

Analytical treatment of neutrino asymmetry equilibration from flavor oscillations in the early universe

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A recent numerical study by A. D. Dolgov, S. H. Hansen, S. Pastor, S. T. Petcov, G. G. Raffelt, and D. V. Semikoz (DHPPRS) [Nucl. Phys. **B632**, 363 (2002)] found that complete or partial equilibrium between all active neutrino flavors can be achieved before the big bang nucleosynthesis epoch via flavor oscillations, if the oscillation parameters are those inferred from the atmospheric and solar neutrino data, and, in some cases, if θ_{13} is also sizable. As such, cosmological constraints on the electron neutrino-antineutrino asymmetry are now applicable in all three neutrino sectors. In the present work, we provide an analytical treatment of the scenarios considered in DHPPRS, and demonstrate that their results are stable even for very large initial asymmetries. The equilibration mechanism can be understood in terms of a Mikheyev-Smirnov-Wolfenstein-like effect for a maximally mixed and effectively monochromatic system. We also comment on the DHPPRS's choices of mixing parameters, and their handling of collisional effects, both of which could impinge on the extent of flavor equilibrium.

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I. INTRODUCTION

One of the open questions in cosmology is the possibility of admitting a large relic neutrino-antineutrino asymmetry. For an ensemble of neutrinos and antineutrinos of flavor α in thermal and chemical equilibrium, this asymmetry may be alternatively expressed in terms of the chemical potential ξ_{ν_α} of the species. As of now, there are no direct observations of the cosmic neutrino background. Thus the existence or otherwise of any sizable ξ can only be established indirectly from requiring consistency with big bang nucleosynthesis (BBN), and from the study of the cosmic microwave background radiation (CMBR) spectrum. Presently, BBN limits the chemical potential in the ν_e sector to be at most of order 1, while bounds on ξ_{ν_μ} and ξ_{ν_τ} are considerably less stringent [1]. A recent combined analysis of BBN and CMBR has generated the constraints $-0.01 < \xi_{\nu_e} < 0.22$, $|\xi_{\nu_\mu, \nu_\tau}| < 2.6$, assuming no neutrino oscillations [2].

The situation changes if one considers also the effects of neutrino oscillations. In a new numerical study by Dolgov, Hansen, Pastor, Petcov, Raffelt, and Semikoz (DHPPRS) [3], it was shown that, for the oscillation parameters inferred from the atmospheric neutrino data and the large mixing angle (LMA) solution of the solar neutrino problem, complete equilibrium between all active flavors is established prior to the onset of BBN at temperature $T \approx 1$ MeV.¹ For other solar neutrino solutions, DHPPRS found that a partial equilibrium is possible if the mixing angle θ_{13} is close to its present experimental limit of $\tan^2 \theta_{13} \leq 0.065$. Clearly, if flavor equilibrium holds, constraints on ξ_{ν_e} will apply to all three flavors.

Central to the DHPPRS study is the inclusion of a highly

nonlinear neutrino self-interaction potential in the evolution equation for the ensemble. Previously, this term was found to give rise to synchronized vacuum oscillations characterized strongly by the initial conditions [5]. This raises some questions: How sensitive are the DHPPRS results to the initial asymmetries? For instance, can the presence of a large asymmetry in the ν_μ or ν_τ sector delay its equilibration with its ν_e counterpart? Why do the final asymmetries exhibit oscillatory behavior in some but not all cases? Asymmetries that oscillate out of phase with each other are certainly not in equilibrium.

To find the answers, we begin with an analysis of two-flavor oscillations involving ν_e in the solar neutrino parameter space. The knowledge gained therefrom will be applied to the three-flavor case, and, in particular, to investigating the role of θ_{13} . Whenever numerical quantities are called for, e.g., the mass squared differences, we shall adopt the values used in DHPPRS. These will be noted at the appropriate points. With the exception of the small mixing angle (SMA) solution, the solar mixing angle is always taken to be maximal in DHPPRS. This provides a motivation for us to examine also the more realistic case of large but not maximal mixing.

II. TWO FLAVORS

A. Preliminary considerations

Consider a two-state system consisting of ν_e and ν_x , where ν_x may be ν_μ , ν_τ , or a linear combination thereof. We parametrize the transformation between the weak and mass eigenstates in vacuum with a mixing angle θ :

$$\begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}, \quad (1)$$

where the states $\nu_{1,2}$ have masses $m_{1,2}$, respectively. The same parametrization applies to the $\bar{\nu}_e \leftrightarrow \bar{\nu}_x$ system.

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¹Equilibration of the neutrino flavors for the atmospheric and LMA oscillation parameters was first suggested in Ref. [4].

For each momentum p , we write down the one-body reduced density matrices ρ ($\bar{\rho}$ for antineutrinos) and express them in terms of the function P_0 (\bar{P}_0) and a ‘‘polarization’’ vector \mathbf{P} ($\bar{\mathbf{P}}$):

$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{ex} \\ \rho_{xe} & \rho_{xx} \end{pmatrix} = \frac{1}{2} [P_0 + \mathbf{P} \cdot \boldsymbol{\sigma}], \quad (2)$$

$$\bar{\rho} = \begin{pmatrix} \bar{\rho}_{ee} & \bar{\rho}_{xe} \\ \bar{\rho}_{ex} & \bar{\rho}_{xx} \end{pmatrix} = \frac{1}{2} [\bar{P}_0 + \bar{\mathbf{P}} \cdot \boldsymbol{\sigma}],$$

where $\mathbf{P} = P_x \mathbf{x} + P_y \mathbf{y} + P_z \mathbf{z}$, and $\boldsymbol{\sigma} = \sigma_x \mathbf{x} + \sigma_y \mathbf{y} + \sigma_z \mathbf{z}$ are the Pauli matrices. In this notation, the ν_e and ν_x distribution functions at p are, respectively,

$$f_{\nu_e} = \frac{1}{2} [P_0 + P_z] f_{\text{eq}}(0), \quad (3)$$

$$f_{\nu_x} = \frac{1}{2} [P_0 - P_z] f_{\text{eq}}(0),$$

for which we have chosen the reference distribution function $f_{\text{eq}}(0)$ to be of Fermi-Dirac form,

$$f_{\text{eq}}(\xi) \equiv \frac{1}{1 + e^{p/T - \xi}}, \quad (4)$$

with chemical potential ξ set to zero for all temperatures T . The distributions $f_{\bar{\nu}_e}$ and $f_{\bar{\nu}_x}$ may be similarly established from Eq. (3) by replacing P_0 and P_z with their antineutrino counterparts. The number density of a particle species ψ follows from taking the integral of f_ψ over all momenta, $n_\psi = (1/2\pi^2) \int f_\psi p^2 dp$. Note that the functions (P_0, \mathbf{P}) and $(\bar{P}_0, \bar{\mathbf{P}})$ carry both momentum and time dependence unless otherwise stated.

