Role of scalar field in the formation of structure in the universe

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(Received 2 May 2002; published 30 July 2002)

A scalar field approach to the Jeans mass calculation is discussed. Considering a massive scalar field arbitrarily coupled to a gravitational background, the stress-energy tensor expectation values are computed in a coherent state. The density matrix is used to represent the expectation values. The energy density and pressure associated with the density perturbations are evaluated. Using these results, the exact expressions for the Jeans length and Jeans mass are evaluated.

DOI: 10.1103/PhysRevD.66.024038 PACS number(s): 04.62.+v, 98.80.Bp, 98.80.Cq

I. INTRODUCTION

The standard theory of cosmological structure formation [1–3] is based on the idea of gravitational instability [4–6], according to which small initial irregularities in the distribution of matter become amplified by the attractive nature of gravity. Gravitational instability is a consequence of small fluctuations in the density [7]. Gravitational instability causes the growth of perturbations in an expanding universe. The gravitational instability of a spatially uniform state of dustlike matter described by classical nonrelativistic equations has been first investigated by Jeans [8,9]. If the mass of a body is larger than some minimum mass called the Jeans mass, the self-gravity of matter will start affecting the structure of the body significantly.

The potential role of the scalar field in cosmology has been well discussed and it is found that quantum fields have a profound influence on the dynamical behavior of the early universe [10–12]. The inflationary universe scenario [13] broaches the question concerning the role of a scalar field in cosmological evolution and particularly of its influence on the development of cosmological inhomogeneities. The influence of quantum fields on the cosmological phase transitions, inflation and particle creation has been investigated by many authors [14–17]. In order to explain nonlinear structures observed today on the scale of galaxies and clusters we require initial perturbations [18]. There are two distinct theories of how the initial seed fluctuations might have arisen [7]. One of these models involved the idea of topological defects created during phase transitions in the early universe. The alternative picture involves the inflationary model of the universe, in which the primordial quantum fluctuations get amplified and evolve to become classical seed perturbations [2,19]. The most natural choice for the seed perturbations is the quantum fluctuations in the scalar field $\phi(x,t)$ driving the inflation [20].

Perturbations in a universe filled by a scalar field minimally coupled to gravity is clearly described by Mukhanov, Feldman, and Brandenberger [18]. They have calculated the growth rates of perturbations and the analysis is applied to study the evolution of fluctuations in inflationary universe models. The gravitational instability of a spatially uniform state of a relativistic scalar field on a time-dependent background is discussed by Khlopov, Malomed, and Zeldovich [6] and the instability is demonstrated to be similar to the Jeans instability. Ferreira and Joyce have studied the structure formation with a self-tuning scalar field and they have explained the effect of scalar field on cosmic microwave background (CMB) [21]. Structure formation within the Lemaitre-Tolman model have been investigated by Krasinki and Hellaby [22]. They have determined how fast the condensations can grow, once they appear in a homogeneous background. In Ref. [23] Wetterich describes the influence of the back reaction of density fluctuations on the cosmological evolution for a homogeneous and isotropic average metric. Density fluctuations of a cosmological quantum real scalar field in a coherent state is studied and the Jeans instability mechanism is generalized in this context by Bianchi, Grasso, and Ruffini [24].

In the present work we consider a massive scalar field ϕ coupled arbitrarily to the gravitational background. A coherent state representation [25] is constructed for each mode of the quantized scalar field in Sec. III and the stress-energy tensor expectation values are computed in the coherent state. In Sec. IV the density matrix [26] is used to represent the expectation values of the stress-energy tensor. A simplified expression is obtained by using the WKB approximation. In Sec. V the energy density and pressure associated with the density perturbations are evaluated. This result is used to evaluate the Jeans mass for the present case in Sec. VI. The results and discussions are presented in Sec. VII.

II. SCALAR FIELD GRAVITATIONALLY COUPLED TO BIANCHI TYPE-I SPACETIME

Consider a massive scalar field ϕ coupled arbitrarily to the gravitational background and described by the Lagrangian density

$$\pounds=\sqrt{-g}\left\{\frac{1}{2}[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-(m^2+\xi R)\phi^2]\right\} \tag{1}$$

with the energy-momentum tensor,

$$T_{\mu\nu}=\partial_\mu\phi\partial_\nu\phi-g_{\mu\nu}L, \tag{2}$$

where $L=(-g)^{-1/2}\pounds$. In the gravitationally coupled case,

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$$T_{\mu\nu}=(1-2\xi)\partial_\mu\phi\partial_\nu\phi+\left(2\xi-\frac{1}{2}\right)g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi$$
$$-2\xi\phi\nabla_\mu\nabla_\nu\phi+2\xi g_{\mu\nu}\phi\Box\phi-\xi G_{\mu\nu}\phi^2+\frac{m^2}{2}g_{\mu\nu}\phi^2. \tag{3}$$

