Quantum field theory of three flavor neutrino mixing and oscillations with CP violation

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(Received 1 May 2002; published 31 July 2002)

We study in detail the quantum field theory of mixing among three generations of Dirac fermions (neutrinos). We construct the Hilbert space for the flavor fields and determine the generators of the mixing transformations. By use of these generators, we recover all the known parametrizations of the three flavor mixing matrix and we find a number of new ones. The algebra of the currents associated with the mixing transformations is shown to be a deformed $su(3)$ algebra, when CP violating phases are present. We then derive the exact oscillation formulas, exhibiting new features with respect to the usual ones. CP and T violation are also discussed.

DOI: 10.1103/PhysRevD.66.025033 PACS number(s): 14.60.Pq, 11.15.Tk, 12.15.Ff

I. INTRODUCTION

In recent years, there has been much progress in the understanding of flavor mixing in quantum field theory (QFT) [1–17]. The original discovery of the unitary inequivalence of the mass and the flavor representations in QFT [1], has prompted further investigations on fermion mixing [2–8] as well as on boson mixing [8–13]. It has emerged that the rich nonperturbative vacuum structure associated with field mixing leads to relevant modification of the flavor oscillation formulas, exhibiting new features with respect to the usual quantum-mechanical ones [18]. Some topologically nontrivial features, such as the occurrence of a geometric (Berry-Anandan) phase in field mixing has been also pointed out [19].

In this paper we study in detail the case of three flavor fermion (neutrino) mixing. This is not a simple extension of the previous results [1–3] since the existence of a CP violating phase in the parametrization of the three flavor mixing matrix introduces novel features which are absent in the two-flavor case. We determine the generators of the mixing transformations and, by use of them, we recover the known parametrizations of the three flavor mixing matrix and find a number of new ones. We construct the flavor Hilbert space, for which the ground state (flavor vacuum) turns out to be a generalized coherent state. We also study the algebraic structure of currents and charges associated with the mixing transformations and we find that, in the presence of CP violation, it is that of a deformed $su(3)$. The construction of the flavor Hilbert space is an essential step in the derivation of exact oscillation formulas, which account for CP violation and reduce to the corresponding quantum-mechanical ones in the relativistic limit.

The paper is organized as follows. In Sec. II we construct the Hilbert space for three flavor mixed fermions. In Sec. III we study the various parametrization of the unitary 3×3 mixing matrix obtained by use of the algebraic generators. In Sec. IV we study the currents and charges for three flavor mixing, which are then used in Sec. V to derive the exact neutrino oscillation formulas. Finally in Sec. VI, CP and T violations in QFT neutrino oscillations are discussed. Section VII is devoted to conclusions. In the Appendixes we give some useful formulas and a discussion of the arbitrary mass parametrization in the expansion of flavor fields as recently reported in [7,8].

II. THREE-FLAVOR FERMION MIXING

We start by considering the following Lagrangian density describing three Dirac fields with a mixed mass term:

$$\mathcal{L}(x)=\overline{\Psi}_f(x)(i\not\partial-\mathsf{M})\Psi_f(x), \qquad (1)$$

where $\Psi_f^T=(\nu_e,\nu_\mu,\nu_\tau)$ and $\mathsf{M}=\mathsf{M}^\dagger$ is the mixed mass matrix.

Among the various possible parametrizations of the mixing matrix for three fields, we choose to work with the following one since it is the familiar parametrization of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [20,21]:

$$\Psi_f(x)=\mathcal{U}\Psi_m(x)=\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}\end{pmatrix}\Psi_m(x), \qquad (2)$$

with $c_{ij}=\cos\theta_{ij}$ and $s_{ij}=\sin\theta_{ij}$, θ_{ij} being the mixing angle between ν_i,ν_j and $\Psi_m^T=(\nu_1,\nu_2,\nu_3)$.

Using Eq. (2) we diagonalize the quadratic form of Eq. (1), which then reduces to the Lagrangian for three Dirac fields, with masses m_1, m_2, and m_3:

$$\mathcal{L}(x)=\bar{\Psi}_m(x)(i\not\partial-\mathsf{M}_d)\Psi_m(x), \tag{3}$$

where $\mathsf{M}_d=\mathrm{diag}(m_1,m_2,m_3)$.

Following Ref. [1] we construct the generator for the mixing transformation (2) and define[1]

$$\nu_\sigma^\alpha(x)\equiv G_{\boldsymbol{\theta}}^{-1}(t)\nu_i^\alpha(x)G_{\boldsymbol{\theta}}(t), \tag{4}$$

where $(\sigma,i)=(e,1),(\mu,2),(\tau,3)$, and

$$G_{\boldsymbol{\theta}}(t)=G_{23}(t)G_{13}(t)G_{12}(t), \tag{5}$$

where

$$G_{12}(t)\equiv\exp[\theta_{12}L_{12}(t)],$$

$$L_{12}(t)=\int d^3x[\nu_1^\dagger(x)\nu_2(x)-\nu_2^\dagger(x)\nu_1(x)], \tag{6}$$

$$G_{23}(t)\equiv\exp[\theta_{23}L_{23}(t)],$$

$$L_{23}(t)=\int d^3x[\nu_2^\dagger(x)\nu_3(x)-\nu_3^\dagger(x)\nu_2(x)], \tag{7}$$

$$G_{13}(t)\equiv\exp[\theta_{13}L_{13}(\delta,t)],$$

$$L_{13}(\delta,t)=\int d^3x[\nu_1^\dagger(x)\nu_3(x)e^{-i\delta}-\nu_3^\dagger(x)\nu_1(x)e^{i\delta}]. \tag{8}$$

It is evident from the above form of the generators that the phase δ is unavoidable for three field mixing, while it can be incorporated in the definition of the fields in the two-flavor case.

The free fields ν_i $(i=1,2,3)$ can be quantized in the usual way [22] (we use $t\equiv x_0$):

[1]Let us consider, for example, the generation of the first row of the mixing matrix $\mathcal{U}$. We have (see also Appendix B) $\partial\nu_e/\partial\theta_{23}=0$; and $\partial\nu_e/\partial\theta_{13}=G_{12}^{-1}G_{13}^{-1}[\nu_1,L_{13}]G_{13}G_{12}=G_{12}^{-1}G_{13}^{-1}e^{-i\delta}\nu_3G_{13}G_{12}$, thus

$$\partial^2\nu_e/\partial\theta_{13}^2=-\nu_e\Rightarrow\nu_e=f(\theta_{12})\cos\theta_{13}+g(\theta_{12})\sin\theta_{13};$$

with the initial conditions [from Eq. (4)]: $f(\theta_{12})=\nu_e|_{\theta_{13}=0}$ and $g(\theta_{12})=\partial\nu_e/\partial\theta_{13}|_{\theta_{13}=0}=e^{-i\delta}\nu_3$. We also have

$$\partial^2 f(\theta_{12})/\partial\theta_{13}^2=-f(\theta_{12})\Rightarrow f(\theta_{12})=A\cos\theta_{12}+B\sin\theta_{12}$$

with the initial conditions $A=\nu_e|_{\theta=0}=\nu_1$ and $B=\partial f(\theta_{12})/\partial\theta_{12}|_{\theta=0}=\nu_2$, and $\theta=(\theta_{12},\theta_{13},\theta_{23})$.

$$\nu_i(x)=\sum_r\int d^3k[u_{\mathbf{k},i}^r(t)\alpha_{\mathbf{k},i}^r+v_{-\mathbf{k},i}^r(t)\beta_{-\mathbf{k},i}^{r\dagger}]e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{9}$$

with $i=1,2,3$, also $u_{\mathbf{k},i}^r(t)=e^{-i\omega_{k,i}t}u_{\mathbf{k},i}^r$, $v_{\mathbf{k},i}^r(t)=e^{i\omega_{k,i}t}v_{\mathbf{k},i}^r$, and $\omega_{k,i}=\sqrt{\mathbf{k}^2+m_i^2}$. The vacuum for the mass eigenstates is denoted by $|0\rangle_m$: $\alpha_{\mathbf{k},i}^r|0\rangle_m=\beta_{\mathbf{k},i}^r|0\rangle_m=0$. The anticommutation relations are the usual ones; the wave function orthonormality and completeness relations are those of Ref. [1].

The main result of Ref. [1] is the unitary inequivalence (in the infinite volume limit) of the vacua for the flavor fields and for the fields with definite masses. There such an inequivalence was proved for the case of two generations; subsequently, in Ref. [5], a rigorous general proof of such inequivalence for any number of generations has been given (see also Ref. [13]). Thus we do not need to repeat here such a proof and we define the the *flavor vacuum* as

$$|0(t)\rangle_f\equiv G_\theta^{-1}(t)|0\rangle_m. \tag{10}$$

The form of this state is considerably more complicated then the one for two generations. When $\delta=0$, the generator G_θ is an element of the $SU(3)$ group (see Sec. IV) and the flavor vacuum is classified as an $SU(3)$ generalized coherent state à la Perelomov [23]. A nonzero CP violating phase introduces an interesting modification of the algebra associated with the mixing transformations Eq. (2): we discuss this in Sec. IV.

