5}{\mu}-\frac{\alpha}{|\alpha|R}$$

where the horizon radius r_H now satisfies

$$r_H^2=\alpha^2+\frac{\mu}{r_H^5}. \tag{29}$$

The decay rate is given by

$$\Gamma\sim e^{-15\pi^4\mu R/112G_{10}}. \tag{30}$$

The post decay evolution of these ten-dimensional theories may be addressed by repeating the analysis of Sec. II A 1 everywhere replacing $d\Omega_6\to d\Omega_5$. The picture is that of an expanding bubble of nothing with surface isometry group $SO(2)\times SO(6)$. Two aspects of the semiclassical instability are important for the discussion in the next section. First of all the analytically continued bounce solution (bubble) constructed in Sec. II A 1 is centered about $\rho=0$ in the plane defining the twist parameter B. It is difficult to imagine an off-axis bounce (centered around $\rho\neq 0$) having the correct asymptotic form to be glued into the decaying spacetime, i.e., $SO(2)$ isometry defined about $\rho=0$. In addition the predecay geometry is translationally invariant along the hyperplane, so one would expect the semi-classical decay to proceed by nucleating bubbles with a uniform distribution along the $\rho=0$ hyperplane. These bubbles will expand off the hyperplane eventually affecting the geometry at all points in the spacetime. For the critically twisted case $B=1/R$ the identifications (26) act trivially on the spacetime and the $U(1)\times SO(6,1)\times SO(2)$ isometry is restored to a full $U(1)\times SO(8,1)$. The $\rho=0$ hyperplane is no longer distinguished and so the geometry will decay by production of spherically symmetric bubbles nucleated throughout the spacetime.

2. Type IIA/B

The $R<\sqrt{\alpha'}$ limit of the twisted circle compactification works out very nicely for the type IIA or IIB starting points. The spectra of these theories has been analyzed in detail in [49,50] and we will recount only a few important aspects of their results. Our purpose is to compare these results with the semiclassical instability described above offering support for our conjecture that these instabilites are related.

For $|B|\neq 0,1/R$ the $(9+1)$-dimensional Lorentz invariance of uncompactified theory is broken to $(6+1)$-dimensional Lorentz invariance by the twisted compactification. These theories contain tachyonic states in the winding sectors for $R<2\alpha'|B|$. Combining this with the limited range of the twist parameter $0\leqslant|B|\leqslant 1/R$, we see that the largest value of R for which the theory is tachyonic is $R=\sqrt{2\alpha'}$ which occurs for the critical twist $|B|=1/R$. When $B=0$ the theory is supersymmetric and there are of course no tachyonic modes. For any $|B|\neq 0$ the lowest mass state is a $(w=1)$ winding mode in the NS+NS+ sector with a negative mass shift due to its angular momentum in the ϕ plane. This arises from a gyromagnetic interaction term in the string Hamiltonian of the form

$$-\frac{2BRw}{\alpha'}(\hat{J}_R-\hat{J}_L) \tag{31}$$

where $\hat{J}_R,\hat{J}_L$ are the angular momentum operators in the ϕ plane. In terms of world-sheet oscillator excitations it is the same tensor fluctuation that in flat space gives rise to the graviton.

For $|B|\neq 0,1/R$ the winding states [closing only up to $n_1=w$ in Eq. (26)] must not only stretch over the circle, but must also stretch to accommodate the arc-length subtended by $2\pi wBR$. This clearly depends on the distance ρ from the hyperplane about which ϕ is defined giving a winding energy contribution to the string mass of the form

$$\delta m^2=\frac{w^2R^2}{\alpha'^2}(1+\rho^2B^2). \tag{32}$$

The tachyonic states in the theory are necessarily winding states and for $B\neq 0$ it is clear that any finite negative mass contribution will be canceled for sufficiently large values of ρ. The tachyonic states are thus effectively localized about $\rho=0$. This fits in nicely with our conjecture relating the semiclassical instability for large R to the tachyonic instability for small R. In both cases the decay seed is localized to the distinguished hyperplane.