Consider a (3+1)-dimensional Bianchi type-I spacetime which is spatially homogeneous with small anisotropy and has the line element

$$ds^2=dt^2-\sum_{i=1}^{3}a_i^2(t)(dx^i)^2 \tag{4}$$

as the background metric. Taking the conformal time transformation, $\partial t=C^{1/2}\partial\eta$ where $C=(a_1a_2a_3)^{2/3}$ and denoting $\partial\phi/\partial t=\dot{\phi}$, we can write the diagonal components of the stress-energy tensor:

$$T_{\eta\eta}=\frac{\dot{\phi}^2}{2C}-\left(2\xi-\frac{1}{2}\right)\left(\sum_{i=1}^{3}\frac{1}{a_i^2}(\partial_i\phi)^2\right)$$
$$+2\xi\frac{\dot{C}}{C^2}\phi\dot{\phi}+\frac{3\xi}{C}\left(\frac{\dot{C}^2}{C^2}+\kappa\right)\phi^2+\left(\frac{m^2}{2}\right)\phi^2 \tag{5}$$

and for $i=1,2,3$:

$$T_{ii}=(1-2\xi)(\partial_i\phi)^2$$
$$-\left(2\xi-\frac{1}{2}\right)\left[\frac{a_i^2}{C}\dot{\phi}^2-a_i^2\left(\sum_{j=1}^{3}\frac{1}{a_j^2}(\partial_j\phi)^2\right)\right]$$
$$+6\frac{\xi}{C}\left(\frac{\ddot{C}}{2C}-\frac{\dot{C}^2}{4C^2}+\kappa\right)\phi^2-a_i^2\left(\frac{m^2}{2}\right)\phi^2. \tag{6}$$

Considering the minimally coupled case, $\xi=0$, we get

$$T_{\eta\eta}=\frac{\dot{\phi}^2}{2C}+\frac{1}{2}\left(\sum_{i=1}^{3}\frac{1}{a_i^2}(\partial_i\phi)^2\right)+\left(\frac{m^2}{2}\right)\phi^2 \tag{7}$$

and

$$T_{ii}=(\partial_i\phi)^2+\frac{1}{2}\left[\frac{a_i^2}{C}\dot{\phi}^2-a_i^2\left(\sum_{j=1}^{3}\frac{1}{a_j^2}(\partial_j\phi)^2\right)\right]$$
$$-a_i^2\left(\frac{m^2}{2}\right)\phi^2. \tag{8}$$

Each mode of the quantized scalar field can be expanded in even and odd parity modes,

$$\phi(x)=(2\pi)^{-3/2}\sum_{\vec{k}}\left[q_{\vec{k}}(\eta)\cos\vec{k}\cdot\vec{x}+q_{-\vec{k}}(\eta)\sin\vec{k}\cdot\vec{x}\right]. \tag{9}$$

Since the background metric is spatially homogeneous we require the quantum state of the system to be also spatially homogeneous. Thus we need consider only the spatially homogeneous modes of the expressions in Eqs. (7) and (8). Substituting the above expression in Eqs. (7) and (8) and applying $(2\pi)^{-3/2}\int d^3x$ to the result yields the spatially averaged components

$$\overline{T}_{\eta\eta}=\frac{1}{32\pi^3C}\sum_{\vec{k}}\left[\left(\frac{\partial q_{\vec{k}}}{\partial\eta}\right)^2+\omega_{\vec{k}}^2(\eta)q_{\vec{k}}^2\right] \tag{10}$$

and

$$\overline{T}_{ii}=\frac{1}{32\pi^3C}a_i^2\sum_{\vec{k}}\left[\left(\frac{\partial q_{\vec{k}}}{\partial\eta}\right)^2+\left(\frac{2k_i^2}{a_i^2}C-\omega_{\vec{k}}^2(\eta)\right)q_{\vec{k}}^2\right], \tag{11}$$

where

$$\omega_{\vec{k}}^2(\eta)=C\left(\sum_{i=1}^{3}\frac{k_i^2}{a_i^2}+m^2\right) \tag{12}$$

and $\Sigma_{\vec{k}}$ extends over both even and odd parity modes.