By use of $G_\theta(t)$, the flavor fields can be expanded as

$$\nu_\sigma(x)=\sum_r\int d^3k[u_{\mathbf{k},i}^r(t)\alpha_{\mathbf{k},\sigma}^r(t)+v_{-\mathbf{k},i}^r(t)\beta_{-\mathbf{k},\sigma}^{r\dagger}(t)]e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{11}$$

with $(\sigma,i)=(e,1),(\mu,2),(\tau,3)$. The flavor annihilation operators are defined as $\alpha_{\mathbf{k},\sigma}^r(t)\equiv G_{\boldsymbol{\theta}}^{-1}(t)\alpha_{\mathbf{k},i}^rG_{\boldsymbol{\theta}}(t)$ and $\beta_{-\mathbf{k},\boldsymbol{\sigma}}^{\mathbf{r}\dagger}(t)\equiv G_{\boldsymbol{\theta}}^{-1}(t)\beta_{-\mathbf{k},i}^{r\dagger}G_{\boldsymbol{\theta}}(t)$. They clearly act as annihilators for the flavor vacuum Eq. (10). For further reference, it is useful to list explicitly the flavor annihilation or creation operators (see also Ref. [1]). In the reference frame $\mathbf{k}=(0,0,|\mathbf{k}|)$ the spins decouple and their form is particularly simple:

$$\alpha_{\mathbf{k},e}^r(t)=c_{12}c_{13}\alpha_{\mathbf{k},1}^r+s_{12}c_{13}[U_{12}^{\mathbf{k}*}(t)\alpha_{\mathbf{k},2}^r+\epsilon^rV_{12}^{\mathbf{k}}(t)\beta_{-\mathbf{k},2}^{r\dagger}]$$
$$+e^{-i\delta}s_{13}[U_{13}^{\mathbf{k}*}(t)\alpha_{\mathbf{k},3}^r+\epsilon^rV_{13}^{\mathbf{k}}(t)\beta_{-\mathbf{k},3}^{r\dagger}], \tag{12}$$

$$\alpha_{\mathbf{k},\mu}^r(t)=(c_{12}c_{23}-e^{i\delta}s_{12}s_{23}s_{13})\alpha_{\mathbf{k},2}^r$$
$$-(s_{12}c_{23}+e^{i\delta}c_{12}s_{23}s_{13})$$
$$\times[U_{12}^{\mathbf{k}}(t)\alpha_{\mathbf{k},1}^r-\epsilon^rV_{12}^{\mathbf{k}}(t)\beta_{-\mathbf{k},1}^{r\dagger}]$$
$$+s_{23}c_{13}[U_{23}^{\mathbf{k}*}(t)\alpha_{\mathbf{k},3}^r+\epsilon^rV_{23}^{\mathbf{k}}(t)\beta_{-\mathbf{k},3}^{r\dagger}], \tag{13}$$

$$\alpha_{\mathbf{k},\tau}^r(t)=c_{23}c_{13}\alpha_{\mathbf{k},3}^r-(c_{12}s_{23}+e^{i\delta}s_{12}c_{23}s_{13})$$
$$\times[U_{23}^{\mathbf{k}}(t)\alpha_{\mathbf{k},2}^r-\epsilon^rV_{23}^{\mathbf{k}}(t)\beta_{-\mathbf{k},2}^{r\dagger}]$$
$$+(s_{12}s_{23}-e^{i\delta}c_{12}c_{23}s_{13})$$
$$\times[U_{13}^{\mathbf{k}}(t)\alpha_{\mathbf{k},1}^r-\epsilon^rV_{13}^{\mathbf{k}}(t)\beta_{-\mathbf{k},1}^{r\dagger}], \tag{14}$$

TABLE I. The values of masses and mixing angles used for plots.

m_1	m_2	m_3	θ_{12}	θ_{13}	θ_{23}	δ
1	200	3000	$\pi/4$	$\pi/4$	$\pi/4$	$\pi/4$

$$\begin{aligned}\beta^r_{-\mathbf{k},e}(t)=&c_{12}c_{13}\beta^r_{-\mathbf{k},1}\\&+s_{12}c_{13}[U^{\mathbf{k}*}_{12}(t)\beta^r_{-\mathbf{k},2}-\epsilon^r V^{\mathbf{k}}_{12}(t)\alpha^{r\dagger}_{\mathbf{k},2}]\\&+e^{i\delta}s_{13}[U^{\mathbf{k}*}_{13}(t)\beta^r_{-\mathbf{k},3}-\epsilon^r V^{\mathbf{k}}_{13}(t)\alpha^{r\dagger}_{\mathbf{k},3}],\end{aligned} \tag{15}$$

$$\begin{aligned}\beta^r_{-\mathbf{k},\mu}(t)=&(c_{12}c_{23}-e^{-i\delta}s_{12}s_{23}s_{13})\beta^r_{-\mathbf{k},2}\\&-(s_{12}c_{23}+e^{-i\delta}c_{12}s_{23}s_{13})\\&\times[U^{\mathbf{k}}_{12}(t)\beta^r_{-\mathbf{k},1}+\epsilon^r V^{\mathbf{k}}_{12}(t)\alpha^{r\dagger}_{\mathbf{k},1}]\\&+s_{23}c_{13}[U^{\mathbf{k}*}_{23}(t)\beta^r_{-\mathbf{k},3}-\epsilon^r V^{\mathbf{k}}_{23}(t)\alpha^{r\dagger}_{\mathbf{k},3}],\end{aligned} \tag{16}$$

$$\begin{aligned}\beta^r_{-\mathbf{k},\tau}(t)=&c_{23}c_{13}\beta^r_{-\mathbf{k},3}-(c_{12}s_{23}+e^{-i\delta}s_{12}c_{23}s_{13})\\&\times[U^{\mathbf{k}}_{23}(t)\beta^r_{-\mathbf{k},2}+\epsilon^r V^{\mathbf{k}}_{23}(t)\alpha^{r\dagger}_{\mathbf{k},2}]\\&+(s_{12}s_{23}-e^{-i\delta}c_{12}c_{23}s_{13})\\&\times[U^{\mathbf{k}}_{13}(t)\beta^r_{-\mathbf{k},1}+\epsilon^r V^{\mathbf{k}}_{13}(t)\alpha^{r\dagger}_{\mathbf{k},1}].\end{aligned} \tag{17}$$

These operators satisfy canonical (anti)commutation relations at equal times. The main difference with respect to their ''naive'' quantum-mechanical counterparts is in the anomalous terms proportional to the V_{ij} factors. In fact, $U^{\mathbf{k}}_{ij}$ and $V^{\mathbf{k}}_{ij}$ are Bogoliubov coefficients defined as:

$$V^{\mathbf{k}}_{ij}(t)=|V^{\mathbf{k}}_{ij}|e^{i(\omega_{k,j}+\omega_{k,i})t},\quad U^{\mathbf{k}}_{ij}(t)=|U^{\mathbf{k}}_{ij}|e^{i(\omega_{k,j}-\omega_{k,i})t} \tag{18}$$

$$\begin{aligned}|U^{\mathbf{k}}_{ij}|=&\left(\frac{\omega_{k,i}+m_i}{2\omega_{k,i}}\right)^{1/2}\left(\frac{\omega_{k,j}+m_j}{2\omega_{k,j}}\right)^{1/2}\\&\times\left(1+\frac{|\mathbf{k}|^2}{(\omega_{k,i}+m_i)(\omega_{k,j}+m_j)}\right)=\cos(\xi^{\mathbf{k}}_{ij})\end{aligned} \tag{19}$$

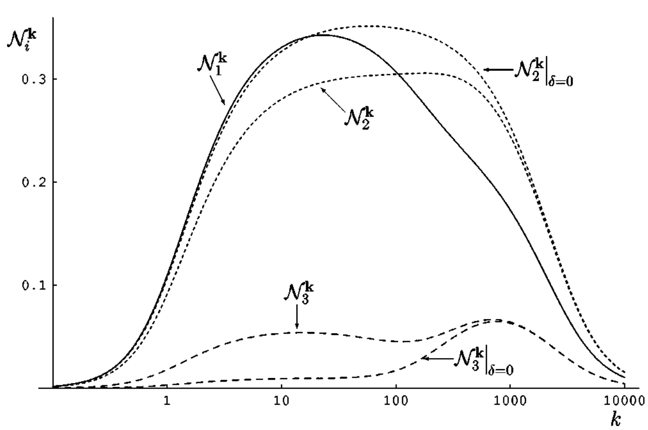

FIG. 1. Plot of the condensation densities $\mathcal{N}^{\mathbf{k}}_i$ in function of $|\mathbf{k}|$ for the values of parameters as in Table I.

$$\begin{aligned}|V^{\mathbf{k}}_{ij}|=&\left(\frac{\omega_{k,i}+m_i}{2\omega_{k,i}}\right)^{1/2}\left(\frac{\omega_{k,j}+m_j}{2\omega_{k,j}}\right)^{1/2}\\&\times\left(\frac{|\mathbf{k}|}{(\omega_{k,j}+m_j)}-\frac{|\mathbf{k}|}{(\omega_{k,i}+m_i)}\right)=\sin(\xi^{\mathbf{k}}_{ij})\end{aligned} \tag{20}$$

$$|U^{\mathbf{k}}_{ij}|^2+|V^{\mathbf{k}}_{ij}|^2=1 \tag{21}$$

where $i,j=1,2,3$ and $j>i$. The following identities hold:

$$\begin{aligned}&V^{\mathbf{k}}_{23}(t)V^{\mathbf{k}*}_{13}(t)+U^{\mathbf{k}*}_{23}(t)U^{\mathbf{k}}_{13}(t)=U^{\mathbf{k}}_{12}(t),\\&V^{\mathbf{k}}_{23}(t)U^{\mathbf{k}*}_{13}(t)-U^{\mathbf{k}*}_{23}(t)V^{\mathbf{k}}_{13}(t)=-V^{\mathbf{k}}_{12}(t)\\&U^{\mathbf{k}}_{12}(t)U^{\mathbf{k}}_{23}(t)-V^{\mathbf{k}*}_{12}(t)V^{\mathbf{k}}_{23}(t)=U^{\mathbf{k}}_{13}(t),\\&U^{\mathbf{k}}_{23}(t)V^{\mathbf{k}}_{12}(t)+U^{\mathbf{k}*}_{12}(t)V^{\mathbf{k}}_{23}(t)=V^{\mathbf{k}}_{13}(t)\\&V^{\mathbf{k}*}_{12}(t)V^{\mathbf{k}}_{13}(t)+U^{\mathbf{k}*}_{12}(t)U^{\mathbf{k}}_{13}(t)=U^{\mathbf{k}}_{23}(t),\\&V^{\mathbf{k}}_{12}(t)U^{\mathbf{k}}_{13}(t)-U^{\mathbf{k}}_{12}(t)V^{\mathbf{k}}_{13}(t)=-V^{\mathbf{k}}_{23}(t),\end{aligned} \tag{22}$$

$$\xi^{\mathbf{k}}_{13}=\xi^{\mathbf{k}}_{12}+\xi^{\mathbf{k}}_{23},\quad \xi^{\mathbf{k}}_{ij}=\arctan(|V^{\mathbf{k}}_{ij}|/|U^{\mathbf{k}}_{ij}|). \tag{23}$$

