Neutralino exchange corrections to the Higgs boson mixings with explicit CP violation

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A calculus for the derivatives of the eigenvalues of the neutralino mass matrix with respect to the CP violating background fields is developed and used to compute the mixings among the CP even and the CP odd Higgs sectors arising from the inclusion of the neutralino sector consisting of the neutralino, the Z boson, and the neutral Higgs bosons ($\chi_i^0-Z-h^0-H^0$) exchange in the loop contribution to the effective potential including the effects of large CP violating phases. Along with the top squark, bottom squark, tau slepton and chargino–W–charged-Higgs-boson (χ^+-W-H^+) contributions computed previously the present analysis completes the one loop corrections to the Higgs boson mass matrix in the presence of large phases. CP violation in the neutral Higgs sector is discussed in the above framework with specific focus on the mixings of the CP even and the CP odd sectors arising from the neutralino sector. It is shown that numerically the effects of the neutralino exchange contribution on the mixings of the CP even and the CP odd sectors are comparable to the effects of the top squark and of the chargino exchange contributions and thus the neutralino exchange contribution must be included for a realistic analysis of mixings in the CP even and the CP odd sectors. Phenomenological implications of these results are discussed.

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I. INTRODUCTION

CP violation in supersymmetric theories via soft supersymmetry (SUSY) breaking parameters has received a considerable degree of attention since the beginning of the formulation of supersymmetric models [1]. Recently, there has been enhanced interest in the investigation of their effects due to the realization that supersymmetric theories may allow for large CP violating phases [2] consistent with the electric dipole moment (EDM) of the electron and of the neutron [3]. Such a situation can arise because of several possibilities, such as the SUSY spectrum being heavy [2], due to internal cancellations [4] and due to the possibility that the CP phases may reside in the third generation and consequently their effects on the first two generation EDMs are suppressed [5]. Of course it is possible that a more unified framework may determine the combination of phases that enter the EDMs to be small [6]. However, we shall investigate here the possibility that the phases are large and the EDM constraints are satisfied by one of the methods discussed above so that the sparticle spectrum is consistent with the naturalness constraints (see, e.g., Ref. [7]). In this case their effects on low energy physics can be quite significant and a number of low energy phenomena have been discussed including the effect of CP phases. These include the effect of CP phases on $g-2$ [8], on dark matter [9], on the trileptonic signal [10], on baryogenesis [11], and on other low energy phenomena [12]. Another area where the effect of CP phases has been discussed is the Higgs sector [13–18]. It is well known that loop corrections to the Higgs boson masses and mixings are important [19]. In fact in the absence of the loop corrections the lightest Higgs boson mass must lie below M_Z which is already exprimentally excluded and it is the presence of the loop corrections that raises its value above M_Z. An interesting phenomenon arises if the loop corrections have CP violating phases. In this case it has been pointed out that a significant mixing can occur between the CP even and the CP odd neutral Higgs sectors of the theory [13]. In Refs. [13–18] the effect of CP phases via the top squark and bottom squark exchanges was carried out. Further, in the work of Ref. [16] it was pointed out that the effect of chargino loop corrections can be quite significant and in fact the CP violating effects from the chargino exchange may even dominate the CP violating effects from the top-squark–bottom-squark exchange for the case of large tan β. It should be noted, however, that the ways of avoiding a large electron or neutron EDM in SUSY models become more fine-tuned when the mercury EDM experimental limit is included [20].

In this paper we give an analysis of the one loop correction to the Higgs boson mass including the neutralino–Z-boson–neutral-Higgs-boson exchange including the CP violating phases. The inclusion of the CP dependent neutralino exchange corrections are more intricate relative to the top-squark–bottom-squark exchanges and the chargino exchanges. This is due to the fact that the top-squark–bottom-squark exchange and the chargino exchange involve diagonalization of only 2×2 squark and chargino mass matrices and thus the evaluation of their contribution can be carried out analytically in a straightforward fashion. For the case of the neutralino exchange the neutralino mass matrix is a 4×4 object and its diagonalization analytically is more intricate and a straightforward technique for the analysis is desirable. In this paper we develop a calculus for the derivatives of the eigenvalues of the neutralino mass matrix to obtain an explicit analytic expression for the neutralino exchange contribution. The outline of the rest of the paper is as follows. In Sec. II we give the Higgs potential and discuss the minimization conditions in the presence of the CP violating phases. In Sec. III we discuss the calculus for the computation of derivatives of the eigenvalues of the neutralino mass matrix. In Sec. IV we use the technique of Sec. III and compute the one loop contributions to the Higgs boson mass matrix from the neutralino–Z–neutral-Higgs-boson exchange. Discussion of the numerical results is given in Sec. V. Conclusions are given in Sec. VI. Some further details of the analysis are given in Appendixes A and B.

