

**QED and the high polarization of the thermal radiation from neutron stars**

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The thermal emission of strongly magnetized neutron-star atmospheres is thought to be highly polarized. However, because of the different orientations of the magnetic field over the surface of the neutron star (NS), it is commonly assumed that the net observed polarization will be significantly reduced as the polarization from different regions will cancel each other. We show that the birefringence of the magnetized QED vacuum decouples the polarization modes in the magnetosphere; therefore, the direction of the polarization follows the direction of the magnetic field up to a large distance from the stellar surface. At this distance, the rays that leave the surface and are destined for our detectors pass through only a small solid angle; consequently, the polarization direction of the emission originating in different regions will tend to align together. The net observed polarization of the thermal radiation of NSs should therefore be very large. Measurement of this polarization will be the first direct evidence of the birefringence of the magnetized vacuum due to QED and a direct probe of behavior of the vacuum at magnetic fields of order of and above the critical QED field of  $4.4 \times 10^{13}$  G. The large observable polarization will also help us learn more about the atmospheric properties of NSs.

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**I. INTRODUCTION**

When strong magnetic fields are present, the atmospheres of NSs have a significantly different opacity in the two polarization eigenmodes [1,2]. As a result, the effective depth to which an observer will be able to see the atmosphere in the two polarizations will be significantly different, and each one will therefore have a different effective temperature. The net effect is that each element on the neutron star (NS) surface emits highly polarized ( $>50\%$ ) thermal radiation at optical through x-ray frequencies [3–5]; at the point of emission, the direction of polarization is correlated with the direction of the magnetic field. However, the magnetic field orientation varies over the surface of a neutron star. When the polarized intensities are then summed, a relatively small net polarization results. Typical values of 5% to 25% are obtained [3,6]. This, however, neglects the effects that QED has on the propagation of photons through the vacuum.

Heyl and Shaviv [7] examined the specific case of purely radial photon trajectories from a rotating neutron star. Since each line of sight is represented by a single trajectory, no depolarization occurs. The net polarization observed is guaranteed to be equal in magnitude to that produced. In this paper, we examine the complementary issue of how photons propagate near the surface of the star. In this case both non-radial trajectories and general relativity have important effects as seen in earlier work [6]. However, no previous work has included a realistic treatment of the optically active magnetosphere of a neutron star [7] to examine the extent of

depolarization of the thermal radiation from the surface of a neutron star.

**II. PHYSICAL BACKGROUND**

In the presence of strong magnetic fields, QED renders the vacuum birefringent—the indices of refraction of the two linear polarization modes differ from each other [8,9]. In spite of over a century of effort, the effect of a magnetic field on the speed of light in vacua has not been detected [10–13].

A convenient formalism to describe the birefringence of a medium and the effects it has on the polarization states is by using the Poincaré space. The latter can describe the polarization state vector  $\mathbf{s}$  (which is the normalized Stokes vector) and a complex linear combination over the vector space perpendicular to the light ray [14].

In Poincaré space, the birefringence can be described by a vector  $\hat{\Omega}$  which points in the direction of the faster moving polarization mode [14] and of which the amplitude is the difference between the wave numbers of the two modes for a particular frequency ( $\Delta k$ ). The faster mode is polarized perpendicular to the direction of  $\mathbf{B}_\perp$ . For weak fields  $B \ll B_{QED} = 4.4 \times 10^{13}$  G, Heyl and Shaviv [7] find

$$|\hat{\Omega}| = \frac{\alpha}{15} \frac{\nu}{c} \left( \frac{B_\perp}{B_{QED}} \right)^2 \quad (1)$$

where  $B_\perp$  is the strength of the magnetic field perpendicular to the propagation direction of the photon,  $\nu$  is the photon frequency and  $c$  is the speed of light in vacuum. The origin of the vector  $\hat{\Omega}$  is purely a QED effect arising from the interaction of the outgoing photons with the virtual electrons of the vacuum. A similar birefringent vector arises when the

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interaction with the plasma electrons is taken into account; however, this will be important only at frequencies below the optical [15,16].