III. STRESS-ENERGY TENSOR EXPECTATION VALUES IN COHERENT STATE

As an alternative to the N representation, we can construct an (over)complete normalized set $|\Gamma_{\vec{k}}\rangle$ of coherent state for each mode of the scalar field. The behavior of the classical scalar field near the cosmological singularity is best followed quantum mechanically by constructing such a representation. Coherent states are defined to be eigenstates of the annihilation operator

$$a_{\vec{k}}|\Gamma_{\vec{k}}\rangle=\Gamma_{\vec{k}}|\Gamma_{\vec{k}}\rangle, \tag{13}$$

where $\Gamma_{\vec{k}}$ is the time-dependent complex number and $a_{\vec{k}}$ is defined by

$$a_{\vec{k}}=-i\frac{d\beta_{\vec{k}}(\eta)}{d\eta}\hat{q}_{\vec{k}}+i\beta_{\vec{k}}(\eta)\hat{p}_{\vec{k}} \quad \text{where} \quad \hat{p}_{\vec{k}}=-i\partial/\partial\hat{q}_{\vec{k}}. \tag{14}$$

The complex function $\beta_{\vec{k}}(\eta)$ is a solution to the classical equation of motion corresponding to the Lagrangian density, Eq. (1), such that

$$\beta_{\vec{k}}^*\frac{d\beta_{\vec{k}}}{d\eta}-\beta_{\vec{k}}\frac{d\beta_{\vec{k}}^*}{d\eta}=i. \tag{15}$$

To completely fix the representation, a boundary condition must be imposed on $\beta_{\vec{k}}(\eta)$. In most models there exists a regime $\eta=\eta_{WKB}$ defined by the WKB condition

$$\omega_{\vec{k}}^{-1}\frac{d\omega_{\vec{k}}}{d\eta}\ll\omega_{\vec{k}} \tag{16}$$

in which we can require

$$\lim_{\eta->\eta_{WKB}}\beta_{\vec{k}}(\eta)=(2\omega_{\vec{k}})^{-1/2}\exp\left(i\int^{\eta}\omega_{\vec{k}}d\bar{\eta}\right) \tag{17}$$

and

$$\lim_{\eta->\eta_{WKB}}\frac{d\beta_{\vec{k}}(\eta)}{d\eta}=i\omega_{\vec{k}}\beta_{\vec{k}}(\eta). \tag{18}$$

Taking the expectation values of the diagonal components in the coherent state we get

$$\langle\overline{T}_{\eta\eta}\rangle_{cs}=\frac{1}{32\pi^3C}\sum_{\vec{k}}\left\{\begin{array}{l}\left[\left(\frac{d\beta_{\vec{k}}^{*}}{d\eta}\right)^2+\omega_{\vec{k}}^2(\eta)\beta_{\vec{k}}^{*2}\right]\Gamma_{\vec{k}}^2\\+\left[\left(\frac{d\beta_{\vec{k}}}{d\eta}\right)^2+\omega_{\vec{k}}^2(\eta)\beta_{\vec{k}}^2\right]\Gamma_{\vec{k}}^{*2}\\+\left[\left|\frac{d\beta_{\vec{k}}}{d\eta}\right|^2+\omega_{\vec{k}}^2(\eta)|\beta_{\vec{k}}|^2\right](2\Gamma_{\vec{k}}^2+1)\end{array}\right\} \tag{19}$$

and

$$\langle\overline{T}_{ii}\rangle_{cs}=\frac{1}{32\pi^3C}a_i^2\sum_{\vec{k}}\left\{\begin{array}{c}\left[\left(\frac{d\beta_{\vec{k}}^{*}}{d\eta}\right)^2+\left(\frac{2k_i^2}{a_i^2}C-\omega_{\vec{k}}^2(\eta)\right)\beta_{\vec{k}}^{*2}\right]\Gamma_{\vec{k}}^2\\+\left[\left(\frac{d\beta_{\vec{k}}}{d\eta}\right)^2+\left(\frac{2k_i^2}{a_i^2}C-\omega_{\vec{k}}^2(\eta)\right)\beta_{\vec{k}}^2\right]\Gamma_{\vec{k}}^{*2}\\+\left[\left|\frac{d\beta_{\vec{k}}}{d\eta}\right|^2+\left(\frac{2k_i^2}{a_i^2}C-\omega_{\vec{k}}^2(\eta)\right)|\beta_{\vec{k}}|^2\right](2\Gamma_{\vec{k}}^2+1)\end{array}\right\}. \tag{20}$$

IV. THE DENSITY MATRIX AND $\langle T_{\mu\nu}^{\vec{k}}\rangle$

The coherent state for the scalar field is the product over modes of the coherent state for each mode. We assume the modes to be noninteracting so that the density matrix for the field is just the product of the density matrices for each mode. Thus we find a density matrix

$$\rho=\int\left(\Pi_{\vec{k}}\frac{d^2\Gamma_{\vec{k}}}{\pi\langle n_{\vec{k}}\rangle}\right)\exp\left(-\sum_{\vec{k}}\frac{|\Gamma_{\vec{k}}|^2}{\langle n_{\vec{k}}\rangle}\right)\times|\{\Gamma_{\vec{k}}\}\rangle\langle\{\Gamma_{\vec{k}}\}|, \tag{21}$$