II. *CP* PHASES AND MINIMIZATION OF HIGGS POTENTIAL

We begin by defining the soft SUSY breaking parameters for the minimal supergravity (MSUGRA) case [21]. Here the low energy physics for the CP conserving case is parametrized by m_0, $m_{1/2}$, A_0, and $\tan\beta$ where m_0 is the universal scalar mass, $m_{1/2}$ is the universalgaugino mass, A_0 is the universal trilinear coupling, and $\tan\beta = v_2/v_1$ is the ratio of the Higgs vacuum expectation values (VEVs), where the VEV of H_2 gives mass to the up quarks and the VEV of H_1 gives mass to the down quarks and the leptons. In the presence of CP violation MSUGRA allows for only two CP violating phases which can be taken to be θ_{μ_0} and α_{A_0}, where θ_{μ_0} is the phase of the Higgs mixing parameter μ_0 and α_{A_0} is the phase of A_0. The analysis of this paper, however, will be more general, valid for the minimal supersymmetric standard model (MSSM) parameter space. The Higgs sector in the MSSM at the one loop level is described by the scalar potential $V(H_1,H_2) = V_0 + \Delta V$ where

$$
\begin{aligned}
V_0 &= m_1^2|H_1|^2 + m_2^2|H_2|^2 + (m_3^2 H_1\cdot H_2 + \text{H.c.}) \\
&\quad + \frac{(g_2^2+g_1^2)}{8}|H_1|^4 + \frac{(g_2^2+g_1^2)}{8}|H_2|^4 - \frac{g_2^2}{2}|H_1\cdot H_2|^2 \\
&\quad + \frac{(g_2^2-g_1^2)}{4}|H_1|^2|H_2|^2 \\
\Delta V &= \frac{1}{64\pi^2}\sum_i c_i(2J_i+1)(-1)^{2J_i}\left[M_i^4(H_1,H_2)\right. \\
&\quad \left.\times\left(\ln\frac{M_i^2(H_1,H_2)}{Q^2} - \frac{3}{2}\right)\right].
\end{aligned}
\tag{1}
$$

Here $m_1^2 = m_{H_1}^2 + |\mu|^2$, $m_2^2 = m_{H_2}^2 + |\mu|^2$, $m_3^2 = |\mu B|$ and $m_{H_{1,2}}$ and B are the soft SUSY breaking parameters, ΔV is the one loop correction to the effective potential [22,23] and includes contributions from all the fields that enter MSSM consisting of the standard model fields and their superpartners, i.e., the sfermions, the gauginos and Higgsinos [23]. The sum over i in Eq. (1) runs over particles with spin J_i and $c_i(2J_i+1)$ counts the degrees of the ith particle, and Q is the renormalization group running scale. It is well known that the one loop corrections to the effective potential can make significant contributions to the Higgs vacuum expectation values in the minimization of the effective potential [23].

In general the effective potential depends on the CP violating phases and its minimization will lead to induced CP violating effects on the Higgs vacuum expectation values [13]. It is found convenient to parametrize the Higgs VEVs in the presence of CP violating effects in the following form

$$
\begin{aligned}
(H_1) &= \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} v_1+\phi_1+i\psi_1 \\ H_1^- \end{pmatrix}, \\
(H_2) &= \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} = \frac{e^{i\theta_H}}{\sqrt{2}}\begin{pmatrix} H_2^+ \\ v_2+\phi_2+i\psi_2 \end{pmatrix}
\end{aligned}
\tag{2}
$$

where θ_H is in general non-vanishing as a consequence of the minimization conditions. Thus the minimization of the potential with respect to the fields $\phi_1,\psi_1,\phi_2,\psi_2$ gives

$$
\begin{aligned}
&\frac{1}{v_2}\left(\frac{\partial\Delta V}{\partial\psi_1}\right)_0 = m_3^2\sin\theta_H \\
&-\frac{1}{v_1}\left(\frac{\partial\Delta V}{\partial\phi_1}\right)_0 = m_1^2 + \frac{g_2^2+g_1^2}{8}(v_1^2-v_2^2) + m_3^2\tan\beta\cos\theta_H .
\end{aligned}
\tag{3}
$$

and

$$
\begin{aligned}
&\frac{1}{v_1}\left(\frac{\partial\Delta V}{\partial\psi_2}\right)_0 = m_3^2\sin\theta_H \\
&-\frac{1}{v_2}\left(\frac{\partial\Delta V}{\partial\phi_2}\right)_0 = m_2^2 - \frac{g_2^2+g_1^2}{8}(v_1^2-v_2^2) + m_3^2\cot\beta\cos\theta_H .
\end{aligned}
\tag{4}
$$

In the above the subscript 0 stands for the fact that we are evaluating the relevant quantities at the point $\phi_1=\phi_2=\psi_1=\psi_2=0$. We note in passing that in Eqs. (3) and (4) only one of the two equations that involve the variation with respect to ψ_1 and ψ_2 is independent [15].

