Geodesically complete nondiagonal inhomogeneous cosmological solutions in dilatonic gravity with a stiff perfect fluid

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New nondiagonal G_2 inhomogeneous cosmological solutions are presented in a wide range of scalar-tensor theories with a stiff perfect fluid as a matter source. The solutions have no big-bang singularity or any other curvature singularities. The dilaton field and the fluid energy density are regular everywhere, too. The geodesic completeness of the solutions is investigated.

All versions of string theory and higher dimensional gravity theories predict the existence of the dilaton field which determines the gravitational "constant" as a variable quantity. The existence of a scalar partner of the tensor graviton may have a serious influence on the space-time structure and important consequences for cosmology and astrophysics. A large amount of research has been done in order to unveil the possible cosmological significance of the dilaton ([1–14], and references therein). With a few exceptions most of the cosmological studies within the scalar-tensor theories were devoted to the homogeneous case. The homogeneous models are good approximations of the present universe. There is, however, no reason to assume that such a regular expansion is also suitable for a description of the early universe. Moreover, as is well known, the present universe is not exactly spacially homogeneous. That is why it is necessary to study inhomogeneous cosmological models. They allow us to investigate a number of long standing questions regarding the occurrence of singularities, the behavior of the solutions in the vicinity of a singularity, and the possibility of our universe arising from generic initial data.

In this work we shall address the question of the occurrence of singularities in inhomogeneous cosmologies within the framework of scalar-tensor theories. As is well known, most of the homogeneous models (both in general relativity and in scalar-tensor theories) predict a universal spacelike big-bang singularity in a finite past. It was, therefore, believed that this would be the usual singularity in general. The inclusion of inhomogeneities drastically changes this point of view. There are inhomogeneous cosmological solutions in general relativity which have no big-bang or any other curvature singularity. The first such solution was discovered by Senovilla in 1990 [15]. Senovilla's solution represents a cylindrically symmetric universe filled with radiation. This solution has a diagonal metric and is also globally hyperbolic and geodesically complete [16]. Senovilla's solution was generalized by Ruiz and Senovilla in Ref. [17] where a large family of singularity-free diagonal G_2 inhomogeneous perfect fluid solutions was found. Nonsingular diagonal inhomogeneous solutions in general relativity describing cylindrically symmetric universes filled with stiff perfect fluid were

found by Patel and Dadhich in Ref. [18]. Other examples of diagonal nonsingular solutions in general relativity can be found in Refs. [19–25].

In Ref. [26], Mars found the first nondiagonal G_2 inhomogeneous cosmological solution of the Einstein equations with stiff perfect fluid as a source. This solution is globally hyperbolic and geodesically complete. Mars's solution was generalized by Griffiths and Bicak in Ref. [27].

Within the framework of scalar-tensor theories there are also inhomogeneous cosmological solutions without big-bang or any other curvature singularity. In Ref. [28], Giovannini derived gravi-dilaton inhomogeneous cosmological solutions with everywhere regular curvature invariants and bounded dilaton in tree-level dilaton driven models. In a subsequent paper [29], it was shown that these solutions describe singularity-free dilaton driven cosmologies. A nondiagonal inhomogeneous cosmological solution with regular curvature invariants and unbounded dilaton in the tree level effective string models was found by Pimentel [30]. Very recently, inhomogeneous cosmological solutions without any curvature singularities were obtained by the author in a wide class of scalar-tensor theories with stiff perfect fluid as a source [31].

In this work we take a further step upwards and present new nondiagonal G_2 inhomogeneous cosmological stiff perfect fluid solutions with no curvature singularities in a wide range of scalar-tensor theories. Scalar-tensor theories (without a cosmological potential) are described by the following action in Jordan (string) frame [32,33]:

$$S = \frac{1}{16\pi G_*} \int d^4x \sqrt{-g}(F(\Phi)R - Z(\Phi)g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi)$$
$$+ S_m[\Psi_m; g_{\mu\nu}]. \tag{1}$$

Here, G_* is the bare gravitational constant and R is the Ricci scalar curvature with respect to the space-time metric $g_{\mu\nu}$. The dynamics of the scalar field Φ depends on the functions $F(\Phi)$ and $Z(\Phi)$. In order for the gravitons to carry positive energy the function $F(\Phi)$ must be positive. The nonnegativity of the energy of the dilaton requires that $2F(\Phi)Z(\Phi) + 3[dF(\Phi)/d\Phi]^2 \geq 0$. The action of matter depends on the material fields Ψ_m and the space-time metric $g_{\mu\nu}$ but does not involve the scalar field Φ in order for the weak equivalence principle to be satisfied.

