S-brane solutions in supergravity theories

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In this paper time-dependent solutions of supergravities with a dilaton and an arbitrary rank antisymmetric tensor field are found. Although the solutions are nonsupersymmetric the equations of motion can be integrated in a simple form. Such supergravity solutions are related to Euclidean or spacelike branes (*S*-branes).

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I. INTRODUCTION

There has been a recent surge of interest in time-dependent solutions in string theory. In [1–4] the question of a stringy resolution of cosmological singularities in time-dependent string orbifolds was discussed. The de Sitter/conformal field theory (dS/CFT) correspondence [5,6] identifies time evolution in de Sitter space with renormalization group flow [7]. Very recently Sen [8,9] (see also [10]) constructed a conformal field theory description of dynamical open string tachyon condensation. For earlier work on time-dependent solutions see [11–16].

Dirichlet branes [17] are extended solitonic objects carrying Ramond-Ramond (RR) charge and therefore the world volume of such a (static) brane includes the time direction. It is a natural question, partly motivated by the dS/CFT correspondence, whether there are Euclidean branes which have a purely spacelike world volume. Euclidean branes were first constructed in [18,19] in type II* theories which are nonunitary theories obtained by timelike *T* duality from the standard type II theories. The simplest starting point for the construction of a Euclidean brane in type II theories is given by considering open strings which satisfy Dirichlet boundary conditions in the time direction [20]. Such a spacelike brane (*S*-brane) only exists for one instant in time.

Another argument for the existence of *S*-branes uses the open string tachyons in unstable *D*-branes or *D*-brane–anti-*D*-brane pairs. (Similar constructions are also possible in field theory [21].) The basic argument for the existence of *S*-branes, illustrated by a specific example, is the following. In type IIA string theory there exists "mismatched" *D*-branes, such as the $D3$-brane, which are unstable and contain a tachyon field. Let us consider the $D3$-brane as our example. The potential of the tachyon field, $U(T)$, resembles a double well; it was argued that the stable $D2$-brane is the tachyonic kink solution of the unstable $D3$ world volume field theory [22]. However, one can imagine a similar notion for the time-dependent case. Suppose the initial data ($t=0$) for the $D3$-brane tachyon field is located at the unstable maximum, $U(0)$, with a small constant positive velocity. Then the tachyon field will roll off from the top of the potential and evolve to the positive minimum at $t=\infty$. During this evolution closed string radiation will be emitted and then will propagate to infinity. Similarly, as a consequence of time-reversal symmetry, the tachyon field will approach the negative minimum at $t=-\infty$. This process can be realized as incoming radiation which excites the tachyon field to the top of the potential barrier. The full picture is a timelike kink in the tachyon field which is an $S2$-brane.

Using the known coupling of the spacetime RR fields to the world volume open string tachyon it was shown that this $S2$-brane carries charge, defined as the integral of the RR field over a surrounding sphere (including the time dimension). The same kind of charge is carried by an ordinary $D2$-brane. In analogy with Sen's identification, this timelike kink can be identified as an $SD2$-brane, i.e. a Dirichlet brane arising from open string with a Dirichlet boundary condition on the time dimension.

Obviously this construction can be generalized to other codimensions, for example to branes as vortices in a brane-antibrane pair. Moreover, a similar discussion for the initial data along the null direction will lead to null branes (*N*-branes).

Both the boundary state and the tachyon picture of the *S*-brane suggests that an Sp-brane [with $(p+1)$-dimensional Euclidean world volume] in d dimensions should have $ISO(p+1)\times SO(d-p-2,1)$ symmetry. The noncompact $SO(d-p-2,1)$ can be interpreted as the *R* symmetry of a Euclidean field theory living on the *S*-brane. In [20] supergravity solutions respecting this symmetry were found in two particular cases. It is the aim of this paper to generalize these *S*-brane solutions to arbitrary form field, codimensions, and dilaton coupling. These solutions are new interesting time-dependent or cosmological solutions of supergravities. Note, however, that the relation of these solutions to the boundary state and tachyon construction of the *S*-brane is quite nontrivial and not well understood at present.