III. CALCULUS FOR DERIVATIVES OF EIGENVALUES OF NEUTRALINO MASS MATRIX

As mentioned in Sec. I, in previous analyzes computations of the CP dependent loop corrections from the top-squark–bottom-squark and from the chargino–W–charged Higgs-boson sectors have been carried out. In these analyses one was able to analytically obtain the eigen values by diagonalizing the 2×2 squark matrices and the 2×2 chargino mass matrix and then differentiate them analytically to obtain the loop correction to the Higgs boson mass matrix. As also pointed out in Sec. I for the neutralino exchange case the situation is more difficult since the neutralino mass matrix is a 4×4 matrix and the analytic solutions for the eigenvalues of the neutralino (mass)^2 matrix are not easily obtained. Here we expand on a technique introduced in Ref. [23] to derive a calculus for the derivatives of the eigenvalues for the neutralino mass matrix. This technique is valid for an arbitrary high order eigenvalue equation. We shall show that quite remarkably even though one cannot analytically solve for the eigenvalues one can analytically solve for the derivatives of the eigenvalues with respect to the background fields in terms of the eigenvalues and the parameters that appear in the eigenvalue equation. To illustrate the procedure we consider an nth order eigenvalue equation

$$F(\lambda)=\mathrm{Det}(M^{\dagger}M-\lambda I)=\lambda^{n}+c^{(n-1)}\lambda^{n-1}+c^{(n-2)}\lambda^{n-2}+\cdots+c^{(1)}\lambda+c^{(0)}=0. \tag{5}$$

Here the coefficients are explicit functions of the background fields

$$\Phi_{\alpha}=\{\phi_1,\phi_2,\psi_1,\psi_2\} \tag{6}$$

while the eigenvalues are implicit functions of the background fields through the satisfaction of the eigenvalue equation. Equation (5) has n eigenvalues which we denote by λ_i $(i=1,2,\ldots,n)$. From Eq. (5) it follows that

$$\frac{\partial\lambda_i}{\partial\Phi_{\alpha}}=-\left(\frac{D_{\alpha}F}{D_{\lambda}F}\right)_{\lambda=\lambda_i} \tag{7}$$

and

$$\frac{\partial^2\lambda_i}{\partial\Phi_{\alpha}\partial\Phi_{\beta}}=\left[-\frac{D_{\alpha}FD_{\beta}FD_{\lambda}^2F}{(D_{\lambda}F)^3}+\frac{D_{\alpha}FD_{\beta}D_{\lambda}F+D_{\beta}FD_{\alpha}D_{\lambda}F}{(D_{\lambda}F)^2}-\frac{D_{\alpha}D_{\beta}F}{D_{\lambda}F}\right]_{\lambda=\lambda_i} \tag{8}$$

where D_{λ} differentiates the λ dependence in F

$$D_{\lambda}F(\lambda)=\frac{dF}{d\lambda} \tag{9}$$

and D_{α} differentiates only the coefficients in Eq. (5), i.e.,

$$D_{\alpha}F=c_{\alpha}^{(n-1)}\lambda^{(n-1)}+c_{\alpha}^{(n-2)}\lambda^{(n-2)}+\cdots+c_{\alpha}^{(1)}\lambda+c_{\alpha}^{(0)}. \tag{10}$$

$D_{\alpha}D_{\beta}F$ are similarly defined where $c_{\alpha}^{(k)}$ etc are replaced with $c_{\alpha\beta}^{(k)}$ where

$$c_{\alpha}^{(k)}=\frac{\partial c^{(k)}}{\partial\Phi_{\alpha}},\quad c_{\alpha\beta}^{(k)}=\frac{\partial^2 c^{(k)}}{\partial\Phi_{\alpha}\partial\Phi_{\beta}} \tag{11}$$

and the derivatives $D_{\alpha}D_{\lambda}$ are defined in an obvious way. We note in passing that D_{α} and D_{λ} commute

$$[D_{\alpha},D_{\lambda}]=0. \tag{12}$$

Equations (7) and (8) are the central equations of our analysis. It is easy to check that for the 2×2 matrix case, e.g., for the top squark and the chargino exchanges, they give exactly the results obtained by explicit differentiation of the eigenvalues. However, now these equations provide us with a technique for analyzing cases where the analytic solutions to the eigenvalues are not available.

IV. NEUTRALINO, *Z* AND NEUTRAL HIGGS LOOP CONTRIBUTIONS

As mentioned above the CP dependent contributions to the Higgs boson masses from top squark and bottom squark exchanges have been discussed at length in the literature [13–18]. More recently the CP dependent chargino–W–charged-Higgs-boson contributions were also discussed [16]. In this work we use the technique discussed in Sec. III to compute the contribution from the neutralino–Z–neutral-Higgs-boson exchange. The loop correction in this subsector is given by

$$\begin{aligned}\Delta V(\chi_i^0,Z,h^0,H^0)&=\frac{1}{64\pi^2}\left[\sum_{i=1}^{4}(-2)M_{\chi_i^0}^4\left(\ln\frac{M_{\chi_i^0}^2}{Q^2}-\frac{3}{2}\right)\right.\\&\quad+3M_Z^4\left(\ln\frac{M_Z^2}{Q^2}-\frac{3}{2}\right)+M_{h^0}^4\ln\left(\frac{M_{h^0}^2}{Q^2}-\frac{3}{2}\right)\\&\quad\left.+M_{H^0}^4\ln\left(\frac{M_{H^0}^2}{Q^2}-\frac{3}{2}\right)\right].\end{aligned} \tag{13}$$