The evolution of \mathbf{P} and $\bar{\mathbf{P}}$ is governed by the quantum kinetic equations (QKEs) [6]

$$\dot{\mathbf{P}} = + \left[\frac{\Delta m^2}{2p} \mathbf{B} - \frac{8\sqrt{2}G_F p}{3m_W^2} E_{ee} \mathbf{z} \right] \times \mathbf{P} + \sqrt{2}G_F (\mathbf{J} - \bar{\mathbf{J}}) \times \mathbf{P}, \quad (5)$$

$$\dot{\bar{\mathbf{P}}} = - \left[\frac{\Delta m^2}{2p} \mathbf{B} - \frac{8\sqrt{2}G_F p}{3m_W^2} E_{ee} \mathbf{z} \right] \times \bar{\mathbf{P}} + \sqrt{2}G_F (\mathbf{J} - \bar{\mathbf{J}}) \times \bar{\mathbf{P}},$$

where $\Delta m^2 = m_2^2 - m_1^2$, $\mathbf{B} = \sin 2\theta \mathbf{x} - \cos 2\theta \mathbf{z}$, G_F is the Fermi constant, m_W is the mass of the W boson, $E_{ee} = (7/60)\pi^2 T^4$ is the electron-positron energy density, and we are assuming, at this stage, that the background medium does not distinguish between ν_μ and ν_τ . Observe that refractive matter effects for this system are entirely CP symmetric—we have dropped the CP asymmetric term proportional to the difference between the charged lepton and

antilepton number densities. This difference is expected to be of the order of the baryon asymmetry, and is negligible in comparison with the CP symmetric background. The last term in Eq. (5) comprising the integrated polarization vectors

$$\mathbf{J} = \frac{1}{2\pi^2} \int \mathbf{P} f_{\text{eq}}(0) p^2 dp, \quad (6)$$

$$\bar{\mathbf{J}} = \frac{1}{2\pi^2} \int \bar{\mathbf{P}} f_{\text{eq}}(0) p^2 dp,$$

arises from neutrino self-interaction [7], and is by definition also time dependent.

We have chosen to ignore the effects of collisions for now, since, for the bulk of the momentum distribution, the neutrino mean free path $\Gamma^{-1} \sim (G_F^2 p T^4)^{-1}$ is larger than an effective oscillation length $\sim 2\pi(V_x^2 + V_z^2)^{-1/2}$, where

$$V_x = \frac{\Delta m^2}{2p} \sin 2\theta, \quad (7)$$

$$V_z = -\frac{\Delta m^2}{2p} \cos 2\theta - \frac{8\sqrt{2}G_F p}{3m_W^2} E_{ee},$$

given the mass squared differences and mixing angles of the various solar neutrino solutions [8]. The evolution of this two-state system is therefore primarily oscillation-driven. We also do not consider repopulation from the background for consistency with the $\Gamma=0$ assumption, and hence $\dot{P}_0 = \dot{\bar{P}}_0 = 0$.

Note that the collisionless approximation strictly does not apply to $\nu_\mu \leftrightarrow \nu_\tau$ oscillations in the atmospheric parameter space. These oscillations become operational at higher temperatures, where frequent ‘‘non-forward’’ scattering on the background medium modifies significantly the system’s dynamics (i.e., the system is ‘‘collision-driven’’). One should also be cautious when dealing with oscillation-driven systems (such as $\nu_e \leftrightarrow \nu_x$)—residual effects from collisions may still be sizable, especially if there is a substantial period of large mixing. This issue will be revisited later in Sec. II D, but let us emphasise at this point that a full treatment of collisional effects on active-active neutrino oscillations is considerably more complicated than is implied in DHPPRS. The fact that both neutrino flavors can now participate in momentum-changing scattering processes gives rise to new terms in the evolution equation in addition to the simple damping of P_x and P_y encountered in the active-sterile case. See, for example, Ref. [9] for details.

Ideally, we would like to compare the distribution functions of the two neutrino species concerned. When thermal and chemical equilibria prevail, these are completely specified by the chemical potentials $\xi_{\nu_e} = -\xi_{\bar{\nu}_e}$, and $\xi_{\nu_x} = -\xi_{\bar{\nu}_x}$, which are related to the neutrino-antineutrino asymmetries $L_{\nu_\alpha} = (n_{\nu_\alpha} - n_{\bar{\nu}_\alpha})/n_\gamma$ via

$$L_{\nu_\alpha} = \frac{1}{12\zeta(3)} (\pi^2 \xi_{\nu_\alpha} + \xi_{\nu_\alpha}^3), \quad (8)$$

where $\alpha = e, x$, $n_\gamma = 2\zeta(3)T^3/\pi^2$ is the photon number density, and ζ is the Riemann zeta function. A non-thermal distribution (e.g., due to oscillations) generally has no well-defined chemical potential. However, if we demand only that equality between the number densities of ν_e and ν_x (and separately, $\bar{\nu}_e$ and $\bar{\nu}_x$) be established for flavor equilibration, then it is sufficient to track the evolution of the difference between L_{ν_e} and L_{ν_x} ,

$$\begin{aligned} L_{\nu_e} - L_{\nu_x} &= \frac{1}{2\pi^2 n_\gamma} \int [(f_{\nu_e} - f_{\bar{\nu}_e}) - (f_{\nu_x} - f_{\bar{\nu}_x})] p^2 dp \\ &= \frac{1}{2\pi^2 n_\gamma} \int (P_z - \bar{P}_z) f_{\text{eq}}(0) p^2 dp \\ &= \frac{1}{n_\gamma} (J_z - \bar{J}_z), \end{aligned} \quad (9)$$

regardless of whether thermal and/or chemical equilibria are in place.

$$\frac{d}{dt} \begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = \begin{bmatrix} 0 & -V_z & 0 \\ V_z & 0 & -V_x \\ 0 & V_x & 0 \end{bmatrix} + \sqrt{2} G_F n_\gamma \begin{pmatrix} 0 & -I_z & I_y \\ I_z & 0 & -I_x \\ -I_y & I_x & 0 \end{pmatrix} \begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = (\mathcal{V} + \mathcal{S}) \mathbf{P}, \quad (11)$$

where V_x and V_z are as defined in Eq. (7). For the antineutrino system, the matrix \mathcal{V} is replaced with $-\mathcal{V}$.