As already observed in Ref. [1] we remark that, in contrast with the case of two-flavor mixing, the condensation densities are now different for particles of different masses:

$$\begin{aligned}\mathcal{N}^{\mathbf{k}}_1=&{}_f\langle 0(t)|N^{\mathbf{k},r}_{\alpha_1}|0(t)\rangle_f={}_f\langle 0(t)|N^{\mathbf{k},r}_{\beta_1}|0(t)\rangle_f\\=&s^2_{12}c^2_{13}|V^{\mathbf{k}}_{12}|^2+s^2_{13}|V^{\mathbf{k}}_{13}|^2,\end{aligned} \tag{24}$$

$$\begin{aligned}\mathcal{N}^{\mathbf{k}}_2=&{}_f\langle 0(t)|N^{\mathbf{k},r}_{\alpha_2}|0(t)\rangle_f={}_f\langle 0(t)|N^{\mathbf{k},r}_{\beta_2}|0(t)\rangle_f\\=&|-s_{12}c_{23}+e^{i\delta}c_{12}s_{23}s_{13}|^2|V^{\mathbf{k}}_{12}|^2+s^2_{23}c^2_{13}|V^{\mathbf{k}}_{23}|^2,\end{aligned} \tag{25}$$

$$\begin{aligned}\mathcal{N}^{\mathbf{k}}_3=&{}_f\langle 0(t)|N^{\mathbf{k},r}_{\alpha_3}|0(t)\rangle_f={}_f\langle 0(t)|N^{\mathbf{k},r}_{\beta_3}|0(t)\rangle_f\\=&|-c_{12}s_{23}+e^{i\delta}s_{12}c_{23}s_{13}|^2|V^{\mathbf{k}}_{23}|^2+|s_{12}s_{23}\\&+e^{i\delta}c_{12}c_{23}s_{13}|^2|V^{\mathbf{k}}_{13}|^2.\end{aligned} \tag{26}$$

We plot in Fig. 1 the condensation densities for sample values of parameters as given in Table I.[2]

III. THE PARAMETRIZATIONS OF THE THREE-FLAVOR MIXING MATRIX

In Sec. II we have studied the generator of the mixing matrix $\mathcal{U}$ of Eq. (2). However, this matrix is only one of the various forms in which a 3×3 unitary matrix can be parameterized. Indeed, the generator Eq. (5) can be used for generating such alternative parametrizations. To see this, let us first define in a more general way the generators G_{ij} including phases for all of them:

[2]Here and in the following plots, we use the same (energy) units for the values of masses and momentum.

$$G_{12}(t)\equiv\exp[\theta_{12}L_{12}(\delta_{12},t)],$$

$$L_{12}(\delta_{12},t)=\int d^3x[\nu_1^\dagger(x)\nu_2(x)e^{-i\delta_{12}}-\nu_2^\dagger(x)\nu_1(x)e^{i\delta_{12}}], \tag{27}$$

$$G_{23}(t)\equiv\exp[\theta_{23}L_{23}(\delta_{23},t)],$$

$$L_{23}(\delta_{23},t)=\int d^3x[\nu_2^\dagger(x)\nu_3(x)e^{-i\delta_{23}}-\nu_3^\dagger(x)\nu_2(x)e^{i\delta_{23}}], \tag{28}$$

$$G_{13}(t)\equiv\exp[\theta_{13}L_{13}(\delta_{13},t)],$$

$$L_{13}(\delta_{13},t)=\int d^3x[\nu_1^\dagger(x)\nu_3(x)e^{-i\delta_{13}}-\nu_3^\dagger(x)\nu_1(x)e^{i\delta_{13}}]. \tag{29}$$

Six different matrices can be obtained by permuting the order of the G_{ij} (useful relations are listed in Appendix A) in Eq. (5). We obtain

$$G_1\equiv G_{23}G_{13}G_{12}$$

$$\mathcal{U}_1=\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13}e^{-i\delta_{12}} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}e^{i\delta_{12}}-s_{23}s_{13}c_{12}e^{i(\delta_{13}-\delta_{23})} & c_{12}c_{23}-s_{23}s_{13}s_{12}e^{-i(\delta_{23}-\delta_{13}+\delta_{12})} & s_{23}c_{13}e^{-i\delta_{23}} \\ -s_{13}c_{12}c_{23}e^{i\delta_{13}}+s_{12}s_{23}e^{i(\delta_{12}+\delta_{23})} & -c_{23}s_{13}s_{12}e^{i(\delta_{13}-\delta_{12})}-s_{23}c_{12}e^{i\delta_{23}} & c_{23}c_{13} \end{pmatrix} \tag{30}$$

$$G_2\equiv G_{23}G_{12}G_{13}$$

$$\mathcal{U}_2=\begin{pmatrix} c_{12}c_{13} & s_{12}e^{-i\delta_{12}} & s_{13}c_{12}e^{-i\delta_{13}} \\ -s_{12}c_{13}c_{23}e^{i\delta_{12}}-s_{23}s_{13}e^{i(\delta_{13}-\delta_{23})} & c_{12}c_{23} & -s_{13}c_{23}s_{12}e^{i(\delta_{12}-\delta_{13})}+s_{23}c_{13}e^{-i\delta_{23}} \\ -s_{13}c_{23}e^{i\delta_{13}}+s_{12}s_{23}c_{13}e^{i(\delta_{12}+\delta_{23})} & -c_{12}s_{23}e^{i\delta_{23}} & c_{23}c_{13}+s_{12}s_{13}s_{23}e^{i(\delta_{12}+\delta_{23}-\delta_{13})} \end{pmatrix} \tag{31}$$

$$G_3\equiv G_{13}G_{23}G_{12}$$

$$\mathcal{U}_3=\begin{pmatrix} c_{12}c_{13}+s_{13}s_{23}s_{12}e^{i(\delta_{12}-\delta_{13}+\delta_{23})} & s_{12}c_{13}e^{-i\delta_{12}}-s_{13}s_{23}c_{12}e^{i(\delta_{23}-\delta_{13})} & s_{13}c_{23}e^{-i\delta_{13}} \\ -s_{12}c_{23}e^{i\delta_{12}} & c_{12}c_{23} & s_{23}e^{-i\delta_{23}} \\ c_{13}s_{23}s_{12}e^{i(\delta_{23}+\delta_{12})}-s_{13}c_{12}e^{i\delta_{13}} & -c_{13}s_{23}c_{12}e^{i\delta_{23}}-s_{12}s_{13}e^{i(\delta_{13}-\delta_{12})} & c_{23}c_{13} \end{pmatrix} \tag{32}$$

$$G_4\equiv G_{13}G_{12}G_{23}$$

$$\mathcal{U}_4=\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13}c_{23}e^{-i\delta_{12}}-s_{13}s_{23}e^{i(\delta_{23}-\delta_{13})} & s_{12}s_{23}c_{13}e^{-i(\delta_{12}+\delta_{23})}+s_{13}c_{23}e^{-i\delta_{13}} \\ -s_{12}e^{i\delta_{12}} & c_{12}c_{23} & s_{23}c_{12}e^{-i\delta_{23}} \\ -c_{12}s_{13}e^{i\delta_{13}} & -c_{13}s_{23}e^{i\delta_{23}}-s_{12}c_{23}s_{13}e^{i(\delta_{13}-\delta_{12})} & c_{23}c_{13}-s_{12}s_{23}s_{13}e^{-i(\delta_{12}+\delta_{23}-\delta_{13})} \end{pmatrix} \tag{33}$$

$$G_5\equiv G_{12}G_{13}G_{23}$$

$$\mathcal{U}_5=\begin{pmatrix} c_{12}c_{13} & s_{12}c_{23}e^{-i\delta_{12}}-s_{13}c_{12}s_{23}e^{-i(\delta_{13}-\delta_{23})} & s_{13}c_{12}c_{23}e^{-i\delta_{13}}+s_{12}s_{23}e^{-i(\delta_{12}+\delta_{23})} \\ -s_{12}c_{13}e^{i\delta_{12}} & c_{12}c_{23}+s_{12}s_{23}s_{13}e^{i(\delta_{12}-\delta_{13}+\delta_{23})} & s_{23}c_{12}e^{-i\delta_{23}}-s_{12}c_{23}s_{13}e^{i(\delta_{12}-\delta_{13})} \\ -s_{13}e^{i\delta_{13}} & -c_{13}s_{23}e^{i\delta_{23}} & c_{23}c_{13} \end{pmatrix} \tag{34}$$

$$G_6\equiv G_{12}G_{23}G_{13}$$

$$\mathcal{U}_6=\begin{pmatrix} c_{12}c_{13}-s_{12}s_{23}s_{13}e^{-i(\delta_{12}+\delta_{23}-\delta_{13})} & s_{12}c_{23}e^{-i\delta_{12}} & c_{12}s_{13}e^{-i\delta_{13}}+s_{12}s_{23}c_{13}e^{-i(\delta_{12}+\delta_{23})} \\ -c_{12}s_{23}s_{13}e^{i(\delta_{13}-\delta_{23})}-s_{12}c_{13}e^{i\delta_{12}} & c_{12}c_{23} & c_{12}s_{23}c_{13}e^{-i\delta_{23}}-s_{12}s_{13}e^{i(\delta_{12}-\delta_{13})} \\ -c_{23}s_{13}e^{i\delta_{13}} & -s_{23}e^{i\delta_{23}} & c_{23}c_{13} \end{pmatrix}. \tag{35}$$

The above matrices are generated for a particular set of initial conditions, namely for those of Eq. (4). The freedom in the choice of the initial conditions reflects the possibility of obtaining other unitary matrices from the above ones by permuting rows and columns and by multiplying row or columns for a phase factor.

We thus can easily recover all the existing parametrizations of the CKM matrix [21,24–29]: the Maiani parametrization [24,27] is obtained from $\mathcal{U}_1$ by setting $\theta_{12}\to\theta$, $\theta_{13}\to\beta$, $\theta_{23}\to\gamma$, $\delta_{12}\to 0$, $\delta_{13}\to 0$, $\delta_{23}\to -\delta$; the Chau-Keung parametrization [24,28] is recovered from $\mathcal{U}_1$ by setting $\delta_{12}\to 0$ and $\delta_{23}\to 0$; the Kobayashi-Maskawa [21,24] is recovered from $\mathcal{U}_5$ by setting $\theta_{12}\to\theta_2$, $\theta_{13}\to\theta_1$, $\theta_{23}\to\theta_3$, $\delta_{12}\to -\delta$, $\delta_{13}\to 0$ and $\delta_{23}\to 0$, $\theta_i\to\frac{3}{2}\pi-\theta_i$, with $i=1,2,3$, and multiply the last column for (-1); the Anselm parametrization [24,29] is obtained from $\mathcal{U}_1$ by setting $\theta_{12}\leftrightarrow\theta_{13}$, then $\delta_{12}\to 0$, $\delta_{13}\to 0$, $\theta_{12}\to\pi+\theta_{12}$, $\theta_{13}\to\pi-\theta_{13}$, $\theta_{23}\to\frac{3}{2}\pi+\theta_{23}$, exchanging the second and third column and multiplying the last row for (-1).

From the above analysis it is clear that a number of new parametrizations of the mixing matrix can be generated and that a clear physical meaning can be attached to each of them, by considering the order in which the generators G_{ij} act and the initial conditions used for getting that particular matrix.