The neutralino mass matrix is given by

$$M_{\chi^0}=\begin{pmatrix}\tilde{m}_1 & 0 & -\frac{g_1}{\sqrt{2}}H_1^0 & \frac{g_1}{\sqrt{2}}H_2^0\\ 0 & \tilde{m}_2 & \frac{g_2}{\sqrt{2}}H_1^0 & -\frac{g_2}{\sqrt{2}}H_2^0\\ -\frac{g_1}{\sqrt{2}}H_1^0 & \frac{g_2}{\sqrt{2}}H_1^0 & 0 & -\mu\\ \frac{g_1}{\sqrt{2}}H_2^0 & -\frac{g_2}{\sqrt{2}}H_2^0 & -\mu & 0\end{pmatrix} \tag{14}$$

where $\mu=|\mu|e^{i\theta_{\mu}}$, $\tilde{m}_1=|\tilde{m}_1|e^{i\xi_1}$ and $\tilde{m}_2=|\tilde{m}_2|e^{i\xi_2}$. We note that in the supersymmetric limit $M_{\chi_i^0}=(0,0,M_Z,M_Z)$ and $(M_{h^0},M_{H^0})=(M_Z,0)$ and consequently in this limit the loop corrections from this subsector vanish. We return now to the full analysis and follow the method described in Ref. [16] to minimize the potential and compute the loop corrections. First we give the determination of θ_H from the minimization constraints including the top squark, the bottom squark, the tau slepton, the chargino and neutralino contributions. One finds that θ_H is given by the equation

$$m_3^2\sin\theta_H=\frac{1}{2}\beta_{h_t}|\mu||A_t|\sin\gamma_t f_1(m^2_{\tilde t_1},m^2_{\tilde t_2})+\frac{1}{2}\beta_{h_b}|\mu||A_b|\sin\gamma_b f_1(m^2_{\tilde b_1},m^2_{\tilde b_2})+\frac{1}{2}\beta_{h_\tau}|\mu||A_\tau|\sin\gamma_\tau f_1(m^2_{\tilde\tau_1},m^2_{\tilde\tau_2})$$
$$-\frac{g_2^2}{16\pi^2}|\mu||\tilde m_2|\sin\gamma_2 f_1(m^2_{\chi_1},m^2_{\chi_2})+\frac{1}{16\pi^2}\sum_{j=1}^{4}\frac{M^2_{\chi_j^0}}{D_j}\left[\ln\left(\frac{M^2_{\chi_j^0}}{Q^2}\right)-1\right](M^4_{\chi_j^0}(-g_2^2|\mu||\tilde m_2|\sin\gamma_2$$
$$-g_1^2|\mu||\tilde m_1|\sin\gamma_1)+M^2_{\chi_j^0}[g_2^2(|\tilde m_1|^2+|\mu|^2)|\tilde m_2||\mu|\sin\gamma_2+g_1^2(|\tilde m_1|^2+|\mu|^2)|\tilde m_1||\mu|\sin\gamma_1]$$
$$+(-g_2^2|\tilde m_1|^2|\mu|^3|\tilde m_2|\sin\gamma_2-g_1^2|\tilde m_2|^2|\mu|^3|\tilde m_1|\sin\gamma_1)) \qquad (15)$$

where

$$2D_j\equiv(D_\lambda F)_{\lambda=\lambda_j}=4M^6_{\chi_j^0}+3aM^4_{\chi_j^0}+2bM^2_{\chi_j^0}+c$$

$$\beta_{h_t}=\frac{3h_t^2}{16\pi^2},\quad \beta_{h_b}=\frac{3h_b^2}{16\pi^2},\quad \beta_{h_\tau}=\frac{3h_\tau^2}{16\pi^2} \qquad (16)$$

$$\gamma_t=\alpha_{A_t}+\theta_\mu,\quad \gamma_b=\alpha_{A_b}+\theta_\mu,\quad \gamma_\tau=\alpha_{A_\tau}+\theta_\mu,\quad \gamma_1=\xi_1+\theta_\mu,\gamma_2=\xi_2+\theta_\mu$$

and where a,b,c are defined in Appendix A and $f_1(u,v)$ is given by

$$f_1(u,v)=-2+\ln\frac{uv}{Q^4}+\frac{v+u}{v-u}\ln\frac{v}{u}. \qquad (17)$$

To construct the mass squared matrix of the Higgs scalars we need to compute the quantity

$$M^2_{\alpha\beta}=\left(\frac{\partial^2 V}{\partial\Phi_\alpha\partial\Phi_\beta}\right)_0=M^{2(0)}_{\alpha\beta}+\Delta M^2_{\alpha\beta} \qquad (18)$$

where $M^{2(0)}_{\alpha\beta}$ is the contribution from V_0 and $\Delta M^2_{\alpha\beta}$ is the contribution from ΔV where $\Phi_\alpha(\alpha=1-4)$ are defined by Eq. (6) and as already mentioned earlier the subscript 0means that we set $\phi_1=\phi_2=\psi_1=\psi_2=0$ after evaluating the mass matrix. The loop contribution $\Delta M^2_{\alpha\beta}$ arising from the neutralino–Z–neutral-Higgs-boson sector is given by

$$\Delta M^2_{\alpha\beta}=\frac{1}{32\pi^2}\mathrm{Str}\left[\frac{\partial M^2}{\partial\Phi_\alpha}\frac{\partial M^2}{\partial\Phi_\beta}\ln\frac{M^2}{Q^2}+M^2\frac{\partial^2M^2}{\partial\Phi_\alpha\partial\Phi_\beta}\left(\ln\frac{M^2}{Q^2}-1\right)\right]_0. \qquad (19)$$