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II. GENERAL *S*-BRANES

In this section we analyze equations governing $S(p\text{-}1)$-branes associated with the charge of a q-form field strength. The system contains a graviton, a q-form field strength, $F_{[q]}$, and a dilaton scalar, ϕ, coupled to the form field with the coupling constant a. This is a general framework which encompasses the bosonic sector of various supergravity theories, coming from a truncation of the low energy limit of *M*-theory and string theories, by a certain choice of the dimension d, the rank of the form field q, and the dilaton coupling a. In the Einstein frame, the action is given by

$$S=\int d^dx\sqrt{-g}\left(R-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-\frac{1}{2q!}\mathrm{e}^{a\phi}F_{[q]}^2\right). \qquad (1)$$

This action is invariant under the following discrete S duality:

$$g_{\mu\nu}\rightarrow g_{\mu\nu},\quad F\rightarrow \mathrm{e}^{-a\phi}*F,\quad \phi\rightarrow-\phi, \qquad (2)$$

where * denotes a d-dimensional Hodge dual. This may be used to construct electric versions of magnetic S-branes and vice versa. The equations of motion, derived from the variation of the action with respect to the individual fields, are

$$R_{\mu\nu}-\frac{1}{2}\partial_\mu\phi\partial_\nu\phi-\frac{\mathrm{e}^{a\phi}}{2(q-1)!}\times\left[F_{\mu\alpha_2\cdots\alpha_q}F_\nu{}^{\alpha_2\cdots\alpha_q}-\frac{q-1}{q(d-2)}F_{[q]}^2g_{\mu\nu}\right]=0, \qquad (3)$$

$$\partial_\mu(\sqrt{-g}\mathrm{e}^{a\phi}F^{\mu\nu_2\cdots\nu_q})=0, \qquad (4)$$

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\partial^\mu\phi)-\frac{a}{2q!}\mathrm{e}^{a\phi}F_{[q]}^2=0. \qquad (5)$$

We study S-branes with a world volume given by a p-dimensional conformally flat space and with a transverse space being the k-dimensional hyperspace $\Sigma_{k,\sigma}$ and $q-k$ dimensional delocalized space. Obviously, in d dimensions, $p=d-q-1$. With this in mind we choose the metric

$$ds^2=-\mathrm{e}^{2A}dt^2+\mathrm{e}^{2B}(dx_1^2+\cdots+dx_p^2)+\mathrm{e}^{2C}d\Sigma_{k,\sigma}^2+\mathrm{e}^{2D}(dy_1^2+\cdots+dy_{q-k}^2), \qquad (6)$$

parametrized by four t-dependent functions $A(t)$,$B(t)$,$C(t)$, and $D(t)$. The hyperspace $\Sigma_{k,\sigma}$ for $\sigma=0,+1,-1$ is the k-dimensional flat space, the sphere, and the hyperbolic space, respectively. They can be described as

$$d\Sigma_{k,\sigma}^2=\bar{g}_{ab}dz^adz^b=\begin{cases} d\psi^2+\sinh^2\psi d\Omega_{k-1}^2, & \sigma=-1,\\ d\psi^2+\psi^2 d\Omega_{k-1}^2, & \sigma=0,\\ d\psi^2+\sin^2\psi d\Omega_{k-1}^2, & \sigma=+1,\end{cases} \qquad (7)$$

satisfying

$$\bar{R}_{ab}=\sigma(k-1)\bar{g}_{ab}. \qquad (8)$$

The metrics above have $SO(k-1,1)$, $ISO(k)$, and $SO(k)$ symmetries, respectively. In [20] in order to have a solution with the correct R symmetry only the case $\sigma=-1$ and hence $SO(k-1,1)$ symmetry was considered. In the following we will discuss all three choices of σ.

