PHYSICAL REVIEW D **66**, 016003 (2002)

Vector manifestation and fate of vector mesons in dense matter

Masayasu Harada
Department of Physics, Nagoya University, Nagoya 464-8602, Japan
and School of Physics, Korea Institute for Advanced Study, Seoul 130-012, Korea

Youngman Kim
School of Physics, Seoul National University, Seoul 151-742, Korea

Mannque Rho
Service de Physique Théorique, CEA Saclay, 91191 Gif-sur-Yvette, France
and School of Physics, Korea Institute for Advanced Study, Seoul 130-012, Korea
(Received 10 January 2002; published 29 July 2002)

We describe the in-medium properties of hadrons in dense matter near chiral restoration using a Wilsonian matching to QCD of an effective field theory with hidden local symmetry at the chiral cutoff Λ. We find that chiral symmetry is restored in vector manifestation in the manner of Harada and Yamawaki at a critical matter density n_c. We express the critical density in terms of QCD correlators in dense matter at the matching scale. In a manner completely analogous to what happens at the critical N_f^c and at the critical temperature T_c, the vector meson mass is found to vanish (in the chiral limit) at chiral restoration. This result provides support for the Brown-Rho scaling predicted a decade ago.

DOI: 10.1103/PhysRevD.66.016003 PACS number(s): 11.30.Rd

Following recent developments on hidden local symmetry [1] and color-flavor locking [2] in the hadronic sector, Brown and Rho proposed [3,4] that the vector manifestation (VM) scenario of Harada and Yamawaki [1] for the realization of chiral symmetry in strongly interacting systems which was shown to be valid for large number of flavors N_f should also be applicable to high-density (or high-temperature) hadronic matter relevant to the interior of compact stars (or relativistic heavy-ion processes) and that, as a consequence, the scaling behavior of vector mesons in medium proposed by Brown and Rho [5] near a chiral restoration critical density n_c (or temperature T_c) follows from the VM. That the vector meson mass vanishes in the chiral limit at the critical temperature T_c in accordance with the VM mode was recently shown to hold by Harada and Sasaki [6]. In this paper, we supply the arguments to suggest that the same phenomenon occurs in density, namely, that, at $n=n_c$, the vector meson mass vanishes in the chiral limit.

We begin by giving a brief summary of the key arguments as to how VM figures in the properties of hadrons in medium.

To study how hadrons behave in a dense (hot) medium starting from normal conditions, one resorts to effective field theories with Lagrangians that have the assumed symmetry properties of QCD. Such Lagrangians are constructed so as to describe low-energy interactions of hadrons in a medium-free vacuum. As one increases the density (temperature), that is, as the scale is changed, the flow of the given theory is not unique even though the symmetries remain unchanged. As shown by Harada and Yamawaki [7], the effective field theory with hidden local symmetry (HLS) [8,9] can flow to two or more different fixed points depending upon how the *parameters* of the Lagrangian are dialed. It turns out that if the *bare* parameters of the Lagrangian are matched in the manner of Wilson to QCD at the chiral scale $\Lambda_\chi \sim 4\pi f_\pi$ (where f_π is the pion decay constant) above the vector meson mass (based on a systematic chiral perturbation with HLS [10–13]), then flow comes out to be unique with a fixed point [7]. This implies that the different flows typically present in all effective field theories, even if consistent with the symmetries of QCD, may not correctly represent QCD dynamics unless the *bare* parameters of the effective Lagrangian are matched at an appropriate scale, say, Λ_χ, with QCD. Most remarkably, though, when the HLS theory is matched with QCD, Wilsonian renormalization group equations (RGEs) show that the mass *parameter* M_ρ of the vector meson and the hidden gauge coupling *parameter* g do flow to zero, together with the pion decay constant f_π going to zero at the chiral restoration point, realizing what is referred to as the vector manifestation, and consequently the vector meson pole mass which is given in terms of the parameters M_ρ and g vanishes at the critical point with decoupling of the multiplet of vector mesons. This has been shown to be what happens at $N_f = N_f^c \sim 5$ for $T=0$ [1] and $T=T_c\sim 250$ MeV for $N_f=3$ [6].

To set up the arguments for the density problem, we consider a system of hadrons in the background of a filled Fermi sea. For the moment, we consider the Fermi sea as merely a *background*, sidestepping the question of how the Fermi sea is formed from a theory defined in a matter-free vacuum. Imagine that mesons—the pion and the ρ meson—are introduced in HLS theory [8,9] with a cutoff set at the scale, say, Λ_χ. Since we are dealing with dense fermionic matter, we will need to introduce the degrees of freedom associated with baryons or alternatively constituent quarks (or quasiquarks). At low density, say, $n<\tilde{n}$, with $\tilde{n}$ being some density greater than n_0, the precise value of which cannot be pinned down at present, we may choose to put the cutoff Λ_0 below the nucleon mass $m_N\sim 1$ GeV but above the ρ mass $m_\rho = 770$ MeV and integrate out all the baryons. In this case, the *bare* parameters of the HLS Lagrangian will depend upon the density n (or equivalently the Fermi momentum P_F) since the baryons that are integrated out carry information about the baryon density through their interactions in the full theory with the baryons within the Fermi sea. Once the baryons are integrated out, we will then be left with the standard HLS Lagrangian theory with the Nambu-Goldstone (NG) and gauge boson fields only *except that the bare parameters of the effective Lagrangian will be density dependent*. It should be noticed that *the cutoff can also be density dependent*. However, in general, the density dependence of the cutoff is not related to those of the bare parameters by the RGEs. For $T>0$ and $n=0$ this difference appears from the ''intrinsic'' temperature dependence introduced in Ref. [6], which was essential for the VM to occur at the chiral restoration point.

As density increases beyond $\tilde{n}$, the fermions will, however, start figuring explicitly, that is, the fermion field will be present below the cutoff $\tilde{\Lambda}$ ($n>\tilde{n}$). The reason is that as the density approaches the chiral restoration point, the constituent quark (called the quasiquark) picture—which seems to be viable even in matter-free space [14]—becomes more appropriate [4] and the quasiquark mass drops rapidly, ultimately vanishing (in the chiral limit) at the critical point. This picture has been advocated by several authors in a related context [15].

We now describe in some detail how the above scenario takes place. As a simple albeit unrealistic case in dense matter, consider the fermionic degrees of freedom to be baryons with a mass scale above the cutoff for all densities up to the chiral restoration density. In this case we can integrate out the baryons and take, as in [1,6,7], the standard HLS model based on the $G_{\rm global}\times H_{\rm local}$ symmetry, where $G=\mathrm{SU}(N_f)_{\rm L}\times\mathrm{SU}(N_f)_{\rm R}$ is the global chiral symmetry and $H=\mathrm{SU}(N_f)_{\rm V}$ is the HLS. When the kinetic term of gauge bosons of $H_{\rm local}$ is ignored, the HLS model is reduced to the nonlinear sigma model based on G/H, with $G_{\rm global}\times H_{\rm local}$ broken down to the diagonal sum which is the flavor symmetry H of G/H. In the HLS model the basic quantities are the gauge bosons $\rho_\mu=\rho_\mu^a T_a$ of the HLS and two $\mathrm{SU}(N_f)$-matrix valued variables $\xi_{\rm L}$ and $\xi_{\rm R}$. They are parametrized as $\xi_{\rm L,R}=e^{i\sigma/F_\sigma}e^{\mp i\pi/F_\pi}$, where $\pi=\pi^a T_a$ denote the pseudoscalar Nambu-Goldstone bosons associated with the spontaneous breaking of G and $\sigma=\sigma^a T_a$ the NG bosons absorbed into the HLS gauge bosons ρ_μ which are identified with the vector mesons. F_π and F_σ are relevant decay constants, and the parameter a is defined as $a\equiv F_\sigma^2/F_\pi^2$. $\xi_{\rm L}$ and $\xi_{\rm R}$ transform as $\xi_{\rm L,R}(x)\to h(x)\xi_{\rm L,R}(x)g^\dagger_{\rm L,R}$, where $h(x)\in H_{\rm local}$ and $g_{\rm L,R}\in G_{\rm global}$. The covariant derivatives of $\xi_{\rm L,R}$ are defined by $D_\mu\xi_{\rm L}=\partial_\mu\xi_{\rm L}-ig\rho_\mu\xi_{\rm L}+i\xi_{\rm L}\mathcal{L}_\mu$, and similarly with the replacement L$\leftrightarrow$R, $\mathcal{L}_\mu\leftrightarrow\mathcal{R}_\mu$, where g is the HLS gauge coupling, and $\mathcal{L}_\mu$ and $\mathcal{R}_\mu$ denote the external gauge fields gauging the $G_{\rm global}$ symmetry. The HLS Lagrangian is given by [8,9]

$$\mathcal{L}=F_\pi^2\mathrm{tr}[\hat{\alpha}_{\perp\mu}\hat{\alpha}_\perp^\mu]+F_\sigma^2\mathrm{tr}[\hat{\alpha}_{\parallel\mu}\hat{\alpha}_\parallel^\mu]+\mathcal{L}_{\rm kin}(\rho_\mu), \tag{1}$$

where $\mathcal{L}_{\rm kin}(\rho_\mu)$ denotes the kinetic term of ρ_μ and

$$\hat{\alpha}^\mu_{\substack{\perp\\ \parallel}}=(D_\mu\xi_{\rm R}\cdot\xi_{\rm R}^\dagger\mp D_\mu\xi_{\rm L}\cdot\xi_{\rm L}^\dagger)/(2i). \tag{2}$$

As stated above, the three parameters of the Lagrangian F_π, F_σ (or a), and g will depend on density. Since Lorentz invariance is broken, a distinction has to be made between the temporal and spatial components of the constants in Eq. (1). We will ignore the difference for the moment. This will be justified below and, in more detail, in Appendix A. For the moment continuing with Eq. (1), we need to match Eq. (1) with QCD to define the *bare* Lagrangian for the effective theory. To determine the *bare* parameters, we set the matching scale at $\Lambda\approx\Lambda_\chi$, below which only the HLS degrees of freedom are present, and extend the Wilsonian matching [12], which was originally proposed for $T=n=0$ in Ref. [12] and extended to nonzero temperature in Ref. [6], to nonzero density. We match the axial-vector and vector current correlators in the HLS with those derived in the operator product expansion (OPE) for QCD at nonzero density. The correlators in the HLS around the matching scale $\mathcal{M}=\Lambda$ (where $\mathcal{M}$ is the renormalization scale[1]) are well described by the same forms as those at $T=n=0$ [12] with the bare parameters having the ''intrinsic'' density dependence[2]

$$\Pi_A^{\rm (HLS)}(Q^2)=\frac{F_\pi^2(\Lambda;n)}{Q^2}-2z_2(\Lambda;n),$$

$$\Pi_V^{\rm (HLS)}(Q^2)=\frac{F_\sigma^2(\Lambda;n)[1-2g^2(\Lambda;n)z_3(\Lambda;n)]}{M_\rho^2(\Lambda;n)+Q^2}-2z_1(\Lambda;n), \tag{3}$$

where $M_\rho^2(\Lambda;n)\equiv g^2(\Lambda;n)F_\sigma^2(\Lambda;n)$ is the bare ρ mass, and $z_{1,2,3}(\Lambda;n)$ are the bare coefficient parameters of the relevant $\mathcal{O}(p^4)$ terms [11,12], all at $\mathcal{M}=\Lambda$. Since the Lorentz non-invariant terms in the current correlators by the OPE are suppressed by some powers of n/Λ^3 (see, e.g., Ref. [17]), we can ignore them from both the hadronic and QCD sectors. (See Appendix A for the justification for the hadronic sector.) Matching the above correlators with those by the OPE in the same way as was done for $T=n=0$ [12], we determine the bare parameters that include what we shall call ''intrinsic'' density dependence, which are then converted into those of the on-shell parameters through the Wilsonian RGEs [1,12]. As a result, the parameters appearing in the hadronic density corrections have intrinsic density dependence.