where

$$|\{\Gamma_{\vec{k}}\}\rangle\equiv\Pi_{\vec{k}}|\Gamma_{\vec{k}}\rangle.$$

The density matrix given by Eq. (21) may be used to evaluate expectation values through $\langle A\rangle=\mathrm{tr}(\rho A)$, where $\langle A\rangle$ is the expectation value of any operator A. Using the density matrix, the stress-tensor expectation values are evaluated as

$$\langle T_{\mu\nu}^{\vec{k}}\rangle=\mathrm{tr}(T_{\mu\nu}^{\vec{k}}\rho_{\vec{k}})=\int d^2\Gamma_{\vec{k}}|\alpha(\Gamma_{\vec{k}})|^2\langle\Gamma_{\vec{k}}|T_{\mu\nu}^{\vec{k}}|\Gamma_{\vec{k}}\rangle, \tag{22}$$

where

$$|\alpha(\Gamma_{\vec{k}})|^2=\frac{1}{\pi\langle n_{\vec{k}}\rangle}e^{-|\Gamma_{\vec{k}}|^2/\langle n_{\vec{k}}\rangle}. \tag{23}$$

Thus

$$\langle T_{\eta\eta}^{\vec{k}}\rangle=\frac{1}{32\pi^3C}(2\langle n_{\vec{k}}\rangle+1)\left[\left|\frac{d\beta_{\vec{k}}}{d\eta}\right|^2+\omega_{\vec{k}}^2(\eta)|\beta_{\vec{k}}|^2\right] \tag{24}$$

and

$$\langle T_{ii}^{\vec{k}}\rangle=\frac{a_i^2}{32\pi^3C}(2\langle n_{\vec{k}}\rangle+1)\times\left[\left|\frac{d\beta_{\vec{k}}}{d\eta}\right|^2+\left(\frac{2k_i^2}{a_i^2}C-\omega_{\vec{k}}^2(\eta)\right)|\beta_{\vec{k}}|^2\right]. \tag{25}$$

The interpretation of $\langle n_{\vec{k}}\rangle$ as a particle number is valid only in a WKB regime defined by $\omega_{\vec{k}}^{-1}(d\omega_{\vec{k}}/d\eta)\ll\omega_{\vec{k}}$. This condition will be valid for modes with wavelength smaller than the Hubble radius (oscillation period much less than the expansion time scale). In such a regime we require the WKB limit Eq. (17) for $\beta_{\vec{k}}$. Evaluating Eqs. (24) and (25) in the

WKB limit we get a simplified expression for the diagonal components of the expectation value of the energy-momentum tensor,

$$\lim_{\eta \to \eta_{WKB}} \langle T^{\vec{k}}_{\eta\eta} \rangle = \frac{1}{16\pi^3 C} (\langle n_{\vec{k}} \rangle + 1/2) \omega_{\vec{k}}(\eta) \tag{26}$$

and

$$\lim_{\eta \to \eta_{WKB}} \langle T^{\vec{k}}_{ii} \rangle = \frac{k_i^2}{16\pi^3 \omega_{\vec{k}}} (\langle n_{\vec{k}} \rangle + 1/2). \tag{27}$$

Using the definition Eq. (12) of $\omega_{\vec{k}}$ and the metric (4) it is clear that the trace of $\langle T^{\vec{k}}_{\mu\nu} \rangle$ is formally zero for a massless scalar field. Regularization of the vacuum stress-energy term may yield a trace anomaly.

V. ENERGY DENSITY AND PRESSURE ASSOCIATED WITH THE PERTURBATIONS

The vanishing of the nondiagonal terms of the expectation values of the components of $T_{\mu\nu}$ allows us to treat the scalar field in complete analogy to a perfect fluid. The similarity of the gravitational instabilities of a free scalar field and dust-like matter was pointed out by Turner [25]. Consider the relation [1,2]

$$\langle T^{\mu\nu} \rangle = (\rho_1 - p_1) u^\mu u^\nu - p_1 g^{\mu\nu}, \tag{28}$$

where $u^\mu = (1,0,0,0)$ and p_1 and ρ_1 are the first order fluctuation amplitudes of the corresponding quantities. The energy density and pressure associated with the perturbation are

$$\rho = \langle T^0_0 \rangle \quad \text{and} \quad p = -1/3 \langle T^i_i \rangle. \tag{29}$$

From the definition of the sound velocity of adiabatic perturbations,

$$v_s^2 = \frac{p_1}{\rho_1} = \frac{k_i^2}{3a_i^2 \left(\sum_{i=1}^{3} \frac{k_i^2}{a_i^2} + m^2 \right)}. \tag{30}$$