With this ansatz, the equation for the form field (4), can easily be solved giving

$$F_{[q]}=b\,\mathrm{vol}(\Sigma_{k,\sigma})\wedge dy_1\wedge\cdots\wedge dy_{q-k}, \qquad (9)$$

where b is the field strength parameter, $\mathrm{vol}(\Sigma_{k,\sigma})$ denotes the unit volume form of the hyperspace $\Sigma_{k,\sigma}$.

The ansatz (6) and (9) has $q-k$ flat directions and takes these directions to be toroidal. The solutions are in some sense "smeared" or delocalized along these directions. Note that from the tachyon picture the appearance of delocalized coordinates is quite natural since the tachyon is localized on a brane. The solutions of [20] can be obtained by setting $k=q$.

To derive the equations for the metric functions A,B,C, and D one calculates first the Ricci tensor for the metric (6), the nonvanishing components being

$$R_{tt}=-p(\ddot{B}+\dot{B}^2-\dot{A}\dot{B})-k(\ddot{C}+\dot{C}^2-\dot{A}\dot{C})-(q-k)(\ddot{D}+\dot{D}^2-\dot{A}\dot{D}), \qquad (10)$$

$$R_{xx}=\mathrm{e}^{2B-2A}[\ddot{B}-\dot{A}\dot{B}+p\dot{B}^2+k\dot{B}\dot{C}+(q-k)\dot{B}\dot{D}], \qquad (11)$$

$$R_{yy}=\mathrm{e}^{2D-2A}[\ddot{D}-\dot{A}\dot{D}+p\dot{B}\dot{D}+k\dot{C}\dot{D}+(q-k)\dot{D}^2], \qquad (12)$$

$$R_{ab}=\{\mathrm{e}^{2C-2A}[\ddot{C}-\dot{A}\dot{C}+p\dot{B}\dot{C}+k\dot{C}^2+(q-k)\dot{C}\dot{D}]+\sigma(k-1)\}\bar{g}_{ab}. \qquad (13)$$

From the expressions for the Ricci tensor, we note that the formulation can be largely simplified once we chose the following gauge condition:

$$-A+pB+kC+(q-k)D=0. \qquad (14)$$

After taking the above gauge, the Einstein equations (3) finally reduce to the following set of equations:

$$-\ddot{A}+\dot{A}^2-p\dot{B}^2-k\dot{C}^2-(q-k)\dot{D}^2-\frac{1}{2}\dot{\phi}^2-\frac{(q-1)b^2}{2(d-2)}\mathrm{e}^{a\phi+2pB}=0, \qquad (15)$$

$$\ddot{B}+\frac{(q-1)b^2}{2(d-2)}\mathrm{e}^{a\phi+2pB}=0, \qquad (16)$$

$$\ddot{C}+\sigma(k-1)\mathrm{e}^{2A-2C}-\frac{pb^2}{2(d-2)}\mathrm{e}^{a\phi+2pB}=0, \qquad (17)$$

$$\ddot{D}-\frac{pb^2}{2(d-2)}e^{a\phi+2pB}=0. \tag{18}$$

Substituting our ansatz into Eq. (5) one obtains the following dilaton equation:

$$\ddot{\phi}+\frac{ab^2}{2}e^{a\phi+2pB}=0, \tag{19}$$

where the dots denote derivatives with respect to t.

Equations (16), (18), and (19) are of similar structure, and it is easy to see that the appropriate combinations of D, B, and ϕ, B obey simple homogeneous equations. Therefore

$$\phi=\frac{a(d-2)}{q-1}B+c_1t+c_2, \tag{20}$$

with constant c_1,c_2. A similar relation can be found for D and B, for which, however, we simply take

$$D=-\frac{p}{q-1}B. \tag{21}$$

It is more convenient to reparametrize $A(t),B(t),C(t),D(t)$ ensuring the gauge (14) choice by two independent functions $f(t),g(t)$ as

