Vanishing magnetic mass in three-dimensional QED with a Chern-Simons term

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We show that at one loop the magnetic mass vanishes at finite temperature in QED in any dimension. In three-dimensional QED, even the zero temperature part can be regularized to zero. We calculate the two-loop contributions to the magnetic mass in three-dimensional QED with a Chern-Simons term and show that it vanishes. We give a simple proof which shows that the magnetic mass vanishes to all orders at finite temperature in this theory. This proof also holds for QED in any dimension.

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I. INTRODUCTION

In an earlier Letter [1], we studied the question of the screening mass in the $(2+1)$-dimensional Abelian Higgs model with a Chern-Simons term as well as in three-dimensional QED (QED_3) with a Chern-Simons term. We showed there that, at one loop, the magnetic mass in QED_3 vanishes. This is quite surprising considering the fact that the Chern-Simons term has associated with it various magnetic phenomena [2,3] and yet the magnetic mass vanishes. In that Letter [1], we formally argued, based on the Ward identity as well as the assumption of analyticity of the amplitudes, that this result holds to all orders. However, as is well known, amplitudes cease to be analytic and infrared divergences, in general, become severe at finite temperature [4], both of which can invalidate a formal argument. Therefore, in this paper we study this question systematically in QED_3 with a Chern-Simons term at finite temperature and give an alternate proof that the magnetic mass indeed vanishes to all orders. It has already been noted [5] that, at two loops, the parity-violating part of the gauge self-energy (correction to the Chern-Simons term) develops an infrared divergence at finite temperature in the absence of a tree level Chern-Simons term. It is for this reason that we study QED_3 with a Chern-Simons term. However, as the one-loop result shows [1], even with a tree level Chern-Simons term, the photon propagator develops a massless pole and, therefore, the question of infrared divergence has to be analyzed carefully. We note that the vanishing of the magnetic mass at finite temperature has already been studied in four-dimensional QED (QED_4) [6]. However, infrared divergences become more severe as we go to lower dimensions. The two-dimensional theory (Schwinger model) is known to be well behaved [7] and, therefore, the $(2+1)$ dimensional theory is the most interesting theory to study from this point of view.

Our results are organized as follows. In Sec. II we show that, at one loop, the magnetic mass vanishes in QED in any dimension at finite temperature. The additional feature of the $(2+1)$ dimensional theory is that even the zero temperature contribution to the magnetic mass can be regularized to zero. We also explicitly show that, at two loops, QED_3 with a Chern-Simons term has a vanishing contribution to the magnetic mass and that there is no infrared divergence present in this amplitude at this order even in the absence of a tree level Chern-Simons term. In Sec. III we prove that the vanishing of the magnetic mass holds to all orders. In Sec. IV we present a brief summary of our results.

II. EXPLICIT CALCULATIONS

Let us consider QED_3 with a Chern-Simons term described by the Lagrangian density

$$L=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{\kappa}{2}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda+\overline{\psi}(i\not{D}-m)\psi \tag{1}$$

where κ is known as the Chern-Simons coefficient and the covariant derivative is defined to be

$$D_\mu\psi=(\partial_\mu-ieA_\mu)\psi. \tag{2}$$

In $2+1$ dimensions, the Chern-Simons term as well as the mass term for the fermion break discrete symmetries such as parity and time reversal [2,3] and, therefore, are intimately connected. Namely, even if there is no Chern-Simons term present at the tree level, it is generated through radiative corrections in a massive fermion theory [8]. Let us note, however, that both these terms are invariant under charge conjugation under which

$$CA_\mu C^{-1}=-A_\mu,\quad C\psi C^{-1}=-\gamma^2\overline{\psi}^T. \tag{3}$$

As a result, the Lagrangian density in Eq. (1) is invariant under charge conjugation and it is the charge conjugation invariance which, for example, makes the amplitudes with an odd number of photons to vanish (Furry's theorem) in this theory. We would like to study the question of the magnetic mass in this theory at finite temperature.