Now, to study the chiral restoration in dense matter, we assume that we can do in the fermionless theory the Wilsonian matching at the critical density n_c for $N_f=3$ assuming

[1]We reserve μ for the chemical potential.

[2]Note that at the level of the *bare* Lagrangian there is no vector–axial-vector mixing as discussed for hot matter by Dey, Eletsky, and Ioffe [16]. At the matching scale, there are no loop corrections. Mixing occurs through hadronic loops when decimation is done.

that $\langle \bar{q}q \rangle$ approaches 0 (continuously) for $n \to n_c$.[3] Then, the axial-vector and vector current correlators given by OPE in the QCD sector approach each other, and will agree at n_c. Then through the Wilsonian matching we require that the correlators in Eq. (3) agree with each other. As in the case of large N_f [1] and in the case of $T \sim T_c$ [6], this agreement can be satisfied also in dense matter if the following conditions are met:

$$g(\Lambda;n) \to_{n\to n_c} 0, \quad a(\Lambda;n) \to_{n\to n_c} 1,$$
$$z_1(\Lambda;n) - z_2(\Lambda;n) \to_{n\to n_c} 0. \tag{4}$$

We show in Appendix A that these conditions remain valid—with a suitable in-medium extension—when the breaking of Lorentz symmetry in the medium is taken into account in the bare Lagrangian.

Next we need to consider how these parameters flow as the scale parameter is varied. The flows are obtained by solving the RGE's for the parameters. The RGEs for the parameters of the HLS theory as the scale $\mathcal{M}$ is varied were derived in Refs. [19,1,7] with the effect of quadratic divergences included. These equations describe the flow of the parameters for a dense system for a *fixed* chemical potential μ (or density n).[4] They show that $a=1$, $g=0$, and $X=1$ with X defined by

$$X \equiv \frac{N_f}{2(4\pi)^2} \frac{\mathcal{M}^2}{F_\pi^2(\mathcal{M})} \tag{5}$$

are fixed points. Thus at $\mu=\mu_c$, given the bare parameters (4) at the matching scale Λ, both g and a flow to the fixed point. The RGEs given in Refs. [19,1,7] then imply that at $\mu=\mu_c$, $g=0$ and $a=1$ remain unchanged as $\mathcal{M}$ is varied. Now what about $F_\pi(\mathcal{M})$, which cannot be fixed by requiring only the agreement between the vector and axial-vector current correlators? As we will discuss in more detail later, in the absence of the hadronic dense-loop corrections $F_\pi(\mathcal{M}=0;\mu_c)=0$ is obtained from the fact that $X=1$ is a fixed point and corresponds to the pion decay constant $f_\pi(\mu_c)=0$. *Thus the chiral transition at high density will coincide with the VM.*

As stated, as density is raised—and in particular near the critical density on which we will focus—we expect the fermionic degrees of freedom to figure explicitly below the cutoff at which the Wilsonian matching is effected. In principle, to account for the fermionic degrees of freedom below the scale $\mathcal{M}=\Lambda(\mu)$ for a given $\mu > \tilde{\mu}$, we may introduce either light baryons with a running mass that drops with increasing density or, more appropriately, constituent quarks with masses scaling with density as suggested by Riska and Brown [15]. We adopt the latter in this paper.

We introduce the quasiquark field ψ below the scale $\Lambda(\mu)$ for $\mu \geqslant \tilde{\mu}$ into the Lagrangian. A chiral Lagrangian for π with the constituent quark (quasiquark) was given in Ref. [20]. In Ref. [9] the quasiquark field, say ψ, is introduced into the HLS Lagrangian in such a way that it transforms homogeneously under the HLS: $\psi \to h(x)\cdot\psi$ where $h(x) \in H_{\text{local}}$. Here we extend the Lagrangian of Ref. [9] to a general one with which we can perform a systematic derivative expansion. Since we are considering the model near the chiral phase transition point where the quasiquark mass is expected to become small, we assign $\mathcal{O}(p)$ to the constituent quark (quasiquark) mass m_q. Furthermore, we assign $\mathcal{O}(p)$ to the chemical potential μ or the Fermi momentum P_F, as we consider that the cutoff is larger than μ even near the phase transition point. Using this counting scheme we can make a systematic expansion in the HLS with the quasiquark included. We should note that this counting scheme is different from the one in the model for the π and baryons given in Ref. [21], where the baryon mass is counted as $\mathcal{O}(1)$. The leading order Lagrangian including one quasiquark field and one antiquasiquark field is counted as $\mathcal{O}(p)$ and given by

$$\delta\mathcal{L}_{Q(1)} = \bar{\psi}(x)(iD_\mu\gamma^\mu + \mu\gamma^0 - m_q)\psi(x) + \bar{\psi}(x)[\kappa\gamma^\mu \hat{\alpha}_{\parallel\mu}(x) + \lambda\gamma_5\gamma^\mu\hat{\alpha}_{\perp\mu}(x)]\psi(x) \tag{6}$$

where $D_\mu\psi = (\partial_\mu - ig\rho_\mu)\psi$ and κ and λ are constants to be specified later. At one-loop level the Lagrangian (6) generates the $\mathcal{O}(p^4)$ contributions including hadronic dense-loop effects as well as divergent effects. The divergent contributions are renormalized by the parameters, and thus the RGEs for three leading order parameters F_π, a, and g [and parameters of the $\mathcal{O}(p^4)$ Lagrangian] are modified from those without a quasiquark field. In addition, we need to consider the renormalization group flow for the quasiquark mass m_q.[5] Calculating one-loop contributions for RGE's in $\mathcal{M}$ for a given μ, we find

$$\mathcal{M}\frac{dF_\pi^2}{d\mathcal{M}} = C[3a^2g^2F_\pi^2 + 2(2-a)\mathcal{M}^2] - \frac{m_q^2}{2\pi^2}\lambda^2 N_c,$$

$$\mathcal{M}\frac{da}{d\mathcal{M}} = -C(a-1)\left[3a(1+a)g^2 - (3a-1)\frac{\mathcal{M}^2}{F_\pi^2}\right] + a\frac{\lambda^2}{2\pi^2}\frac{m_q^2}{F_\pi^2}N_c,$$

[3] We are assuming that the transition is not strongly first order. The quasiquark degrees of freedom introduced later make sense only within the same hypothesis. There is nothing at present that invalidates our assumption, but if the transition were proven to be strongly first order, some of the arguments used in this paper might need qualification. We note that, in the presence of the current quark mass, the quark condensate is believed to decrease rapidly but continuously around the "phase transition" point [18].

[4] We are using density n and chemical potential μ interchangeably. In the case of nearly massless quasiquarks near chiral restoration, $\mu \approx P_F$.

[5] The constants κ and λ will also run such that, at $\mu=\mu_c$, $\kappa=\lambda=1$ while, at $\mu<\mu_c$, $\kappa \neq \lambda$. The running will be small near n_c, so we will ignore their running here.

$$\mathcal{M}\frac{dg^2}{d\mathcal{M}}=-C\frac{87-a^2}{6}g^4+\frac{N_c}{6\pi^2}g^4(1-\kappa)^2,$$

$$\mathcal{M}\frac{dm_q}{d\mathcal{M}}=-\frac{m_q}{8\pi^2}[(C_\pi-C_\sigma)\mathcal{M}^2-m_q^2(C_\pi-C_\sigma)+M_\rho^2C_\sigma-4C_\rho], \quad (7)$$

where $C=N_f/[2(4\pi)^2]$ and

$$C_\pi\equiv\left(\frac{\lambda}{F_\pi}\right)^2\frac{N_f^2-1}{2N_f},$$

$$C_\sigma\equiv\left(\frac{\kappa}{F_\sigma}\right)^2\frac{N_f^2-1}{2N_f},$$

$$C_\rho\equiv g^2(1-\kappa)^2\frac{N_f^2-1}{2N_f}.$$

For $\mu>\tilde{\mu}$ at which the quasiquarks enter, the cutoff will be different from that without. However the matching conditions (4) will remain the same. Now Eq. (7) shows that $(g,a)=(0,1)$ is a fixed point only when $m_q=0$. Since $m_q=0$ itself is a fixed point of the RGE for m_q, $(g,a,m_q)=(0,1,0)$ is a fixed point of the coupled RGEs for g, a, and m_q. Furthermore, and most importantly, $X=1$ *becomes the fixed point of the RGE for X* [7]. This means that at the fixed point, $F_\pi(0)=0$ [see Eq. (5)]. What does this mean in dense matter? To see what this means, we note that for $T=\mu=0$, this $F_\pi(0)=0$ condition is satisfied for a given number of flavors $N_f^{\rm cr}\sim 5$ through the Wilsonian matching [1]. For $N_f=3$, $\mu=0$, and $T\neq 0$, this condition is never satisfied due to thermal hadronic corrections [6]. Remarkably, as we show in Appendix B, for $N_f=3$, $T=0$, and $\mu=\mu_c$, it turns out that dense hadronic corrections vanish up to $\mathcal{O}(p^6)$ corrections. Therefore the fixed point $X=1$ [i.e., $F_\pi(0)=0$] does indeed signal chiral restoration at the critical density.

Let us here focus on what happens to hadrons at and very near the critical point μ_c. This problem can be easily addressed with the machinery developed above. To do this we define, following [6], the "on-shell" quantities

$$F_\pi=F_\pi(\mathcal{M}=0;\mu),$$

$$g=g(\mathcal{M}=M_\rho(\mu);\mu),$$

$$a=a(\mathcal{M}=M_\rho(\mu);\mu), \quad (8)$$

where M_ρ is determined from the "on-shell condition"

$$M_\rho^2=M_\rho^2(\mu)=a(\mathcal{M}=M_\rho(\mu);\mu)\times g^2(\mathcal{M}=M_\rho(\mu);\mu)F_\pi^2(\mathcal{M}=M_\rho(\mu);\mu). \quad (9)$$

Then, the parameter M_ρ in this paper is renormalized in such a way that it becomes the pole mass at $\mu=0$.

We first look at the "on-shell" pion decay constant f_π. At $\mu=\mu_c$, it is given by

$$f_\pi(\mu_c)\equiv f_\pi(\mathcal{M}=0;\mu_c)=F_\pi(0;\mu_c)+\Delta(\mu_c) \quad (10)$$

where Δ is the dense hadronic contribution arising from fermion loops involving Eq. (6). As we shall show explicitly in Appendix B, up to $\mathcal{O}(p^6)$ in the power counting, $\Delta(\mu_c)=0$ at the fixed point $(g,a,m_q)=(0,1,0)$. Thus

$$f_\pi(\mu_c)=F_\pi(0;\mu_c)=0. \quad (11)$$

Since

$$F_\pi^2(0;\mu_c)=F_\pi^2(\Lambda;\mu_c)-\frac{N_f}{2(4\pi)^2}\Lambda^2, \quad (12)$$

and at the matching scale Λ, $F_\pi^2(\Lambda;\mu_c)$ is given by a QCD correlator at $\mu=\mu_c$, μ_c can be computed from

$$F_\pi^2(\Lambda;\mu_c)=\frac{N_f}{2(4\pi)^2}\Lambda^2. \quad (13)$$

Note that in free space this is the equation that determines $N_f^c\sim 5$ [1]. In order for this equation to have a solution at the critical density, it is necessary that $F_\pi^2(\Lambda;\mu_c)/F_\pi^2(\Lambda;0)\sim 3/5$. We do not have at present a reliable estimate of the density dependence of the QCD correlator to verify this condition but a decrease of F_π of this order in the medium looks quite reasonable.