In a nonrelativistic regime $k_i / a_i \ll m$ and we can write

$$v_s^2 = \frac{1}{3} \frac{k_i^2}{a_i^2 m^2}. \tag{31}$$

For the Bianchi type-I spacetime with scale factors a_i let $V = a_1 a_2 a_3$ be the "volume scale factor" [14]. Then the mean scale factor $\overline{a} \propto V^{1/3}$ and $C \propto \overline{a}^2$. Let us put $\overline{a}$ instead of a_i in Eq. (31). Then,

$$v_s^2 = \frac{1}{3} \frac{k_i^2}{\overline{a}^2 m^2}. \tag{32}$$

Rewriting the metric for the Bianchi type-I spacetime, in Eq. (4), using spherical polar coordinates,

$$\begin{aligned} ds^2 = c^2 dt^2 - R_1^2(t) \frac{dr^2}{(1 - kr^2)} - R_2^2(t) r^2 d\theta^2 \\ - R_3^2(t) r^2 \sin^2\theta d\phi^2 \end{aligned} \tag{33}$$

with

$$\begin{aligned} \frac{R_1^2(t)}{(1-kr^2)} &= a_1^2(t), \\ R_2^2(t) r^2 &= a_2^2(t), \\ R_3^2(t) r^2 \sin^2\theta &= a_3^2(t). \end{aligned} \tag{34}$$

Taking $c = 1$ we get

$$\begin{aligned} R^0{}_0 &= \frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\ddot{R}_3}{R_3}, \\ R^1{}_1 &= \frac{\ddot{R}_1}{R_1} + \frac{\dot{R}_1 \dot{R}_2}{R_1 R_2} + \frac{\dot{R}_1 \dot{R}_3}{R_1 R_3} + \frac{2k}{R_1^2}, \\ R^2{}_2 &= \frac{\ddot{R}_2}{R_2} + \frac{\dot{R}_1 \dot{R}_2}{R_1 R_2} + \frac{\dot{R}_2 \dot{R}_3}{R_2 R_3} + \frac{2k}{R_1^2} - \frac{1}{r^2}\left[\frac{1}{R_1^2} - \frac{1}{R_2^2}\right], \\ R^3{}_3 &= \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_1 \dot{R}_3}{R_1 R_3} + \frac{\dot{R}_2 \dot{R}_3}{R_2 R_3} + \frac{2k}{R_1^2} - \frac{1}{r^2}\left[\frac{1}{R_1^2} - \frac{1}{R_2^2}\right], \\ R &= 2\left[\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\ddot{R}_3}{R_3}\right] + 2\left[\frac{\dot{R}_1 \dot{R}_2}{R_1 R_2} + \frac{\dot{R}_1 \dot{R}_3}{R_1 R_3} + \frac{\dot{R}_2 \dot{R}_3}{R_2 R_3}\right] \\ &\quad + \frac{6k}{R_1^2} - \frac{2}{r^2}\left[\frac{1}{R_1^2} - \frac{1}{R_2^2}\right]. \end{aligned} \tag{35}$$

The Einstein equation $G^\mu_\nu = k T^\mu_\nu$ with $k = 8\pi$ and $T^\mu_\nu = \mathrm{diag}(\rho, -p_r, -p_\theta, -p_\phi)$ then yields the set of equations

$$\begin{aligned} 8\pi\rho &= -\left[\frac{\dot{R}_1 \dot{R}_2}{R_1 R_2} + \frac{\dot{R}_1 \dot{R}_3}{R_1 R_3} + \frac{\dot{R}_2 \dot{R}_3}{R_2 R_3}\right] - \frac{3k}{R_1^2} + \frac{1}{r^2}\left[\frac{1}{R_1^2} - \frac{1}{R_2^2}\right], \\ 8\pi p_r &= -\left[\frac{\ddot{R}_2}{R_2} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_2 \dot{R}_3}{R_2 R_3} + \frac{k}{R_1^2}\right] + \frac{1}{r^2}\left[\frac{1}{R_1^2} - \frac{1}{R_2^2}\right], \\ 8\pi p_\theta &= -\left[\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_1 \dot{R}_3}{R_1 R_3} + \frac{k}{R_1^2}\right], \\ 8\pi p_\phi &= -\left[\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\dot{R}_1 \dot{R}_2}{R_1 R_2} + \frac{k}{R_1^2}\right]. \end{aligned} \tag{36}$$

Let $\overline{R}$ be the mean scale factor of the Bianchi type-I universe with metric in Eq. (33). Then

$$H=\frac{\dot{\bar{R}}}{\bar{R}} \quad \text{and} \quad \frac{\ddot{\bar{R}}}{\bar{R}}=-q(t)[H(t)]^2 \tag{37}$$

and Eq. (36) becomes

$$8\pi\rho=-3\left[\frac{\dot{\bar{R}}^2}{\bar{R}}+\frac{k}{\bar{R}^2}\right], \tag{38}$$

$$8\pi p=-\left[2\frac{\ddot{\bar{R}}}{\bar{R}}+\frac{\dot{\bar{R}}^2}{\bar{R}}+\frac{k}{\bar{R}^2}\right].$$