$$A=kg-\frac{p}{q-1}f,\quad B=f,\quad C=g-\frac{p}{q-1}f,$$

$$D=-\frac{p}{q-1}f. \tag{22}$$

Consequently, the equations of motion reduce to

$$\ddot{f}+\frac{(q-1)b^2}{2(d-2)}e^{\chi f+ac_1t+ac_2}=0, \tag{23}$$

$$\ddot{g}+\sigma(k-1)e^{2(k-1)g}=0, \tag{24}$$

$$\frac{p}{q-1}\ddot{f}-k\ddot{g}+k(k-1)\dot{g}^2-\frac{(d-2)\chi}{2(q-1)}\dot{f}^2-\frac{1}{2}c_1^2$$
$$-\frac{ac_1(d-2)}{(q-1)}\dot{f}-\frac{(q-1)b^2}{2(d-2)}e^{\chi f+ac_1t+ac_2}=0, \tag{25}$$

where the parameter χ is defined as

$$\chi=2p+\frac{a^2(d-2)}{q-1}. \tag{26}$$

The terms linear in t can be absorbed into f by defining

$$f(t)=h(t)-\frac{ac_1}{\chi}t-\frac{ac_2}{\chi}. \tag{27}$$

In terms of h the equations of motion become

$$\ddot{h}+\frac{(q-1)b^2}{2(d-2)}e^{\chi h}=0, \tag{28}$$

$$\ddot{g}+\sigma(k-1)e^{2(k-1)g}=0, \tag{29}$$

$$\frac{p}{q-1}\ddot{h}-k\ddot{g}+k(k-1)\dot{g}^2-\frac{(d-2)\chi}{2(q-1)}\dot{h}^2-\frac{pc_1^2}{\chi}$$
$$-\frac{(q-1)b^2}{2(d-2)}e^{\chi h}=0. \tag{30}$$

In fact, Eqs. (28), (29), and (30) are equivalent to the two first order equations

$$\dot{h}^2+\frac{(q-1)b^2}{(d-2)\chi}e^{\chi h}=\alpha^2, \tag{31}$$

$$\dot{g}^2+\sigma e^{2(k-1)g}=\beta^2, \tag{32}$$

provided the integration constants α and β satisfy

$$\frac{pc_1^2}{\chi}+\frac{(d-2)\chi\alpha^2}{2(q-1)}-k(k-1)\beta^2=0. \tag{33}$$

These equations can easily be integrated and the solution in terms of f and g are given by

$$f(t)=\frac{2}{\chi}\ln\left(\frac{\alpha}{\cosh\left[\frac{\chi\alpha}{2}(t-t_0)\right]}\right)+\frac{1}{\chi}\ln\left(\frac{(d-2)\chi}{(q-1)b^2}\right)$$
$$-\frac{ac_1}{\chi}t-\frac{ac_2}{\chi}, \tag{34}$$

$$g(t)=\begin{cases}\frac{1}{k-1}\ln\left(\frac{\beta}{\sinh[(k-1)\beta(t-t_1)]}\right), & \sigma=-1.\\ \pm\beta(t-t_1), & \sigma=0.\\ \frac{1}{k-1}\ln\left(\frac{\beta}{\cosh[(k-1)\beta(t-t_1)]}\right), & \sigma=+1.\end{cases} \tag{35}$$

Superficially it might seem that the solution depends on six parameters t_0,t_1,c_1,c_2,b,β. However, it is possible to eliminate two of them. First, β can be eliminated by rescaling $t\rightarrow\beta^{-1}t$ together with suitable scaling for coordinates $\{x,y\}$, and secondly, t_1 can be set to zero by a shift of t. Hence the solution depends on four parameters.

In Table I we list the values of parameters for eleven-dimensional supergravity and types IIA and IIB theories in ten dimensions where the dilaton coupling is $a=(5-q)/2$. The corresponding S-branes are not entirely independent; the discrete S-duality (2) relates them in pairs. These electric-magnetic pairs are indicated in parentheses.