Before proceeding, observe that in the absence of the self-interaction term \mathcal{S} , the vector \mathbf{P} would simply exhibit the usual ‘‘matter-suppressed’’ precession around the vector $\mathbf{V} = V_x \mathbf{x} + V_z \mathbf{z}$ at a rate

$$\omega_\mathcal{V} = \sqrt{V_x^2 + V_z^2}, \quad (12)$$

and gradually give way to vacuum oscillations as the electron-positron energy density drops off with the expansion of the universe. If, hypothetically, only the \mathcal{S} term is present, \mathbf{P} would precess around \mathbf{I} at a rate

$$\omega_\mathcal{S} = \sqrt{2} G_F n_\gamma |\mathbf{I}|, \quad (13)$$

provided that \mathbf{I} changes sufficiently slowly with time (to be justified later).

In the following, we shall work under the assumption that the inequality

$$\omega_\mathcal{V} \ll \omega_\mathcal{S} \quad (14)$$

is maintained for all momenta at all times. This condition translates roughly into requiring that

$$1 \gg \frac{8p}{3m_W^2 n_\gamma |\mathbf{I}|} \approx 2 \times 10^{-9} \frac{y}{|\mathbf{I}|} \left(\frac{T}{\text{MeV}} \right)^2 \quad (15)$$

B. Derivation of a collective evolution equation

Let us now examine the time development of the ‘‘normalized’’ vector $\mathbf{I} \equiv (\mathbf{J} - \bar{\mathbf{J}})/n_\gamma$, for which we construct from Eqs. (5) and (6) an evolution equation

$$\begin{aligned} \dot{\mathbf{I}} &= \mathbf{B} \times \frac{1}{2\pi^2 n_\gamma} \int \frac{\Delta m^2}{2p} (\mathbf{P} + \bar{\mathbf{P}}) f_{\text{eq}}(0) p^2 dp - \mathbf{z} \\ &\times \frac{1}{2\pi^2 n_\gamma} \int \frac{8\sqrt{2} G_F p}{3m_W^2} E_{ee}(\mathbf{P} + \bar{\mathbf{P}}) f_{\text{eq}}(0) p^2 dp. \end{aligned} \quad (10)$$

Equilibration between the two neutrino species will necessarily imply $I_z = 0$. Note, however, that the converse is not generally true.

Equation (10) is exact; our task is to find approximate solutions for \mathbf{P} and $\bar{\mathbf{P}}$. To this end, we first rewrite the evolution equation (5) in matrix form:

for $T > T_r$, and

$$1 \gg \frac{\Delta m^2}{2\sqrt{2} G_F p n_\gamma |\mathbf{I}|} \approx 0.12 \frac{y^{-1}}{|\mathbf{I}|} \left(\frac{|\Delta m^2|}{\text{eV}^2} \right) \left(\frac{T}{\text{MeV}} \right)^{-4} \quad (16)$$

for $T < T_r$, where $y \equiv p/T$, and

$$T_r \approx 19.8 y^{-1/3} \left(\frac{|\Delta m^2|}{\text{eV}^2} \right)^{1/6} \text{ MeV} \quad (17)$$

is the temperature at which the vacuum and electron-positron background terms become equal in magnitude. The quantity $|\mathbf{I}|$ may be regarded crudely as a measurement of the degree of alignment of the individual polarization vectors \mathbf{P} and $\bar{\mathbf{P}}$, such that if all neutrinos and antineutrinos are in flavor eigenstates (i.e., $P_{x,y} \approx \bar{P}_{x,y} \approx 0$), the magnitude of \mathbf{I} equals the difference in the ν_e and ν_x neutrino-antineutrino asymmetries. It turns out that initial fulfilment of the requirement (14) tends to preserve the alignment until the self-interaction term significantly weakens with the expansion of the universe through n_γ . Hence, assuming all \mathbf{P}^i and $\bar{\mathbf{P}}^i$, where the superscript ‘‘i’’ denotes initial, to be in the \mathbf{z} direction, the reader may substitute in Eqs. (15) and (16) the relation $|\mathbf{I}| \approx |\mathbf{I}^i| \approx |L_{\nu_e}^i - L_{\nu_x}^i|$, and see that these conditions are always met by the bulk of the momentum distribution for the Δm^2 's

concerned, provided that the disparity between the initial asymmetries is, say, $\geq 10^{-5}$ in magnitude.²

We solve the three coupled differential equations (11) by transforming to an instantaneous diagonal basis defined as $\mathbf{Q}=\mathcal{U}\mathbf{P}$, in which the evolution equation takes the form

$$\dot{\mathbf{Q}}=[\mathcal{U}(\mathcal{V}+\mathcal{S})\mathcal{U}^{-1}-\mathcal{U}\dot{\mathcal{U}}^{-1}]\mathbf{Q}, \quad (18)$$

with $\mathcal{U}(\mathcal{V}+\mathcal{S})\mathcal{U}^{-1}=\text{Diag}(k_1, k_2, k_3)$, where the eigenvalue k_1 is identically zero, and $k_{2,3}$ are two imaginary numbers of equal magnitude but opposite signs. The associating eigenvectors $\mathbf{q}_{1,2,3}$ in terms of the original coordinates $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ constitute the columns of the transformation matrix

$$\mathcal{U}^{-1}=\mathcal{U}^\dagger=(\mathbf{q}_1|\mathbf{q}_2|\mathbf{q}_3), \quad (19)$$

where \mathbf{q}_1 is real, and $\mathbf{q}_{2,3}$ form a complex conjugate (c.c.) pair. In the limit $\omega_\nu \ll \omega_S$, the eigenvalues are

$$k_1=0, \quad k_2=k_3^* \approx i\omega_S + \mathcal{O}(\omega_\nu), \quad (20)$$

and the real eigenvector \mathbf{q}_1 is well approximated by

$$\begin{aligned} \mathbf{q}_1 &\approx \frac{1}{\sqrt{I_x^2 + I_y^2 + I_z^2}} \begin{pmatrix} I_x \\ I_y \\ I_z \end{pmatrix} \\ &+ [\mathcal{O}(\omega_\nu/\omega_S)\mathbf{q}_2^0 + \text{c.c.}] \\ &\equiv \hat{\mathbf{I}} + [\mathcal{O}(\omega_\nu/\omega_S)\mathbf{q}_2^0 + \text{c.c.}], \end{aligned} \quad (21)$$

where $\{\mathbf{q}_1^0, \mathbf{q}_2^0, \mathbf{q}_3^0\}$ are the set of eigenvectors in the limit $\omega_\nu=0$. We shall not reproduce here the exact forms of the remaining two complex conjugate eigenvectors, but simply point out that \mathbf{q}_2 and \mathbf{q}_3 together sweep out a plane perpendicular to \mathbf{q}_1 .