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As a matter source we consider a stiff perfect fluid with equation of state $p = \rho$.

The general form of the solutions is given by

$$ds^2 = F^{-1}(\Phi(t)) \Big[e^{\gamma a^2 r^2} \cosh(2at)(-dt^2 + dr^2)$$

$$+ r^2 \cosh(2at) d\phi^2 + \frac{1}{\cosh(2at)} (dz + ar^2 d\phi)^2 \Big],$$

$$8\pi G_* \rho = f(\lambda) \frac{a^2(\gamma-1) F^3[\Phi(t)] e^{-\gamma a^2 r^2}}{\cosh(2at)}, \tag{2}$$

$$u_\mu = F^{-1/2}(\Phi(t)) e^{(1/2)\gamma a^2 r^2} \cosh^{1/2}(2at) \delta^0_\mu.$$

The solution depends on three parameters — a, γ ($\gamma > 1$) and λ. The range of the coordinates is

$$-\infty < t, z < \infty, \quad 0 \leq r < \infty, \quad 0 \leq \phi \leq 2\pi. \tag{3}$$

The explicit form of the functions $\Phi(t)$ and $f(\lambda)$, and the range of the parameter λ depend on the particular scalar tensor theory. These solutions can be generated[1] from the general relativistic Mars's solution [26] using the solution generating methods developed in Ref. [31]. Below we consider the explicit form of the general solution for some particular scalar-tensor theories.

Barker's theory. Barker's theory is described by the functions $F(\Phi) = \Phi$ and $Z(\Phi) = (4 - 3\Phi)/2\Phi(\Phi-1)$.

In the case of Barker's theory the explicit forms of the functions $\Phi(t)$ and $f(\lambda)$ are

$$\Phi^{-1}(t) = 1 - \lambda \cos^2(a\sqrt{\gamma-1} t), \tag{4}$$

$$f(\lambda) = 1 - \lambda, \tag{5}$$

where the range of λ is $0 < \lambda < 1$. This range can be extended to $0 \leq \lambda \leq 1$. For $\lambda = 0$ and $\lambda = 1$ we obtain the Mars's solution and gravi-dilaton vacuum solution, respectively. That is why we consider only $0 < \lambda < 1$. It should be noted that the range of the parameter λ is crucial for the curvature invariants. It is easy to see that the gravi-dilaton vacuum solution corresponding to $\lambda = 1$ has divergent curvature invariants because of the conformal factor $\Phi^{-1}(t) = \sin^2(a\sqrt{\gamma-1} t)$.

Brans-Dicke theory. Brans-Dicke theory is described by the functions $F(\Phi) = \Phi$ and $Z(\Phi) = \omega/\Phi$, where ω is a constant parameter. Here we consider the case $\omega > -3/2$. The explicit form of the functions $\Phi(t)$ and $f(\lambda)$ in the Brans-Dicke case is the following:

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[1]Some of the solutions were first obtained by solving the corresponding system of partial differential equations for nondiagonal G_2 cosmologies [34].

$$\Phi^{-1/2}(t) = \lambda \exp\left(a \sqrt{\frac{\gamma-1}{3+2\omega}} t \right) + (1-\lambda)$$

$$\times \exp\left(-a \sqrt{\frac{\gamma-1}{3+2\omega}} t \right), \tag{6}$$

$$f(\lambda) = 4\lambda(1-\lambda). \tag{7}$$

Here the range of the parameter λ is $0 < \lambda < 1$. The solution exists for $\lambda = 0$ and $\lambda = 1$, too. In these cases, however, we obtain a gravi-dilaton vacuum solution which is just Pimentel's solution [30]. That is the reason we do not consider these limiting values of λ. The solution is invariant under the transformations $\lambda \leftrightarrow 1 - \lambda$ and $t \leftrightarrow -t$. In this generalized sense, we can say that the solution is even in time.