Computation of the 4×4 Higgs boson mass matrix in the basis of Eq. (6) gives

$$\begin{pmatrix} M_Z^2c_\beta^2+M_A^2s_\beta^2+\Delta_{11} & -(M_Z^2+M_A^2)s_\beta c_\beta+\Delta_{12} & \Delta_{13}s_\beta & \Delta_{13}c_\beta\\ -(M_Z^2+M_A^2)s_\beta c_\beta+\Delta_{12} & M_Z^2s_\beta^2+M_A^2c_\beta^2+\Delta_{22} & \Delta_{23}s_\beta & \Delta_{23}c_\beta\\ \Delta_{13}s_\beta & \Delta_{23}s_\beta & (M_A^2+\Delta_{33})s_\beta^2 & (M_A^2+\Delta_{33})s_\beta c_\beta\\ \Delta_{13}c_\beta & \Delta_{23}c_\beta & (M_A^2+\Delta_{33})s_\beta c_\beta & (M_A^2+\Delta_{33})c_\beta^2\end{pmatrix} \qquad (20)$$

where $c_\beta(s_\beta)=\cos\beta(\sin\beta)$ and m_A^2 is given by

$$m_A^2=(\sin\beta\cos\beta)^{-1}\Bigg(-m_3^2\cos\theta+\frac{1}{2}\beta_{h_t}|A_t||\mu|\cos\gamma_t f_1(m^2_{\tilde{t}_1},m^2_{\tilde{t}_2})+\frac{1}{2}\beta_{h_b}|A_b||\mu|\cos\gamma_b f_1(m^2_{\tilde{b}_1},m^2_{\tilde{b}_2})$$
$$+\frac{1}{2}\beta_{h_\tau}|A_\tau||\mu|\cos\gamma_\tau f_1(m^2_{\tilde{\tau}_1},m^2_{\tilde{\tau}_2})+\frac{g_2^2}{16\pi^2}|\tilde{m}_2||\mu|\cos\gamma_2 f_1(m^2_{\chi_1^+},m^2_{\chi_2^+})-\frac{1}{16\pi^2}\sum_{j=1}^{4}\frac{M^2_{\chi_j}}{D_j}\left[\ln\left(\frac{M^2_{\chi_j}}{Q^2}\right)-1\right][M^4_{\chi_j}$$
$$\times(-g_2^2|\mu||\tilde{m}_2|\cos\gamma_2-g_1^2|\mu||\tilde{m}_1|\cos\gamma_1)+M^2_{\chi_j}(g_2^2(|\tilde{m}_1|^2+|\mu|^2)|\mu||\tilde{m}_2|\cos\gamma_2+g_1^2(|\tilde{m}_2|^2+|\mu|^2)|\mu||\tilde{m}_1|\cos\gamma_1)$$
$$-g_2^2|\tilde{m}_1|^2|\mu|^3|\tilde{m}_2|\cos\gamma_2-g_1^2|\tilde{m}_2|^2|\mu|^3|\tilde{m}_1|\cos\gamma_1]\Bigg). \tag{21}$$

The first term in the second brace on the right-hand side of Eq. (21) is the tree term, while the second, the third, the fourth and the fifth terms come from the top squark, bottom squark, tau slepton and chargino exchange contributions. The remaining contributions in Eq. (21) arise from the neutralino sector. The Δ's appearing in Eq. (20) can be decomposed as follows:

$$\Delta_{\alpha\beta}=\Delta_{\alpha\beta\tilde{t}}+\Delta_{\alpha\beta\tilde{b}}+\Delta_{\alpha\beta\tilde{\tau}}+\Delta_{\alpha\beta\chi^+}+\Delta_{\alpha\beta\chi^0} \tag{22}$$

where $\Delta_{\alpha\beta\tilde{t}}$ is the contribution from the top squark (and top) exchange in the loops, $\Delta_{\alpha\beta\tilde{b}}$ is the contribution from the bottom squark (and bottom) exchange in the loops, $\Delta_{\alpha\beta\tilde{\tau}}$ is the contribution from the tau slepton (and tau) exchange, $\Delta_{\alpha\beta\chi^+}$ is the contribution from the chargino (and W and charged Higgs boson) exchange in the loops, and $\Delta_{\alpha\beta\chi^0}$ is the contribution arising from the neutralino (and Z and neutral Higgs boson exchange) in the loops. The computations of $\Delta_{\alpha\beta\tilde{t}}$, $\Delta_{\alpha\beta\tilde{b}}$, $\Delta_{\alpha\beta\tilde{\tau}}$, and $\Delta_{\alpha\beta\chi^+}$ have been given before and are not reproduced here. We compute here only the $\Delta_{\alpha\beta\chi^0}$ arising from the $(\chi_i^0-Z-h^0-H^0)$ exchange. The $\Delta_{\alpha\beta\chi^0}$ are listed below:

$$\Delta_{11\chi^0}=-\frac{1}{16\pi^2}\sum_{j=1}^{4}M^2_{\chi_j}\left[\ln\left(\frac{M^2_{\chi_j}}{Q^2}\right)-1\right]\Bigg\{-\frac{(a_1M^6_{\chi_j}+b_1M^4_{\chi_j}+c_1M^2_{\chi_j}+d_1)^2(12M^4_{\chi_j}+6aM^2_{\chi_j}+2b)}{D_j^3}$$
$$+\frac{2(a_1M^6_{\chi_j}+b_1M^4_{\chi_j}+c_1M^2_{\chi_j}+d_1)(3a_1M^4_{\chi_j}+2b_1M^2_{\chi_j}+c_1)}{D_j^2}\Bigg\}-\frac{1}{16\pi^2}\sum_{j=1}^{4}\frac{(a_1M^6_{\chi_j}+b_1M^4_{\chi_j}+c_1M^2_{\chi_j}+d_1)^2}{D_j^2}\ln\left(\frac{M^2_{\chi_j}}{Q^2}\right)$$
$$+\frac{3}{128\pi^2}(g_1^2+g_2^2)^2v_1^2\ln\left(\frac{M_Z^2}{Q^2}\right)-\frac{1}{32\pi^2}\Bigg(\frac{1}{16}\frac{A_0^2}{(M^2_{H^0}-M^2_{h^0})^2}f_2(M^2_{H^0},M^2_{h^0})-\frac{1}{16}(g_1^2+g_2^2)^2v_1^2\ln\frac{M^2_{H^0}M^2_{h^0}}{Q^4}$$
$$-\frac{1}{8}(g_1^2+g_2^2)\frac{v_1A_0}{(M^2_{H^0}-M^2_{h^0})}\ln\frac{M^2_{H^0}}{M^2_{h^0}}\Bigg) \tag{23}$$

where D_j is defined in Eq. (16) and f_2 is defined by

$$f_2(u,v)=-2+\frac{v+u}{v-u}\ln\frac{v}{u} \tag{24}$$

$$\Delta_{22\chi^0}=-\frac{1}{16\pi^2}\sum_{j=1}^{4}M^2_{\chi_j}\left[\ln\left(\frac{M^2_{\chi_j}}{Q^2}\right)-1\right]\Bigg\{-\frac{(a_2M^6_{\chi_j}+b_2M^4_{\chi_j}+c_2M^2_{\chi_j}+d_2)^2(12M^4_{\chi_j}+6aM^2_{\chi_j}+2b)}{D_j^3}$$
$$+\frac{2(a_2M^6_{\chi_j}+b_2M^4_{\chi_j}+c_2M^2_{\chi_j}+d_2)(3a_2M^4_{\chi_j}+2b_2M^2_{\chi_j}+c_2)}{D_j^2}\Bigg\}$$
$$-\frac{1}{16\pi^2}\sum_{j=1}^{4}\frac{(a_2M^6_{\chi_j}+b_2M^4_{\chi_j}+c_2M^2_{\chi_j}+d_2)^2}{D_j^2}\ln\left(\frac{M^2_{\chi_j}}{Q^2}\right)+\frac{3}{128\pi^2}(g_1^2+g_2^2)^2v_2^2\ln\left(\frac{M_Z^2}{Q^2}\right)$$
$$-\frac{1}{32\pi^2}\Bigg(\frac{1}{16}\frac{B_0^2}{(M^2_{H^0}-M^2_{h^0})^2}f_2(M^2_{H^0},M^2_{h^0})-\frac{1}{16}(g_1^2+g_2^2)^2v_2^2\ln\frac{M^2_{H^0}M^2_{h^0}}{Q^4}-\frac{1}{8}(g_1^2+g_2^2)\frac{v_2B_0}{(M^2_{H^0}-M^2_{h^0})}\ln\frac{M^2_{H^0}}{M^2_{h^0}}\Bigg) \tag{25}$$