Next we compute the ρ pole mass near μ_c. The details of the calculation are given in Appendix C. Here we just quote the result. With the inclusion of the fermionic dense loop terms, the pole mass, for $M_\rho,m_q\ll P_F$, is of the form

$$m_\rho^2(\mu)=M_\rho^2(\mu)+g^2G(\mu), \quad (14)$$

$$G(\mu)=\frac{\mu^2}{2\pi^2}\left[\frac{1}{3}(1-\kappa)^2+N_c(N_fc_{V1}+c_{V2})\right]. \quad (15)$$

At $\mu=\mu_c$, we have $g=0$ and $a=1$ so that $M_\rho(\mu)=0$ and since $G(\mu_c)$ is nonsingular $m_\rho=0$. Thus the fate of the ρ meson at the critical density is the same as that at the critical temperature. This is our main result. It is noted [1] that although the conditions for $g(\Lambda;n)$ and $a(\Lambda;n)$ in Eq. (4) coincided with Georgi's vector limit [22],[6] the VM should be distinguished from Georgi's vector realization [22].

So far we have focused on the critical density at which the Wilsonian matching clearly determines $g=0$ and $a=1$, without knowing much about the details of the current correlators. Here we consider how the parameters flow as functions of the chemical potential μ. In the low-density region, we expect that the "intrinsic" density dependence of the bare parameters is small. If we ignore the intrinsic density effect, we may then resort to the Morley-Kislinger (MK) theorem 2

[6]It was suggested in [23] that the Georgi vector limit was relevant to chiral restoration. Here we note that the chiral transition involves *both* the vector limit and the vanishing of the pion decay constant. A nonzero pion decay constant with the vector limit is not consistent with low-energy theorems.

[24] (sketched and referred to for definiteness as the "MK theorem" in Appendix D), which states that, given a RGE in terms of $\mathcal{M}$, one can simply trade in μ for $\mathcal{M}$ for dimensionless quantities and for dimensionful quantities with suitable calculable additional terms. The results are

$$\mu\frac{dF_\pi^2}{d\mu}=-2F_\pi^2+C[3a^2g^2F_\pi^2+2(2-a)\mathcal{M}^2]-\frac{m_q^2}{2\pi^2}\lambda^2N_c,$$

$$\mu\frac{da}{d\mu}=-C(a-1)\left[3a(1+a)g^2-(3a-1)\frac{\mu^2}{F_\pi^2}\right]+a\frac{\lambda^2}{2\pi^2}\frac{m_q^2}{F_\pi^2}N_c,$$

$$\mu\frac{dg^2}{d\mu}=-C\frac{87-a^2}{6}g^4+\frac{N_c}{6\pi^2}g^4(1-\kappa)^2,$$

$$\mu\frac{dm_q}{d\mu}=-m_q-\frac{m_q}{8\pi^2}[(C_\pi-C_\sigma)\mu^2-m_q^2(C_\pi-C_\sigma)+M_\rho^2C_\sigma-4C_\rho], \tag{16}$$

where F_π, a, g, etc., are understood as $F_\pi(\mathcal{M}=\mu;\mu)$, $a(\mathcal{M}=\mu;\mu)$, $g(\mathcal{M}=\mu;\mu)$, and so on.

It should be stressed that the MK theorem presumably applies in the given form to "fundamental theories" such as QED but not without modifications to effective theories such as the one we are considering. The principal reason is that there is a change of relevant degrees of freedom from above Λ, where QCD variables are relevant, to below Λ, where hadronic variables figure. Consequently, we do not expect Eq. (16) to apply in the vicinity of μ_c. Specifically, near the critical point, the "intrinsic" density dependence of the bare theory will become indispensable and the naive application of Eq. (16) should break down. One can see this clearly in the following example. The condition $g(\mathcal{M}=\mu_c;\mu_c)=0$ that follows from the matching condition (3) would imply, when Eq. (16) is naively applied, that $g(\mu)=0$ for *all* μ. This is obviously incorrect.[7] Near the critical density the "intrinsic" density dependence should be included in the RGE: Noting that Eq. (16) is for, e.g., $g(\mathcal{M}=\mu;\mu)$, we can write down the RGE for g corrected by the "intrinsic" density dependence as

$$\mu\frac{d}{d\mu}g(\mu;\mu)=\mathcal{M}\frac{\partial}{\partial\mathcal{M}}g(\mathcal{M};\mu)\bigg|_{\mathcal{M}=\mu}+\mu\frac{\partial}{\partial\mu}g(\mathcal{M};\mu)\bigg|_{\mathcal{M}=\mu}, \tag{17}$$

where the first term on the right-hand side reproduces Eq. (16) and the second term appears due to the "intrinsic" density dependence. Note that $g=0$ is a fixed point when the second term is neglected [this follows from Eq. (16)], and the presence of the second term makes $g=0$ be no longer the fixed point of Eq. (17).[8] The second term can be determined from QCD through Wilsonian matching. However, we do not presently have a reliable estimate of the μ dependence of the QCD correlators. Analyzing the μ dependence away from the critical density in detail requires a lot more work, so we relegate this issue to a later publication.

The Wilsonian matching of the correlators at $\Lambda=\Lambda_\chi$ allows one to see how the ρ mass scales very near the critical density (or temperature). For this purpose, it suffices to look at the intrinsic density dependence of M_ρ. We find that close to μ_c

$$M_\rho^2(\Lambda;\mu)\sim\frac{\langle\bar{q}q(\mu)\rangle^2}{F_\pi^2(\Lambda;\mu)\Lambda^2}, \tag{18}$$

which implies that

$$\frac{m_\rho^\star}{m_\rho}\sim\frac{\langle\bar{q}q\rangle^\star}{\langle\bar{q}q\rangle}. \tag{19}$$

Here the star denotes density dependence. Note that Eq. (19) is consistent with the "Nambu scaling" or more generally with sigma-model scaling. How this scaling fares with nature is discussed in [4].

The following observations can be drawn from this work.

The *parameters* of Brown-Rho (BR) scaling Lagrangian [5] can be identified with those of the HLS Lagrangian that are Wilsonian matched at the matching scale and flow to the fixed point $(g,a,m_q)=(0,1,0)$ with increasing density. An interesting question arises here: How is the BR scaling which is related to Landau Fermi liquid interaction at normal matter density (reviewed in [4]) interpreted in terms of the HLS flow?

It seems plausible that the density-dependent vector meson mass that arises via a Higgs mechanism in the color-flavor locking discussed in [3,4] refers to $M_\rho(\mu)$, namely, the part that reflects what was interpreted in [4] as "sliding vacuum." This is the Lorentz-invariant piece of the mass. The physical pole mass should also contain the dense loop corrections that take into account the velocity $v\neq1$.

The "intrinsic" density dependence that is governed by the Wilsonian matching with QCD and the VM fixed points, as in the case of the "intrinsic" temperature dependence discussed by Harada and Sasaki [6], is mostly if not completely missing in most of the model descriptions published in the literature. For instance, the prescription of replacing m_V by $m_V^\star$ near chiral restoration in the Rapp-Wambach approach as described in [26,27]—which seemed *ad hoc* at the time those papers were written—reflects what is lacking in the Rapp-

[7] The RGEs (16) were recently studied in [25] with the nucleons incorporated as explicit fermionic degrees of freedom.

[8] The condition $g(\mu_c;\mu_c)=0$ follows from the fixed point of the RGE in $\mathcal{M}$, but it is not a fixed point of the RGE in μ.

Wambach formulation near chiral restoration and may be justified by the "sliding vacuum" effect.

The notion of density dependence of the cutoff and the Morley-Kislinger procedure invoked here in the low-density region imply that the cutoff used in effective field theories should drop as density is increased. This supports the early suggestion of Adami and Brown [28] that the cutoff in the in-medium Nambu-Jona-Lasinio (NJL) model should be density dependent.

To summarize, we have shown that the vector manifestation is realized in dense matter at the chiral restoration with the vector meson mass m_ρ going to zero in the chiral limit. Thus the VM is *universal* in the sense that it occurs at N_f^c for $T=\mu=0$, at T_c for $N_f<N_F^c$ and $\mu=0$, and at μ_c for $T=0$ and $N_f<N_f^c$. This scenario is characterized by the common feature that, at the chiral transition, the longitudinal component of the ρ meson joins the pion into a degenerate multiplet, a scenario which differs from the standard sigma-model scenario. Since the gauge coupling constant g is to go to zero near the critical point, the dropping-mass vector meson will become sharper with vanishing width as suggested in [1], a phenomenon that cannot be accessed by a strong-coupling theory valid at low density that involves an expanding width [29].

We are indebted to Gerry Brown for useful comments. We are also grateful for discussions with Hyun Kyu Lee and Koichi Yamawaki. Two of us (M.H. and M.R.) acknowledge the hospitality of the Korea Institute for Advanced Study where this work was done. The work of M.H. is supported in part by a Grant-in-Aid for Scientific Research (A) No. 12740144 and that of Y.K. by the BK21 project of the Ministry of Education.

APPENDIX A: EFFECTS OF LORENTZ SYMMETRY BREAKING

The discussion in the main text was made without explicit consideration of the effect of Lorentz symmetry breaking inherent in a dense medium. In this appendix, we examine if and how the Lorentz symmetry breaking affects the conditions (4) at n_c. Modulo the intrinsic density dependence of the bare parameters, we obtain the general conditions—which are an extension of the conditions in Eq. (4) to a system without Lorentz invariance—by simply requiring that the axial-vector and vector current correlators in the HLS agree, $G^{T,L}_{V(HLS)}=G^{T,L}_{A(HLS)}$, at n_c without considering the matching to OPE in QCD. A more complete analysis that includes the matching to OPE in QCD with Lorentz symmetry breaking which is needed to describe processes away from the critical point will be reported elsewhere.