For the $S3$-brane of IIB supergravity the five-form field strength should be self-dual, which is not ensured by our previous ansatz. Therefore we solve this case separately. By self-duality $F_{[5]}^2=0$ the equation of motion for the dilaton field in the gauge (14) becomes

$$\ddot{\phi}=0. \tag{36}$$

TABLE I. Parameters of S-branes.

	M-theory		Type II string theories								
	$S5$	$S2$	$S6$	NS $S5$	$S5$	$S4$	$[S3]$	$S2$ $(*S4)$	$S1$ $(*S5)$	NS$S1$ $(*$NS $S5)$	S0 $(*S6)$
d	11	11	10	10	10	10	10	10	10	10	10
q	4	7	2	3	3	4	5	6	7	7	8
a	0	0	3/2	-1	1	1/2	0	-1/2	-1	1	-3/2
p	6	3	7	6	6	5	4	3	2	2	1
χ	12	6	32	16	16	32/3	8	32/5	16/3	16/3	32/7

In fact, the dilaton coupling with form field $F_{[5]}$ is absent in IIB theory, $a=0$, so the dilaton field can be set to a constant. Following an analogous calculation we found that the self-dual five-form field should be

$$F_{[5]}=\frac{b}{\sqrt{2}}(1+*)\mathrm{vol}(\Sigma_{k,\sigma})\wedge dy_1\wedge\cdots\wedge dy_{5-k}. \quad (37)$$

The $S3$-brane solution is therefore given by setting $a=0$ and $c_1=0$ and the metric can be directly read from the general expressions of solutions given in this section by using the values of parameters in Table I.

A. Hyperbolic transverse space

In this section we will discuss the form of the metric in special limiting cases. In order to simplify notation we set $t_1=0$ and $\beta=1$ by a shift and rescaling discussed before.

The asymptotic region is at $t\to 0$ where the radius of the $\Sigma_{k,-1}$ diverges. Defining $u=[(k-1)t]^{-1/(k-1)}$, near $t=0,u=\infty$ the metric becomes

$$ds^2_{t\to 0}\sim \mathrm{e}^{-2pf_0/(q-1)}(-du^2+u^2d\Sigma^2_{k,-1}+d\vec{y}^{\,2}_{q-k})+\mathrm{e}^{2f_0}d\vec{x}^{\,2}_p, \quad (38)$$

with

$$f_0=\frac{2}{\chi}\ln\left(\frac{\alpha}{\cosh\left(\frac{\chi\alpha}{2}t_0\right)}\right)+\frac{1}{\chi}\ln\left(\frac{(d-2)\chi}{(q-1)b^2}\right)-\frac{ac_2}{\chi}. \quad (39)$$

The large t, near-brane behavior is given by

$$\begin{aligned}ds^2_{t\to\infty}\sim{}& \mathrm{e}^{-2pf_1/(q-1)}\mathrm{e}^{[2p/(q-1)][\alpha+(ac_1/\chi)]t}(-2^{2k/(k-1)}\\ &\times \mathrm{e}^{-kt}dt^2+2^{2/(k-1)}\mathrm{e}^{-t}d\Sigma^2_{k,-1}+d\vec{y}^{\,2}_{q-k})\\ &+\mathrm{e}^{2f_1}\mathrm{e}^{-2[\alpha+(ac_1/\chi)]t}d\vec{x}^{\,2}_p, \end{aligned} \quad (40)$$

with

$$f_1=\frac{2}{\chi}\ln\alpha+\alpha t_0-\frac{2}{\chi}\ln 2+\frac{1}{\chi}\ln\left(\frac{(d-2)\chi}{(q-1)b^2}\right)-\frac{ac_2}{\chi}. \quad (41)$$

Even though the Ricci scalar tends to zero in this region the geometry is singular because for example the coefficient of $d\vec{x}^{\,2}_p$ vanishes.