Throughout this paper, we will use the imaginary time formalism [4,9,10] to study the finite temperature effects in this theory. Therefore, we will consider the theory in the Euclidean space. In such a theory, the self-energy of the photon is independent of the gauge fixing parameter and, in a covariant gauge, can be parametrized to all orders as [1]

$$\Pi_{\mu\nu}=P_{\mu\nu}\Pi_{\mathrm{T}}+Q_{\mu\nu}\Pi_{\mathrm{L}}+\epsilon_{\mu\nu\lambda}p_\lambda\Pi_{\mathrm{odd}} \tag{4}$$

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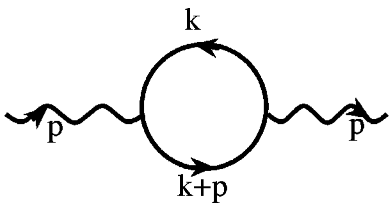

FIG. 1. One-loop diagram for photon self-energy.

where

$$P_{\mu\nu}=\tilde{\delta}_{\mu\nu}-\frac{\tilde{p}_\mu\tilde{p}_\nu}{\tilde{p}^2},\quad Q_{\mu\nu}=\frac{p^2}{\tilde{p}^2}\bar{u}_\mu\bar{u}_\nu \tag{5}$$

with

$$\tilde{\delta}_{\mu\nu}=\delta_{\mu\nu}-u_\mu u_\nu,\quad \tilde{p}_\mu=p_\mu-(u\cdot p)u_\mu,$$
$$\bar{u}_\mu=u_\mu-\frac{u\cdot p}{p^2}p_\mu. \tag{6}$$

Here u_μ represents the velocity of the heat bath which, in the rest frame, takes the form $u_\mu=(1,0,0)$.

There are several things to note here. First, Π_{T} and Π_{L} lead, respectively, to the transverse and the longitudinal masses for the photon. While the longitudinal mass is responsible for the screening of charges, it is the transverse mass which is related to the magnetic mass of the photon. Second, the presence of the parity odd term, in the self-energy, is a consequence of the fact that parity is violated in this theory. From the form of the tensors in Eq. (5), it is easy to see that we can write, in general, the magnetic mass for the photon (which is defined in the static limit) as

$$\Pi_{\mathrm{T}}(0)=\frac{1}{D-2}\delta_{ij}\Pi_{ij}(0)$$
$$=\frac{1}{D-2}\delta_{ij}\Pi^{\mathrm{PC}}_{ij}(0),\quad D>2 \tag{7}$$

where D represents the number of space-time dimensions. Namely, the magnetic mass is determined completely from the parity-conserving part of the self-energy and does not depend on the parity-violating structure.

We note that in the imaginary time formalism, the tree level fermion propagator has the form

$$S^{(0)}(p)=\frac{1}{\not{p}+m}=\frac{-\not{p}+m}{p^2+m^2}. \tag{8}$$

where, in the Euclidean space, we work with

$$\gamma_0=i\sigma_3,\quad \gamma_1=i\sigma_1,\quad \gamma_2=i\sigma_2 \tag{9}$$

and

$$p_0=(2n+1)\pi T=\frac{(2n+1)\pi}{\beta}. \tag{10}$$

With these, we note that the photon self-energy in QED at one loop, in an arbitrary dimension, takes the form (see Fig. 1)

$$\Pi^{(1)}_{\mu\nu}(p)=e^2\int\frac{d^Dk}{(2\pi)^D}\mathrm{tr}\,\gamma_\mu S^{(0)}(k+p)\gamma_\nu S^{(0)}(k)$$
$$=\frac{e^2}{\beta}\sum_n\int\frac{d^{D-1}k}{(2\pi)^{D-1}}\mathrm{tr}\,\gamma_\mu S^{(0)}(k+p)\gamma_\nu\times S^{(0)}(k). \tag{11}$$

In D-dimensional Euclidean space, the trace over the gamma matrices takes the form

$$\mathrm{tr}\,\gamma_\mu\gamma_\nu=-2^{[D/2]}\delta_{\mu\nu}=-C(D)\delta_{\mu\nu}, \tag{12}$$

where $[D/2]$ represents the floor of $D/2$. Evaluating the Dirac trace and performing the sum over the discrete frequencies, we obtain in the static limit

$$\Pi^{(1)}_{\mathrm{T}}(0)=\delta_{ij}\Pi^{(1)}_{ij}(0)=-\frac{C(D)e^2}{2}\int\frac{d^{D-1}k}{(2\pi)^{D-1}}\times\left((D-1)+\frac{\vec{k}^2}{\omega_k}\frac{\partial}{\partial\omega_k}\right)\left(\frac{\tanh\left(\frac{\beta\omega_k}{2}\right)}{\omega_k}\right)$$
$$=-\frac{C(D)e^2}{2^{D-1}\pi^{(D-1)/2}\Gamma\left(\frac{D-1}{2}\right)}\int_0^\infty dk\,k^{D-2}\times\left((D-1)+\frac{k^2}{\omega_k}\frac{\partial}{\partial\omega_k}\right)\left(\frac{\tanh\left(\frac{\beta\omega_k}{2}\right)}{\omega_k}\right) \tag{13}$$