1. Polarization tensors

We start by summarizing the polarization tensors used here and also in the succeeding appendixes. In hot and/or dense matter, the polarization tensors are no longer restricted to be Lorentz covariant, but should be $O(3)$ covariant. Thus we need four independent symmetric $O(3)$ tensors. Here we adopt the following form [30]:

$$\begin{aligned}
P_T^{\mu\nu} &\equiv g_i^\mu\left(\delta_{ij}-\frac{\vec{p}_i\vec{p}_j}{\bar{p}^2}\right)g_j^\nu \\
&=(g^{\mu\alpha}-u^\mu u^\alpha)\left(-g_{\alpha\beta}-\frac{p^\alpha p^\beta}{\bar{p}^2}\right) \\
&\quad\times(g^{\beta\nu}-u^\beta u^\nu), \\
P_L^{\mu\nu} &\equiv -\left(g^{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\right)-P_T^{\mu\nu} \\
&=\left(g^{\mu\alpha}-\frac{p^\mu p^\alpha}{p^2}\right)u_\alpha\frac{p^2}{\bar{p}^2}u_\beta \\
&\quad\times\left(g^{\beta\nu}-\frac{p^\beta p^\nu}{p^2}\right), \\
P_C^{\mu\nu} &\equiv \frac{1}{\sqrt{2}\bar{p}}\left[\left(g^{\mu\alpha}-\frac{p^\mu p^\alpha}{p^2}\right)u_\alpha p^\nu\right. \\
&\quad\left.+p^\mu u_\beta\left(g^{\beta\nu}-\frac{p^\beta p^\nu}{p^2}\right)\right] \\
P_D^{\mu\nu} &\equiv \frac{p^\mu p^\nu}{p^2},
\end{aligned} \tag{A1}$$

where $p^\mu=(p_0,\vec{p})$ is the four-momentum and $\bar{p}\equiv|\vec{p}|$. The rest frame of the medium is indicated by

$$u^\mu=(1,\vec{0}). \tag{A2}$$

The polarization tensors satisfy the following identities [30]:

$$\begin{aligned}
&P_T^{\mu\nu}P_{T\mu\nu}=2,\quad g_{\mu\nu}P_T^{\mu\nu}=-2,\quad u_\mu P_T^{\mu\nu}=0,\\
&P_L^{\mu\nu}P_{L\mu\nu}=1,\quad g_{\mu\nu}P_L^{\mu\nu}=-1,\quad u_\mu u_\nu P_L^{\mu\nu}=\frac{\bar{p}^2}{p^2},\\
&P_C^{\mu\nu}P_{C\mu\nu}=-1,\quad g_{\mu\nu}P_C^{\mu\nu}=0,\\
&u_\mu u_\nu P_C^{\mu\nu}=-\sqrt{2}\frac{p_0\bar{p}}{p^2}.
\end{aligned} \tag{A3}$$

2. Axial-vector correlator

As argued in Sec. 5 of Ref. [18], the vector correlator receives at the chiral restoration point an important contribution from quasiquark loop diagrams. Such a contribution cannot in general be expressed by a local effective Lagrangian in which quasiquarks are absent. In the present work, we are considering the HLS model that includes the quasiquarks as explicit degrees of freedom near the critical point. Therefore we consider it reasonable to assume that the *bare* HLS theory we are concerned with can be expressed by a *local* Lagrangian with the nonlocal quasiquark contribution appearing at the later stage of Wilsonian decimation.

The bare HLS Lagrangian in hot and/or dense matter is generally expected to include the effect of Lorentz noninvariance. The Lagrangian density valid to $\mathcal{O}(p^4)$ relevant to the axial-vector current correlator can be written as

$$\mathcal{L}_{(A)}=[(F^t_{\pi,\text{bare}})^2u_\mu u_\nu+F^t_{\pi,\text{bare}}F^s_{\pi,\text{bare}}(g_{\mu\nu}-u_\mu u_\nu)]\text{tr}[\hat{\alpha}^\mu_\perp\hat{\alpha}^\nu_\perp]+[2z^L_{2,\text{bare}}u_\mu u_\alpha g_{\nu\beta}+z^T_{2,\text{bare}}(g_{\mu\alpha}g_{\nu\beta}-2u_\mu u_\alpha g_{\nu\beta})]\text{tr}[\hat{\mathcal{A}}^{\mu\nu}\hat{\mathcal{A}}^{\alpha\beta}], \tag{A4}$$

where $F^t_{\pi,\text{bare}}$ and $F^s_{\pi,\text{bare}}$ denote the *bare* parameters associated with the temporal and spatial pion decay constants. The rest frame of the medium is specified by u^μ as in Eq. (A2). The parameters $z^L_{2,\text{bare}}$ and $z^T_{2,\text{bare}}$ correspond in medium to the vacuum parameter $z_{2,\text{bare}}$ [11,12] at $T=\mu=0$, and $\hat{\mathcal{A}}^{\mu\nu}$ is defined by

$$\hat{\mathcal{A}}_{\mu\nu}\equiv\frac{1}{2}[\xi_\text{R}\mathcal{R}_{\mu\nu}\xi^\dagger_\text{R}-\xi_\text{L}\mathcal{L}_{\mu\nu}\xi^\dagger_\text{L}], \tag{A5}$$

where $\mathcal{R}_{\mu\nu}$ and $\mathcal{L}_{\mu\nu}$ are the field-strength tensors of the external gauge fields $\mathcal{R}_\mu$ and $\mathcal{L}_\mu$:

$$\mathcal{L}_{\mu\nu}=\partial_\mu\mathcal{L}_\nu-\partial_\nu\mathcal{L}_\mu-i[\mathcal{L}_\mu,\mathcal{L}_\nu],$$
$$\mathcal{R}_{\mu\nu}=\partial_\mu\mathcal{R}_\nu-\partial_\nu\mathcal{R}_\mu-i[\mathcal{R}_\mu,\mathcal{R}_\nu]. \tag{A6}$$

Note that in the bare theory $\hat{\alpha}^\mu_\perp$ is expanded as

$$\hat{\alpha}^\mu_\perp=\mathcal{A}^\mu+\frac{\partial_\mu\pi}{F^t_{\pi,\text{bare}}}+\cdots, \tag{A7}$$

where $\mathcal{A}_\mu=(\mathcal{R}_\mu-\mathcal{L}_\mu)/2$.

We define the axial-vector current correlator $G^{\mu\nu}_A(p)$ by

$$i\int d^4x e^{ipx}\langle 0|T\,J^a_{5\mu}(x)J^b_{5\nu}(0)|0\rangle=\delta^{ab}G^{\mu\nu}_A(p), \tag{A8}$$

and decompose it into

$$G^{\mu\nu}_A(p)=P^{\mu\nu}_LG^L_A(p)+P^{\mu\nu}_TG^T_A(p). \tag{A9}$$

It follows from the bare HLS Lagrangian, Eq. (A4), and Fig. 1 that

$$G^{\mu\nu}_{A(\text{HLS})}(p)=\frac{\tilde{\Gamma}^\mu_{A,\text{bare}}\tilde{\Gamma}^\nu_{A,\text{bare}}}{-[p_0^2-v^2_\text{bare}\overline{p}^2]}+[(F^t_{\pi,\text{bare}})^2u^\mu u^\nu+F^t_{\pi,\text{bare}}F^s_{\pi,\text{bare}}(g^{\mu\nu}-u^\mu u^\nu)]-2z^L_{2,\text{bare}}p^2P^{\mu\nu}_L-2(z^L_{2,\text{bare}}p_0^2-z^T_{2,\text{bare}}\overline{p}^2)P^{\mu\nu}_T, \tag{A10}$$

where v_bare is the bare pion velocity related to $F^t_{\pi,\text{bare}}$ and $F^s_{\pi,\text{bare}}$ by

$$v^2_\text{bare}=\frac{F^s_{\pi,\text{bare}}}{F^t_{\pi,\text{bare}}}, \tag{A11}$$

FIG. 1. Tree-level contributions to the axial-vector (vector) current correlator. The dashed line denotes the pion for the axial-vector correlator or the ρ meson for the vector correlator.

and

$$\tilde{\Gamma}^\mu_{A,\text{bare}}\equiv[F^t_{\pi,\text{bare}}u_\mu u_\alpha+F^s_{\pi,\text{bare}}(g_{\mu\alpha}-u_\mu u_\alpha)]p^\alpha. \tag{A12}$$

To obtain the $P^{\mu\nu}_L$ and $P^{\mu\nu}_T$ terms in Eq. (A10), we have used the following identities:

$$(u\cdot p)^2g^{\mu\nu}+p^2u^\mu u^\nu-(u\cdot p)(u^\mu p^\nu+p^\mu u^\nu)=-p^2P^{\mu\nu}_L-p_0^2P^{\mu\nu}_T=(p^2g^{\mu\nu}-p^\mu p^\nu)-\overline{p}^2P^{\mu\nu}_T. \tag{A13}$$

Now, by using the identities Eq. (A3), we obtain from Eq. (A9) and Eq. (A10),

$$G^L_{A(\text{HLS})}(p)=\frac{p^2F^t_{\pi,\text{bare}}F^s_{\pi,\text{bare}}}{-[p_0^2-v^2_\text{bare}\overline{p}^2]}-2p^2z^L_{2,\text{bare}} \tag{A14}$$

$$G^T_{A(\text{HLS})}(p)=-F^t_{\pi,\text{bare}}F^s_{\pi,\text{bare}}-2(p_0^2z^L_{2,\text{bare}}-\overline{p}^2z^T_{2,\text{bare}}). \tag{A15}$$

3. Vector correlator

The bare Lagrangian density valid to $\mathcal{O}(p^4)$ relevant to the vector correlator is

$$\mathcal{L}_{(V)}=[(F^t_{\sigma,\text{bare}})^2u_\mu u_\nu+F^t_{\sigma,\text{bare}}F^s_{\sigma,\text{bare}}(g_{\mu\nu}-u_\mu u_\nu)]\times\text{tr}[\hat{\alpha}^\mu_\parallel\hat{\alpha}^\nu_\parallel]+[2z^L_{1,\text{bare}}u_\mu u_\alpha g_{\nu\beta}+z^T_{1,\text{bare}}(g_{\mu\alpha}g_{\nu\beta}-2u_\mu u_\alpha g_{\nu\beta})]\text{tr}[\hat{\mathcal{V}}^{\mu\nu}\hat{\mathcal{V}}^{\alpha\beta}]+[2z^L_{3,\text{bare}}u_\mu u_\alpha g_{\nu\beta}+z^T_{3,\text{bare}}(g_{\mu\alpha}g_{\nu\beta}-2u_\mu u_\alpha g_{\nu\beta})]\text{tr}[V^{\mu\nu}\hat{\mathcal{V}}^{\alpha\beta}]+\left[-\frac{1}{g^2_{L,\text{bare}}}u_\mu u_\alpha g_{\nu\beta}-\frac{1}{2g^2_{T,\text{bare}}}(g_{\mu\alpha}g_{\nu\beta}-2u_\mu u_\alpha g_{\nu\beta})\right]\text{tr}[V^{\mu\nu}V^{\alpha\beta}], \tag{A16}$$

where $\hat{\mathcal{V}}^{\mu\nu}$ is defined by

$$\hat{\mathcal{V}}_{\mu\nu}\equiv\frac{1}{2}[\xi_\text{R}\mathcal{R}_{\mu\nu}\xi^\dagger_\text{R}+\xi_\text{L}\mathcal{L}_{\mu\nu}\xi^\dagger_\text{L}]. \tag{A17}$$

$F^t_{\sigma,\text{bare}}$ and $F^s_{\sigma,\text{bare}}$ denote the bare parameters associated with the temporal and spatial components of the decay constants of the σ (i.e., the longitudinal ρ). We define the matter extension of the parameter a as

$$a^t \equiv \left(\frac{F^t_{\sigma,\mathrm{bare}}}{F^t_{\pi,\mathrm{bare}}} \right)^2, \quad a^s \equiv \left(\frac{F^s_{\sigma,\mathrm{bare}}}{F^s_{\pi,\mathrm{bare}}} \right)^2. \tag{A18}$$

$z^L_{1,\mathrm{bare}}$ and $z^T_{1,\mathrm{bare}}$ in Eq. (A16) correspond in medium to $z_{1,\mathrm{bare}}$ and $z^L_{3,\mathrm{bare}}$; $z^T_{3,\mathrm{bare}}$ to $z_{3,\mathrm{bare}}$; and $g_{L,\mathrm{bare}}$ and $g_{T,\mathrm{bare}}$ to g_{bare}.