For a system behaving like dust [2] $p=0$ and $\rho=\rho_0(R_0^3/R^3)$ where the suffix ''0'' denotes the corresponding quantity in the present epoch. Using the above equations we can write

$$\rho=\frac{3}{8\pi G}\left(H^2+\frac{k}{\bar{R}^2}\right), \tag{39}$$

where $3k/8\pi G\bar{R}^2$ is the contribution due to anisotropy, ρ_{AN} [14].

VI. JEANS MASS

Let us consider the simple case with $a_1=a_2=a_3=\bar{a}$ and $k=0$. Then the equation that tells us how or whether gravitational instability leads to the growth of condensation in the expanding universe [1–4] is

$$\ddot{\delta}+2\frac{\dot{\bar{a}}}{\bar{a}}\dot{\delta}+\left(\frac{v_s^2k^2}{\bar{a}^2}-4\pi G\rho\right)\delta=0, \tag{40}$$

where $\delta=\rho_1/\rho$, the density contrast parameter.

During most phases of the expansion of the universe, we can approximate the expansion factor by $\bar{a}(t)\propto t^n$ with a suitable n which is less than unity [3]. For the matter-dominated case, the scale factor $\bar{a}(t)\propto t^{2/3}$ and for the spatially flat $k=0$ case,

$$\rho=\frac{1}{6\pi G t^2}. \tag{41}$$

For a general specific heat ratio γ, pressure varies as p^{γ} and the speed of sound is

$$v_s=\left(\frac{\gamma p}{\rho}\right)^{1/2}=\left(\frac{\gamma\rho^{\gamma}}{\rho}\right)^{1/2}\propto\rho\,\frac{\gamma-1}{2}. \tag{42}$$

But Eq. (41) implies that

$$t^{-2}\propto\rho. \tag{43}$$

Equations (42) and (43) show that

$$v_s\propto t^{1-\gamma}. \tag{44}$$

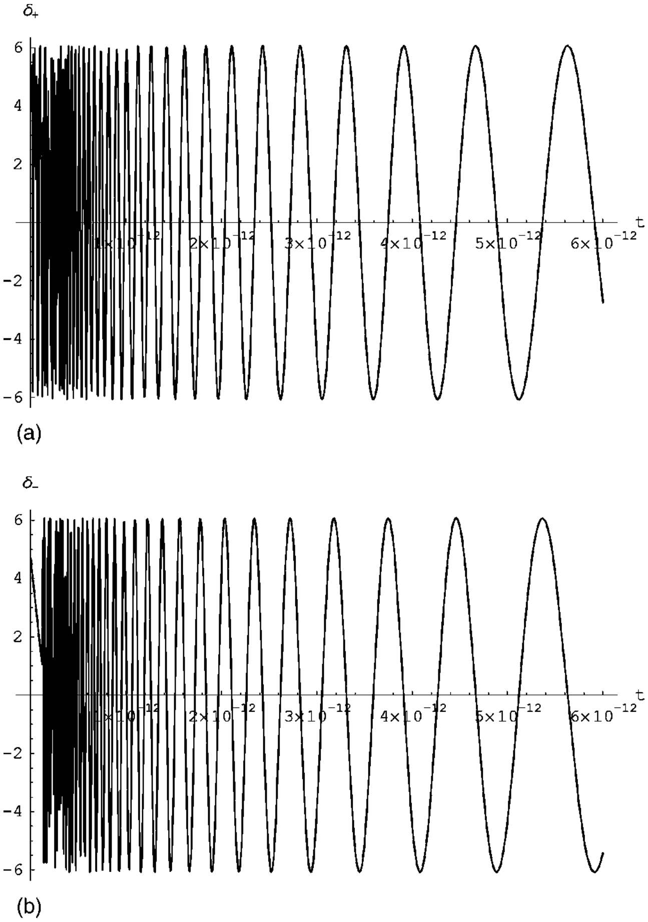

FIG. 1. (a) Oscillating mode δ_+. (b) Oscillating mode δ_-.

Equation (40) takes the form

$$\ddot{\delta}+\frac{4}{3t}\dot{\delta}+\left(\frac{\Lambda^2}{t^{2\gamma-2/3}}-\frac{2}{3t^2}\right)\delta=0, \tag{45}$$

where

$$\Lambda^2=t^{2\gamma-2/3}\frac{v_s^2k^2}{\bar{a}^2} \tag{46}$$

and γ is the specific heat ratio.