where we have defined $k=|\vec{k}|$ and $\omega_k=\sqrt{k^2+m^2}$. We can now identify the temperature-dependent part of Eq. (13) to be

$$\Pi^{(1)(\beta)}_{\mathrm{T}}(0)=\frac{C(D)e^2}{2^{D-2}\pi^{(D-1)/2}\Gamma\left(\frac{D-1}{2}\right)}\int_0^\infty dk\,k^{D-2}\times\left((D-1)+\frac{k^2}{\omega_k}\frac{\partial}{\partial\omega_k}\right)\left(\frac{n_F(\omega_k)}{\omega_k}\right), \tag{14}$$

where

$$n_F(\omega_k)=\frac{1}{e^{\beta\omega_k}+1} \tag{15}$$

is the Fermi-Dirac distribution function. The expression on the right-hand side of Eq. (14) is easily seen to vanish: namely,

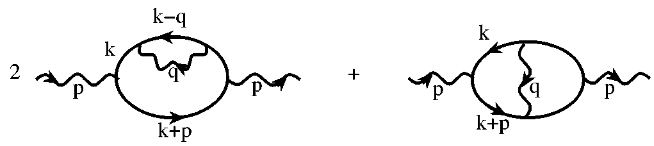

FIG. 2. Two-loop diagrams for photon self-energy.

$$\Pi_{\rm T}^{(1)(\beta)}(0)=\frac{C(D)e^2}{2^{D-2}\pi^{(D-1)/2}\Gamma\left(\frac{D-1}{2}\right)}\int_m^\infty d\omega_k\frac{\partial}{\partial\omega_k}\times\left((\omega_k^2-m^2)^{(D-1)/2}\frac{n_F(\omega_k)}{\omega_k}\right)=0,\quad D\geqslant 2. \tag{16}$$

This shows that, at finite temperature, the one-loop correction to the magnetic mass vanishes in any dimension $D \geqslant 2$. We note that, at one loop, there is no infrared divergence since we are considering a massive fermion. In general, the zero temperature part in Eq. (13) has an ultraviolet divergence. However, in $(2+1)$ dimensions, there is the added interesting feature that the zero temperature part can be regularized to zero within the framework of Pauli-Villars regularization or dimensional regularization or a gauge invariant projection method [2]. We also note that the one-loop result is completely independent of the presence or absence of a tree level Chern-Simons term in $2+1$ dimensions and, therefore, holds even for pure QED_3. Furthermore, let us note that while this has been an exact result at one loop, it can also be easily checked within the hard thermal loop approximation [11] where

$$\Pi_{\rm T\,(HTL)}^{(1)(\beta)}(0)\simeq -C(D)e^2\int\frac{d^{D-1}k}{(2\pi)^{D-1}}\frac{n_F(\omega_k)}{\omega_k}\times\left((D-3)+\frac{\vec{k}^2\vec{p}^2}{(\vec{k}\cdot\vec{p})^2}\right)=0. \tag{17}$$

We will show next that the magnetic mass also vanishes at two loops in QED_3 with a Chern-Simons term. The two-loop photon self-energy diagrams (see Fig. 2) can be obtained from the one-loop box diagrams by connecting the photon lines in all possible ways. We note that at tree level, the photon propagator, in a covariant gauge, has the form

$$D_{\mu\nu}^{(0)}(p)=\frac{1}{p^2+\kappa^2}\left[\left(\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)-\kappa\epsilon_{\mu\nu\lambda}\frac{p_\lambda}{p^2}\right]+\xi\frac{p_\mu p_\nu}{(p^2)^2} \tag{18}$$

where ξ is the gauge fixing parameter and $p_0=2\pi nT=2\pi n/\beta$. Furthermore, since the four photon amplitude at one loop (the sum of the box diagrams) is gauge invariant, it vanishes when it is contracted with the momentum associated with a photon line. Therefore, the only two terms from the photon propagator in Eq. (18) that can contribute to the two-loop self-energy are the $\delta_{\mu\nu}$ and the $\epsilon_{\mu\nu\lambda}$ terms. With these simplifications in mind, we obtain from the two-loop photon self-energy, in the static limit,

$$\Pi_{\rm T}^{(2)}(0)=4e^4\int\frac{d^3q}{(2\pi)^3}\frac{1}{q^2+\kappa^2}\times\int\frac{d^3k}{(2\pi)^3}\frac{\partial}{\partial k_i}\left[\frac{k_i}{(k^2+m^2)^2}-\frac{k_i[q^2+4m(\kappa-m)]}{(k^2+m^2)^2[(k+q)^2+m^2]}\right]=0. \tag{19}$$