Let us first obtain a general form of the vector current correlator $G_V^{\mu\nu}$, defined in an analogous way to $G_A^{\mu\nu}$ in Eq. (A8). By using the vector meson propagator $iD_{\mu\nu}$, the V-$\mathcal{V}$ two-point function $\Pi_{V\parallel}$ and the $\mathcal{V}$-$\mathcal{V}$ two-point function $\Pi_\parallel$, we can express $G_V^{\mu\nu}$ in HLS theory as

$$G^{\mu\nu}_{V(\mathrm{HLS})} = \Pi^{\mu\alpha}_{V\parallel} iD_{\alpha\beta} \Pi^{\beta\nu}_{V\parallel} + \Pi^{\mu\nu}_\parallel . \tag{A19}$$

The vector meson propagator $iD_{\mu\nu}$ is related to the V-V two-point function $\Pi_V^{\mu\nu}$ by

$$i(D^{-1})^{\mu\nu} = \Pi^{\mu\nu}_V . \tag{A20}$$

It is convenient to decompose $\Pi_V^{\mu\nu}$ into the following four independent pieces:

$$\Pi^{\mu\nu}_V = g^{\mu\nu}\Pi^S_V + P^{\mu\nu}_L \Pi^L_V + P^{\mu\nu}_T \Pi^T_V + P^{\mu\nu}_C \Pi^C_V , \tag{A21}$$

in terms of which the vector meson propagator is given by

$$\begin{aligned} -iD^{\mu\nu} = {} & P^{\mu\nu}_L \frac{\Pi^S_V}{\Pi^S_V(\Pi^L_V - \Pi^S_V) - (\Pi^C_V)^2/2} + P^{\mu\nu}_T \frac{1}{\Pi^T_V - \Pi^S_V} \\ & + P^{\mu\nu}_C \frac{\Pi^C_V}{\Pi^S_V(\Pi^L_V - \Pi^S_V) - (\Pi^C_V)^2/2} \\ & + P^{\mu\nu}_D \frac{\Pi^L_V - \Pi^S_V}{\Pi^S_V(\Pi^L_V - \Pi^S_V) - (\Pi^C_V)^2/2}. \end{aligned} \tag{A22}$$

Similarly to $\Pi_V^{\mu\nu}$, the two-point functions $\Pi_{V\parallel}$ and $\Pi_\parallel$ can be decomposed as

$$\begin{aligned} \Pi^{\mu\nu}_{V\parallel} &= g^{\mu\nu}\Pi^S_{V\parallel} + P^{\mu\nu}_L \Pi^L_{V\parallel} + P^{\mu\nu}_T \Pi^T_{V\parallel} + P^{\mu\nu}_C \Pi^C_{V\parallel} , \\ \Pi^{\mu\nu}_{\parallel} &= g^{\mu\nu}\Pi^S_{\parallel} + P^{\mu\nu}_L \Pi^L_{\parallel} + P^{\mu\nu}_T \Pi^T_{\parallel} + P^{\mu\nu}_C \Pi^C_{\parallel} . \end{aligned} \tag{A23}$$

With Eqs. (A22) and (A23), $G^{\mu\nu}_{V(\mathrm{HLS})}$ reads

$$\begin{aligned} G^{\mu\nu}_{V(\mathrm{HLS})} = {} & P^{\mu\nu}_T G^T_{V(\mathrm{HLS})} + P^{\mu\nu}_L G^L_{V(\mathrm{HLS})} \\ & + P^{\mu\nu}_C G^C_{V(\mathrm{HLS})} + P^{\mu\nu}_D G^D_{V(\mathrm{HLS})} , \end{aligned} \tag{A24}$$

where

$$\begin{aligned} G^T_{V(\mathrm{HLS})} &= \frac{-(\Pi^T_{V\parallel} - \Pi^S_{V\parallel})^2}{\Pi^T_V - \Pi^S_V} + \Pi^T_\parallel - \Pi^S_\parallel , \\ G^L_{V(\mathrm{HLS})} &= \frac{1}{\Pi^S_V(\Pi^L_V - \Pi^S_V) - (\Pi^C_V)^2/2} \\ & \quad \times \Big[-\Pi^S_V(\Pi^L_{V\parallel} - \Pi^S_{V\parallel})^2 + \Pi^C_V \Pi^C_{V\parallel}(\Pi^L_{V\parallel} - \Pi^S_{V\parallel}) \\ & \quad - \frac{1}{2}(\Pi^L_V - \Pi^S_V)(\Pi^C_{V\parallel})^2 \Big] + (\Pi^L_\parallel - \Pi^S_\parallel), \\ G^C_{V(\mathrm{HLS})} &= \frac{1}{\Pi^S_V(\Pi^L_V - \Pi^S_V) - (\Pi^C_V)^2/2} \\ & \quad \times [-\Pi^C_V\{(\Pi^S_{V\parallel})^2 - \Pi^S_{V\parallel}\Pi^L_{V\parallel} - (\Pi^C_{V\parallel})^2/2\} \\ & \quad - \Pi^S_V \Pi^C_{V\parallel}(\Pi^L_{V\parallel} - \Pi^S_{V\parallel}) \\ & \quad - (\Pi^L_V - \Pi^S_V)\Pi^S_{V\parallel}\Pi^C_{V\parallel}] + \Pi^C_\parallel , \\ G^D_{V(\mathrm{HLS})} &= \frac{1}{\Pi^S_V(\Pi^L_V - \Pi^S_V) - (\Pi^C_V)^2/2} \times [\Pi^C_V \Pi^S_{V\parallel}\Pi^C_{V\parallel} \\ & \quad - \Pi^S_V(\Pi^C_{V\parallel})^2/2 - (\Pi^L_V - \Pi^S_V)(\Pi^S_{V\parallel})^2] + \Pi^S_\parallel . \end{aligned} \tag{A25}$$

The requirement for the current conservation is that $G^C_{V(\mathrm{HLS})}$ and $G^D_{V(\mathrm{HLS})}$ vanish. We can easily see that

$$G^C_{V(\mathrm{HLS})} = C^D_{V(\mathrm{HLS})} = 0, \tag{A26}$$

when the following conditions are satisfied:[9]

$$\begin{aligned} \Pi^S_V &= \Pi^S_\parallel = -\Pi^S_{V\parallel} , \\ \Pi^C_V &= \Pi^C_\parallel = -\Pi^C_{V\parallel} . \end{aligned} \tag{A27}$$

Then, $G^T_{V(\mathrm{HLS})}$ and $G^L_{V(\mathrm{HLS})}$ can be rewritten as

$$\begin{aligned} G^T_{V(\mathrm{HLS})} &= \frac{-(\Pi^T_{V\parallel} + \Pi^S_V)^2}{\Pi^T_V - \Pi^S_V} + \Pi^T_\parallel - \Pi^S_V , \\ G^L_{V(\mathrm{HLS})} &= \frac{1}{\Pi^S_V(\Pi^L_V - \Pi^S_V) - (\Pi^C_V)^2/2} \\ & \quad \times \Big[-\Pi^S_V(\Pi^L_{V\parallel} + \Pi^S_V)^2 \\ & \quad - \frac{1}{2}(\Pi^C_V)^2(\Pi^L_V + \Pi^S_V + 2\Pi^L_{V\parallel}) \Big] + (\Pi^L_\parallel - \Pi^S_V). \end{aligned} \tag{A28}$$

Now, using the bare Lagrangian (A16), we find (here and below, the subscript "bare" is omitted to simplify writing)

$$\Pi^S_V = \Pi^S_\parallel = -\Pi^S_{V\parallel} = \frac{p_0^2}{p^2}(F^t_\sigma)^2 - \frac{\overline{p}^2}{p^2}(F^t_\sigma F^s_\sigma),$$

[9]Two conditions in Eq. (A27) are actually satisfied by the contributions obtained from the bare Lagrangian in Eq. (A16).

$$\Pi_V^C = \Pi_\parallel^C = -\Pi_{V\parallel}^C = \sqrt{2}\frac{p_0\bar{p}}{p^2}F_\sigma^t(F_\sigma^t - F_\sigma^s), \tag{A29}$$

and

$$\begin{aligned}
\Pi_V^L &= \frac{p_0^2+\bar{p}^2}{p^2}F_\sigma^t(F_\sigma^t-F_\sigma^s)+\frac{p^2}{g_L^2},\\
\Pi_V^T &= \frac{p_0^2}{p^2}F_\sigma^t(F_\sigma^t-F_\sigma^s)+\frac{p_0^2}{g_L^2}-\frac{\bar{p}^2}{g_T^2},\\
\Pi_{V\parallel}^L &= -\frac{p_0^2+\bar{p}^2}{p^2}F_\sigma^t(F_\sigma^t-F_\sigma^s)-p^2 z_3^L,\\
\Pi_{V\parallel}^T &= -\frac{p_0^2}{p^2}F_\sigma^t(F_\sigma^t-F_\sigma^s)-(p_0^2 z_3^L-\bar{p}^2 z_3^T),\\
\Pi_\parallel^L &= \frac{p_0^2+\bar{p}^2}{p^2}F_\sigma^t(F_\sigma^t-F_\sigma^s)-2p^2 z_1^L,\\
\Pi_\parallel^T &= \frac{p_0^2}{p^2}F_\sigma^t(F_\sigma^t-F_\sigma^s)-2(p_0^2 z_1^L-\bar{p}^2 z_1^T).
\end{aligned} \tag{A30}$$

Finally, the results are

$$\begin{aligned}
G_{V(\mathrm{HLS})}^L &= \frac{p^2F_\sigma^tF_\sigma^s(1-2g_L^2z_3^L)}{-[p_0^2-(F_\sigma^s/F_\sigma^t)\bar{p}^2-M_v^2]}-2p^2z_1^L+\mathcal{O}(p^4),\\
G_{V(\mathrm{HLS})}^T &= \frac{F_\sigma^tF_\sigma^s}{-[p_0^2-(g_L^2/g_T^2)\bar{p}^2-M_v^2]}\\
&\quad\times[p_0^2-(g_L^2/g_T^2)\bar{p}^2-2g_L^2(p_0^2z_3^L-\bar{p}^2z_3^T)]\\
&\quad-2(p_0^2z_1^L-\bar{p}^2z_1^T)+\mathcal{O}(p^4),
\end{aligned} \tag{A31}$$

in which we have dropped terms proportional to $(p^2 z_3^L)^2$ and $(p_0^2 z_3^L - \bar{p}^2 z_3^T)^2$, which are of higher order in the present counting scheme, together with other $\mathcal{O}(p^4)$ terms that require $\mathcal{O}(p^6)$ Lagrangian density. In the above expressions M_v is the bare mass at rest frame:

$$M_v^2 \equiv g_L^2 F_\sigma^t F_\sigma^s. \tag{A32}$$

It should be noticed that, in the rest frame, $G_{V(\mathrm{HLS})}^L$ is equal to $G_{V(\mathrm{HLS})}^T$, and that both the longitudinal and transverse modes of the vector meson have the same bare mass.