Equation (18) is not yet soluble; the term $\mathcal{U}\dot{\mathcal{U}}^{-1}$ contains off-diagonal elements. However, these may be set to zero to facilitate calculations since the condition

$$\chi_{ij} \equiv \left| \frac{(\mathcal{U}\dot{\mathcal{U}}^{-1})_{ij}}{k_i - k_j} \right| \sim \frac{1}{\omega_S} \frac{1}{|\mathbf{I}|} \left| \frac{d\mathbf{I}}{dt} \right| \ll 1, \quad (22)$$

where $i, j=1, 2, 3$ and $i \neq j$, is always satisfied in the $\omega_\nu \ll \omega_S$ limit. This can be seen from Eq. (10), in which any one component of $\dot{\mathbf{I}}$ is at most of order $\omega_\nu |\mathbf{I}|$, such that $\chi_{ij} \sim \omega_\nu/\omega_S \ll 1$ is consistent with earlier assumptions.

The formal solution to the now decoupled evolution equation (18) is

$$\mathbf{Q} \approx \text{Diag} \left(1, e^{i\int_{t_i}^t \omega_S dt'}, e^{-i\int_{t_i}^t \omega_S dt'} \right) \mathbf{Q}^i, \quad (23)$$

or equivalently in the original $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ basis,

$$\begin{aligned} \mathbf{P} &\approx \mathcal{U}^{-1} \text{diag} \left(1, e^{i\int_{t_i}^t \omega_S dt'}, e^{-i\int_{t_i}^t \omega_S dt'} \right) \mathcal{U}^i \mathbf{P}^i \\ &= (\mathbf{q}_1|\mathbf{q}_2|\mathbf{q}_3) \begin{pmatrix} \mathbf{q}_1^i \cdot \mathbf{P}^i \\ e^{i\int_{t_i}^t \omega_S dt'} \mathbf{q}_2^{i*} \cdot \mathbf{P}^i \\ e^{-i\int_{t_i}^t \omega_S dt'} \mathbf{q}_3^{i*} \cdot \mathbf{P}^i \end{pmatrix}, \end{aligned} \quad (24)$$

where the superscript or subscript ‘‘i’’ denotes initial. Observe that the terms $e^{i\int \omega_S dt}$ and $e^{-i\int \omega_S dt}$ lead to rapid oscillations which average to zero over the characteristic time scale of $\mathbf{q}_{1,2,3}$. We retain only the time-averaged component and adopt the identity (21), whereupon \mathbf{P} becomes

$$\mathbf{P} \approx (\mathbf{P}^i \cdot \hat{\mathbf{I}}^i) \hat{\mathbf{I}} + \mathcal{O}(\omega_\nu/\omega_S) \hat{\mathbf{I}} + [\mathcal{O}(\omega_\nu/\omega_S) \mathbf{q}_2^0 + \text{c.c.}], \quad (25)$$

which, to the lowest order in ω_ν/ω_S , is a product of two quantities $\mathbf{P}^i \cdot \hat{\mathbf{I}}^i$ and $\hat{\mathbf{I}}$ carrying, respectively, the momentum and the time dependences of the original function. Equation (25) pertains also to $\bar{\mathbf{P}}$, save for the replacement of \mathbf{P}^i with $\bar{\mathbf{P}}^i$, and ω_ν with $-\omega_\nu$. Substituting into Eq. (10), we obtain for the collective vector \mathbf{I} a simplified evolution equation,

$$\begin{aligned} \dot{\mathbf{I}} &\approx \frac{1}{|\mathbf{I}|} \frac{1}{2\pi^2 n_\nu} \left[\mathbf{B} \int \frac{\Delta m^2}{2p} (\mathbf{P}^i + \bar{\mathbf{P}}^i) \cdot \hat{\mathbf{I}}^i f_{\text{eq}}(0) p^2 dp \right. \\ &\quad \left. - \mathbf{z} \int \frac{8\sqrt{2} G_{FP}}{3m_W^2} E_{ee} (\mathbf{P}^i + \bar{\mathbf{P}}^i) \cdot \hat{\mathbf{I}}^i f_{\text{eq}}(0) p^2 dp \right] \times \mathbf{I}, \end{aligned} \quad (26)$$

to the lowest order in ω_ν/ω_S .

The non-dissipative character of \mathbf{I} 's evolution is immediately manifest in Eq. (26). Furthermore, the dynamics of the neutrino-antineutrino ensemble can be completely and simply determined from the initial conditions and from known external factors, independently of the evolution of the ensemble *per se*. Synchronized vacuum oscillations in multi-momentum systems subject to intense self-interactions was discovered numerically in Ref. [5], and recently reinterpreted in Ref. [10]. Equation (26) is essentially a generalization of the physical picture developed in Ref. [10] for pure vacuum oscillations. We opted to conduct a more systematic, first-principles derivation here for pedagogy.

We take as the initial condition that all neutrinos and antineutrinos are in flavor eigenstates such that \mathbf{P}^i and $\bar{\mathbf{P}}^i$ are parallel to $\hat{\mathbf{I}}^i$, and

$$\mathbf{P}^i \cdot \hat{\mathbf{I}}^i \approx \frac{f_{\nu_e}^i - f_{\nu_x}^i}{f_{\text{eq}}(0)}, \quad \bar{\mathbf{P}}^i \cdot \hat{\mathbf{I}}^i \approx \frac{f_{\bar{\nu}_e}^i - f_{\bar{\nu}_x}^i}{f_{\text{eq}}(0)}, \quad (27)$$

up to a common sign. Two additional assumptions of thermal as well as chemical equilibria, that is,

²The case of equal initial asymmetries has *a priori* no bearing for the purpose of the present work, although it could lead to some very interesting phenomena.