The solution of Eq. (45) for $\gamma>\frac{4}{3}$ is

$$\delta_{\pm}\propto t^{-1/6}J_{\mp 5/6\nu}\left(\frac{\Lambda t^{-\nu}}{\nu}\right), \tag{47}$$

where $J_n(x)$ are Bessel functions of the first kind and $\nu=\gamma-\frac{4}{3}>0$. The Bessel function $J_n(x)$ oscillates for $x\gg 1$ as shown Figs. 1(a) and 1(b). For $x<1$, the solutions δ_+ and δ_- behave as in Figs. 2(a) and 2(b). Both the growing as well as damped modes are present. It is evident from Fig. 2 that the growing modes dominate over the decaying modes.

The critical condition $x=1$ gives

$$t^{1/3}\approx\Lambda. \tag{48}$$

Equations (46) and (48) together imply that

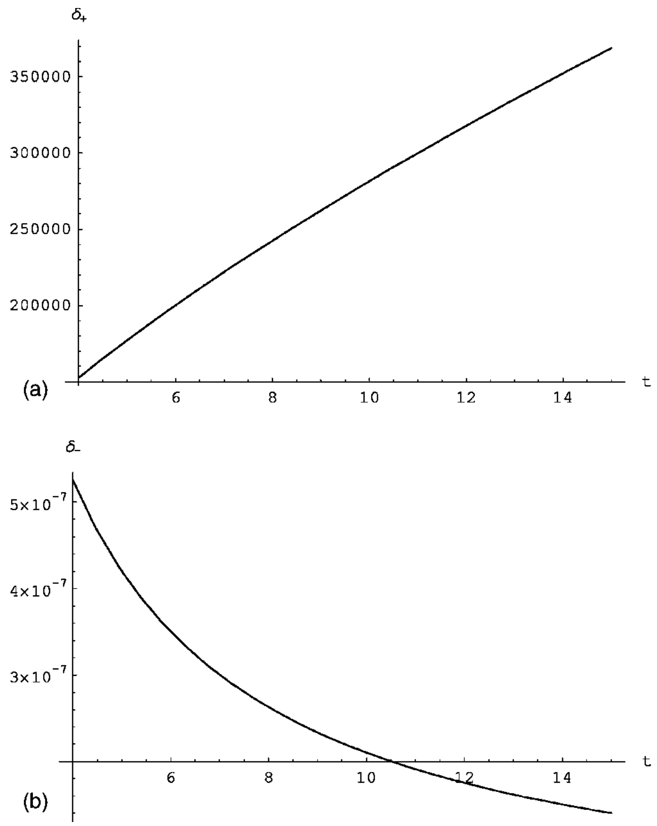

FIG. 2. (a) Growing mode δ_+. (b) Decaying mode δ_-.

$$t^{-2}\sim 6\pi G\rho\sim\frac{v_s^2k^2}{\bar{a}^2} \tag{49}$$

which corresponds to the classical Jeans criterion. Substituting the expression for sound velocity in Eq. (49) we get the classical Jeans length for the perturbations as

$$\lambda_J=2\pi/K_J \quad \text{where} \quad K_J^2\sim m\sqrt{\pi G\rho}, \tag{50}$$

where $K=k/\bar{a}$. Perturbations for which the wave number is smaller than the Jeans wave number can grow to form different structures in the universe. The instability growth rate monotonically falls off when K^2 increases from 0 to K_J^2. The above expression for the Jeans wave number K_J well agrees with the results of similar calculations [6,24,27]. The Jeans mass for the perturbations is then given by

$$M_J=\frac{4}{3}\pi\rho\left(\frac{2\pi}{K_J}\right)^3=\frac{32}{3}\pi^{13/4}\rho^{1/4}\left(\frac{1}{m\sqrt{G}}\right)^{3/2} \tag{51}$$

$$=\frac{32}{3}\pi^{13/4}\rho^{1/4}\left(\frac{m_{pl}}{m}\right)^{3/2}=10^2\rho^{1/4}\left(\frac{m_{pl}}{m}\right)^{3/2}.$$

The fluctuations will have a chance to grow under its self-gravitation if the mass is greater than M_J. This implies that the perturbation for which the mass of the fluctuating matter is greater than M_J, may grow under its self-gravitation to form a galaxy.

VII. DISCUSSIONS AND CONCLUSIONS

The theory of gravitational perturbations in an expanding universe is used to describe the growth of structure in the universe. Khlopov and co-workers have discussed the gravitational instability of the free scalar field and the Jeans wave number is obtained from the solution of the dispersion relation [18]. Jetzer and Scialom have obtained the expression for the Jeans wave number starting from the general relativistic equations and solving the dispersion relation [27]. In the present work the same result is obtained by a different approach. The scalar field approach to the Jeans mass calculation is discussed. The application of the classical Jeans theory to the scalar field is conditioned by the vanishing of the expectation values of the nondiagonal components of the energy-momentum tensor. The scalar field is treated in complete analogy to a perfect fluid and the energy density and pressure associated with the gravitational perturbations are evaluated. The exact expressions of the Jeans length and the Jeans mass for the perturbations are obtained.