4. The equality $G_{V(\mathrm{HLS})}^{T,L} = G_{A(\mathrm{HLS})}^{T,L}$ at n_c

At the chiral phase transition point, the axial-vector and vector current correlators must agree with each other: $G_{A(\mathrm{HLS})}^L = G_{V(\mathrm{HLS})}^L$ and $G_{A(\mathrm{HLS})}^T = G_{V(\mathrm{HLS})}^T$. Imposing this condition, we obtain

$$\begin{aligned}
&\frac{p^2F_\pi^tF_\pi^s}{-[p_0^2-(F_\pi^s/F_\pi^t)\bar{p}^2]}-2p^2z_2^L\\
&\quad=\frac{p^2F_\sigma^tF_\sigma^s(1-2g_L^2z_3^L)}{-[p_0^2-(F_\sigma^s/F_\sigma^t)\bar{p}^2-M_L^2]}-2p^2z_1^L,
\end{aligned} \tag{A33}$$

$$\begin{aligned}
&-F_\pi^tF_\pi^s-2(p_0^2z_2^L-\bar{p}^2z_2^T)\\
&\quad=\frac{F_\sigma^tF_\sigma^s[p_0^2-(g_L^2/g_T^2)\bar{p}^2-2g_L^2(p_0^2z_3^L-\bar{p}^2z_3^T)]}{-[p_0^2-(g_L^2/g_T^2)\bar{p}^2-M_L^2]}\\
&\quad\quad-2(p_0^2z_1^L-\bar{p}^2z_1^T).
\end{aligned} \tag{A34}$$

Now, we see that the above equalities are satisfied for any values of p_0 and $\bar{p}$ around the matching scale only if the following conditions are met:

$$\begin{aligned}
&a^t = \left(\frac{F_\sigma^t}{F_\pi^t}\right)^2 = 1, \quad a^s = \left(\frac{F_\sigma^s}{F_\pi^s}\right)^2 = 1,\\
&g_L = 0, \quad g_T = 0,\\
&z_2^L = z_1^L, \quad z_2^T = z_1^T.
\end{aligned} \tag{A35}$$

These conditions are the Lorentz-noninvariant version of the vector manifestation conditions, Eq. (4), discussed in the main text.

APPENDIX B: PION DECAY CONSTANT

In this appendix, we show how the dense hadronic corrections to the pion decay constant vanish at $\mu = \mu_c$. In this and the next appendixes we neglect possible effects of Lorentz symmetry breaking in the bare theory of the HLS: We assume that the bare HLS Lagrangian possesses the Lorentz invariance and that the dominant effects of Lorentz symmetry breaking come from the quasiquark dense-loop correction. The relevant Lagrangian involving quasiquark fields at $\mathcal{O}(p)$ is given in Eq. (6). Here we also include the following $\mathcal{O}(p^2)$ Lagrangian:

$$\begin{aligned}
\delta\mathcal{L}_{Q(2A)} &= \bar{\psi}\{c_{A1}\mathrm{tr}[\hat{\alpha}_{\perp\mu}\hat{\alpha}_\perp^\mu]+c_{A2}\hat{\alpha}_{\perp\mu}\hat{\alpha}_\perp^\mu\}\psi\\
&\quad+\bar{\psi}c_{A3}[\hat{\alpha}_{\perp\mu},\hat{\alpha}_{\perp\nu}][\gamma^\mu,\gamma^\nu]\psi,
\end{aligned} \tag{B1}$$

where the c's are constants of mass dimension -1.[10] The $\mathcal{O}(p^2)$ Lagrangian in Eq. (B1) generates the $\mathcal{O}(p^5)$ corrections to the first term of the leading order HLS Lagrangian in Eq. (1) at one loop. Since the relevant diagrams are tadpoles, all the divergent corrections are proportional to m_q. Then the RGE for F_π is not changed at the critical density where m_q vanishes.

Our convention and notation in this and succeeding appendixes are $p^\mu = (p_0, \vec{p})$, $\bar{p} \equiv |\vec{p}|$, $\omega_M(\bar{p}) \equiv \sqrt{M^2 + \bar{p}^2}$ in

[10] For $N_f = 2$, the c_{A1} term and c_{A2} term are not independent. We can set, e.g., $c_{A2} = 0$ without loss of generality.

free space, $\omega(\bar{p})\equiv\omega_0(\bar{p})=\bar{p}$, and for the pion $\tilde{\omega}(\bar{p})\equiv\tilde{\omega}_0(\bar{p})=v(\bar{p})\bar{p}$ where $v(\bar{p})$ is the pion velocity. The rest frame of the medium will be indicated by $u^\mu=(1,\vec{0})$ as in Eq. (A2).

In terms of the axial-vector-axial-vector two-point functions $\Pi_\perp^{\mu\nu}$, the temporal and spatial components of the pion decay constant are given by

$$f_\pi^t=\frac{1}{\tilde{F}}\left.\frac{u_\mu\Pi_\perp^{\mu\nu}(p_0,\vec{p})p_\nu}{p_0}\right|_{p_0=\tilde{\omega}},$$

$$f_\pi^s=\frac{1}{\tilde{F}}\left.\frac{-p^\alpha(g_{\alpha\mu}-u_\alpha u_\mu)\Pi_\perp^{\mu\nu}(p_0,\vec{p})p_\nu}{\bar{p}^2}\right|_{p_0=\tilde{\omega}},\qquad \text{(B2)}$$

where $\tilde{F}$ is the π wave function renormalization constant. According to the analysis of Ref. [21] in dense matter, this $\tilde{F}$ is nothing but f_π^t:

$$\tilde{F}=f_\pi^t. \qquad \text{(B3)}$$

We wish to compute the $f_\pi^{t,s}$ in HLS including the quasiquark terms. In HLS, the correlator tensors are

$$\Pi_\perp^{\mu\nu}(p_0,\vec{p})=g^{\mu\nu}F_\pi^2+2z_2(g^{\mu\nu}p^2-p^\mu p^\nu)+\bar{\Pi}_\perp^{\mu\nu}(p_0,\vec{p}), \qquad \text{(B4)}$$

where $\bar{\Pi}_\perp^{\mu\nu}(p_0,\vec{p})$ denotes the hadronic dense/thermal corrections we are interested in. On shell for the pion, we have

$$\Pi_\perp^{tt}(\tilde{\omega},\bar{p})=F_\pi^2-2z_2\bar{p}^2+\bar{\Pi}_\perp^{tt}(\tilde{\omega},\bar{p}),$$

$$\Pi_\perp^{ts}(\tilde{\omega},\bar{p})=2z_2 v\bar{p}^2+\bar{\Pi}_\perp^{ts}(\tilde{\omega},\bar{p}),$$

$$\Pi_\perp^{ss}(\tilde{\omega},\bar{p})=-F_\pi^2-2z_2v^2\bar{p}^2+\bar{\Pi}_\perp^{ss}(\tilde{\omega},\bar{p}), \qquad \text{(B5)}$$

where

$$\Pi_\perp^{tt}(p_0,\bar{p})\equiv u_\mu\Pi_\perp^{\mu\nu}(p_0,\vec{p})u_\nu,$$

$$\Pi_\perp^{ts}(p_0,\bar{p})\equiv\frac{1}{\bar{p}}u_\mu\Pi_\perp^{\mu\nu}(p_0,\vec{p})(g_{\nu\alpha}-u_\nu u_\alpha)p^\alpha$$
$$=\frac{1}{\bar{p}}p^\alpha(g_{\alpha\mu}-u_\alpha u_\mu)\Pi_\perp^{\mu\nu}(p_0,\vec{p})u_\nu$$
$$=\Pi_\perp^{st}(p_0,\bar{p}),$$

$$\Pi_\perp^{ss}(p_0,\bar{p})\equiv\frac{1}{\bar{p}^2}p^\alpha(g_{\alpha\mu}-u_\alpha u_\mu)\Pi_\perp^{\mu\nu}(p_0,\vec{p})\times(g_{\nu\beta}-u_\nu u_\beta)p^\beta, \qquad \text{(B6)}$$

and v denotes the pion velocity in a dense medium:

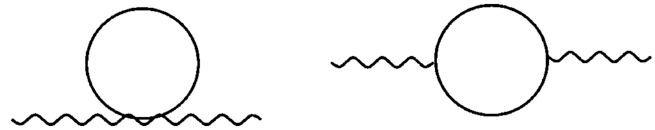

FIG. 2. The hadronic dense (loop) corrections to the vector-vector or axial-vector-axial-vector two-point functions. The solid lines denote quasiquarks.

$$v^2=1-\frac{1}{F_\pi^2}[\bar{\Pi}_\perp^{tt}(\omega,\bar{p})+2\bar{\Pi}_\perp^{ts}(\omega,\bar{p})+\bar{\Pi}_\perp^{ss}(\omega,\bar{p})]. \qquad \text{(B7)}$$

In this expression, we have replaced $\tilde{\omega}$ by ω, since the difference is of higher order.

Substituting Eq. (B5) into Eq. (B2), we obtain

$$[f_\pi^t]^2=\left[\Pi_\perp^{tt}(\tilde{\omega},\bar{p})+\frac{1}{v}\Pi_\perp^{ts}(\tilde{\omega},\bar{p})\right]_{p_0=\tilde{\omega}}$$
$$=F_\pi^2+\bar{\Pi}_\perp^{tt}(\omega,\bar{p})+\bar{\Pi}_\perp^{ts}(\omega,\bar{p})+\mathcal{O}(p^4), \qquad \text{(B8)}$$

$$f_\pi^t f_\pi^s=[-v\Pi_\perp^{ts}(\tilde{\omega},\bar{p})-\Pi_\perp^{ss}(\tilde{\omega},\bar{p})]_{p_0=\tilde{\omega}}$$
$$=F_\pi^2-\bar{\Pi}_\perp^{ts}(\omega,\bar{p})-\bar{\Pi}_\perp^{ss}(\omega,\bar{p})+\mathcal{O}(p^4). \qquad \text{(B9)}$$

This expression is consistent with the relation $v^2=f_\pi^s/f_\pi^t$.