Theory with "conformal" coupling. The theory with "conformal" coupling is described by the functions $F(\Phi) = 1 - \frac{1}{6}\Phi^2$ and $Z(\Phi) = 1$. In this case we have

$$F^{-1}[\Phi(t)] = 1 + 4\lambda(1-\lambda) \sinh^2\left(a \sqrt{\frac{\gamma-1}{3}} t \right), \tag{8}$$

$$f(\lambda) = (1-2\lambda)^2. \tag{9}$$

The range of the parameter λ is $0 < \lambda \leq 1/2$. For $\lambda = 1/2$ we obtain a gravi-dilaton vacuum solution which is well behaved and can be included as a limiting case.

$Z(\Phi) = (\Omega^2 - 3\Phi)/2\Phi^2$ theory. Here we consider the scalar-tensor theory described by the functions $F(\Phi) = \Phi$ and $Z(\Phi) = (\Omega^2 - 3\Phi)/2\Phi^2$ where $\Omega > 0$. The explicit forms of $\Phi(t)$ and $f(\lambda)$ are

$$\Phi^{-1}(t) = \left(1 + \frac{1}{\Omega} a\sqrt{\gamma-1} t \right)^2 + \lambda, \tag{10}$$

$$f(\lambda) = \lambda. \tag{11}$$

Here, the range of the parameter is $0 < \lambda < \infty$.

$Z(\Phi) = \frac{1}{2}(\Phi^2 - 3\Phi + 3)/\Phi(\Phi-1)$ theory. The theory with $F(\Phi) = \Phi$ and $Z(\Phi) = \frac{1}{2}(\Phi^2 - 3\Phi + 3)/\Phi(\Phi-1)$ possesses the following solution:

$$\Phi^{-1}(t) = \frac{\lambda^2}{\lambda^2 + (1-\lambda^2)\sin^2(\lambda a\sqrt{\gamma-1} t)}, \tag{12}$$

$$f(\lambda) = \lambda^2. \tag{13}$$

In order for the dilaton field in this solution to have positive energy we should restrict the range of the parameter λ to $0 < \lambda < 1$.

Using the solution generating methods developed in Ref. [31] we can generate nondiagonal G_2 inhomogeneous cosmological solutions in many other scalar-tensor theories different from those considered above. However, the solutions we have presented here are expressed in a closed analytic form and they are also representative and cover a wide range of the possible behaviors of the scalar-tensor solutions which can be generated from Mars's solution.

Let us consider the main properties of the found solutions. The metric functions, the gravitational scalar (the dilaton), and the fluid energy density are everywhere regular. The space-times described by our solutions have no big-bang nor any other curvature singularity—the curvature invariants $I_1 = C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta}$, $I_2 = R_{\mu\nu}R^{\mu\nu}$, and $I_3 = R^2$ are regular everywhere. The solution possesses a two-dimensional Abelian group of isometries inherited from the seed Mars's solution and generated by the Killing vectors $\partial/\partial z$ and $\partial/\partial\phi$. In addition, the metrics have a well defined axis of symmetry and the elementary flatness condition [35] is satisfied. Since the presented solutions are conformally related to the Mars's solution, the spacetimes described by them are globally hyperbolic. In fact, the global hyperbolicity can be proved independently as a consequence of the proof of the geodesic completeness presented below.

The existence of two Killing vectors gives rise to two constants of motion along the geodesics

$$K = F^{-1}[\Phi(t)]\left[\cosh(2at)r^2\frac{d\phi}{ds}\right.$$
$$\left.+\frac{ar^2}{\cosh(2at)}\left(\frac{dz}{ds}+ar^2\frac{d\phi}{ds}\right)\right], \quad (14)$$

$$L = \frac{F^{-1}[\Phi(t)]}{\cosh(2at)}\left(\frac{dz}{ds}+ar^2\frac{d\phi}{ds}\right). \quad$$