IV. CURRENTS AND CHARGES FOR THREE-FLAVOR FERMION MIXING

In this section we study the currents associated with the Lagrangians Eqs. (3) and (1). To this end, let us consider the transformations acting on the triplet of free fields with different masses Ψ_m, in the line of Ref. [4].

$\mathcal{L}$ is invariant under global $U(1)$ phase transformations of the type $\Psi_m'=e^{i\alpha}\Psi_m$: as a result, we have the conservation of the Noether charge $Q=\int d^3x I^0(x)$ [with $I^\mu(x)=\bar{\Psi}_m(x)\gamma^\mu\Psi_m(x)$] which is indeed the total charge of the system (i.e. the total lepton number).

Consider then the $SU(3)$ global transformations acting on Ψ_m:

$$\Psi_m'(x)=e^{i\alpha_j F_j}\Psi_m(x), \quad j=1,2,\ldots,8, \tag{36}$$

with α_j real constants, $F_j=\frac{1}{2}\lambda_j$ being the generators of $SU(3)$ and λ_j the Gell-Mann matrices [20].

The Lagrangian is not generally invariant under Eq. (36) and we obtain, by use of the equations of motion,

$$\delta\mathcal{L}(x)=i\alpha_j\bar{\Psi}_m(x)[F_j,\mathsf{M}_d]\Psi_m(x)=-\alpha_j\partial_\mu J^\mu_{m,j}(x)$$

$$J^\mu_{m,j}(x)=\bar{\Psi}_m(x)\gamma^\mu F_j\Psi_m(x), \quad j=1,2,\ldots,8. \tag{37}$$

It is useful to list explicitly the eight currents (we suppress spacetime dependence):

$$J^\mu_{m,1}=\frac{1}{2}[\bar{\nu}_1\gamma^\mu\nu_2+\bar{\nu}_2\gamma^\mu\nu_1],$$

$$J^\mu_{m,2}=-\frac{i}{2}[\bar{\nu}_1\gamma^\mu\nu_2-\bar{\nu}_2\gamma^\mu\nu_1]$$

$$J^\mu_{m,3}=\frac{1}{2}[\bar{\nu}_1\gamma^\mu\nu_1-\bar{\nu}_2\gamma^\mu\nu_2],$$

$$J^\mu_{m,4}=\frac{1}{2}[\bar{\nu}_1\gamma^\mu\nu_3+\bar{\nu}_3\gamma^\mu\nu_1]$$

$$J^\mu_{m,5}=-\frac{i}{2}[\bar{\nu}_1\gamma^\mu\nu_3-\bar{\nu}_3\gamma^\mu\nu_1],$$

$$J^\mu_{m,6}=\frac{1}{2}[\bar{\nu}_2\gamma^\mu\nu_3+\bar{\nu}_3\gamma^\mu\nu_2]$$

$$J^\mu_{m,7}=-\frac{i}{2}[\bar{\nu}_2\gamma^\mu\nu_3-\bar{\nu}_3\gamma^\mu\nu_2],$$

$$J^\mu_{m,8}=\frac{1}{2\sqrt{3}}[\bar{\nu}_1\gamma^\mu\nu_1+\bar{\nu}_2\gamma^\mu\nu_2-2\bar{\nu}_3\gamma^\mu\nu_3]. \tag{38}$$

The related charges $Q_{m,j}(t)\equiv\int d^3x\, J^0_{m,j}(x)$ satisfy the $su(3)$ algebra $[Q_{m,j}(t),Q_{m,k}(t)]=if_{jkl}Q_{m,l}(t)$. Note that only two of the above charges are time independent, namely $Q_{m,3}$ and $Q_{m,8}$. We can thus define the combinations

$$Q_1\equiv\frac{1}{3}Q+Q_{m,3}+\frac{1}{\sqrt{3}}Q_{m,8}, \tag{39}$$

$$Q_2\equiv\frac{1}{3}Q-Q_{m,3}+\frac{1}{\sqrt{3}}Q_{m,8}, \tag{40}$$

$$Q_3\equiv\frac{1}{3}Q-\frac{2}{\sqrt{3}}Q_{m,8}, \tag{41}$$

$$Q_i=\sum_r\int d^3k(\alpha^{r\dagger}_{\mathbf{k},i}\alpha^r_{\mathbf{k},i}-\beta^{r\dagger}_{-\mathbf{k},i}\beta^r_{-\mathbf{k},i}), \quad i=1,2,3. \tag{42}$$

These are nothing but the Noether charges associated with the non-interacting fields ν_1, ν_2, and ν_3: in the absence of mixing, they are the flavor charges, separately conserved for each generation.

As already observed in Sec. II, in the case when CP is conserved ($\delta=0$), the mixing generator Eq. (5) is an element of the $SU(3)$ group and can be expressed in terms of the above charges as

$$G_{\boldsymbol{\theta}}(t)|_{\delta=0}=e^{i2\theta_{23}Q_{m,7}(t)}e^{i2\theta_{13}Q_{m,5}(t)}e^{i2\theta_{12}Q_{m,2}(t)}. \tag{43}$$

Following Ref. [4] we can now perform the $SU(3)$ transformations on the flavor triplet Ψ_f and obtain another set of currents for the flavor fields:

$$\Psi_f'(x)=e^{i\alpha_j F_j}\Psi_f(x), \quad j=1,2,\ldots,8, \tag{44}$$

which leads to

$$\delta\mathcal{L}(x)=i\alpha_j\bar{\Psi}_f(x)[F_j,\mathsf{M}]\Psi_f(x)=-\alpha_j\partial_\mu J^\mu_{f,j}(x),$$

$$J^{\mu}_{f,j}(x)=\bar{\Psi}_f(x)\gamma^{\mu}F_j\Psi_f(x),\quad j=1,2,\ldots,8. \tag{45}$$

Alternatively, the same currents can be obtained by applying on the $J^{\mu}_{m,j}(x)$ the mixing generator Eq. (5):

$$J^{\mu}_{f,j}(x)=G^{-1}_{\theta}(t)J^{\mu}_{m,j}(x)G_{\theta}(t),\quad j=1,2,\ldots,8. \tag{46}$$

The related charges $Q_{f,j}(t) \equiv \int d^3x\, J^0_{f,j}(x)$ still close the $su(3)$ algebra. Due to the off-diagonal (mixing) terms in the mass matrix M, $Q_{f,3}(t)$ and $Q_{f,8}(t)$ are time dependent. This implies an exchange of charge between ν_e, ν_μ and ν_τ, resulting in the flavor oscillations.

In accordance with Eqs. (44)–(41), we define the *flavor charges* for mixed fields as

$$Q_e(t)\equiv\frac{1}{3}Q+Q_{f,3}(t)+\frac{1}{\sqrt{3}}Q_{f,8}(t), \tag{47}$$

$$Q_\mu(t)\equiv\frac{1}{3}Q-Q_{f,3}(t)+\frac{1}{\sqrt{3}}Q_{f,8}(t), \tag{48}$$

$$Q_\tau(t)\equiv\frac{1}{3}Q-\frac{2}{\sqrt{3}}Q_{f,8}(t), \tag{49}$$

with $Q_e(t)+Q_\mu(t)+Q_\tau(t)=Q$. These charges have a simple expression in terms of the flavor ladder operators:

$$Q_\sigma(t)=\sum_r\int d^3k[\alpha^{r\dagger}_{\mathbf{k},\sigma}(t)\alpha^{r}_{\mathbf{k},\sigma}(t)-\beta^{r\dagger}_{-\mathbf{k},\sigma}(t)\beta^{r}_{-\mathbf{k},\sigma}(t)], \tag{50}$$

with $\sigma=e,\mu,\tau$,because of the connection with the Noether charges of Eq. (42) via the mixing generator: $Q_\sigma(t)=G^{-1}_{\theta}(t)Q_iG_{\theta}(t)$. Notice also that the operator $\Delta Q_\sigma(t)\equiv Q_\sigma(t)-Q_i$ with $(\sigma,i)=(e,1),(\mu,2),(\tau,3)$ describes how much the mixing violates the (lepton) charge conservation for a given generation.

Let us now come back to the algebra of the currents including CP violating phases. To this end, we consider a generalization of the Gell-Mann matrices (we use a tilde for denoting the modified quantities including phases):

$$\tilde{\lambda}_1=\begin{pmatrix}0 & e^{i\delta_2} & 0\\ e^{-i\delta_2} & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\quad \tilde{\lambda}_2=\begin{pmatrix}0 & -ie^{i\delta_2} & 0\\ ie^{-i\delta_2} & 0 & 0\\ 0 & 0 & 0\end{pmatrix},$$

$$\tilde{\lambda}_3=\begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\end{pmatrix},\quad \tilde{\lambda}_4=\begin{pmatrix}0 & 0 & e^{-i\delta_5}\\ 0 & 0 & 0\\ e^{i\delta_5} & 0 & 0\end{pmatrix}$$

$$\tilde{\lambda}_5=\begin{pmatrix}0 & 0 & -ie^{-i\delta_5}\\ 0 & 0 & 0\\ ie^{i\delta_5} & 0 & 0\end{pmatrix},\quad \tilde{\lambda}_6=\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & e^{i\delta_7}\\ 0 & e^{-i\delta_7} & 0\end{pmatrix},$$

$$\tilde{\lambda}_7=\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & -ie^{i\delta_7}\\ 0 & ie^{-i\delta_7} & 0\end{pmatrix},\quad \tilde{\lambda}_8=\frac{1}{\sqrt{3}}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -2\end{pmatrix}. \tag{51}$$

These are normalized as the Gell-Mann matrices: $\mathrm{tr}(\lambda_j\lambda_k)=2\delta_{jk}$. We define as usual the algebraic generators:

$$\tilde{F}_j=\frac{1}{2}\tilde{\lambda}_j,\quad j=1,\ldots,8. \tag{52}$$

The above generators Eq. (52) do not close the $su(3)$ algebra unless the condition $\delta_2+\delta_5+\delta_7=0$ is imposed [cf. Eq. (53)], i.e. if one of the three phases is fixed in terms of the remaining two. Such a request is clearly incompatible with the parametrizations of the mixing matrices of Secs. II and III [cf., e.g., the discussion after Eq. (35); we have the correspondence $\{\delta_2,\delta_5,\delta_7\}\leftrightarrow\{\delta_{12},\delta_{13},\delta_{23}\}$]. For the matrix (2), as already observed, it implies $\delta=0$.