Jeans considered the problem of the formation of galaxies in the universe as a process involving the interplay of gravitational attraction and the pressure force acting on a mass of nonrelativistic fluid. So long as the pressure forces are negligible an overdense region is expected to accrete material from its surroundings by the gravitational attraction and thus becomes even more dense. The denser it becomes the more it will accrete, resulting in an instability which can ultimately cause the collapse of a fluctuation to a gravitationally bound object. The instability is the first step to an understanding of where the structure in the galaxy distribution came from; it grew by gravity out of smaller structures that existed earlier. The knowledge of Jeans wavelength $\lambda_J=2\pi/K_J$ provides an estimate of the size of the objects which can be formed by gravitational collapse. The present calculations show that the perturbations for which the mass of the fluctuating matter is greater than the Jeans mass M_J given by Eq. (51) have the prospects to grow under its self-gravitation to form different structures.

Briefly, quantum fluctuations in an expanding universe can lead to energy density perturbations. It is usually assumed that there exist small primordial perturbations which slowly increase in amplitude due to the gravitational instability to form the structures we observe at the present time on the scales of galaxies and galaxy clusters. The simple criterion needed to decide whether a fluctuation will grow with time is that the typical length scale of a fluctuation should be greater than the Jeans length λ_J for the fluid.

ACKNOWLEDGMENTS

The authors would like to thank Professor Varun Sahni and Professor Daniel Sudarsky for useful discussions. One of us would like to thank UGC, N. Delhi for financial support. V.C.K. acknowledges the Associateship of IUCAA, Pune.

[1] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (Wiley, New York, 1972).
[2] J.V. Narlikar, *Introduction to Cosmology* (Cambridge University Press, Cambridge, England, 1993).
[3] T. Padmanabhan, *Structure Formation in the Universe* (Cambridge University Press, Cambridge, England, 1993).
[4] P.J.E. Peebles, *Principles of Physical Cosmology* (Princeton University Press, Princeton, NJ, 1993).
[5] J.A. Peacock, *Cosmological Physics* (Cambridge University Press, Cambridge, England, 1999).
[6] M.Yu. Khlopov, B.A. Malomed, and Ya.B. Zeldovich, Mon. Not. R. Astron. Soc. **215**, 575 (1985).
[7] P. Coles, *The Routledge Critical Dictionary of the New Cosmology* (Routledge Inc., New York, 1999).
[8] J.H. Jeans, *Astronomy and Cosmology* (Cambridge University Press, Cambridge, England, 1929).
[9] W.B. Bonnor, Mon. Not. R. Astron. Soc. **117**, 104 (1957).
[10] H. Ford and D.J. Toms, Phys. Rev. D **25**, 1510 (1982).
[11] B.L. Hu and Y. Zhang, Phys. Rev. D **37**, 2151 (1988).
[12] L. Dolan and R. Jackiw, Phys. Rev. D **9**, 2904 (1974).
[13] A.D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood Academic Publishers, Chur, Switzerland, 1990).
[14] E.W. Kolb and M.S. Turner, *The Early Universe* (Addison-Wesley, Reading, MA, 1990).
[15] I.L. Buchbinder and S.D. Odintsov, Class. Quantum Grav. **2**, 721 (1985).
[16] G. Congola and I.L. Shapiro, Class. Quantum Grav. **15**, 787 (1998).
[17] M. Joy and V.C. Kuriakose, Phys. Rev. D **62**, 104017 (2000).
[18] V.F. Mukhanov, H.A. Feldman, and R.H. Brandenberger, Phys. Rep. **215**, 203 (1992).
[19] J. Bardeen, P. Steinhardt, and M.S. Turner, Phys. Rev. D **28**, 679 (1983).
[20] R.H. Brandenberger, Rev. Mod. Phys. **57**, 1 (1985).
[21] P.G. Ferreira and M. Joyce, Phys. Rev. Lett. **79**, 4740 (1997).
[22] A. Krasinski and C. Hellaby, Phys. Rev. D **65**, 023501 (2002).
[23] C. Wetterich, astro-ph/0111166.
[24] M. Bianchi, D. Grasso, and R. Ruffini, Astron. Astrophys. **231**, 301 (1990).
[25] M.S. Turner, Phys. Rev. D **28**, 1243 (1983).
[26] B.K. Berger, Phys. Rev. D **12**, 368 (1975).
[27] P. Jetzer and D. Scialom, Phys. Rev. D **55**, 7440 (1997).