This result holds for both the zero temperature as well as the finite temperature parts and we note that this is true even when $\kappa=0$ and, therefore, the vanishing of the magnetic mass, at two loops, holds even for pure QED_3. This has to be contrasted with the behavior of the parity-violating part of the self-energy which has an infrared divergence when $\kappa\to 0$ [5]. Therefore, we see that unlike the parity-violating part of the self-energy, the parity-conserving part has a better infrared divergence behavior. It is worth pointing out here that this result can also be obtained in a simple manner using the Ward identities of the theory. Let us also note that the Chern-Simons term does have a nontrivial contribution to the parity conserving part of the self-energy, $\Pi_{ij}^{\rm PC}$ [1]. However, surprisingly, it does not contribute to the magnetic mass (7).

III. VANISHING OF MAGNETIC MASS TO ALL ORDERS

Normally, an all orders proof of a result in a gauge theory is simplified enormously through the use of Ward identities. However, at finite temperature, the non-analyticity of the amplitudes at the origin in the energy-momentum space leads to difficulties [4,12]. For example, let us consider the N-point photon amplitude which would satisfy a relation of the form

$$p_{\alpha,\mu_\alpha}\Gamma_{\mu_1,\cdots,\mu_N}(p_1,\ldots,p_N)=0,\quad \mu_\alpha=0,1,2. \tag{20}$$

In the static limit where all the external energies vanish, on the other hand, we can write the amplitude as $\Gamma_{(m,n)}$ with $m+n=N$ where m represents the number of time indices while n corresponds to the number of space indices. In this case, the Ward identity (20) takes the form

$$p_{\alpha,i_\alpha}\Gamma_{(m,i_1,\ldots,i_n)}(\ldots,p_1,\ldots,p_n)=0,\quad i_\alpha=1,2 \tag{21}$$

and one can formally argue that, for small p_{α,i_α}, the amplitude behaves like [13,14]

$$\Gamma_{(m,i_1,\ldots,i_n)}(\ldots,p_1,\ldots,p_n)\sim O(p_1\ldots p_n). \tag{22}$$

This behavior is, in fact, explicitly seen at one loop in the two-point and the four-point functions for the photon in the static limit [15]. In the long-wavelength limit where the spatial components of the momenta vanish, on the other hand, we have much more limited information. Furthermore, even if we know the behavior of the amplitudes in the static and the long wave limits independently, this information is not

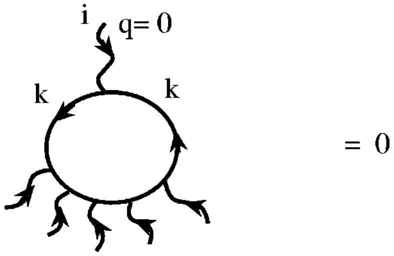

FIG. 3. Vanishing of an amplitude with one spacelike photon index carrying zero energy and momentum.

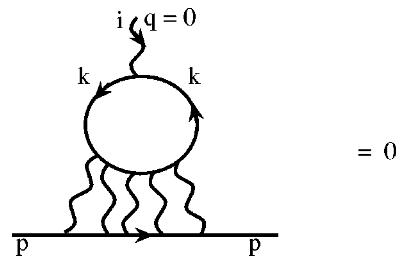

FIG. 4. Vanishing of a vertex diagram where the external photon with a space index and zero energy momentum is attached to an internal fermion line that is not a continuation of the external fermion lines.

very useful in constructing higher loop amplitudes, where the energy and momenta of the internal photon lines are integrated over all possible values.

Therefore, we pursue the following strategy in proving, to all orders, that the magnetic mass vanishes at finite temperature. First, let us look at the tree level propagator for the fermion in Eq. (8) and note that

$$\frac{\partial S^{(0)}(p)}{\partial p_\mu} = -S^{(0)}(p)\gamma_\mu S^{(0)}(p). \tag{23}$$

Namely, we see that, much like at zero temperature [13], at finite temperature, differentiating the tree level fermion propagator is equivalent to introducing a tree level photon vertex with zero energy momentum (up to the coupling). This can also be rewritten in the more familiar form

$$\frac{\partial (S^{(0)})^{-1}(p)}{\partial p_\mu} = \gamma_\mu \tag{24}$$

which says that, at the tree level, differentiating the fermion two-point function gives rise to a vertex with zero energy and momentum, up to the coupling constant.