The $\tilde{F}_j$ satisfy a deformed $su(3)$ algebra with deformed commutation relations given by

$$\begin{aligned}
&[\tilde{F}_2,\tilde{F}_5]=\frac{i}{2}\tilde{F}_7e^{-i\Delta(\tilde{F}_3-\sqrt{3}\tilde{F}_8)},\\
&[\tilde{F}_2,\tilde{F}_7]=-\frac{i}{2}\tilde{F}_5e^{-i\Delta(\tilde{F}_3+\sqrt{3}\tilde{F}_8)},\quad [\tilde{F}_5,\tilde{F}_7]=\frac{i}{2}\tilde{F}_2e^{2i\Delta\tilde{F}_3}\\
&[\tilde{F}_1,\tilde{F}_4]=\frac{i}{2}\tilde{F}_7e^{-i\Delta(\tilde{F}_3-\sqrt{3}\tilde{F}_8)},\\
&[\tilde{F}_1,\tilde{F}_7]=-\frac{i}{2}\tilde{F}_4e^{-i\Delta(\tilde{F}_3+\sqrt{3}\tilde{F}_8)},\quad [\tilde{F}_4,\tilde{F}_7]=\frac{i}{2}\tilde{F}_1e^{2i\Delta\tilde{F}_3}\\
&[\tilde{F}_1,\tilde{F}_5]=-\frac{i}{2}\tilde{F}_6e^{-i\Delta(\tilde{F}_3-\sqrt{3}\tilde{F}_8)},\\
&[\tilde{F}_1,\tilde{F}_6]=\frac{i}{2}\tilde{F}_5e^{-i\Delta(\tilde{F}_3+\sqrt{3}\tilde{F}_8)},\quad [\tilde{F}_5,\tilde{F}_6]=-\frac{i}{2}\tilde{F}_1e^{2i\Delta\tilde{F}_3}\\
&[\tilde{F}_2,\tilde{F}_4]=\frac{i}{2}\tilde{F}_6e^{-i\Delta(\tilde{F}_3-\sqrt{3}\tilde{F}_8)},\\
&[\tilde{F}_2,\tilde{F}_6]=-\frac{i}{2}\tilde{F}_4e^{-i\Delta(\tilde{F}_3+\sqrt{3}\tilde{F}_8)},\quad [\tilde{F}_4,\tilde{F}_6]=\frac{i}{2}\tilde{F}_2e^{2i\Delta\tilde{F}_3}
\end{aligned} \tag{53}$$

where $\Delta\equiv\delta_2+\delta_5+\delta_7$. The other commutators are the usual $su(3)$ ones. For $\Delta=0$, the $su(3)$ algebra is recovered.

It is useful to look at the deformed algebra in terms of the raising and lowering operators, defined as [20]

$$\tilde{T}_{\pm}\equiv\tilde{F}_1\pm i\tilde{F}_2,\quad \tilde{U}_{\pm}\equiv\tilde{F}_6\pm i\tilde{F}_7,\quad \tilde{V}_{\pm}\equiv\tilde{F}_4\pm i\tilde{F}_5. \tag{54}$$

We also define

$$\tilde{T}_3\equiv\tilde{F}_3,\quad \tilde{U}_3\equiv\frac{1}{2}(\sqrt{3}\tilde{F}_8-\tilde{F}_3),\quad \tilde{V}_3\equiv\frac{1}{2}(\sqrt{3}\tilde{F}_8+\tilde{F}_3). \tag{55}$$

Then the only deformed commutators are the following ones:

$$[\widetilde{T}_+,\widetilde{V}_-]=-\widetilde{U}_-e^{2i\Delta\widetilde{U}_3},\quad [\widetilde{T}_+,\widetilde{U}_+]=\widetilde{V}_+e^{-2i\Delta\widetilde{V}_3},$$
$$[\widetilde{U}_+,\widetilde{V}_-]=\widetilde{T}_-e^{2i\Delta\widetilde{T}_3}. \tag{56}$$

V. NEUTRINO OSCILLATIONS

The oscillation formulas are obtained by taking expectation values of the above charges on the (flavor) neutrino state. Consider for example an initial electron neutrino state defined as $|\nu_e\rangle\equiv\alpha^{r\dagger}_{\mathbf{k},e}(0)|0\rangle_f$ (for a discussion on the correct definition of flavor states see Refs. [2,3,11]). Working in the Heisenberg picture, we obtain

$$\mathcal{Q}^{\rho}_{\mathbf{k},\sigma}(t)\equiv\langle\nu_\rho|Q_\sigma(t)|\nu_\rho\rangle-{}_f\langle0|Q_\sigma(t)|0\rangle_f$$
$$=|\{\alpha^r_{\mathbf{k},\sigma}(t),\alpha^{r\dagger}_{\mathbf{k},\rho}(0)\}|^2+|\{\beta^{r\dagger}_{-\mathbf{k},\sigma}(t),\alpha^{r\dagger}_{\mathbf{k},\rho}(0)\}|^2, \tag{57}$$

$$\mathcal{Q}^{\bar{\rho}}_{\mathbf{k},\sigma}(t)\equiv\langle\bar{\nu}_\rho|Q_\sigma(t)|\bar{\nu}_\rho\rangle-{}_f\langle0|Q_\sigma(t)|0\rangle_f$$
$$=-|\{\beta^r_{\mathbf{k},\sigma}(t),\beta^{r\dagger}_{\mathbf{k},\rho}(0)\}|^2-|\{\alpha^{r\dagger}_{-\mathbf{k},\sigma}(t),\beta^{r\dagger}_{\mathbf{k},\rho}(0)\}|^2, \tag{58}$$

where $|0\rangle_f\equiv|0(0)\rangle_f$. Overall charge conservation is obviously ensured at any time: $\mathcal{Q}_{\mathbf{k},e}(t)+\mathcal{Q}_{\mathbf{k},\mu}(t)+\mathcal{Q}_{\mathbf{k},\tau}(t)=1$. We remark that the expectation value of Q_σ cannot be taken on vectors of the Fock space built on $|0\rangle_m$, as shown in Refs. [2,3,11]. Also we observe that ${}_f\langle0|Q_\sigma(t)|0\rangle_f\neq0$, in contrast with the two-flavor case [3,8]. We introduce the following notation:

$$\Delta^{\mathbf{k}}_{ij}\equiv\frac{\omega_{k,j}-\omega_{k,i}}{2},\quad \Omega^{\mathbf{k}}_{ij}\equiv\frac{\omega_{k,i}+\omega_{k,j}}{2}.$$

Then the oscillation (in time) formulas for the flavor charges, on an initial electron neutrino state, follow as

$$\mathcal{Q}^e_{\mathbf{k},e}(t)=1-\sin^2(2\theta_{12})\cos^4\theta_{13}[|U^{\mathbf{k}}_{12}|^2\sin^2(\Delta^{\mathbf{k}}_{12}t)+|V^{\mathbf{k}}_{12}|^2\sin^2(\Omega^{\mathbf{k}}_{12}t)]-\sin^2(2\theta_{13})\cos^2\theta_{12}[|U^{\mathbf{k}}_{13}|^2\sin^2(\Delta^{\mathbf{k}}_{13}t)$$
$$+|V^{\mathbf{k}}_{13}|^2\sin^2(\Omega^{\mathbf{k}}_{13}t)]-\sin^2(2\theta_{13})\sin^2\theta_{12}[|U^{\mathbf{k}}_{23}|^2\sin^2(\Delta^{\mathbf{k}}_{23}t)+|V^{\mathbf{k}}_{23}|^2\sin^2(\Omega^{\mathbf{k}}_{23}t)], \tag{59}$$

$$\mathcal{Q}^e_{\mathbf{k},\mu}(t)=2J_{CP}[|U^{\mathbf{k}}_{12}|^2\sin(2\Delta^{\mathbf{k}}_{12}t)-|V^{\mathbf{k}}_{12}|^2\sin(2\Omega^{\mathbf{k}}_{12}t)+(|U^{\mathbf{k}}_{12}|^2-|V^{\mathbf{k}}_{13}|^2)\sin(2\Delta^{\mathbf{k}}_{23}t)+(|V^{\mathbf{k}}_{12}|^2-|V^{\mathbf{k}}_{13}|^2)\sin(2\Omega^{\mathbf{k}}_{23}t)$$
$$-|U^{\mathbf{k}}_{13}|^2\sin(2\Delta^{\mathbf{k}}_{13}t)+|V^{\mathbf{k}}_{13}|^2\sin(2\Omega^{\mathbf{k}}_{13}t)]+\cos^2\theta_{13}\sin\theta_{13}[\cos\delta\sin(2\theta_{12})\sin(2\theta_{23})$$
$$+4\cos^2\theta_{12}\sin\theta_{13}\sin^2\theta_{23}][|U^{\mathbf{k}}_{13}|^2\sin^2(\Delta^{\mathbf{k}}_{13}t)+|V^{\mathbf{k}}_{13}|^2\sin^2(\Omega^{\mathbf{k}}_{13}t)]$$
$$-\cos^2\theta_{13}\sin\theta_{13}[\cos\delta\sin(2\theta_{12})\sin(2\theta_{23})-4\sin^2\theta_{12}\sin\theta_{13}\sin^2\theta_{23}][|U^{\mathbf{k}}_{23}|^2\sin^2(\Delta^{\mathbf{k}}_{23}t)+|V^{\mathbf{k}}_{23}|^2\sin^2(\Omega^{\mathbf{k}}_{23}t)]$$
$$+\cos^2\theta_{13}\sin(2\theta_{12})[(\cos^2\theta_{23}-\sin^2\theta_{23}\sin^2\theta_{13})\sin(2\theta_{12})+\cos\delta\cos(2\theta_{12})\sin\theta_{13}\sin(2\theta_{23})]$$
$$\times[|U^{\mathbf{k}}_{12}|^2\sin^2(\Delta^{\mathbf{k}}_{12}t)+|V^{\mathbf{k}}_{12}|^2\sin^2(\Omega^{\mathbf{k}}_{12}t)], \tag{60}$$