$$\begin{aligned} f_{\nu_e}^i &\simeq f_{\text{eq}}(\xi_{\nu_e}^i), & f_{\bar{\nu}_e}^i &\simeq f_{\text{eq}}(-\xi_{\nu_e}^i), \\ f_{\nu_x}^i &\simeq f_{\text{eq}}(\xi_{\nu_x}^i), & f_{\bar{\nu}_x}^i &\simeq f_{\text{eq}}(-\xi_{\nu_x}^i), \end{aligned} \quad (28)$$

then allow for a further approximation of Eq. (26) as

$$\begin{aligned} \dot{\mathbf{I}} &\simeq \frac{1}{|\mathbf{I}|} \frac{1}{2\pi^2 n_\gamma} \left\{ \mathbf{B} \int \frac{\Delta m^2}{2p} [f_{\text{eq}}(\xi_{\nu_e}^i) + f_{\text{eq}}(-\xi_{\nu_e}^i) \right. \\ &\quad \left. - f_{\text{eq}}(\xi_{\nu_x}^i) - f_{\text{eq}}(-\xi_{\nu_x}^i)] p^2 dp \right. \\ &\quad \left. - \mathbf{z} \int \frac{8\sqrt{2}G_F p}{3m_W^2} E_{ee} [f_{\text{eq}}(\xi_{\nu_e}^i) + f_{\text{eq}}(-\xi_{\nu_e}^i) \right. \\ &\quad \left. - f_{\text{eq}}(\xi_{\nu_x}^i) - f_{\text{eq}}(-\xi_{\nu_x}^i)] p^2 dp \right\} \times \mathbf{I}. \end{aligned} \quad (29)$$

The integrals are now in a form opportune for the exploitation of these very useful identities:

$$\int [f_{\text{eq}}(\xi) - f_{\text{eq}}(-\xi)] p^2 dp = \frac{T^3}{3} (\pi^2 \xi + \xi^3),$$

$$\int [f_{\text{eq}}(\xi) + f_{\text{eq}}(-\xi)] p dp = \frac{T^2}{6} (\pi^2 + 3\xi^2),$$

$$\int [f_{\text{eq}}(\xi) + f_{\text{eq}}(-\xi)] p^3 dp = \frac{T^4}{60} (7\pi^4 + 30\pi^2 \xi^2 + 15\xi^4). \quad (30)$$

These, together with the conservation of the collective vector \mathbf{I} ,

$$\begin{aligned} |\mathbf{I}| &\simeq |\dot{\mathbf{I}}| \simeq \frac{1}{2\pi^2 n_\gamma} \left| \int [(f_{\nu_e}^i - f_{\bar{\nu}_e}^i) - (f_{\nu_x}^i - f_{\bar{\nu}_x}^i)] p^2 dp \right| \\ &\simeq \frac{T^3}{6\pi^2 n_\gamma} \left| \pi^2 (\xi_{\nu_e}^i - \xi_{\nu_x}^i) + (\xi_{\nu_e}^i{}^3 - \xi_{\nu_x}^i{}^3) \right|, \end{aligned} \quad (31)$$

permit us to recast Eq. (29) into a more illuminating and readily soluble form,

$$\dot{\mathbf{I}} \simeq \frac{3}{2} \frac{\tilde{y}(\xi_{\nu_e}^i{}^2 - \xi_{\nu_x}^i{}^2)}{|\pi^2(\xi_{\nu_e}^i - \xi_{\nu_x}^i) + (\xi_{\nu_e}^i{}^3 - \xi_{\nu_x}^i{}^3)|} \left(\frac{\Delta m^2}{2\tilde{p}} \mathbf{B} - \frac{8\sqrt{2}G_F \tilde{p}}{3m_W^2} E_{ee} \mathbf{z} \right) \times \mathbf{I}, \quad (32)$$

where

$$\tilde{y} = \frac{\tilde{p}}{T} \equiv \sqrt{\pi^2 + \frac{1}{2}(\xi_{\nu_e}^i{}^2 + \xi_{\nu_x}^i{}^2)} \quad (33)$$

represents some ‘‘average’’ momentum. Equations (32) and (33) will form the basis of the discussions to follow.³

C. Discussions

Equation (32) has a straightforward interpretation. Consider the terms inside the parentheses. These are identically the vacuum and electron-positron background terms found in a typical single momentum evolution equation [Eq. (5) minus self-interaction], and control the ensemble’s ‘‘collective’’ matter-affected mixing angle,

$$\sin 2\theta_c = \frac{V_x}{\sqrt{V_x^2 + V_z^2}} \Big|_{p=\tilde{p}}, \quad (34)$$

where V_x and V_z are evaluated for $p = \tilde{p}$.

The characteristic momentum \tilde{p} defined in Eq. (33) reflects, to some extent, the initial configuration of the ensemble. In addition to augmenting the neutrino-antineutrino asymmetry, a large positive chemical potential for a neutrino flavor tends to skew the distribution towards higher momenta, while leaving the spectrum of its less abundant antineutrino virtually intact. A large negative chemical potential has the opposite effect. Hence the net result for \tilde{p} is that it follows more or less the trends of the more abundant species, and thereby grows with $|\xi^i|$. For initial chemical potentials satisfying the constraint

$$\xi_{\nu_e}^i{}^2 + \xi_{\nu_x}^i{}^2 \leq 2\pi^2, \quad (35)$$

the value of $\tilde{p} \simeq \pi T$ originates from the mismatch between the averages $\langle p \rangle$ and $\langle 1/p \rangle^{-1}$ taken over the function $d^2 f_{\text{eq}}(\xi)/d\xi^2$ evaluated at $\xi = 0$.

³The method used to derive Eq. (32) from the QKEs is equally valid had a CP asymmetric background been included. In fact, even if the background strongly differentiates between neutrinos and antineutrinos, the resulting evolution equation would still predict identical behaviors for both CP partners, contrary to the case when self-interaction is absent. Unfortunately, this scenario is of little interest for the early universe. Perhaps it might find application in a supernova environment.

The quantity exhibiting by far the strongest dependence on the initial conditions is the collective matter-affected oscillation length, or equivalently, the collective effective mass squared difference,

$$\Delta m_{\text{eff}}^2 = \kappa 2\tilde{p} \sqrt{V_x^2 + V_z^2}|_{p=\tilde{p}}, \quad (36)$$

where

$$\kappa = \frac{3}{2} \frac{\tilde{y}(\xi_{\nu_e}^{i2} - \xi_{\nu_x}^{i2})}{|\pi^2(\xi_{\nu_e}^i - \xi_{\nu_x}^i) + (\xi_{\nu_e}^{i3} - \xi_{\nu_x}^{i3})|}. \quad (37)$$

For instance, Δm_{eff}^2 vanishes for $\xi_{\nu_e}^i = -\xi_{\nu_x}^i$, and oscillations are switched off completely, as pointed out in DHPPRS. (Note again that the present formulation is invalid for identical initial asymmetries.)