B. Flat transverse space

The asymptotic region near $t\to 0$ the metric becomes

$$ds^2_{t\to 0}\sim \mathrm{e}^{-2pf_0/(q-1)}(-dt^2+d\Sigma^2_{k,0}+d\vec{y}^{\,2}_{q-k})+\mathrm{e}^{2f_0}d\vec{x}^{\,2}_p. \quad (42)$$

The large t, near-brane behavior is given by

$$\begin{aligned}ds^2_{t\to\infty}\sim{}& \mathrm{e}^{-2pf_1/(q-1)}\mathrm{e}^{[2p/(q-1)][\alpha+(ac_1/\chi)]t}\\ &\times(-\mathrm{e}^{\pm 2kt}dt^2+\mathrm{e}^{\pm 2t}d\Sigma^2_{k,0}+d\vec{y}^{\,2}_{q-k})\\ &+\mathrm{e}^{2f_1}\mathrm{e}^{-2[\alpha+(ac_1/\chi)]t}d\vec{x}^{\,2}_p. \end{aligned} \quad (43)$$

C. Spherical transverse space

The metric in the asymptotic region near $t\to 0$ becomes

$$ds^2_{t\to 0}\sim \mathrm{e}^{-2pf_0/(q-1)}(-dt^2+d\Sigma^2_{k,+1}+d\vec{y}^{\,2}_{q-k})+\mathrm{e}^{2f_0}d\vec{x}^{\,2}_p. \quad (44)$$

The large t, near-brane behavior is given by

$$\begin{aligned}ds^2_{t\to\infty}\sim{}& \mathrm{e}^{-2pf_1/(q-1)}\mathrm{e}^{[2p/(q-1)][\alpha+(ac_1/\chi)]t}\\ &\times(-2^{2k/(k-1)}\mathrm{e}^{-kt}dt^2+2^{2/(k-1)}\mathrm{e}^{-t}d\Sigma^2_{k,+1}\\ &+d\vec{y}^{\,2}_{q-k})+\mathrm{e}^{2f_1}\mathrm{e}^{-2[\alpha+(ac_1/\chi)]t}d\vec{x}^{\,2}_p. \end{aligned} \quad (45)$$

III. STATIC SOLUTIONS

In this section we will briefly describe the application of the ansatz and gauge we used in the previous section to the case of static solutions. As it turns out these solutions are related (for a different choice of gauge) to the general black brane [25] solutions found in [23] (see also [24,26]).

The ansatz for the static solution is given by

$$\begin{aligned}ds^2={}&\mathrm{e}^{2A}dr^2+\mathrm{e}^{2B}(-dt^2+dx_1^2+\cdots+dx^2_{p-1})\\ &+\mathrm{e}^{2C}d\Sigma^2_{k,\sigma}+\mathrm{e}^{2D}(dy_1^2+\cdots+dy^2_{q-k}),\end{aligned} \quad (46)$$

with the same metric for $\Sigma_{k,\sigma}$ given by Eq. (7) and field strength (9), but in this case all functions $A(r)$,$B(r)$,$C(r)$, and $D(r)$ depend only on the radius coordinate r. Using the gauge condition (14) the equations of motion become (where primes now denote derivatives with respect to r)

$$-A''+A'^2-pB'^2-kC'^2-(q-k)D'^2-\frac{1}{2}\phi'^2$$
$$+\frac{(q-1)b^2}{2(d-2)}e^{a\phi+2pB}=0, \qquad (47)$$

$$B''-\frac{(q-1)b^2}{2(d-2)}e^{a\phi+2pB}=0, \qquad (48)$$

$$C''-\sigma(k-1)e^{2A-2C}$$
$$+\frac{pb^2}{2(d-2)}e^{a\phi+2pB}=0, \qquad (49)$$

$$D''+\frac{pb^2}{2(d-2)}e^{a\phi+2pB}=0. \qquad (50)$$

Again ϕ must be related to the function B as follows:

$$\phi=\frac{a(d-2)}{q-1}B+c_1r+c_2. \qquad (51)$$

Using the same relations as in Eq. (22) the equations of motion can be reduced to two first order differential equations for $f(r),g(r)$. The solutions are given by