$$\mathcal{Q}^e_{\mathbf{k},\tau}(t)=-2J_{CP}[|U^{\mathbf{k}}_{12}|^2\sin(2\Delta^{\mathbf{k}}_{12}t)-|V^{\mathbf{k}}_{12}|^2\sin(2\Omega^{\mathbf{k}}_{12}t)+(|U^{\mathbf{k}}_{12}|^2-|V^{\mathbf{k}}_{13}|^2)\sin(2\Delta^{\mathbf{k}}_{23}t)+(|V^{\mathbf{k}}_{12}|^2-|V^{\mathbf{k}}_{13}|^2)\sin(2\Omega^{\mathbf{k}}_{23}t)$$
$$-|U^{\mathbf{k}}_{13}|^2\sin(2\Delta^{\mathbf{k}}_{13}t)+|V^{\mathbf{k}}_{13}|^2\sin(2\Omega^{\mathbf{k}}_{13}t)]-\cos^2\theta_{13}\sin\theta_{13}[\cos\delta\sin(2\theta_{12})\sin(2\theta_{23})$$
$$-4\cos^2\theta_{12}\sin\theta_{13}\cos^2\theta_{23}][|U^{\mathbf{k}}_{13}|^2\sin^2(\Delta^{\mathbf{k}}_{13}t)+|V^{\mathbf{k}}_{13}|^2\sin^2(\Omega^{\mathbf{k}}_{13}t)]$$
$$+\cos^2\theta_{13}\sin\theta_{13}[\cos\delta\sin(2\theta_{12})\sin(2\theta_{23})+4\sin^2\theta_{12}\sin\theta_{13}\cos^2\theta_{23}][|U^{\mathbf{k}}_{23}|^2\sin^2(\Delta^{\mathbf{k}}_{23}t)$$
$$+|V^{\mathbf{k}}_{23}|^2\sin^2(\Omega^{\mathbf{k}}_{23}t)]+\cos^2\theta_{13}\sin(2\theta_{12})[(\sin^2\theta_{23}-\sin^2\theta_{13}\cos^2\theta_{23})\sin(2\theta_{12})$$
$$-\cos\delta\cos(2\theta_{12})\sin\theta_{13}\sin(2\theta_{23})][|U^{\mathbf{k}}_{12}|^2\sin^2(\Delta^{\mathbf{k}}_{12}t)+|V^{\mathbf{k}}_{12}|^2\sin^2(\Omega^{\mathbf{k}}_{12}t)], \tag{61}$$

where we used the relations (22) and (23). We also introduced the Jarlskog factor J_{CP} defined as [30]

$$J_{CP}\equiv\mathrm{Im}(u_{i\alpha}u_{j\beta}u^*_{i\beta}u^*_{j\alpha}), \tag{62}$$

where the u_{ij} are the elements of mixing matrix $\mathcal{U}$ and $i\neq j,\alpha\neq\beta$. In the parametrization Eq. (2), J_{CP} is given by

$$J_{CP}=\frac{1}{8}\sin\delta\sin(2\theta_{12})\sin(2\theta_{13})\cos\theta_{13}\sin(2\theta_{23}). \tag{63}$$

Evidently, J_{CP} vanishes if $\theta_{ij}=0,\pi/2$ and/or $\delta=0,\pi$: all CP–violating effects are proportional to it.

The above oscillation formulas are exact. The differences with respect to the usual formulas for neutrino oscillations are in the energy dependence of the amplitudes and in the

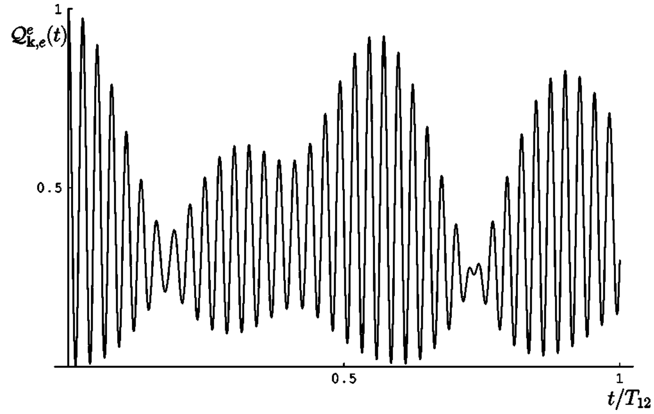

FIG. 2. Plot of the QFT oscillation formula: $\mathcal{Q}^e_{\mathbf{k},e}(t)$ as a function of time for $k=55$ and parameters as in Table I.

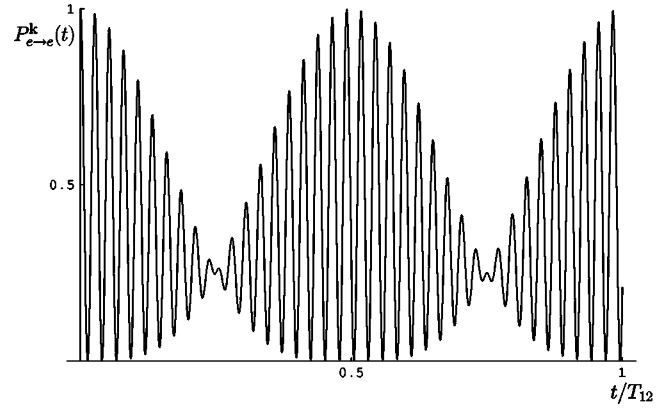

FIG. 3. Plot of the QM oscillation formula: $P^{\mathbf{k}}_{e\to e}(t)$ as a function of time for $k=55$ and parameters as in Table I.

additional oscillating terms. For $|\mathbf{k}|\gg\sqrt{m_1 m_2}$, we have $|U^{\mathbf{k}}_{ij}|^2\to 1$ and $|V^{\mathbf{k}}_{ij}|^2\to 0$ and the traditional (Pontecorvo) oscillation formulas are approximately recovered. Indeed, for sufficiently small time arguments, a correction to the Pontecorvo formula is present even in the relativistic limit.

In Appendix B the oscillation formulas for the flavor charges on an initial electron anti-neutrino state are given. We plot in Figs. 2 and 4 the QFT oscillation formulas $\mathcal{Q}^e_{\mathbf{k},e}(t)$ and $\mathcal{Q}^e_{\mathbf{k},\mu}(t)$ as a function of time, and in Figs. 3 and 5 the corresponding Pontecorvo oscillation formulas $P^{\mathbf{k}}_{e\to e}(t)$ and $P^{\mathbf{k}}_{e\to\mu}(t)$. The time scale is in T_{12} units, where $T_{12}=\pi/\Delta^{\mathbf{k}}_{12}$ is, for the values of parameters of Table I, the largest oscillation period.

VI. *CP* AND *T* VIOLATIONS IN NEUTRINO OSCILLATIONS

In this section we consider the oscillation induced CP and T violation in the context of the present QFT framework. Let us first briefly recall the situation in QM:[3] there, the CP asymmetry between the probabilities of two conjugate neutrino transitions, due to CPT invariance and unitarity of the mixing matrix, is given as [25]

$$\hat{\Delta}^{\rho\sigma}_{CP}(t)\equiv P_{\nu_\sigma\to\nu_\rho}(t)-P_{\bar{\nu}_\sigma\to\bar{\nu}_\rho}(t), \tag{64}$$

where $\sigma,\rho=e,\mu,\tau$. The T violating asymmetry can be obtained in a similar way as [25]

$$\hat{\Delta}^{\rho\sigma}_{T}(t)\equiv P_{\nu_\sigma\to\nu_\rho}(t)-P_{\nu_\rho\to\nu_\sigma}(t)$$
$$=P_{\nu_\sigma\to\nu_\rho}(t)-P_{\nu_\sigma\to\nu_\rho}(-t). \tag{65}$$

The relationship $\hat{\Delta}^{\rho\sigma}_{CP}(t)=\hat{\Delta}^{\rho\sigma}_{T}(t)$ is a consequence of CPT invariance.

The corresponding quantities in QFT have to be defined in the framework of the previous section, i.e. as expectation values of the flavor charges on states belonging to the flavor Hilbert space. We thus have, for the CP violation,

$$\Delta^{\rho\sigma}_{CP}(t)\equiv\mathcal{Q}^{\rho}_{\mathbf{k},\sigma}(t)+\mathcal{Q}^{\bar{\rho}}_{\mathbf{k},\sigma}(t) \tag{66}$$

$$=|\{\alpha^r_{\mathbf{k},\sigma}(t),\alpha^{r\dagger}_{\mathbf{k},\rho}(0)\}|^2+|\{\beta^{r\dagger}_{-\mathbf{k},\sigma}(t),\alpha^{r\dagger}_{\mathbf{k},\rho}(0)\}|^2$$
$$-|\{\alpha^{r\dagger}_{-\mathbf{k},\sigma}(t),\beta^{r\dagger}_{\mathbf{k},\rho}(0)\}|^2-|\{\beta^{r}_{\mathbf{k},\sigma}(t),\beta^{r\dagger}_{\mathbf{k},\rho}(0)\}|^2. \tag{67}$$

We have

$$\sum_\sigma \Delta^{\rho\sigma}_{CP}=0,\quad \rho,\sigma=e,\mu,\tau, \tag{68}$$

which follows from the fact that $\Sigma_\sigma Q_\sigma(t)=Q$ and $\langle\nu_\rho|Q|\nu_\rho\rangle=1$ and $\langle\overline{\nu_\rho}|Q|\overline{\nu_\rho}\rangle=-1$.

We can calculate the CP asymmetry Eq. (66) for a specific case, namely for the transition $\nu_e\to\nu_\mu$. We obtain

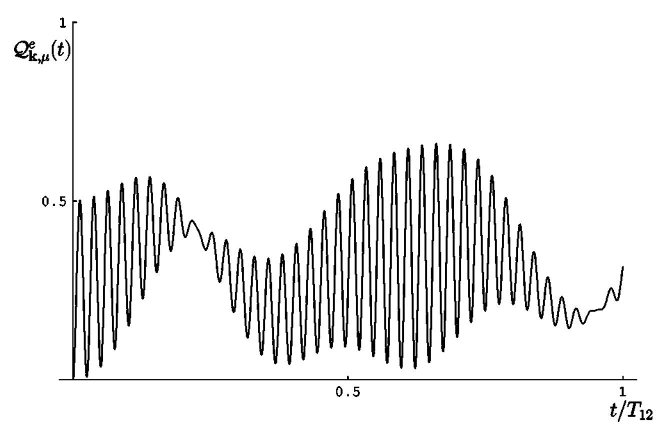

FIG. 4. Plot of the QFT oscillation formula: $\mathcal{Q}^e_{\mathbf{k},\mu}(t)$ as a function of time for $k=55$ and parameters as in Table I.

[3] We use here a ‘‘caret’’ for QM quantities. For notational simplicity, we also suppress momentum indices where unnecessary.