An approximate solution to Eq. (32) can be established in the adiabatic limit by mapping \mathbf{I} onto an instantaneous diagonal basis (in this case, isomorphic to an instantaneous mass basis) and setting the time derivative of the transformation matrix to zero. The resulting expression for the variable I_z is

$$I_z \simeq \left(c2\theta_c c2\theta_c^i + s2\theta_c s2\theta_c^i \cos \int_{t_i}^t \frac{\Delta m_{\text{eff}}^2}{2\tilde{p}} dt' \right) I_z^i, \quad (38)$$

where $c2\theta = \cos 2\theta$, $s2\theta = \sin 2\theta$, subject to the validity of the adiabatic condition

$$\gamma \equiv \left| \frac{V_z \dot{V}_x - V_x \dot{V}_z}{\kappa (V_x^2 + V_z^2)^{3/2}} \right|_{p=\tilde{p}} \leq 1, \quad (39)$$

at all times. For maximal mixing and the matter density profile at hand, γ is a maximum at $T = T_r$ [Eq. (17) with $y = \tilde{y}$], at which it takes on a particularly simple form:

$$\gamma|_{\theta=45^\circ} \simeq 1.5 \times 10^{-5} \kappa^{-1} \left(\frac{|\Delta m^2|}{\text{eV}^2} \right)^{-1/2}, \quad (40)$$

where we have used the relation $dT/dt \simeq -5.44T^3/m_{\text{pl}}$, and $m_{\text{pl}} \simeq 1.22 \times 10^{22}$ MeV is the Planck mass.

Equation (38) predicts for maximal mixing a Mikheyev-Smirnov-Wolfenstein- (MSW-)like effect, transforming I_z from I_z^i to 0 (plus some small amplitude oscillations) when vacuum oscillations overcome refractive matter effects (i.e., when $c2\theta_c \simeq c2\theta$). The temperature at which this transition takes place is given roughly by T_r in Eq. (17) with $y = \tilde{y}$. For initial chemical potentials satisfying the constraint (35), this turns out to be $\simeq 2.6$ MeV for $\Delta m^2 = 4.5 \times 10^{-5}$ eV² (LMA). Extremely large initial $|\xi^i|$'s can lower T_r somewhat, but the connection is weak. As an illustration, the setting of $\xi_{\nu_e}^i = 0$ and $\xi_{\nu_x}^i = 10$ gives, for the same Δm^2 , $T_r \simeq 1.9$ MeV. In the cases of the LOW ($\Delta m^2 = 1 \times 10^{-7}$ eV²) and the Vacuum ($\Delta m^2 = 8 \times 10^{-11}$ eV²) solutions, the temperatures are $\simeq 0.9$ MeV and $\simeq 0.3$ MeV, respectively, for reasonable $|\xi^i|$'s. Evidently, only the LMA transition can take place well ahead of BBN.

For the oscillation parameters of the SMA solution ($\Delta m^2 = 7 \times 10^{-6}$ eV², $\sin 2\theta = 0.05$), I_z remains close to its initial value even after the “transition” at $T_r \simeq 1.9$ MeV, since both the vacuum oscillation and matter refraction terms are predominantly in the negative \mathbf{z} direction for $\Delta m^2 > 0$, and the usual MSW resonance condition cannot be satisfied. This has an important implication: in the absence of substantial collisional damping and other effects not considered here, any equilibration of two neutrino species in a purely two-flavor scenario is, strictly speaking, an accident of maximal mixing.

A second deciding factor on the efficacy of flavor equilibration is the adiabaticity of the transition from matter-suppressed to vacuum oscillations. Unlike that encountered in, for instance, solar neutrino analyses, the adiabaticity parameter γ of Eq. (39) is strongly dependent on the initial conditions. Take for concreteness the case of $\xi_{\nu_e}^i = 0$. The reader may verify that for the LMA solution, the condition (40) is always true if $|\xi_{\nu_x}^i| \gtrsim 0.01$, but can be badly violated at the transition point for the LOW Δm^2 unless $|\xi_{\nu_x}^i| \gtrsim 0.1$.

In the case of maximal mixing, violation of adiabaticity at the transition point generally results in large amplitude “post-transition” oscillations about the equilibrium point at an angular frequency roughly equal to $\Delta m_{\text{eff}}^2/2\tilde{p}$. Naturally, this is quite a separate phenomenon from true equilibration. On the other hand, an adiabaticity parameter that evaluates to infinity at all times (because, for example, $\kappa \rightarrow 0$) signifies that there is no transition at all. From the perspective of equilibrating two vastly different asymmetries, the requirement of $|\xi_{\nu_x}^i| \gtrsim 0.01$ (assuming $\xi_{\nu_e}^i = 0$) in the LMA case for a smooth transition is, by definition, not a major concern. We therefore dwell no further on this topic, but simply point out to the interested reader that there is an existing body of works devoted to non-adiabatic transitions in the solar interior [11]. The almost exponential density profile of the Sun is of course very different from the matter density profile considered here. This will alter the post-transition oscillation frequency and amplitude somewhat, but not the essential physics of non-adiabatic effects.

D. Large but not maximal mixing

Previously, we saw that equilibration of ν_e and ν_x in the absence of collisional effects is peculiar to maximal mixing. In particular, if Δm^2 corresponds to that of the LMA solution to the solar neutrino problem, equilibrium is complete before BBN. However, the real LMA solution encompasses a range of mixing parameters that are merely large, but not maximal [8]. If we had used instead $\sin 2\theta \simeq 0.88$, Eq. (38) would predict for this scenario only a $\sim 50\%$ reduction in I_z^i , i.e., a partial equilibrium.

The question now is this: can collisions save the scene? A full treatment of active-active collisions may be complicated. But for the purpose of exploring the possibilities, we may want to try including in the evolution equation (5) a simple active-sterile style damping term $-D(P_x + P_y)$ [and $-D(\bar{P}_x + \bar{P}_y)$ for the antineutrinos], where $D = (1/2)\lambda G_{\text{FP}}^2 T^4$, and λ is a positive number reflecting the

relative amplitudes of the available inelastic scattering and elastic scattering processes that distinguish between the two flavors.