To compute Eqs. (B8) and (B9), we need to compute hadronic dense fluctuation (loop) terms given in Fig. 2 with Eqs. (6) and (B1). The results are

$$\Pi_{\perp(1)}^{tt}(p_0,\bar{p})=-\lambda^2N_c[2\bar{A}_0-(4m_q^2+\bar{p}^2)\bar{B}_0(p_0,\bar{p})+\bar{B}^{tt}(p_0,\bar{p})],$$

$$\Pi_{\perp(1)}^{ts}(p_0,\bar{p})=-\lambda^2N_c[p_0\bar{p}\bar{B}_0(p_0,\bar{p})+\bar{B}^{ts}(p_0,\bar{p})],$$

$$\Pi_{\perp(1)}^{ss}(p_0,\bar{p})=\lambda^2N_c[2\bar{A}_0-(4m_q^2-p_0^2)\bar{B}_0(p_0,\bar{p})-\bar{B}^{ss}(p_0,\bar{p})],$$

$$\Pi_{\perp(2)}^{tt}(p_0,\bar{p})=-4N_c(N_fc_{A1}+c_{A2})m_q\bar{A}_0,$$

$$\Pi_{\perp(2)}^{ts}(p_0,\bar{p})=0,$$

$$\Pi_{\perp(2)}^{ss}(p_0,\bar{p})=4N_c(N_fc_{A1}+c_{A2})m_q\bar{A}_0, \qquad \text{(B10)}$$

where the subscript (n) with $n=1,2$ represents the contribution from the $\mathcal{O}(p^n)$ Lagrangian and

$$\bar{A}_0\equiv-\int\frac{d^4k}{i(2\pi)^4}\Delta_D(k),$$

$$\bar{B}_0(p_0,\bar{p})\equiv\int\frac{d^4k}{i(2\pi)^4}[\Delta_D(k)\Delta_F(k-p)+\Delta_D(k-p)\Delta_F(k)],$$

$$\bar{B}^{\mu\nu}(p_0,\bar{p}) \equiv \int \frac{d^4k}{i(2\pi)^4}(2k-p)^\mu(2k-p)^\nu \times[\Delta_D(k)\Delta_F(k-p) + \Delta_D(k-p)\Delta_F(k)]. \tag{B11}$$

Here $\Delta_D(k)$ and $\Delta_F(k)$ are given by

$$\Delta_D(k) \equiv i\frac{\pi}{\omega_{m_q}(\bar{k})}\delta(k_0-\omega_{m_q}(\bar{k}))\theta(P_F-\bar{k}),$$
$$\Delta_F(k) \equiv \frac{1}{k^2-m_q^2+i\epsilon}, \tag{B12}$$

with P_F being the Fermi momentum of the quasiquark. Now an explicit calculation gives

$$-2\bar{A}_0 = \frac{1}{4\pi^2}\left[P_F\omega_F - m_q^2\ln\frac{P_F+\omega_F}{m_q}\right],$$
$$\bar{B}_0(\bar{p},\bar{p}) = 0,$$
$$\bar{B}^{tt}(\bar{p},\bar{p}) = \frac{1}{4\pi^2}\left[-P_F\omega_F + m_q^2\ln\frac{P_F+\omega_F}{m} + P_F^2\ln\frac{\omega_F+P_F}{\omega_F-P_F}\right],$$
$$\bar{B}^{ts}(p_0,\bar{p}) = \frac{p_0}{\bar{p}}[\bar{B}_S - \bar{B}^{tt}(p_0,\bar{p})],$$
$$\bar{B}^{ss}(p_0,\bar{p}) = -\left(1+\frac{p_0^2}{\bar{p}^2}\right)\bar{B}_S + \frac{p_0^2}{\bar{p}^2}\bar{B}^{tt}(p_0,\bar{p}),$$
$$\bar{B}_S \equiv \frac{p_\mu p_\nu}{p^2}\bar{B}^{\mu\nu}(p_0,\bar{p}) = -2\bar{A}_0. \tag{B13}$$

Substituting Eq. (B10) with Eq. (B13) into Eqs. (B8) and (B9), we find that there are no hadronic dense loop contributions at $\mathcal{O}(p^4)$ from the $\mathcal{O}(p)$ Lagrangian (6):

$$\delta_{(1)}[f_\pi^t]^2 = \bar{\Pi}_{\perp(1)}^{tt}(\bar{p},\bar{p}) + \bar{\Pi}_{\perp(1)}^{ts}(\bar{p},\bar{p}) = 0,$$
$$\delta_{(1)}[f_\pi^t f_\pi^s] = -\bar{\Pi}_{\perp(1)}^{ts}(\bar{p},\bar{p}) - \bar{\Pi}_{\perp(1)}^{ss}(\bar{p},\bar{p}) = 0. \tag{B14}$$

As for contributions at $\mathcal{O}(p^5)$ from Eq. (B1), the results are

$$\delta_{(2)}[f_\pi^t]^2 = -4N_c(N_f c_{A1}+c_{A2})m_q\bar{A}_0, \tag{B15}$$
$$\delta_{(2)}[f_\pi^t f_\pi^s] = -4N_c(N_f c_{A1}+c_{A2})m_q\bar{A}_0. \tag{B16}$$

In the small mass limit $m_q \ll P_F$, the corrections in Eqs. (B15) and (B16) vary as

$$\delta_{(2)}[f_\pi^t]^2 = \delta_{(2)}[f_\pi^t f_\pi^s] \xrightarrow[m_q\ll P_F]{} N_c(N_f c_{A1}+c_{A2})\frac{m_q P_F^2}{2\pi^2}. \tag{B17}$$

Thus at the fixed point $(g,a,m_q)=(0,1,0)$, the hadronic dense-loop corrections vanish. This means that we have no contributions to $f_\pi^{t,s}$ from dense-loop terms at the critical point. Similarly there are no hadronic dense-loop corrections to the pion velocity, so we recover $v=1$ at the critical point.

APPENDIX C: VECTOR MESON MASS

In this appendix, we give details of the derivation of Eq. (14) which represents the hadronic dense corrections to the vector meson mass. The relevant piece of Lagrangian that is additional to Eq. (6) is of $\mathcal{O}(p^2)$ and has the form

$$\delta\mathcal{L}_{Q(2V)} = \bar{\psi}\{c_{V1}\mathrm{tr}[\hat{\alpha}_{\parallel\mu}\hat{\alpha}_\parallel^\mu] + c_{V2}\hat{\alpha}_{\parallel\mu}\hat{\alpha}_\parallel^\mu\}\psi + \bar{\psi}c_{V3}[\hat{\alpha}_{\parallel\mu},\hat{\alpha}_{\parallel\nu}][\gamma^\mu,\gamma^\nu]\psi, \tag{C1}$$

where c_{V1}, c_{V2}, and c_{V3} are constants of mass dimension -1.[11] The tadpole diagrams from the Lagrangian in Eq. (C1) generate the $\mathcal{O}(p^5)$ corrections to the second term in Eq. (1). The divergent corrections which are proportional to m_q modify the RGE for F_σ, and thus a. At the critical density these corrections vanish since $m_q\to 0$ for $\mu\to\mu_c$.

From the Lagrangian in Eqs. (6) and (C1) the vector-vector two-point function $\Pi_V^{\mu\nu}$ gets the contributions

$$\Pi_V^{\mu\nu(\mathrm{tree})}(p) = F_\sigma^2 g^{\mu\nu} - \frac{1}{g^2}(g^{\mu\nu}p^2 - p^\mu p^\nu), \tag{C2}$$

$$\bar{\Pi}_{V(1)}^{\mu\nu}(p) = -(1-\kappa)^2[(g^{\mu\nu}p^2-p^\mu p^\nu)\bar{B}_0(p_0,\bar{p}) + 2g^{\mu\nu}\bar{A}_0 + \bar{B}^{\mu\nu}(p_0,\bar{p})], \tag{C3}$$

$$\Pi_{V(2)}^{\mu\nu}(p_0,\vec{p}) = 2N_c g^{\mu\nu}(N_f c_{V1}+c_{V2})(-2\bar{A}_0). \tag{C4}$$

As before, the subscript (n) for $n=1,2$ represents the contribution from the $\mathcal{O}(p^n)$ Lagrangian.

Let us define the vector meson mass through the general form of the vector meson propagator at one-loop level. In HLS at one-loop level, $\Pi_V^{\mu\nu}(p_0,\vec{p})$, which is related to the inverse propagator as in Eq. (A20), can be written as

$$\Pi_V^{\mu\nu}(p_0,\vec{p}) = F_\sigma^2 g^{\mu\nu} - \frac{1}{g^2}(g^{\mu\nu}p^2 - p^\mu p^\nu) + \bar{\Pi}_V^{\mu\nu}(p_0,\vec{p}), \tag{C5}$$

where $\bar{\Pi}_V^{\mu\nu}(p_0,\vec{p})$ denotes the hadronic dense/thermal corrections. Then the components in Eq. (A21) take the following form:

[11] For $N_f=2$, the c_{V1} term and c_{V2} term are not independent. We can set, e.g., $c_{V2}=0$ without loss of generality.

$$\Pi_V^S(p_0;\bar{p})=F_\sigma^2+\bar{\Pi}_V^S(p_0;\bar{p}),$$

$$\Pi_V^L(p_0;\bar{p})=\frac{1}{g^2}p^2+\bar{\Pi}_V^L(p_0;\bar{p}),$$

$$\Pi_V^T(p_0;\bar{p})=\frac{1}{g^2}p^2+\bar{\Pi}_T^L(p_0;\bar{p}),$$

$$\Pi_V^C(p_0;\bar{p})=\bar{\Pi}_V^C(p_0;\bar{p}). \tag{C6}$$

Since the $\Pi_V^C(p_0;\bar{p})$ part does not include $\mathcal{O}(1)$ contributions, we can neglect $\Pi_V^C(p_0;\bar{p})$ in the denominator of the propagator in Eq. (A22). Then the propagator of the field V_μ is expressed as

$$-iD^{\mu\nu}=P_L^{\mu\nu}\frac{1}{p^2/g^2-F_\sigma^2+[\bar{\Pi}_V^L(p_0;\bar{p})-\bar{\Pi}_V^S(p_0;\bar{p})]}+P_T^{\mu\nu}\frac{1}{p^2/g^2-F_\sigma^2+[\bar{\Pi}_V^T(p_0;\bar{p})-\bar{\Pi}_V^S(p_0;\bar{p})]}+P_C^{\mu\nu}\frac{\bar{\Pi}_V^C(p_0;\bar{p})}{F_\sigma^2(p^2/g^2-F_\sigma^2)}+P_D^{\mu\nu}\frac{1}{F_\sigma^2+\bar{\Pi}_V^S(p_0;\bar{p})}. \tag{C7}$$

The vector meson pole mass obtained from the pole of the longitudinal propagator at its rest frame is

$$m_{\rho L}^2-M_\rho^2=-g^2\mathrm{Re}[\bar{\Pi}_V^L(p_0=M_\rho;\bar{p}=0)-\bar{\Pi}_V^S(p_0=M_\rho;\bar{p}=0)], \tag{C8}$$

where $M_\rho=gF_\sigma$ is the tree-level mass, and Re denotes the real part. Here m_ρ is replaced by M_ρ in the loop corrections, since the difference is of higher order. When one uses the transverse component, on the other hand, the vector meson pole mass is given by

$$m_{\rho T}^2-M_\rho^2=-g^2\mathrm{Re}[\bar{\Pi}_V^T(p_0=M_\rho;\bar{p}=0)-\bar{\Pi}_V^S(p_0=M_\rho;\bar{p}=0)]. \tag{C9}$$

Consider now the $\mathcal{O}(p^4)$ correction summarized in Eq. (C3). Decomposing it into four components as in Eq. (A21), we get

$$\bar{\Pi}_{V(1)}^S(p_0,\bar{p})=0,$$

$$\bar{\Pi}_{V(1)}^C(p_0,\bar{p})=0,$$

$$\bar{\Pi}_{V(1)}^L(p_0,\bar{p})=-(1-\kappa)^2[-p^2\bar{B}_0(p_0,\bar{p})+\bar{B}_L(p_0,\bar{p})],$$

$$\bar{\Pi}_{V(1)}^T(p_0,\bar{p})=-(1-\kappa)^2[-p^2\bar{B}_0(p_0,\bar{p})+\bar{B}_T(p_0,\bar{p})]. \tag{C10}$$