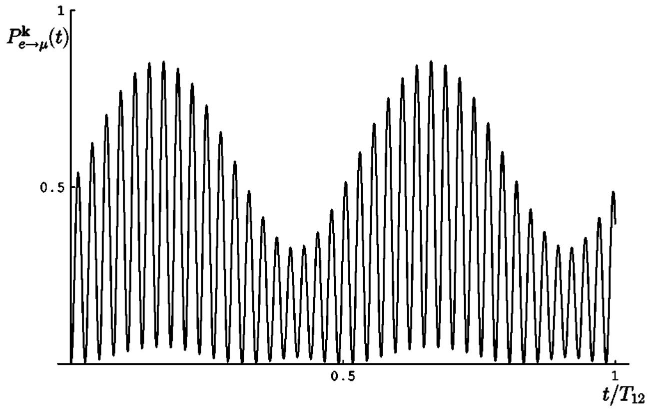

FIG. 5. Plot of the QM oscillation formula: $P^{\mathbf{k}}_{e\to\mu}(t)$ as a function of time for $k=55$ and parameters as in Table I.

$$\Delta^{e\mu}_{CP}(t)=4J_{CP}[|U^{\mathbf{k}}_{12}|^2\sin(2\Delta^{\mathbf{k}}_{12}t)-|V^{\mathbf{k}}_{12}|^2\sin(2\Omega^{\mathbf{k}}_{12}t)$$
$$+(|U^{\mathbf{k}}_{12}|^2-|V^{\mathbf{k}}_{13}|^2)\sin(2\Delta^{\mathbf{k}}_{23}t)$$
$$+(|V^{\mathbf{k}}_{12}|^2-|V^{\mathbf{k}}_{13}|^2)\sin(2\Omega^{\mathbf{k}}_{23}t)-|U^{\mathbf{k}}_{13}|^2\sin(2\Delta^{\mathbf{k}}_{13}t)$$
$$+|V^{\mathbf{k}}_{13}|^2\sin(2\Omega^{\mathbf{k}}_{13}t)], \tag{69}$$

and $\Delta^{e\tau}_{CP}(t)=-\Delta^{e\mu}_{CP}(t)$. As already observed for oscillation formulas, high-frequency oscillating terms and Bogoliubov coefficients in the oscillation amplitudes appear in Eq. (69) as a QFT correction to the QM formula.

The definition of the QFT analogue of the T-violating quantity Eq. (65) is more delicate. Indeed, defining Δ_T as $\Delta^{e\mu}_T\equiv\mathcal{Q}^e_\mu(t)-\mathcal{Q}^\mu_e(t)$ does not seem to work, since we obtain $\Delta^{e\mu}_T-\Delta^{e\mu}_{CP}\neq 0$ in contrast to CPT conservation.

A more consistent definition of the time-reversal violation in QFT is then

$$\Delta^{\rho\sigma}_T(t)\equiv\mathcal{Q}^\rho_{\mathbf{k},\sigma}(t)-\mathcal{Q}^\rho_{\mathbf{k},\sigma}(-t), \quad \rho,\sigma=e,\mu,\tau. \tag{70}$$

With such definition, the equality $\Delta^{\rho\sigma}_T(t)=\Delta^{\rho\sigma}_{CP}(t)$ follows from $\mathcal{Q}^\rho_{\mathbf{k},\sigma}(-t)=-\mathcal{Q}^{\bar{\rho}}_{\mathbf{k},\sigma}(t)$.

We plot in Fig. 6 the CP asymmetry Eq. (69) for sample values of the parameters as in Table I. In Fig. 7 the corre-

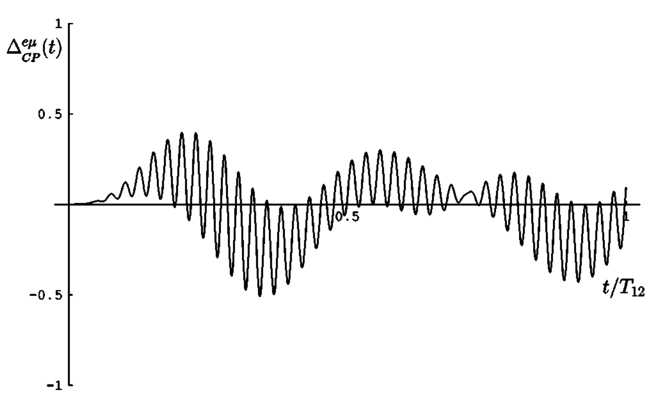

FIG. 6. Plot of the QFT CP asymmetry $\Delta^{e\mu}_{CP}(t)$, as a function of time for $k=55$ and parameters as in Table I.

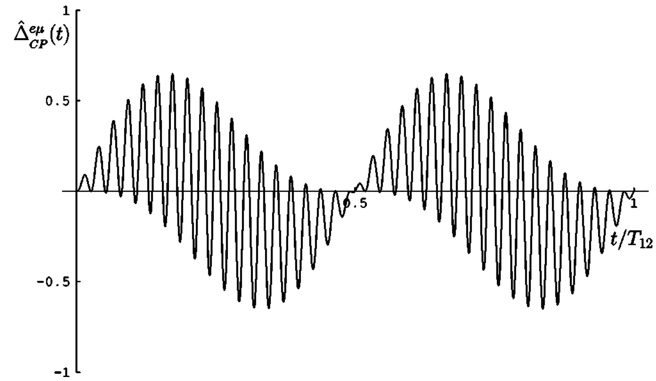

FIG. 7. Plot of the QM CP asymmetry $\hat{\Delta}^{e\mu}_{CP}(t)$, as a function of time for $k=55$ and parameters as in Table I.

sponding standard QM quantity is plotted for the same values of parameters.

VII. CONCLUSIONS

In this paper we have discussed the mixing of (Dirac) fermionic fields in quantum field theory for the case of three flavors with CP violation. We constructed the flavor Hilbert space and studied the currents and charges for mixed fields (neutrinos). The algebraic structure associated with the mixing for the case of three generation turned out to be that of a deformed $su(3)$ algebra, when a CP violating phase is present.

We have then derived all the known parametrization of the three flavor mixing matrix and a number of new ones. We have shown that these parametrizations actually reflect the group theoretical structure of the generator of the mixing transformations.

By use of the flavor Hilbert space, we have calculated the exact QFT oscillation formulas, a generalization of the usual QM Pontecorvo formulas. The comparison between the exact oscillation formulas and the usual ones has been explicitly exhibited for sample values of the neutrino masses and mixings. CP and T violation induced by neutrino oscillations have also been discussed.

We remark that the corrections introduced by the present formalism to the usual Pontecorvo formulas are in principle experimentally testable. The fact that these corrections may be quantitatively below the experimental accuracy reachable at the present state of the art in the detection of the neutrino oscillations does not justify neglecting them in the analysis of the particle mixing and oscillation mechanism. The exact oscillation formulas here derived are the result of a mathematically consistent analysis which cannot be ignored in a correct treatment of the field mixing phenomenon. As we have seen above, our formalism accounts for all the known parametrizations of the mixing matrix and explains their origin and their reciprocal relations, thus unifying the phenomenological proposals scattered in the literature where such parametrizations have been presented. Moreover, our formalism clearly points to the truly nonperturbative character of the particle mixing phenomenon. A lot of physics must be there waiting to be discovered.

ACKNOWLEDGMENTS

The present research has been carried out in the framework of the ESF network COSLAB and we acknowledge partial financial support also by INFN, INFM and EPSRC.

APPENDIX A: ANTI-NEUTRINO OSCILLATION FORMULAS

If we consider an initial electron anti-neutrino state defined as $|\overline{\nu}_e\rangle \equiv \beta^{r\dagger}_{\mathbf{k},e}(0)|0\rangle_f$, we obtain the anti-neutrino oscillation formulas as

$$\mathcal{Q}^{\bar{e}}_{\mathbf{k},e}(t) = -\mathcal{Q}^{e}_{\mathbf{k},e}(t), \tag{A1}$$

$$\begin{aligned}\mathcal{Q}^{\bar{e}}_{\mathbf{k},\mu}(t) = 2J_{CP}[&|U^{\mathbf{k}}_{12}|^2\sin(2\Delta^{\mathbf{k}}_{12}t) - |V^{\mathbf{k}}_{12}|^2\sin(2\Omega^{\mathbf{k}}_{12}t) + (|U^{\mathbf{k}}_{12}|^2 - |V^{\mathbf{k}}_{13}|^2)\sin(2\Delta^{\mathbf{k}}_{23}t)\\
&+(|V^{\mathbf{k}}_{12}|^2 - |V^{\mathbf{k}}_{13}|^2)\sin(2\Omega^{\mathbf{k}}_{23}t) - |U^{\mathbf{k}}_{13}|^2\sin(2\Delta^{\mathbf{k}}_{13}t) + |V^{\mathbf{k}}_{13}|^2\sin(2\Omega^{\mathbf{k}}_{13}t)]\\
&-\cos^2\theta_{13}\sin\theta_{13}[\cos\delta\sin(2\theta_{12})\sin(2\theta_{23}) + 4\cos^2\theta_{12}\sin\theta_{13}\sin^2\theta_{23}][|U^{\mathbf{k}}_{13}|^2\sin^2(\Delta^{\mathbf{k}}_{13}t)\\
&+|V^{\mathbf{k}}_{13}|^2\sin^2(\Omega^{\mathbf{k}}_{13}t)] + \cos^2\theta_{13}\sin\theta_{13}[\cos\delta\sin(2\theta_{12})\sin(2\theta_{23}) - 4\sin^2\theta_{12}\sin\theta_{13}\sin^2\theta_{23}]\\
&\times[|U^{\mathbf{k}}_{23}|^2\sin^2(\Delta^{\mathbf{k}}_{23}t) + |V^{\mathbf{k}}_{23}|^2\sin^2(\Omega^{\mathbf{k}}_{23}t)] - \cos^2\theta_{13}\sin(2\theta_{12})[(\cos^2\theta_{23} - \sin^2\theta_{23}\sin^2\theta_{13})\sin(2\theta_{12})\\
&+\cos\delta\cos(2\theta_{12})\sin\theta_{13}\sin(2\theta_{23})][|U^{\mathbf{k}}_{12}|^2\sin^2(\Delta^{\mathbf{k}}_{12}t) + |V^{\mathbf{k}}_{12}|^2\sin^2(\Omega^{\mathbf{k}}_{12}t)],\end{aligned} \tag{A2}$$