The affinely parametrized causal geodesics satisfy

$$F^{-1}[\Phi(t)]\left\{e^{\gamma a^2 r^2}\cosh(2at)\left[\left(\frac{dt}{ds}\right)^2-\left(\frac{dr}{ds}\right)^2\right]\right.$$
$$\left.-\frac{L^2\cosh(2at)}{F^{-2}[\Phi(t)]}-\frac{(K-Lar^2)^2}{r^2F^{-2}[\Phi(t)]\cosh(2at)}\right\} = \epsilon, \quad$$
$$(15)$$

where $\epsilon = 0$ and 1 for null and timelike geodesics, respectively. Taking into account Eqs. (14) and (15) the geodesic equations for t and r can be written in the following form:

$$\frac{d}{ds}\left(F^{-1}[\Phi(t)]e^{\gamma a^2 r^2}\cosh(2at)\frac{dt}{ds}\right)$$
$$= F[\Phi(t)]e^{-\gamma a^2 r^2}\cosh^{-1}(2at)M(t,r)\partial_t M(t,r), \quad (16)$$

$$\frac{d}{ds}\left(F^{-1}[\Phi(t)]e^{\gamma a^2 r^2}\cosh(2at)\frac{dr}{ds}\right)$$
$$= -F[\Phi(t)]e^{-\gamma a^2 r^2}\cosh^{-1}(2at)M(t,r)\partial_r M(t,r), \quad$$
$$(17)$$

where

$$M(t,r) = F^{-1/2}[\Phi(t)]e^{(1/2)\gamma a^2 r^2}\cosh^{1/2}(2at)$$
$$\times\left[\epsilon+\frac{L^2\cosh(2at)}{F^{-1}[\Phi(t)]}\right.$$
$$\left.+\frac{(K-Lar^2)^2}{r^2F^{-1}[\Phi(t)]\cosh(2at)}\right]^{1/2}. \quad (18)$$

To demonstrate the geodesic completeness of our metric, we have to show that all nonspacelike (i.e., causal) geodesics can be extended to arbitrary values of the affine parameter. We shall consider only future directed geodesics. The past directed geodesics can be treated analogously.

First we consider null geodesics with $K = L = 0$. For them we have $dt/ds = |dr/ds|$ and

$$\frac{d}{ds}\left(F^{-1}[\Phi(t)]e^{\gamma a^2 r^2}\cosh(2at)\frac{dt}{ds}\right) = 0. \quad (19)$$

After integrating we obtain

$$\frac{dt}{ds} = C\{F^{-1}[\Phi(t)]e^{\gamma a^2 r^2}\cosh(2at)\}^{-1}, \quad (20)$$

where $C > 0$ is a constant. Taking into account that for each of our solutions there exists a constant B such that

$$0 < B \leq F^{-1}[\Phi(t)] \quad (21)$$

for arbitrary values of t and fixed parameter λ, we obtain

$$\frac{dt}{ds} = \left|\frac{dr}{ds}\right| \leq \frac{C}{B}. \quad (22)$$

Therefore the geodesics under consideration are complete.

Now let us turn to the general case when at least one of the constants ϵ, K or L is different from zero. Here we shall use a method similar to that for diagonal metrics described in Ref. [16]. Let us parametrize dt/ds and dr/ds by writing

$$\frac{dt}{ds} = \frac{F[\Phi(t)]e^{-\gamma a^2 r^2}}{\cosh(2at)}M(t,r)\cosh(v), \quad (23)$$

$$\frac{dr}{ds} = \frac{F[\Phi(t)]e^{-\gamma a^2 r^2}}{\cosh(2at)}M(t,r)\sinh(v). \quad (24)$$

Substituting these expressions in the equations for t and r we obtain

$$\frac{dv}{ds} = -\frac{F[\Phi(t)]e^{-\gamma a^2 r^2}}{\cosh(2at)}[\partial_t M(t,r)\sinh(v)$$
$$+\partial_r M(t,r)\cosh(v)] \quad (25)$$

or, equivalently,

$$\frac{dv}{ds} = -\frac{1}{2M(t,r)}\{\Gamma_+(t,r)e^v+\Gamma_-(t,r)e^{-v}\}, \quad (26)$$

where

$$\Gamma_+(t,r) = \epsilon \left[a\tanh(2at) + \frac{1}{2} \frac{d\ln\{F^{-1}[\Phi(t)]\}}{dt} + \gamma a^2 r \right]$$
$$+ \frac{(K-Lar^2)^2}{r^2 F^{-1}[\Phi(t)]\cosh(2at)} \left[\gamma a^2 r - \frac{1}{r} \right.$$
$$\left. - \frac{2Lar}{K-Lar^2} \right] + \frac{L^2\cosh(2at)}{F^{-1}[\Phi(t)]} [2a\tanh(2at)$$
$$+ \gamma a^2 r],$$

$$\Gamma_-(t,r) = \epsilon \left[-a\tanh(2at) - \frac{1}{2} \frac{d\ln\{F^{-1}[\Phi(t)]\}}{dt} + \gamma a^2 r \right]$$
$$+ \frac{(K-Lar^2)^2}{r^2 F^{-1}[\Phi(t)]\cosh(2at)} \left[\gamma a^2 r - \frac{1}{r} \right.$$
$$\left. - \frac{2Lar}{K-Lar^2} \right] + \frac{L^2\cosh(2at)}{F^{-1}[\Phi(t)]} [-2a\tanh(2at)$$
$$+ \gamma a^2 r]. \tag{27}$$