$$f(r)=\frac{2}{\chi}\ln\left(\frac{\alpha}{\sinh\left[\frac{\chi\alpha}{2}(r-r_0)\right]}\right)$$
$$+\frac{1}{\chi}\ln\left(\frac{(d-2)\chi}{(q-1)b^2}\right)-\frac{ac_1}{\chi}r-\frac{ac_2}{\chi}, \qquad (52)$$

$$g(r)=\begin{cases}\frac{1}{k-1}\ln\left(\frac{\beta}{\cosh[(k-1)\beta(r-r_1)]}\right), & \sigma=-1.\\ \pm\beta(r-r_1), & \sigma=0.\\ \frac{1}{k-1}\ln\left(\frac{\beta}{\sinh[(k-1)\beta(r-r_1)]}\right), & \sigma=+1.\end{cases} \qquad (53)$$

After rescaling and shifts the solution will depend on four parameters (r_0,c_1,c_2,b). It is instructive to compare the $\sigma=+1$ case to the fully localized, $k=q$, three-parameter $(\rho_0,\bar{c}_1,\bar{c}_2)$ solutions found in [23,24] for type II theories in ten dimensions.[1] A closer analysis shows that these solutions are indeed equivalent after a coordinate transformation

$$r=\frac{1}{8-p}\ln\left(\frac{1+(\rho_0/\rho)^{8-p}}{1-(\rho_0/\rho)^{8-p}}\right), \qquad (54)$$

and a specific value of c_2,

$$c_2=\frac{4(p-4)}{p(8-p)}\ln\left(\frac{b}{\kappa(8-p)}\sinh[(p-8)\kappa r_0]\right). \qquad (55)$$

The relation of parameters is

$$\bar{c}_1=-\frac{c_1}{8-p}, \qquad (56)$$

$$\bar{c}_2=\coth[(p-8)\kappa r_0], \qquad (57)$$

$$\rho_0=2^{1/(p-8)}\exp\left[\frac{p}{4(p-4)}c_2\right], \qquad (58)$$

where

$$\kappa^2=\frac{2(9-p)}{8-p}-\frac{pc_1^2}{16(8-p)}. \qquad (59)$$

In [24] the static solutions for $\sigma=+1$ were interpreted as supergravity solutions corresponding to coincident brane-antibrane pairs. Note that whether this interpretation is correct is not clear *a priori* since one would not expect to have a static time-independent solution for an object which is unstable and decays. It is, however, tempting to speculate that the time-dependent solutions we have found could describe exactly such a process.

IV. CONCLUSION

In this paper we have constructed new time-dependent solutions in supergravities. For transverse spaces which are hyperbolic these solutions generalize the ones found in [20] to arbitrary codimension, rank of field strength, and dilaton coupling. These solutions are expected to be supergravity realizations of *S*-branes, Euclidean branes which only exist at an instant in time. Although the solutions are not supersymmetric the field equations can be integrated (for the hyperbolic as well as the flat and spherical case). One motivation for considering *S*-branes was the role which Euclidean branes play in the dS/CFT correspondence and the role of holography in comparison to AdS/CFT [27–30]. It would be very interesting to explore the role the solutions in this paper might play in this context. Relatedly it is an interesting question whether the solutions in this paper have a cosmological interpretation and if an *S*-brane can be used to get a nonsingular connection between big crunch and big bang cosmologies.

The same gauge and ansatz can be used to find static solutions. We showed that these solutions are equivalent to the ones found in [23,24]. We have speculated that the time dependent solutions could be realizations of a brane-antibrane annihilation process. It would be very interesting to explore this relation further. Furthermore, given the relation of brane-antibrane systems to fluxbranes [31–44], it might be possible that the time-dependent solutions describe the dynamical evolution of fluxbranes. We leave this question for future work.

[1]The most general solutions in [24] actually contain a fourth parameter $\bar{c}_3$ which seems unrelated to the parameter c_2 here. Moreover, please also note our notation of p has value one different from the convention in [24].

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