By using the formulas at rest

$$\bar{B}_T(p_0,\bar{p}=0)=\bar{B}_L(p_0,\bar{p}=0)=\frac{2}{3}\bar{B}_S+\frac{p_0^2-4m_q^2}{3}\bar{B}_0(p_0,\bar{p}=0) \tag{C11}$$

and

$$\bar{B}_0(p_0,\bar{p}=0)=\frac{1}{2}\int\frac{d^3\vec{k}}{(2\pi)^3}\frac{\theta(P_F-\bar{k})}{\omega_{m_q}(\bar{k})}\frac{1}{p_0^2-4\omega^2(\bar{k})+i\epsilon}=\frac{1}{8\pi^2}\left[-\ln\frac{P_F+\omega_F}{m_q}+\frac{1}{2}\sqrt{\frac{4m_q^2-p_0^2-i\epsilon}{-p_0^2-i\epsilon}}\times\ln\frac{\omega_F\sqrt{4m_q^2-p_0^2-i\epsilon}+P_F\sqrt{-p_0^2-i\epsilon}}{\omega_F\sqrt{4m_q^2-p_0^2-i\epsilon}-P_F\sqrt{-p_0^2-i\epsilon}}\right], \tag{C12}$$

we obtain the corrections to the vector meson pole mass as

$$\delta_{(1)}m_{\rho L}^2=\delta_{(1)}m_{\rho T}^2=\frac{2}{3}g^2(1-\kappa)^2[\bar{B}_S-(M_\rho^2+2m_q^2)\times\mathrm{Re}\bar{B}_0(p_0=M_\rho,0)]. \tag{C13}$$

When we take $M_\rho,m_q\ll P_F$ limit, the above expression becomes

$$\delta_{(1)}m_{\rho L}^2|_{M_\rho,m_q\ll P_F}=\delta_{(1)}m_{\rho T}^2|_{M_\rho,m_q\ll P_F}=\frac{g^2}{6\pi^2}(1-\kappa)^2P_F^2. \tag{C14}$$

Next, let us include the higher order correction $[\mathcal{O}(p^5)]$. From the corrections summarized in Eq. (C4), we obtain

$$\bar{\Pi}_{V(2)}^S(p_0,\bar{p})=2N_c(N_fc_{V1}+c_{V2})\bar{B}_S,$$

$$\bar{\Pi}_{V(2)}^C(p_0,\bar{p})=\bar{\Pi}_{V(2)}^L(p_0,\bar{p})=\bar{\Pi}_{V(2)}^T(p_0,\bar{p})=0. \tag{C15}$$

Then, the corrections to the vector meson pole masses are

$$\delta_{(2)}m_{\rho L}^2=\delta_{(2)}m_{\rho T}^2=g^2 2N_c(N_fc_{V1}+c_{V2})\bar{B}_S. \tag{C16}$$

In the $M_\rho,m_q\ll P_F$ limit, this expression is reduced to

$$\delta_{(2)}m_{\rho L}^2|_{M_\rho,m_q\ll P_F}=\delta_{(2)}m_{\rho T}^2|_{M_\rho,m_q\ll P_F}=\frac{g^2}{2\pi^2}N_c(N_fc_{V1}+c_{V2})P_F^2. \tag{C17}$$

Note that up to $\mathcal{O}(p^6)$ corrections the longitudinal and transverse pole masses are the same. This is the reason for the Lorentz invariant structure of the ρ mass in Eq. (14).

APPENDIX D: MK THEOREM

In this appendix we sketch how to go from the RGEs in $\mathcal{M}$ to the RGEs in μ [24]. As noted in the text, the reasoning is applicable to fundamental theories such as QED or QCD (in the weak-coupling sector), but not without modifications to effective theories such as HLS except perhaps for low temperature or low density.

Denote the renormalized thermodynamic potential $\Omega_R(h_i^R, F_\pi^R, m_q^R, \mu, \mathcal{M})$ where h_i stands generically for $h_1 = g$, $h_2 = a$, and $h_3 = \lambda$ [that is, $\Omega(h_i)$ stands for $\Omega(h_1, h_2, h_3)$], μ is the chemical potential, and $\mathcal{M}$ is the renormalization scale parameter. The RG invariance condition that $\mathcal{M}(d/d\mathcal{M})\Omega_R = 0$ gives

$$\left[\mathcal{M}\frac{\partial}{\partial \mathcal{M}} + \beta(h_i^R)\frac{\partial}{\partial h_i^R} - m_q^R \gamma_m \frac{\partial}{\partial m_q^R} - F_\pi^R \gamma_f \frac{\partial}{\partial F_\pi^R}\right] \times \Omega_R(h_i^R, F_\pi^R, m_q^R, \mu, \mathcal{M}) = 0, \tag{D1}$$

where

$$\beta(h_i^R) = \mathcal{M}\frac{\partial h_i^R}{\partial \mathcal{M}},$$
$$\gamma_m = -\frac{1}{m_q^R}\mathcal{M}\frac{\partial m_q^R}{\partial \mathcal{M}},$$
$$\gamma_f = -\frac{1}{F_\pi^R}\mathcal{M}\frac{\partial F_\pi}{\partial \mathcal{M}}. \tag{D2}$$

Since the thermodynamic potential (or pressure) has a mass dimension 4, it should satisfy the identity

$$\left[\mathcal{M}\frac{\partial}{\partial \mathcal{M}} + \mu\frac{\partial}{\partial \mu} + m_q^R \frac{\partial}{\partial m_q^R} + F_\pi^R \frac{\partial}{\partial F_\pi^R}\right] \times \Omega_R(h_i^R, F_\pi^R, m_q^R, \mu, \mathcal{M}) = 4\Omega_R(h_i^R, F_\pi^R, m_q^R, \mu, \mathcal{M}). \tag{D3}$$

By combining Eqs. (D1) and (D3), we can obtain

$$\left[\mu\frac{\partial}{\partial \mu} - \sum_i \beta(h_i^R)\frac{\partial}{\partial h_i^R} + m_q^R(1+\gamma_{m_q^R})\frac{\partial}{\partial m_q^R} + F_\pi^R(1+\gamma_f)\frac{\partial}{\partial F_\pi^R} - 4\right] \times \Omega_R(h_i^R, F_\pi^R, m_q^R, \mu, \mathcal{M}) = 0. \tag{D4}$$

In the low-density region we expect that the "intrinsic" density dependence of the bare theory is small, and thus we introduce the following ansatz:

$$\Omega_R(h_i^R, F_\pi^R, m_q^R, \mu, \mathcal{M}) = \mu^4 \overline{\Omega}_R(h_i^R, F_\pi^R, m_q^R, \mu, \mathcal{M}). \tag{D5}$$

Then $\overline{\Omega}_R$ satisfies

$$\left[\mu\frac{\partial}{\partial \mu} - \sum_i \beta(h_i(\mu))\frac{\partial}{\partial h_i(\mu)} + m_q(\mu)(1+\gamma_m)\frac{\partial}{\partial m_q(\mu)} + F_\pi(\mu)[1+\gamma_f(\mu)]\frac{\partial}{\partial F_\pi(\mu)}\right] \times \overline{\Omega}_R(h_i(\mu), F_\pi(\mu), m_q(\mu), \mu, \mathcal{M}) = 0, \tag{D6}$$

where

$$\mu\frac{\partial h_i(\mu)}{\partial \mu} = \beta_i(h_i(\mu)),$$
$$\mu\frac{\partial m_q(\mu)}{\partial \mu} = -[1+\gamma_m(h_i(\mu))]m_q(\mu),$$
$$\mu\frac{\partial F_\pi(\mu)}{\partial \mu} = -[1+\gamma_f(h_i(\mu))]F_\pi(\mu), \tag{D7}$$

with the conditions

$$h_i(\mu)|_{\mu=\mathcal{M}} = h_i^R, \tag{D8}$$

etc. From Eqs. (D6) and (D7), Eqs. (16) follow. For instance, we have for F_π

$$\mu\frac{\partial F_\pi^2}{\partial \mu} = -2F_\pi^2\left[1 - \frac{1}{2F_\pi^2}\mu\frac{\partial F_\pi^2}{\partial \mu}\right] = -2F_\pi^2 + C[3a^2g^2F_\pi^2 + 2(2-a)\mu^2] - \frac{m_q^2}{2\pi^2}\lambda^2 N_c. \tag{D9}$$

[1] M. Harada and K. Yamawaki, Phys. Rev. Lett. **86**, 757 (2001).
[2] C. Wetterich, Phys. Rev. D **64**, 036003 (2001); J. Berges and C. Wetterich, Phys. Lett. B **512**, 85 (2001).
[3] G.E. Brown and M. Rho, talk given at INPC2001, Berkeley, California, 2001, nucl-th/0101015.
[4] G.E. Brown and M. Rho, Phys. Rep. **363**, 55 (2002).
[5] G.E. Brown and M. Rho, Phys. Rev. Lett. **66**, 2720 (1991).
[6] M. Harada and C. Sasaki, Phys. Lett. B **537**, 280 (2002).
[7] M. Harada and K. Yamawaki, Phys. Rev. Lett. **87**, 152001 (2001).
[8] M. Bando, T. Kugo, S. Uehara, K. Yamawaki, and T. Yanagida, Phys. Rev. Lett. **54**, 1215 (1985).

[9] M. Bando, T. Kugo, and K. Yamawaki, Phys. Rep. **164**, 217 (1988).
[10] M. Harada and K. Yamawaki, Phys. Lett. B **297**, 151 (1992).
[11] M. Tanabashi, Phys. Lett. B **316**, 534 (1993).
[12] M. Harada and K. Yamawaki, Phys. Rev. D **64**, 014023 (2001).
[13] M. Harada and K. Yamawaki, Phys. Rep. (to be published).
[14] A.P. Szczepaniak and E.S. Swenson, Phys. Rev. Lett. **87**, 072001 (2001).
[15] D.O. Riska and G.E. Brown, Nucl. Phys. **A679**, 577 (2001).
[16] M. Dey, V.L. Eletsky, and B.L. Ioffe, Phys. Lett. B **252**, 620 (1990).
[17] T. Hatsuda and S.H. Lee, Phys. Rev. C **46**, 34 (1992).
[18] G.E. Brown and M. Rho, Phys. Rep. **269**, 333 (1996).
[19] M. Harada and K. Yamawaki, Phys. Rev. Lett. **83**, 3374 (1999).
[20] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).
[21] U.G. Meissner, J.A. Oller, and A. Wirzba, Ann. Phys. (N.Y.) **297**, 27 (2002).
[22] H. Georgi, Phys. Rev. Lett. **63**, 1917 (1989).
[23] G.E. Brown and M. Rho, Phys. Lett. B **338**, 301 (1994); M. Rho, ''Chiral Symmetry Restoration as the Georgi Vector Limit,'' hep-ph/9501403.
[24] P.D. Morley and M.B. Kislinger, Phys. Rep. **51**, 63 (1979).
[25] Y. Kim and H.K. Lee, Phys. Rev. C **62**, 037901 (2000).
[26] G.E. Brown, G.Q. Li, R. Rapp, M. Rho, and J. Wambach, Acta Phys. Pol. B **29**, 2309 (1998).
[27] Y. Kim, R. Rapp, G.E. Brown, and M. Rho, Phys. Rev. C **62**, 015202 (2000).
[28] C. Adami and G.E. Brown, Z. Phys. A **340**, 93 (1991).
[29] See, e.g., R. Rapp and J. Wambach, Adv. Nucl. Phys. **25**, 1 (2000).
[30] See, e.g., J.I. Kapusta, *Finite-Temperature Field Theory* (Cambridge University Press, Cambridge, England, 1989), p. 71.