$$\Delta_{12\chi^0} = -\frac{1}{16\pi^2}\sum_{j=1}^{4} M_{\chi_j}^2\left[\ln\left(\frac{M_{\chi_j}^2}{Q^2}\right)-1\right]$$
$$\times\left\{-\frac{(a_1M_{\chi_j}^6+b_1M_{\chi_j}^4+c_1M_{\chi_j}^2+d_1)(a_2M_{\chi_j}^6+b_2M_{\chi_j}^4+c_2M_{\chi_j}^2+d_2)(12M_{\chi_j}^4+6aM_{\chi_j}^2+2b)}{D_j^3}\right.$$
$$+\frac{(a_1M_{\chi_j}^6+b_1M_{\chi_j}^4+c_1M_{\chi_j}^2+d_1)(3a_2M_{\chi_j}^4+2b_2M_{\chi_j}^2+c_2)}{D_j^2}$$
$$\left.+\frac{(a_2M_{\chi_j}^6+b_2M_{\chi_j}^4+c_2M_{\chi_j}^2+d_2)(3a_1M_{\chi_j}^4+2b_1M_{\chi_j}^2+c_1)}{D_j^2}\right\}$$
$$-\frac{1}{16\pi^2}\sum_{j=1}^{4}\frac{(a_1M_{\chi_j}^6+b_1M_{\chi_j}^4+c_1M_{\chi_j}^2+d_1)(a_2M_{\chi_j}^6+b_2M_{\chi_j}^4+c_2M_{\chi_j}^2+d_2)}{D_j^2}\ln\left(\frac{M_{\chi_j}^2}{Q^2}\right)$$
$$+\frac{3}{128\pi^2}(g_1^2+g_2^2)^2v_1v_2\ln\left(\frac{M_Z^2}{Q^2}\right)-\frac{1}{32\pi^2}\left(\frac{1}{16}\frac{A_0B_0}{(M_{H^0}^2-M_{h^0}^2)^2}f_2(M_{H^0}^2,M_{h^0}^2)-\frac{1}{16}(g_1^2+g_2^2)^2v_1v_2\ln\frac{M_{H^0}^2M_{h^0}^2}{Q^4}\right.$$
$$\left.-\frac{1}{16}(g_1^2+g_2^2)\frac{v_1B_0+v_2A_0}{(M_{H^0}^2-M_{h^0}^2)}\ln\frac{M_{H^0}^2}{M_{h^0}^2}\right) \tag{26}$$

$$\Delta_{13\chi^0} = -\frac{1}{16\pi^2}\sum_{j=1}^{4} M_{\chi_j}^2\left[\ln\left(\frac{M_{\chi_j}^2}{Q^2}\right)-1\right]\frac{1}{\sin\beta}$$
$$\times\left\{-\frac{(a_1M_{\chi_j}^6+b_1M_{\chi_j}^4+c_1M_{\chi_j}^2+d_1)(a_3M_{\chi_j}^6+b_3M_{\chi_j}^4+c_3M_{\chi_j}^2+d_3)(12M_{\chi_j}^4+6aM_{\chi_j}^2+2b)}{D_j^3}\right.$$
$$+\frac{(a_1M_{\chi_j}^6+b_1M_{\chi_j}^4+c_1M_{\chi_j}^2+d_1)(3a_3M_{\chi_j}^4+2b_3M_{\chi_j}^2+c_3)}{D_j^2}$$
$$\left.+\frac{(a_3M_{\chi_j}^6+b_3M_{\chi_j}^4+c_3M_{\chi_j}^2+d_3)(3a_1M_{\chi_j}^4+2b_1M_{\chi_j}^2+c_1)}{D_j^2}\right\}$$
$$-\frac{1}{16\pi^2}\sum_{j=1}^{4}\frac{1}{\sin\beta}\frac{(a_1M_{\chi_j}^6+b_1M_{\chi_j}^4+c_1M_{\chi_j}^2+d_1)(a_3M_{\chi_j}^6+b_3M_{\chi_j}^4+c_3M_{\chi_j}^2+d_3)}{D_j^2}\ln\left(\frac{M_{\chi_j}^2}{Q^2}\right) \tag{27}$$

$$\Delta_{23\chi^0} = -\frac{1}{16\pi^2}\sum_{j=1}^{4} M_{\chi_j}^2\left[\ln\left(\frac{M_{\chi_j}^2}{Q^2}\right)-1\right]\frac{1}{\cos\beta}$$
$$\times\left(-\frac{(a_3'M_{\chi_j}^6+b_3'M_{\chi_j}^4+c_3'M_{\chi_j}^2+d_3')(a_2M_{\chi_j}^6+b_2M_{\chi_j}^4+c_2M_{\chi_j}^2+d_2)(12M_{\chi_j}^4+6aM_{\chi_j}^2+2b)}{D_j^3}\right.$$
$$+\frac{(a_3'M_{\chi_j}^6+b_3'M_{\chi_j}^4+c_3'M_{\chi_j}^2+d_3')(3a_2M_{\chi_j}^4+2b_2M_{\chi_j}^2+c_2)}{D_j^2}$$
$$\left.+\frac{(a_2M_{\chi_j}^6+b_2M_{\chi_j}^4+c_2M_{\chi_j}^2+d_2)(3a_3'M_{\chi_j}^4+2b_3'M_{\chi_j}^2+c_3')}{D_j^2}\right)$$

$$-\frac{1}{16\pi^2}\sum_{j=1}^{4}\frac{1}{\cos\beta}\frac{(a_3'M_{\chi_j}^6+b_3'M_{\chi_j}^4+c_3'M_{\chi_j}^2+d_3')(a_2M_{\chi_j}^6+b_2M_{\chi_j}^4+c_2M_{\chi_j}^2+d_2)}{D_j^2}\ln\left(\frac{M_{\chi_j}^2}{Q^2}\right) \tag{28}$$