$$\begin{aligned}\mathcal{Q}^{\bar{e}}_{\mathbf{k},\tau}(t) = -2J_{CP}[&|U^{\mathbf{k}}_{12}|^2\sin(2\Delta^{\mathbf{k}}_{12}t) - |V^{\mathbf{k}}_{12}|^2\sin(2\Omega^{\mathbf{k}}_{12}t) + (|U^{\mathbf{k}}_{12}|^2 - |V^{\mathbf{k}}_{13}|^2)\sin(2\Delta^{\mathbf{k}}_{23}t)\\
&+(|V^{\mathbf{k}}_{12}|^2 - |V^{\mathbf{k}}_{13}|^2)\sin(2\Omega^{\mathbf{k}}_{23}t) - |U^{\mathbf{k}}_{13}|^2\sin(2\Delta^{\mathbf{k}}_{13}t) + |V^{\mathbf{k}}_{13}|^2\sin(2\Omega^{\mathbf{k}}_{13}t)]\\
&+\cos^2\theta_{13}\sin\theta_{13}[\cos\delta\sin(2\theta_{12})\sin(2\theta_{23}) - 4\cos^2\theta_{12}\sin\theta_{13}\cos^2\theta_{23}][|U^{\mathbf{k}}_{13}|^2\sin^2(\Delta^{\mathbf{k}}_{13}t)\\
&+|V^{\mathbf{k}}_{13}|^2\sin^2(\Omega^{\mathbf{k}}_{13}t)] - \cos^2\theta_{13}\sin\theta_{13}[\cos\delta\sin(2\theta_{12})\sin(2\theta_{23}) + 4\sin^2\theta_{12}\sin\theta_{13}\cos^2\theta_{23}]\\
&\times[|U^{\mathbf{k}}_{23}|^2\sin^2(\Delta^{\mathbf{k}}_{23}t) + |V^{\mathbf{k}}_{23}|^2\sin^2(\Omega^{\mathbf{k}}_{23}t)] - \cos^2\theta_{13}\sin(2\theta_{12})[(\sin^2\theta_{23} - \sin^2\theta_{13}\cos^2\theta_{23})\sin(2\theta_{12})\\
&-\cos\delta\cos(2\theta_{12})\sin\theta_{13}\sin(2\theta_{23})][|U^{\mathbf{k}}_{12}|^2\sin^2(\Delta^{\mathbf{k}}_{12}t) + |V^{\mathbf{k}}_{12}|^2\sin^2(\Omega^{\mathbf{k}}_{12}t)].\end{aligned} \tag{A3}$$

APPENDIX B: USEFUL FORMULAS FOR THE GENERATION OF THE MIXING MATRIX

In deriving the $\mathcal{U}_i$ mixing matrices of Secs. II and III, we use the following relationships:

$$[\nu^\alpha_1(x), L_{12}] = \nu^\alpha_2(x)e^{-i\delta_{12}}, \quad [\nu^\alpha_1(x), L_{23}] = 0,$$

$$[\nu^\alpha_1(x), L_{13}] = \nu^\alpha_3(x)e^{-i\delta_{13}}, \tag{B1}$$

$$[\nu^\alpha_2(x), L_{12}] = -\nu^\alpha_1(x)e^{i\delta_{12}},$$

$$[\nu^\alpha_2(x), L_{23}] = \nu^\alpha_3(x)e^{-i\delta_{23}}, \quad [\nu^\alpha_2(x), L_{13}] = 0, \tag{B2}$$

$$[\nu^\alpha_3(x), L_{12}] = 0, \quad [\nu^\alpha_3(x), L_{23}] = -\nu^\alpha_2(x)e^{i\delta_{23}},$$

$$[\nu^\alpha_3(x), L_{13}] = -\nu^\alpha_1(x)e^{i\delta_{13}}, \tag{B3}$$

and

$$G^{-1}_{23}(t)\nu^\alpha_1(x)G_{23}(t) = \nu^\alpha_1(x), \tag{B4}$$

$$G^{-1}_{13}(t)\nu^\alpha_1(x)G_{13}(t) = \nu^\alpha_1(x)c_{13} + \nu^\alpha_3(x)e^{-i\delta_{13}}s_{13}, \tag{B5}$$

$$G^{-1}_{12}(t)\nu^\alpha_1(x)G_{12}(t) = \nu^\alpha_1(x)c_{12} + \nu^\alpha_2(x)e^{-i\delta_{12}}s_{12}, \tag{B6}$$

$$G^{-1}_{23}(t)\nu^\alpha_2(x)G_{23}(t) = \nu^\alpha_2(x)c_{23} + \nu^\alpha_3(x)e^{-i\delta_{23}}s_{23}, \tag{B7}$$

$$G^{-1}_{13}(t)\nu^\alpha_2(x)G_{13}(t) = \nu^\alpha_2(x), \tag{B8}$$

$$G^{-1}_{12}(t)\nu^\alpha_2(x)G_{12}(t) = \nu^\alpha_2(x)c_{12} - \nu^\alpha_1(x)e^{i\delta_{12}}s_{12}, \tag{B9}$$

$$G^{-1}_{23}(t)\nu^\alpha_3(x)G_{23}(t) = \nu^\alpha_3(x)c_{23} - \nu^\alpha_2(x)e^{i\delta_{23}}s_{23}, \tag{B10}$$

$$G_{13}^{-1}(t)\nu_3^\alpha(x)G_{13}(t)=\nu_3^\alpha(x)c_{13}-\nu_1^\alpha(x)e^{i\delta_{13}}s_{13}, \tag{B11}$$

$$G_{12}^{-1}(t)\nu_3^\alpha(x)G_{12}(t)=\nu_3. \tag{B12}$$

APPENDIX C: ARBITRARY MASS PARAMETRIZATION AND PHYSICAL QUANTITIES

In Refs. [6,7] it was noticed that expanding the flavor fields in the same basis as the (free) fields with definite masses [cf. Eq. (11)] is actually a special choice, and that a more general possibility exists. In other words, in the expansion Eq. (11) one could use eigenfunctions with arbitrary masses μ_σ, and therefore not necessarily the same as the masses which appear in the Lagrangian. On this basis, the authors of Refs. [6,7] have generalized the Blasone-Vitiello (BV) formalism by writing the flavor fields as

$$\nu_\sigma(x)=\sum_r \int d^3k[u^r_{\mathbf{k},\sigma}\tilde{\alpha}^r_{\mathbf{k},\sigma}(t)+v^r_{-\mathbf{k},\sigma}\tilde{\beta}^{r\dagger}_{-\mathbf{k},\sigma}(t)]e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{C1}$$

where u_σ and v_σ are the helicity eigenfunctions with mass μ_σ. We denote by a tilde the generalized flavor operators introduced in Refs. [6,7] in order to distinguish them from the ones in the BV formalism, Eq. (11). The expansion Eq. (C1) is more general than the one in Eq. (11) since the latter corresponds to the particular choice $\mu_e\equiv m_1,\mu_\mu\equiv m_2,\mu_\tau\equiv m_3$. Of course, the flavor fields in Eq. (C1) and Eq. (11) are the same fields. The relation, given in Refs. [6,7], between the general flavor operators and the BV ones is

$$\begin{pmatrix}\tilde{\alpha}^r_{\mathbf{k},\sigma}(t)\\ \tilde{\beta}^{r\dagger}_{-\mathbf{k},\sigma}(t)\end{pmatrix}=J^{-1}_{\mu_\sigma}(t)\begin{pmatrix}\alpha^r_{\mathbf{k},\sigma}(t)\\ \beta^{r\dagger}_{-\mathbf{k},\sigma}(t)\end{pmatrix}J_{\mu_\sigma}(t)\quad , \tag{C2}$$

$$J_{\mu_\sigma}(t)=\prod_{\mathbf{k},r}\exp\left\{i\sum_{(\sigma,j)}\xi^{\mathbf{k}}_{\sigma,j}[\alpha^{r\dagger}_{\mathbf{k},\sigma}(t)\beta^{r\dagger}_{-\mathbf{k},\sigma}(t)+\beta^r_{-\mathbf{k},\sigma}(t)\alpha^r_{\mathbf{k},\sigma}(t)]\right\}, \tag{C3}$$

with $(\sigma,j)=(e,1),(\mu,2),(\tau,3),\xi^{\mathbf{k}}_{\sigma,j}\equiv(\chi^{\mathbf{k}}_\sigma-\chi^{\mathbf{k}}_j)/2$ and $\cot\chi^{\mathbf{k}}_\sigma=|\mathbf{k}|/\mu_\sigma,\cot\chi^{\mathbf{k}}_j=|\mathbf{k}|/m_j$. For $\mu_\sigma\equiv m_j$, one has $J_{\mu_\sigma}(t)=1$.

As already noticed in Ref. [3], the flavor charge operators are the Casimir operators for the Bogoliubov transformation (C2), i.e. they are free from arbitrary mass parameters: $\tilde{Q}_\sigma(t)=Q_\sigma(t)$. This is obvious also from the fact that they can be expressed in terms of flavor fields (see Ref. [8]).

Physical quantities should not carry any dependence on the μ_σ: in the two–flavor case, it has been shown [3] that the expectation values of the flavor charges on the neutrino states are free from the arbitrariness. For three generations, the question is more subtle due to the presence of the CP violating phase. Indeed, in Ref. [7] it has been found that the corresponding generalized quantities depend on the arbitrary mass parameters.

In order to understand better the nature of such a dependence, we consider the identity

$$\begin{aligned}\langle\tilde{\psi}|\tilde{Q}_\sigma(t)|\tilde{\psi}\rangle&=\langle\psi|J(0)Q_\sigma(t)J^{-1}(0)|\psi\rangle\\&=\langle\psi|Q_\sigma(t)|\psi\rangle+\langle\psi|[J(0),Q_\sigma(t)]J^{-1}(0)|\psi\rangle.\end{aligned} \tag{C4}$$

valid on any vector $|\psi\rangle$ of the flavor Hilbert space (at $t=0$). From the explicit expression for $J(0)$ we see that the commutator $[J(0),Q_\sigma(t)]$ vanishes for $\mu_\rho=m_j$, $(\rho,j)=(e,1),(\mu,2),(\tau,3)$.

It is thus tempting to define the (effective) physical flavor charges as:

$$\begin{aligned}\tilde{Q}^{phys}_\sigma(t)&\equiv Q_\sigma(t)-J^{-1}(0)[J(0),Q_\sigma(t)]\\&=J^{-1}(0)Q_\sigma(t)J(0),\end{aligned} \tag{C5}$$

such that, for example,

$$\langle\tilde{\nu}_\rho|\tilde{Q}^{phys}_\sigma(t)|\tilde{\nu}_\rho\rangle=\langle\nu_\rho|Q_\sigma(t)|\nu_\rho\rangle. \tag{C6}$$

It is clear that the operator $\tilde{Q}^{phys}_\sigma(t)$ does depend on the arbitrary mass parameters and this dependence is such to compensate the one arising from the flavor states. The choice of physical quantities (flavor observables) as those not depending on the arbitrary mass parameters is here adopted, although different possibilities are explored by other authors, see Refs. [7,12,13].

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