In order for the geodesics to be complete dt/ds and dr/ds have to remain finite for finite values of the affine parameter. In fact, it is sufficient to consider only dt/ds, since dr/ds and dt/ds are related via Eq. (15). The derivatives $d\phi/ds$ and dz/ds are regular functions of t and r, and the only problem we could have appear when r approaches $r=0$ for $K\neq 0$. We shall show, however, that r cannot become zero for $K\neq 0$.

First we consider geodesics with increasing r (i.e., $v > 0$). In this case it is not difficult to see that the term

$$\frac{F[\Phi(t)]e^{-\gamma a^2 r^2}}{\cosh(2at)} M(t,r) \tag{28}$$

in Eq. (23) cannot become singular (for increasing r). Therefore, dt/ds could become singular only for v. We shall show, however, that v cannot become singular for finite values of the affine parameter. For increasing r, v cannot diverge since for large t (large r) the derivative dv/ds becomes negative. Indeed, for all exact solutions presented here, there exists a constant $B_1 > 0$ such that

$$\left| \frac{d\ln\{F^{-1}[\Phi(t)]\}}{dt} \right| < B_1 \tag{29}$$

for arbitrary t and fixed λ.

Therefore, as can be seen from Eqs. (27), the terms associated with the constant ε, K, and L are all positive for large values of t. As a consequence we obtain that the functions $\Gamma_+(t,r)$ and $\Gamma_-(t,r)$ are positive, i.e., $dv/ds < 0$ for large t (large r).[2]

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[2]In fact, the function $\Gamma_-(t,r)$ is exponentially small compared with $\Gamma_+(t,r)$ and may not be considered.

In the second case, when r decreases ($v<0$), the problem comes from $r=0$ when $K\neq 0$. The geodesics with $K=0$ can reach the axis $r=0$ without problems and then continue with $dr/ds>0(v>0)$. When $K\neq 0$, v cannot diverge for finite values of the affine parameter. This follows from the fact that the derivative dv/ds becomes positive for small r (large t) as can be seen from Eqs. (27) and (26), taking into account that $\Gamma_+(t,r)$ is exponentially suppressed compared with $\Gamma_-(t,r)$. The positiveness of the derivative dv/ds when the geodesics are close to the axis $r=0$ prevents the radial coordinate from collapsing too quickly and reaching the axis. The fact that r cannot become zero for $K\neq 0$ may be seen more explicitly as follows. When r approaches zero the dominant term is that associated with K and the other terms can be ignored. So, for small r the geodesics behave as null geodesics with $L=0$:

$$\frac{dt}{ds} = \frac{F[\Phi(t)]e^{-\gamma a^2 r^2}}{\cosh(2at)} M(r)\cosh(v), \tag{30}$$

$$\frac{dr}{ds} = \frac{F[\Phi(t)]e^{-\gamma a^2 r^2}}{\cosh(2at)} M(r)\sinh(v), \tag{31}$$

$$\frac{dv}{ds} = -\frac{F[\Phi(t)]e^{-\gamma a^2 r^2}}{\cosh(2at)} \partial_r M(r)\cosh(v), \tag{32}$$

where $M(r) = (|K|/r)e^{(1/2)\gamma a^2 r^2}$. Hence, we obtain the orbit equation

$$\frac{dr}{dv} = -\frac{M(r)}{\partial_r M(r)}\tanh(v). \tag{33}$$

Integrating, we have

$$e^{-(1/2)\gamma a^2 r^2} r = C_1\cosh(v), \tag{34}$$

where $C_1 > 0$ is a constant. Since $\cosh(v)\geq 1$, r cannot become zero.