One can go even further and analyze the perturbative spectrum for the critical case $|B|=1/R$. Naively the argument based on Eq. (32) would seem to again imply localization of twisted states. However a careful treatment of string quantization reveals that for a critical twist the shift in normal ordering constant restores the zero mode structure in the ϕ plane [49,50]. The tachyons are no longer localized to the $\rho=0$ hyperplane in accord with the delocalization of semiclassical bubble production for the large radius critically twisted circle. This result is not suprising insofar as we can consider the critically twisted case in terms of trivial circle compactification with a reversal of spin structure on the S^1_R. One can argue that the $R \to 0$ limit of type IIA or IIB on a critically twisted (Scherk-Schwarz) can be described by type 0A or 0B on $M^9 \times S^1_{2R\to 0}$ which are better described by the T-dual type 0B or 0A theories on M^{10} [51]. This leads one to pose the following question. Suppose we start with type 0A string theory on M^{10} which has its usual flat-space tachyon. We wish to connect this tachyon to a semiclassical instability. There appear to be two candidates. Either M theory on a critically twisted circle or type IIB string theory on a critically twisted circle of vanishing radius. Though both instabilities lead to the annihilation of spacetime, the first proceeds via an eleven-dimensional bubble geometry while the latter proceeds via a ten-dimensional bubble geometry. This essentially becomes a question of limits. Though M^{10} resembles in many ways $M^9 \times S^1_{R\to\infty}$ there are global distinctions (for example quantization conditions). If we are interested in the type 0A tachyon in strictly M^{10} then we should identify it with the M-theory instability since the limiting theory is fully ten-dimensional (the compactification radius playing the role of the coupling). The Scherk-Schwarz compactification of type IIB only approaches 0A on M^{10} as $M^9 \times S^1_{R\to\infty}$. However one may still suppose that a similar ambiguity would hold for type 0A on $M^9 \times S^1_R$ where R is finite and nonzero. In this case either M theory on $M^9 \times S^{SS}_{R^{11}} \times S^1_R$ or type IIB on $M^9 \times S^{SS}_{\alpha'/2R}$ would seem to work. However the latter does not T-dualize to type 0A on $M^9 \times S^1_R$, but rather to Type 0A on $M^9 \times \tilde{S}^{SS}_{\alpha'/2R}$ [52]. Again the appropriate instability is the M-theory one.

3. Heterotic $SO(32)/E8\times E8$

The twisted circle semiclassical instability is generic. Any theory with a gravity sector formulated on this background will decay by nucleating (possibly deformed) bubbles of nothing. If the twisted circle radius is small enough the semiclassical calculation is no longer valid and we believe the same underlying instability should emerge in whatever description becomes appropriate. We can push this further and say that since the semiclassical instability is generic then, at least for twisted circle compactifications, the corresponding tachyonic instabilities should be generic. For theories with gauge degrees of freedom this can be a nontrivial issue.

Consider taking either of the supersymmetric heterotic theories and compactifying on a twisted circle. The nontrivial boundary conditions for arbitrary B will affect both bosons and fermions winding around the compact circle, but should leave massless the nonwinding gauge bosons of the heterotic theory. The gauge symmetry should thus remain unbroken. It would be natural then to expect that the tachyonic instability will be neutral under the heterotic gauge symmetry. We expect the spectrum of heterotic strings on a twisted circle to be very similar to the spectrum of type II strings on a twisted circle barring the usual differences between the spectra of the free theories. A cursory investigation of the spectrum of these theories in [50] supports this picture. In particular the negative mass contributions attributable to the gyromagnetic interaction term renders the lowest NS+NS+ states tachyonic. This gauge neutral tachyonic state is exactly what we expect from our conjecture relating it to the semiclassical instability. The story changes considerably if we include a Wilson line along the twisted circle which breaks the heterotic gauge symmetry to some subgroup. In this case the semiclassical instability is associated with a gauge symmetry breaking compactification and we expect the corresponding tachyons to transform nontrivially under the unbroken gauge group. The details of the spectrum are currently under investigation.

The critical case without a Wilson line poses an interesting problem for the heterotic strings. Consider setting $|B| = 1/R$ and sending the compactification radius to zero. By the interpolating orbifold argument the resulting description can be described in terms of a nonsupersymmetric string theory on M^{10} that is the T dual of the result of twisting the orginal theory by $(-1)^{F_s}$. Surveying the table of consistent nonsupersymmetric heterotic strings in Sec. II A 6 we find that there is no flat space theory with gauge group $E8\times E8$. In addition the theory with gauge group $SO(32)$ exhibits a tachyon which transforms nontrivially under the gauge group in contradiction to the expectations outlined above. The resolution of this puzzle is what makes the heterotic case so interesting. The two supersymmetric heterotic string theories are actually invariant under a twist by $(-1)^{F_s}$ as pointed out in [53]. Combined with the self-duality of these theories under T duality, the result is that the $R\to 0$ limit of a critically twisted compactification of these theories returns the original theory on $M^9 \times S^1_{\alpha'/R\to\infty}$. In this case the association is not between a semiclassical instability on one hand and a tachyonic instability on the other, but rather between semiclassical instabilities both large and small compactification radii. This case represents the single exception we have found to the strong form of our conjecture. If we include a Wilson line along the critically twisted circle, then the $R\to 0$ limit can be described by the T-duals of the nonsupersymmetric heterotic theories and the delocalized semi-classical instabilities descend to the bulk tachyons.

Our conjecture then implies that the fate of these theories after condensation of the closed string tachyon involves the annihilation of spacetime.

This poses a resolution to an issue raised by Suyama [54,55]. Working under the assumption that condensation of the tachyon in the nonsupersymmetric heterotic theories takes the theories to a stable supersymmetric background the only choices for the end point involve the gauge symmetries $SO(32), E8\times E8$. It was pointed out that condensation of a tachyon transforming nontrivially under a gauge symmetry should reduce the rank of the gauge group. However, the ranks of the gauge symmetries for the nonsupersymmetric heterotic strings are already as small or smaller than in the two supersymmetric cases. Our conclusion, i.e. that condensation of the tachyon leads to an annihilation of the spacetime, avoids this puzzle entirely.