Let us next note that we can write the one-loop N-point photon amplitude as

$$\begin{aligned}\Gamma_{\mu_1,\ldots,\mu_N}(q_1,\ldots,q_N) &= \int \frac{d^3k}{(2\pi)^3}\tilde{\Gamma}_{\mu_1,\ldots,\mu_N}(k;q_1,\ldots,q_N)\\ &= \frac{1}{\beta}\sum_n \int \frac{d^2k}{(2\pi)^2}\tilde{\Gamma}_{\mu_1,\ldots,\mu_N}(k;q_1,\ldots,q_N)\end{aligned} \tag{25}$$

where, as mentioned earlier, $k_0=(2n+1)\pi/\beta$. Using the relations in Eqs. (23) and (24), we see that the amplitude with an additional external photon with a space index and carrying zero energy and momentum is obtained to be (see Fig. 3)

$$\begin{aligned}\Gamma_{i,\mu_1,\ldots,\mu_N}(0,q_1,\ldots,q_N) &= -\frac{e}{\beta}\sum_n \int \frac{d^2k}{(2\pi)^2}\frac{\partial}{\partial k_i}\tilde{\Gamma}_{\mu_1,\ldots,\mu_N}\\ &\quad\times(k;q_1,\ldots,q_N)=0, \quad N\geqslant 2.\end{aligned} \tag{26}$$

This shows that an $N+1$ point amplitude with a photon line carrying a space index and zero energy momentum vanishes for $N\geqslant 2$, independent of the values of the energy and momenta of the other photon lines. The restriction, $N\geqslant 2$, comes from the ultraviolet convergence of the integrand and, at one loop, we have no infrared divergence problem since the fermion is massive. We note that the odd point photon amplitudes vanish by charge conjugation invariance (Furry's theorem) for any value of the external energy and momentum, but this result shows that even point photon amplitudes also vanish when one of the external photon lines has a space index and carries zero energy and momentum. It follows now, from this as well as charge conjugation invariance, that any fermion loop with an external photon carrying a space index and zero energy momentum gives a vanishing contribution in a complicated diagram such as Fig. 4, where the external photon is attached to an internal fermion line that is not a continuation of the external fermion lines.

As a result, using Eqs. (23), (24), it can be shown in a straightforward diagrammatic manner that, to all orders, we can write the three-point photon-fermion-fermion vertex with the photon carrying a space index and zero energy-momentum as

$$\Gamma_i(p,-p,0) = e\,\frac{\partial S^{-1}(p)}{\partial p_i}, \tag{27}$$

where Γ_i and $S^{-1}(p)$ represent, respectively, the vertex and the fermion self-energy to all orders. This is like the zero temperature Ward identity [13], but holds for a space index, which is what we will need for our proof.

With these, let us look at the photon self-energy to all orders given by the Schwinger-Dyson equation, Fig. 5, which leads to

$$\begin{aligned}\Pi_{\mathrm{T}}(0) &= \frac{e}{\beta}\sum_n \int \frac{d^2k}{(2\pi)^2}\mathrm{tr}\,\gamma_i S(k)\Gamma_i(k,-k,0)S(k)\\ &= -\frac{e^2}{\beta}\sum_n \int \frac{d^2k}{(2\pi)^2}\frac{\partial}{\partial k_i}[\mathrm{tr}\,\gamma_i S(k)].\end{aligned} \tag{28}$$

The finite temperature part of this integrand is well behaved and, being a total divergence, vanishes, namely,

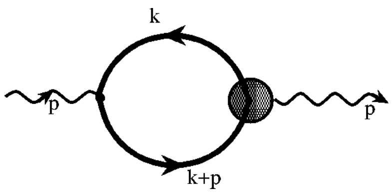

FIG. 5. Schwinger-Dyson relation for photon self-energy. The internal heavy lines represent the full fermion propagator while the blob represents the complete vertex.

$$\Pi_{T}^{(\beta)}(0)=0 \tag{29}$$

to all orders. Since the loop involves massive fermions, there is no problem of infrared divergence in this case. The temperature independent part of this expression has a linear divergence which, as we mentioned in Sec. II can be regularized to zero in $2+1$ dimensions. We note here that the vanishing of the finite temperature part to all orders appears naturally, in this description, to hold in any dimension.

IV. SUMMARY

In this paper we have shown that, at finite temperature, the magnetic mass vanishes at one loop in QED in any dimension. In $2+1$ dimensions, in addition, the zero temperature part can be regularized to zero. In $2+1$ dimension, this result is independent of the presence of a tree level Chern-Simons term. We have calculated and shown explicitly that this result also holds at two loops. We have given a simple proof to show that the magnetic mass vanishes to all orders at finite temperature in this theory. This result also holds for QED in any dimension.

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