Using the result of Kubo and Nagata [17] to describe the polarization evolution in a dielectric birefringent medium, Heyl and Shaviv [7] have shown that this vacuum birefringence will couple the evolution of the photon polarization to the magnetic field direction through the equation

$$\frac{\partial \mathbf{s}}{\partial x_3} = \hat{\Omega} \times \mathbf{s}, \quad (2)$$

where  $x_3$  is the distance along the direction of propagation,  $\mathbf{s}$  is the normalized Stokes vector, and  $\hat{\Omega}$  is the birefringent vector previously described.

Close to the neutron star,  $\hat{\Omega}$  is large, and  $\mathbf{s}$  rotates quickly around  $\hat{\Omega}$ . Thus, as the direction of the magnetic field changes,  $\hat{\Omega}$  will rotate and  $\mathbf{s}$  will follow it adiabatically. Far enough from the NS, the amplitude of  $\hat{\Omega}$  will fall and  $\mathbf{s}$  will not be able to follow  $\hat{\Omega}$  any longer. The condition for adiabaticity to hold is [7]

$$l(\Delta k) = |\hat{\Omega}/(\nabla \ln|\hat{\Omega}|)| \geq 0.5 \quad (3)$$

where  $l$  is the scale length of the magnetic field.

If one assumes that the field surrounding the star has a dipolar geometry, the polarization states evolve adiabatically if

$$\begin{aligned} r \lesssim r_{\text{pl}} &\equiv \left( \frac{\alpha}{45} \frac{\nu}{c} \right)^{1/5} \left( \frac{\mu}{B_{\text{QED}}} \sin \beta \right)^{2/5} \\ &\approx 1.2 \times 10^7 \left( \frac{\mu}{10^{30} \text{ G cm}^3} \right)^{2/5} \left( \frac{\nu}{10^{17} \text{ Hz}} \right)^{1/5} \\ &\quad \times (\sin \beta)^{2/5} \text{ cm}, \end{aligned} \quad (4)$$

where  $r$  is the distance from the center of the star,  $\mu$  is the magnetic dipole moment of the neutron star, and  $\beta$  is the angle between the dipole axis and the line of sight;  $r_{\text{pl}}$  is the polarization-limiting radius, borrowing terminology from the study of radio pulsars [15].

Physically, the adiabatic regime is appropriate as long as  $\Delta n \cdot k \cdot l \geq 1$ , where  $\Delta n$  is the difference between the indices of refraction,  $k$  is the wave vector and  $l \sim r$  is the distance scale over which the physical variables change. In other words, adiabaticity requires that over the physical length scale of the problem, the two modes develop a significant phase difference between them. The coupling or polarization limiting radius therefore does not depend on the rate at which  $\hat{\Omega}$  changes direction. The direction (and rate of change in direction) of  $\hat{\Omega}$  will, however, determine the polarization left beyond  $r_{\text{pl}}$ . Since  $\hat{\Omega}$  is in the 1-2 plane describing linear polarizations, the direction of  $\hat{\Omega}$  at  $r_{\text{pl}}$  will determine the linear polarization component of  $\mathbf{s}$  while the rate of change of  $\hat{\Omega}$  will determine the circular component.