Introduction of this “toy” damping term has a simple consequence for the collective evolution equation (32): there is now an extra term of the form

$$-\tilde{D}(I_x + I_y), \quad (41)$$

with $\tilde{D} = [27\zeta(3)/2\pi^2]\lambda G_F^2 T^5$ for $|\xi^i| \leq 1$. For $\Delta m_{\text{eff}}^2/2\tilde{p} \geq \tilde{D}$, the corresponding solution is similar to Eq. (38), but with a twist:

$$I_z \approx e^{-\int_0^t \Omega dt'} \cos 2\theta_c \cos 2\theta_c^i I_z^i + \text{damped oscillations}, \quad (42)$$

where $\Omega \approx \sin^2 2\theta_c \tilde{D}$ may be interpreted as an “instantaneous equilibration rate” [12]. Clearly, Ω scales with the amount of mixing, causes the exponential in Eq. (42) to decrease with time, and thus contributes to equilibrating the two flavors concerned. This equilibrating power is especially useful if a partial equilibrium has already been reached via an MSW-like effect. However, the extent of the equilibrium is now dependent on the nature of the scattering processes. As a crude estimate, the setting of $\lambda = 1$ will give $\exp[\dots] \approx 0.16$ for $\sin 2\theta \approx 0.88$ at $T \approx 1$ MeV. Thus, together with the MSW-like transition, I_z may still be able to reach a value of $\approx 0.08 I_z^i$ prior to BBN.

Before closing this section, let us stress again that it is not clear at this stage if a proper treatment of active-active collisions will indeed lead to an outcome similar to that afforded to us by the simple active-sterile picture. The latter can only serve as a rough guide.

III. THREE FLAVORS

We shall not derive from first principles a three-flavor analogue of Eq. (32). Such an exercise is perhaps not worthwhile since the collisionless limit is strictly not applicable at higher temperatures. Furthermore, a complete set of QKEs for three active flavors incorporating all necessary collision terms has yet to be written down in a user-friendly form, and until we know how to handle its better established two-flavor counterpart, we shall not dwell on the fine details of the three-flavor case.

However, granted that the role of the self-interaction term is to cause the ensemble to behave in an effectively monochromatic manner, a qualitative, or even semi-quantitative, picture is still available if we suppose that, like the two-flavor case, the matter-affected mixing structure of the three-flavor system is determined by the single mode Hamiltonian (in flavor space),

$$\frac{1}{2p} U \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U^\dagger - \frac{8\sqrt{2}G_F p}{3m_W^2} \begin{pmatrix} E_{ee} + E_{\mu\mu} & 0 & 0 \\ 0 & E_{\mu\mu} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (43)$$

substituted with $p = \tilde{p} \approx \pi T$ for small $|\xi^i|$'s. Here, the quantity $E_{\mu\mu} = 4m_\mu(m_\mu T/2\pi)^{3/2} \exp(-m_\mu/T)$ is the muon-antimuon energy density, with m_μ as the muon mass, $m_{1,2,3}^2$ are the squared masses of the three mass eigenstates, and the transformation between the weak and mass bases is parametrized with three Euler angles,

$$U = \begin{pmatrix} 1 & 0 & 0 \\ c_{23} & s_{23} & 0 \\ -s_{23} & c_{23} & 0 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (44)$$

where $c_{ij} = \cos \theta_{ij}$, and $s_{ij} = \sin \theta_{ij}$ for $ij = 12, 23, 13$. Under this scheme, we identify the atmospheric and solar neutrino oscillation parameters as

$$\Delta m_{\text{atm}}^2 \equiv m_3^2 - m_2^2, \quad \theta_{\text{atm}} \equiv \theta_{23}, \\ \Delta m_{\text{sun}}^2 \equiv m_2^2 - m_1^2, \quad \theta_{\text{sun}} \equiv \theta_{12}, \quad (45)$$

and the third angle θ_{13} is subject to the constraint $\tan^2 \theta_{13} \leq 0.065$ from a combined analysis of the solar, atmospheric and CHOOZ data [8].

The Hamiltonian (43) is simple to decipher, thanks to the inherent mass hierarchy $\Delta m_{\text{atm}}^2 \gg \Delta m_{\text{sun}}^2$ (where $\Delta m_{\text{atm}}^2 \approx 3 \times 10^{-3} \text{ eV}^2$, and $\Delta m_{\text{sun}}^2 \leq 10^{-4} \text{ eV}^2$), and the fact that each flavor receives from the background medium a different contribution to their effective masses. The evolution of the three-flavor system follows essentially the dynamics of three separate and effectively two-flavor subsystems, and the present parametrization of the transformation matrix U turns out to be very convenient for their description [13]. If all three mixing angles are nonzero, we have exactly three potentially equilibrating transitions.

(i) The first occurs at $T \approx 12$ MeV, when $\Delta m_{\text{atm}}^2/2\tilde{p} \approx (8\sqrt{2}G_F \tilde{p}/3m_W^2)E_{\mu\mu}$, and $\nu_\mu \leftrightarrow \nu_\tau$ oscillations cease to be matter-suppressed. In a collisionless environment, the subsystem would undergo an MSW-like transformation which turns ν_μ and ν_τ into the states ν_x and ν_y ,

$$\nu_\mu \rightarrow \nu_x \equiv \frac{1}{\sqrt{2}}(\nu_\mu - \nu_\tau), \\ \nu_\tau \rightarrow \nu_y \equiv \frac{1}{\sqrt{2}}(\nu_\mu + \nu_\tau), \quad (46)$$

assuming $\theta_{\text{atm}} = 45^\circ$. Collisions play the role of breaking these states into an incoherent 1:1 mixture of ν_μ and ν_τ .

(ii) The second transition involves the states

$$\nu_e \rightarrow c_{13}\nu_e - s_{13}\nu_y, \quad \nu_y \rightarrow s_{13}\nu_e + c_{13}\nu_y, \quad (47)$$

and can only be realized for a nonzero θ_{13} . This happens when $(\Delta m_{\text{atm}}^2 + c_{12}^2 \Delta m_{\text{sun}}^2)/2\tilde{p} \approx (8\sqrt{2}G_F \tilde{p}/3m_W^2)(E_{ee}$

+ $E_{\mu\mu}/2$), at $T \approx 5.2$ MeV. (The factor 1/2 accompanying $E_{\mu\mu}$ comes from the fact that the state ν_y is only “half-sensitive” to the muon-antimuon background.) In the absence of collisions, Eq. (38) suggests this transition to be quite impotent, even when θ_{13} is at its upper limit. However, noting all the caveats regarding active-active collisions, a heuristic approach to collisional damping in the style of Sec. II D can yield for $\tan^2 \theta_{13} \approx 0.065$ at $T \approx 1$ MeV an I_z that is less than 1% of its original value by Eq. (42). Thus, near equilibrium between L_{ν_e} and L_{ν_y} prior to BBN is very probable.