A notable exception is the $H_{SO(16)\times SO(16)}$ theory which exhibits no tachyonic instability. This theory may be obtained as the T dual of a Scherk-Schwarz orbifold of either $H^{susy}_{SO(32)}$ or $H^{susy}_{E8\times E8}$ with appropriate Wilson lines. One expects a semiclassical instability towards spacetime annihilation, however in this case it is unclear to what the semiclassical instability descends. However this case is unique in that this particular Scherk-Schwarz compactification generates a positive cosmological constant. For all other Scherk-Schwarz compactifications the cosmological constant is negative driving the theory towards compactification and thus towards the tachyonic regime. In this case the positive cosmological constant generates a potential which pushes the theory towards decompactification thereby restoring the supersymmetric background with which we began.

III. CONCLUSIONS

Our aim in this paper has largely been twofold. On the one hand we have taken seriously the idea that semiclassical gravitational instabilities in supersymmetry breaking compactifications may in certain limits reduce to perturbative instabilities signaled by the appearance of a closed string tachyon. In making this identification it is then natural to identify the end point of condensation of the tachyon with the end point of the semiclassical instability. In every case that we have considered the end point involves an annihilation of the metric degrees of freedom. We have further made a case for the naturalness of such a catastrophic fate by comparing these theories to those theories exhibiting open string tachyons for which extensive evidence has been presented. In both cases the corresponding degrees of freedom are annihilated.

On the other hand we have used this connection between semiclassical instabilities and tachyons to explore a possible web of dualities involving nonsupersymetric string theories. In particular the eleven-dimensional origins of many nonsupersymmetric ten-dimensional string backgrounds has been conjectured and the overall picture appears to hang together quite nicely. Our discussion of the limits of the Fabinger-Horava theory constitute to our knowledge the first attempt to extend the type 0A or M-theory relation of Bergman and Gaberdiel [23] to the heterotic theories [56].

A by product of our arguments is that Scherk-Schwarz compactification is not a very useful tool for constructing phenomenological SUSY breaking theories. In this the usual problem has been that the radius R of the compactification circle would tend to zero because of the potential that develops at one loop (and higher) [57,58]. Thus the system approaches the tachyonic regime. However one might imagine that this modulus is stabilized either by classical flux terms or by some nonperturbative quantum effect. One would of course want this stabilization to occur at some $R > \sqrt{\alpha'}$, in order to avoid having a tachyon (and also usually to get smaller than string scale SUSY breaking). However at such radii the semiclassical instability of Witten that we discussed extensively in this paper takes over [59]. It may be possible of course that the fluxes (or quantum effects) are such as to stabilize the radius at a large enough value such that the semiclassical decay lifetime is larger than the age of the universe, but this strikes us as being somewhat unnatural.

There are a number of outstanding issues associated with our conclusions. First of all a quantitative description of the condensation of closed string tachyons in a vein similar to the open string case would put all of these speculations on a much firmer footing. The bounce solutions for Scherk-Schwarz orbifolds with Wilson lines are under consideration. These might shed light on a nonperturbative framework for the ''other'' nonsupersymmetric heterotic theories [60–62]. Subtleties associated with the zero-radius limit of Scherk-Schwarz compactifications are unresolved but under current investigation [40].

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spacetime is then identified as Euclidean type IIB by a simple spin structure argument. Though this conclusion is reminescent of what we will discuss in this paper, the interpretation of the final state of the theory as type IIB on a boundary is not entirely clear to us.

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[17] The example here is the shifted and unshifted reductions to ten dimensions of the critically twisted circle background first discussed in [14]. These reductions will be discussed in more detail in the next section.

[18] We would like to thank Steve Giddings for discussion on this point.

[19] By type IIA strings we mean a theory of closed oriented world sheets with the standard type IIA GSO constraints quantized in this background.

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[30] More precisely, the $H^{susy}_{E8\times E8}$ makes use of the left-moving current algebra in two independent sets, e.g., in the free fermionic construction the current algebra is carried by two independent sets of 16 left moving fermions. The $H^{susy}_{SO(32)}$ on the other hand makes use of a single set of 32 left moving fermions.

[31] In the free-fermionic construction the world-sheet fields are divided into 8 left and 8 right-moving bosons $(\partial X^\mu,\widetilde{\partial X^\mu})$, 8 right-moving fermions $\widetilde{\psi^\mu}$, and two sets of 16 left-moving fermions (ψ_A,ψ_B). The $H^{susy}_{E8\times E8}$ GSO constraints involve $\exp(i\pi F_A)=\exp(i\pi F_B)=\exp(i\pi\widetilde{F})=+1$, whereas the $H_{E8\times SO(16)}$ construction uses $\exp(i\pi F_A)=\exp(i\pi F_B+i\pi\widetilde{F})=+1$, where $F_A, F_B, \widetilde{F}$ are the world-sheet fermion numbers for $\psi_A, \psi_B, \widetilde{\psi^\mu}$, respectively.

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