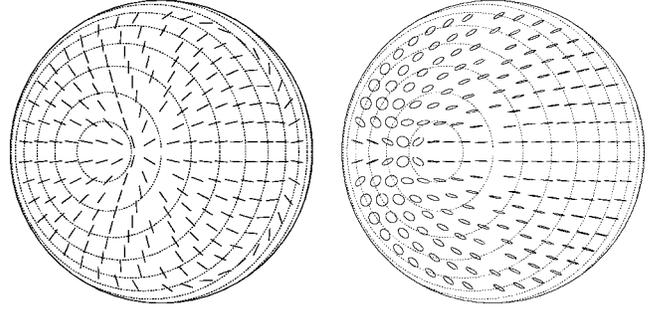


FIG. 1. The apparent polarized image of a neutron star overlaid on the apparent image of the NS. The left panel depicts the observed map of polarization directions if one assumes that the surface emits only in the ordinary mode and neglects the vacuum birefringence induced by QED. The right panel shows the polarization map including birefringence for a frequency of  $\nu = \mu_{30}^{-2} 10^{17}$  Hz. The ellipses and short lines describe the polarization of a light ray originating from the surface element beneath them. The lines and the major axes of the ellipses point towards the direction of the linear component of the polarization direction. The minor to major axis ratio provides the amount of circular polarization ( $s_3$ ). In both maps, the large dashed curves are lines of constant magnetic latitude (separated by  $15^\circ$ ). The observer's line of sight makes an angle of  $30^\circ$  with the dipole axis. For comparison, the net linear polarization on the left is 13% while it is 70% on the right. In a more realistic NS, the values for x-ray frequencies would be reduced to 6–10% and 35–55%, respectively [21], since the intrinsic polarization of each element is not 100% but smaller.

Heuristically, because of vacuum birefringence in the magnetosphere, the observed polarization direction from a surface element is correlated with the direction of the magnetic field far from the stellar surface. At this distance, the bundle of rays that will eventually be detected passes through a small solid angle. Over this small solid angle, the direction of the magnetic field varies little, so the observed polarization from different parts also varies little and a large net polarization results. Furthermore, this heuristic picture predicts that because stars with smaller radii generally result in smaller ray bundles, smaller stars will exhibit a larger net polarization.

This heuristic picture is borne out by detailed calculations. To calculate the process accurately, one calculates the photon trajectory both in spacetime and on the Poincaré sphere in the context of general relativity (GR). First, we incorporate light bending (as is given, for example, by Page [18]). Second, we have to use the result of Pineault [19] who showed that along the bent light rays present in GR, the polarization direction rotates in such a way that it keeps fixed orientation with respect to the normal to the trajectory plane, remaining perpendicular to the wave vector. Additionally, GR distorts the dipole magnetic field [20] near the star.

### III. RESULTS

A sample result of the integration of the polarization is given in Fig. 1, where the apparent polarization at infinity is depicted together with the direction that the polarization

would have had if the effect of QED polarization alignment would have been artificially switched off. Clearly, the effect of the adiabatic evolution of polarization close to the neutron star is to align the polarization vectors such that they sum mostly coherently. A large net polarization could therefore be maintained. Moreover, when the modes couple close to the neutron star (for low frequencies or weak magnetic fields) a circular component of the polarization is generated. However, this component averages to zero if the magnetic field has cylindrical symmetry (for example, if only a centered magnetic dipole moment exists).

If we wish to find the net polarization that an observer will measure, we need to know the intensity as a function of angle and energy of each surface element. For that, detailed atmospheric models are needed; this is beyond the scope of this article. Instead, we will simplify the process by assuming that all surface elements emit radiation isotropically (into their upper hemisphere), with the same flux, and only in one polarization. In real atmospheres, the emission is angle dependent and surface elements closer to the poles are somewhat hotter and thus emit more radiation. The emission from a surface element is not completely polarized but polarized fractions greater than 50% are typical [4,5]. We neglect these complications and defer them to a more detailed analysis since we mainly wish to show at this point that QED effects are important. The inclusion of a detailed atmosphere still results with the same conclusion that a high net polarization is obtained [21].

The results when assuming an isothermal surface with an isotropic emission are given in Fig. 2. Plotted are the net observed polarized fractions as a function of frequency for two different angles between the line of sight and the dipole axis,  $\beta=30^\circ$  and  $\beta=60^\circ$ , each assuming three different radii for the neutron star.  $\mu_{30}$  is the magnetic dipole moment of the NS measured in units of  $10^{30}$  G cm<sup>3</sup> to which the frequency is normalized.

The extent of polarization increases dramatically with increasing frequency which results from the increase in  $r_{\text{pl}}$  with energy. At low energies, QED is unimportant and we recover the earlier result [6] that more compact stars (smaller values of  $R/M$ ) exhibit less polarization. A distant observer sees a larger fraction of the surface of a more compact star due to the general relativistic bending of the photon trajectories.