(iii) The third transition,

$$\begin{aligned} c_{13}\nu_e - s_{13}\nu_y &\rightarrow c_{12}(c_{13}\nu_e - s_{13}\nu_y) - s_{12}\nu_x, \\ \nu_x &\rightarrow s_{12}(c_{13}\nu_e - s_{13}\nu_y) + c_{12}\nu_x, \end{aligned} \quad (48)$$

was described in detail in Sec. II for $\theta_{13}=0$. In the case of a sizable θ_{13} , equilibrium between L_{ν_e} and L_{ν_y} should already be partially accomplished by this time. A second equilibrating transition between ν_e and ν_x at this point will bring L_{ν_e} , L_{ν_μ} and L_{ν_τ} even more in line.

Note that we have been very specific with the labelling of ν_x and ν_y . This actually has an interesting consequence. Consider the case of a maximal θ_{sun} and a vanishing θ_{13} . We may attempt to use a simple “counting” method to establish crudely the final asymmetries given a set of initial conditions. For example, for $L_{\nu_e}^i = L_{\nu_\tau}^i = 0$, and $L_{\nu_\mu}^i = 0.1$, the first transition distributes the asymmetry in ν_μ equally amongst ν_μ and ν_τ such that $L_{\nu_\mu} \approx L_{\nu_\tau} \approx 0.05$. The second transition is not present. At the third transition, the asymmetry carried by $\nu_x = (\nu_\mu - \nu_\tau)/\sqrt{2}$ is shared evenly with ν_e , leading to $L_{\nu_e} \approx L_{\nu_x} \approx 0.025$. However, the decoupled state $\nu_y = (\nu_\mu + \nu_\tau)/\sqrt{2}$ still has an asymmetry of 0.05. Thus the real L_{ν_μ} and L_{ν_τ} should be ≈ 0.0375 . This simple counting exercise serves to illustrate an interesting point: without a finite θ_{13} , equilibrium between L_{ν_e} and the asymmetries of ν_μ and ν_τ cannot be quite exact.

IV. CONCLUSION

We have given an analytical treatment to the neutrino asymmetry equilibration scenarios considered in the numerical studies of DHPPRS [3]. In a two-flavor study of the solar neutrino oscillation parameters, we demonstrated that the equilibration mechanism is based upon a collective adiabatic MSW-like transformation between ν_e and ν_x (and between

$\bar{\nu}_e$ and $\bar{\nu}_x$), where ν_x may be ν_μ , ν_τ , or a linear combination thereof, for synchronized $\nu_e \leftrightarrow \nu_x$ and $\bar{\nu}_e \leftrightarrow \bar{\nu}_x$ oscillations. The transition temperature is determined, as usual, by Δm_{sun}^2 and by a characteristic momentum $\tilde{p} \approx \pi T$ for small initial asymmetries (c.f. $\langle p \rangle \approx 3.15 T$ for a Fermi-Dirac distribution with zero chemical potential). Thus from the sizes of the various possible Δm_{sun}^2 's alone, transition prior to BBN can only be achieved in the case of the LMA solution.

It turns out that the characteristic momentum \tilde{p} does in fact grow with the initial chemical potentials of the ensemble, and extremely large $|\xi^i|$'s can in principle delay the equilibrating transition. However, the dependence is weak. For $|\xi^i|$ not exceeding ~ 4 , the shift in the transition temperature is virtually unobservable.

A second concern for the LMA solution is the extent of the L_{ν_e}/L_{ν_x} equilibrium. We showed in the present work that, when the mixing parameter is chosen to be maximal (as was done in DHPPRS), complete equilibration of these asymmetries can always be achieved irrespective of collisional effects, and for most initial conditions. (The latter can in principle alter the adiabaticity of the transition.) For large but not maximal mixing, however, the extent of the equilibrium will depend on how one handles collisions on active-active neutrino oscillations, and rigorous work on this front is still wanting. Nonetheless, heuristic considerations of collisional damping suggest that full equilibrium before BBN is most likely.

For other solutions of the solar neutrino problem, we demonstrated in a semi-quantitative three-flavor analysis that a partial equilibration of L_{ν_e} and L_{ν_y} , where ν_y is some other linear combination of ν_μ and ν_τ , due to a sizable θ_{13} is possible in principle. Again, the extent of this equilibrium hinges on the treatment of scattering processes with the background, and is therefore subject to the usual caveats regarding active-active collisions. Investigations on this topic are currently underway.

(Note: See also Ref. [14] for a similar explanation of the DHPPRS results.)

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- [1] H. Kang and G. Steigman, Nucl. Phys. **B372**, 494 (1992).
 [2] S.H. Hansen, G. Mangano, A. Melchiorri, G. Miele, and O. Pisanti, Phys. Rev. D **65**, 023511 (2002).
 [3] A.D. Dolgov, S.H. Hansen, S. Pastor, S.T. Petcov, G.G. Raffelt, and D.V. Semikoz, Nucl. Phys. **B632**, 363 (2002).
 [4] C. Lunardini and A.Yu. Smirnov, Phys. Rev. D **64**, 073006

- (2001).
 [5] S. Samuel, Phys. Rev. D **48**, 1462 (1993).
 [6] G. Sigl and G. Raffelt, Nucl. Phys. **B406**, 423 (1993).
 [7] J. Pantaleone, Phys. Lett. B **287**, 128 (1992).
 [8] See, for example, M.C. González-García, M. Maltoni, C. Peña-Garay, and J.W.F. Valle, Phys. Rev. D **63**, 033005 (2001).

- [9] B.H.J. McKellar and M.J. Thomson, Phys. Rev. D **49**, 2710 (1994); G. Raffelt, G. Sigl, L. Stodolsky, Phys. Rev. Lett. **70**, 2363 (1993).
- [10] S. Pastor, G. Raffelt, D.V. Semikoz, Phys. Rev. D **65**, 053011 (2002).
- [11] See, for example, W.C. Haxton, Phys. Rev. D **35**, 2352 (1987); S.T. Petcov, Phys. Lett. B **406**, 355 (1997).
- [12] R.R. Volkas and Y.Y.Y. Wong, Phys. Rev. D **62**, 093024 (2000).
- [13] A particularly nice way of treating three-flavor (resonant) transitions can be found in T.K. Kuo and J. Pantaleone, Phys. Rev. D **35**, 3432 (1987).
- [14] K.N. Abazajian, J.F. Beacom, and N.F. Bell, astro-ph/0203442.