From the proof of the geodesic completeness it follows that every maximally extended null geodesic intersects any of the hypersurfaces $t=\text{const}$. According to Ref. [36], this a sufficient condition that the hypersurfaces $t=\text{const}$ are global Cauchy surfaces. Therefore, the solutions are globally hyperbolic.

We have explicitly proven the geodesic completeness of the solutions using their particular properties. The geodesic completeness can be proved independently by considering the solutions from a more general point of view. In Ref. [37] (see also Ref. [38]), Fernandez-Jambrina presented a general theorem providing wide sufficient conditions for an orthogonally transitive cylindrical space-time to be geodesically complete. It can be verified that the solutions presented here satisfy all conditions in the Fernandez-Jambrina theorem and therefore they are geodesically complete.

New diagonal solutions can be obtained from Eq. (2) as a limiting case. Taking $a \to 0$ and keeping $a^2 \gamma = \beta$ fixed, we obtain the following diagonal inhomogeneous cosmological scalar-tensor solutions:

$$ds^2 = F^{-1}[\Phi(t)][e^{\beta r^2}(-dt^2 + dr^2) + r^2 d\phi^2 + dz^2],$$

$$8\pi G_* \rho = \beta f(\lambda) e^{-\beta r^2} F^3[\Phi(t)],$$

$$u_\mu = F^{-1/2}[\Phi(t)] e^{(1/2)\beta r^2} \delta_\mu^0, \tag{35}$$

where $a\sqrt{\gamma - 1}$ should be replaced by $\sqrt{\beta}$ in the explicit formulas for $F^{-1}[\Phi(t)]$.

We have proven that the solutions presented in the present paper are geodesically complete. This result is not in contradiction with the well-known singularity theorems because in our case the strong energy condition is violated in the Jordan frame. This can be explicitly seen by calculating the components of the Ricci tensor. All components are bounded except for $R_{tr} = -r\gamma a^2 \partial_t \ln\{F[\Phi(t)]\}$. Therefore, for large enough r, one can always find timelike and null vectors ν^μ such that $R_{\mu\nu} \nu^\mu \nu^\nu < 0$, i.e., the strong energy condition is violated. However, the situation is different in the Einstein frame. The Einstein frame metric $g_{\mu\nu}^E$ is just the Mars's metric and it is geodesically complete as we have already mentioned. Since the energy conditions are satisfied in the Einstein frame it remains to see which other conditions of the singularity theorems are violated. The space-time described by the metric $g_{\mu\nu}^E$ does not contain closed trapped surfaces. In order to prove this we will employ the techniques of differential geometry described in Refs. [39] and [40]. Let us consider a closed spacelike surface $\mathcal{S}$ and suppose that it is trapped. Since the surface is compact it must have a point q where r reaches its maximum. Let us denote $r_{\max} = R$ on a constant time hypersurface $t = T$. For the traces of both null second fundamental forms at q, it can be shown that (see Refs. [39] and [40])

$$K_{\mathcal{S}}^+ \big|_q \geq \frac{e^{-(1/2)\gamma a^2 R^2}}{\sqrt{2}R\cosh^{1/2}(2aT)} > 0,$$

$$\tag{36}$$

$$K_{\mathcal{S}}^- \big|_q \leq -\frac{e^{-(1/2)\gamma a^2 R^2}}{\sqrt{2}R\cosh^{1/2}(2aT)} < 0.$$

The traces have opposite signs so that there are no trapped surfaces.

Our solutions are stiff perfect fluid cosmologies and, therefore, the natural question which arises is what happens if the fluid is not stiff. In this case, however, the situation is much more complicated. In contrast to the stiff fluid case, the dilaton-matter sector does not possess nontrivial symmetries which allow us to generate new solutions from known ones. The only way to find exact solutions is to attack directly the corresponding system of coupled partial differential equations. This question is currently under investigation.

Summarizing, in this work we have presented new nondiagonal G_2 inhomogeneous stiff perfect fluid cosmological solutions in a wide range of scalar-tensor theories. The found solutions have no big-bang nor any other curvature singularity. The gravitational scalar (dilaton) and fluid energy density (pressure) are regular everywhere, too. Moreover, the solutions are globally hyperbolic and geodesically complete. To the best of our knowledge, these solutions are the first examples of nonsingular G_2 inhomogeneous perfect fluid scalar-tensor cosmologies with a nondiagonal metric.

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