At higher energies where  $r_{\text{pl}} \gg r$ , the opposite trend is evident. Since stars with larger radii subtend at larger solid angle at  $r_{\text{pl}}$ , the extent of polarization decreases as the stellar radius increases. This is the opposite of the trend expected from GR alone. Additionally, the extent of the polarization also depends on the angle between the line of sight and the dipole axis ( $\beta$ ), which reflects the dependence of  $r_{\text{pl}}$  on  $\beta$ .

#### IV. DISCUSSION

QED has various effects on the emission from neutron stars. For example, it was shown that it should be properly taken into account when calculating the atmospheric opacity [4,22]. Here, it was shown that QED has a startling effect on the polarization of photons while traversing the neutron-star

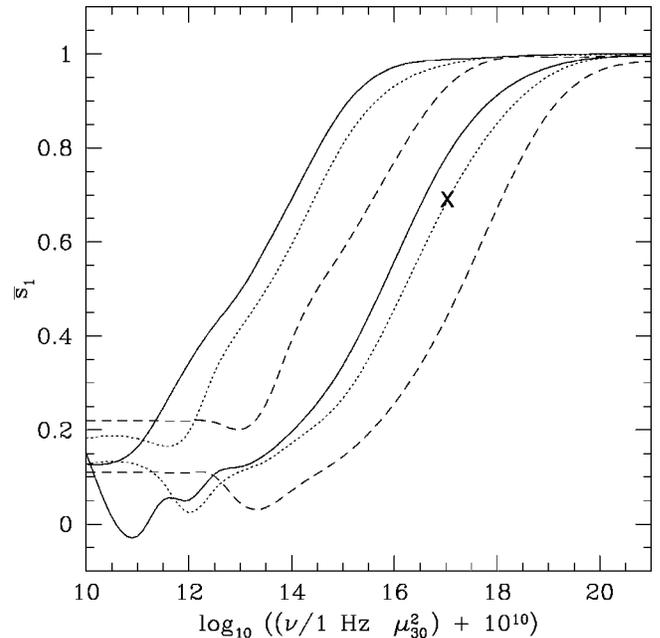


FIG. 2. The net polarization to be observed as a function of frequency for three different NS radii (solid line—6 km; dotted line—10 km; dashed line—18 km) and two observer magnetic colatitudes (upper three curves— $\beta=60^\circ$ ; lower three curves— $\beta=30^\circ$ ). The graphs assume that the surface has a uniform temperature and the emissivity is spherically symmetric. The case depicted in the previous figure is marked by an “X.” It should be compared with the low frequency limit of the curve, for which QED is unimportant.

magnetosphere. Gnedin, Pavlov and Shibano [23] noted that QED may be important in reprocessing radiation in accreting neutron stars.

The intrinsic polarization of the radiation emitted thermal from the surface of a neutron star with a magnetic field stronger than a few times  $10^{11}$  G is very high [3–5,21] (50% to 80% in optical through x-ray bands). Thus, at the peak of their thermal emission  $\sim 10^{18}$  Hz, average pulsars should be highly polarized. Even at optical wavelengths, their net polarization should be significantly higher due to QED alignment. In the optical, the extent of the polarization is strongly sensitive to the angle that the magnetic dipole makes with the line of sight. Furthermore, in agreement with the heuristic picture presented earlier, stars with smaller radii (but the same magnetic dipole moment) exhibit larger net polarizations. The polarized fraction in the optical increases by nearly a factor of two as the radius of the star shrinks from 18 km down to 6 km.

Currently, optical or UV polarimetry of neutron stars could be obtained in principle. However, since the thermal radiation of even the closest neutron star is very faint, it requires very long exposures on even the largest instruments available. In the long run, x-ray polarimetry will probably be more favorable since the thermal emission peaks at x rays.

Since the intrinsic polarization is not wiped out, more information can be extracted about neutron stars and their

atmospheres. This may allow the measurement of  $B$ ,  $M/R$  and possibly the basic composition of their atmosphere which affects the spectral and polarization properties (for example, whether they are rich in H, He or Fe).

The last experiment to have x-ray polarimetry capability was more than 20 years ago aboard the OSO-8 satellite [24,25]. Clearly, a new x-ray polarimeter will be most rewarding and is long overdue [26–28]. The measurement of strong polarization could prove fundamental QED physics in the extreme-field regime.

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