$$\Delta_{33\chi^0}=-\frac{1}{16\pi^2}\sum_{j=1}^{4}M_{\chi_j}^2\left[\ln\left(\frac{M_{\chi_j}^2}{Q^2}\right)-1\right]\left(-\frac{(a_3'M_{\chi_j}^6+b_3'M_{\chi_j}^4+c_3'M_{\chi_j}^2+d_3')^2(12M_{\chi_j}^4+6aM_{\chi_j}^2+2b)}{D_j^3}\frac{1}{\cos^2\beta}\right.$$
$$\left.+\frac{2(a_3'M_{\chi_j}^6+b_3'M_{\chi_j}^4+c_3'M_{\chi_j}^2+d_3')(3a_3'M_{\chi_j}^4+2b_3'M_{\chi_j}^2+c_3')}{D_j^2}\frac{1}{\cos^2\beta}\right)$$
$$-\frac{1}{16\pi^2}\sum_{j=1}^{4}\frac{1}{\cos^2\beta}\frac{(a_3'M_{\chi_j}^6+b_3'M_{\chi_j}^4+c_3'M_{\chi_j}^2+d_3')^2}{D_j^2}\ln\left(\frac{M_{\chi_j}^2}{Q^2}\right). \tag{29}$$

The parameters a,b,c and the derivatives a_i,b_i,c_i,d_i ($i=1,2$, etc.) that appear in Eqs. (23)–(29) are defined in Appendixes A and B. Equations (23)–(29) constitute the main new theoretical results of this paper. These results along with the computations of $\Delta_{\alpha\beta\tilde{t}}$, $\Delta_{\alpha\beta\tilde{b}}$, $\Delta_{\alpha\beta\tilde{\tau}}$ and $\Delta_{\alpha\beta\chi^+}$ give a complete determination of the CP dependent one loop contributions to the Higgs boson masses and mixings. As has been noted beforeit is preferable to work with a 3×3 matrix rather than the 4×4 matrix of Eq. (20). The desired 3×3 matrix can be obtained from Eq. (20) by going to the basis

$$\psi_{1D}=\sin\beta\psi_1+\cos\beta\psi_2,\quad \psi_{2D}=-\cos\beta\psi_1+\sin\beta\psi_2. \tag{30}$$

In this basis the field ψ_{2D} is the zero mass Goldstone boson and decouples while the remaining (mass)^2 matrix in the basis ϕ_1, ϕ_2, ψ_{1D} is given by

$$M_{Higgs}^2=\begin{pmatrix} M_Z^2c_\beta^2+M_A^2s_\beta^2+\Delta_{11} & -(M_Z^2+M_A^2)s_\beta c_\beta+\Delta_{12} & \Delta_{13}\\ -(M_Z^2+M_A^2)s_\beta c_\beta+\Delta_{12} & M_Z^2s_\beta^2+M_A^2c_\beta^2+\Delta_{22} & \Delta_{23}\\ \Delta_{13} & \Delta_{23} & (M_A^2+\Delta_{33})\end{pmatrix}. \tag{31}$$

We label the eigenvalues for this case $m_{H_1}^2,m_{H_2}^2,m_{H_3}^2$ corresponding to the eigenstates H_1,H_2,H_3. These eigenstates are in general admixtures of the CP even and the CP odd states due to the mixing generated by Δ_{13} and Δ_{23}. Thus the CP even-odd mixings arise from Δ_{13} and Δ_{23} and these are nonvanishing only in the presence of CP violation and vanish when the phases go to zero. In this limit one recovers the usual result of two distinct (one CP even and the other CP odd) Higgs boson sectors. We note in passing that Δ_{33} also vanishes in the limit when the CP phases go to zero. This was also the behavior that was observed when the contributions from the top squark, bottom squark, tau slepton and chargino exchanges were considered. Since the main point of this work is to study the phenomenon of CP even-odd mixing the main focus of our analysis is the computation of Δij and specifically of Δ_{13} and Δ_{23} which are the basic sources of mixings between the CP even and the CP odd sectors. We order the eigenvalues of Eq. (31) in such a way that in the limit of no CP violation one has $(m_{H_1},m_{H_2},m_{H_3})\to(m_H,m_h,m_A)$ and $(H_1,H_2,H_3)\to(H,h,A)$ where (h,H) are (light, heavy) CP even Higgs and A is the CP odd Higgs boson in the absence of CP violation.

V. DISCUSSION OF THE NEUTRALINO EXCHANGE CONTRIBUTION TO *CP* EVEN *CP* ODD HIGGS BOSON MIXING

The analytical results given above are quite general as they apply to the MSSM parameter space. However, the MSSM parameter space is quite large. Thus for a numerical study of the CP effects including those from the neutralino sector we will work with a constrained set of parameters consisting of the parameter space m_0, $m_{1/2}$, m_A, $|A_0|$, $\tan\beta$, θ_μ, α_{A_0}, ξ_1, ξ_2 and ξ_3. Starting with these all other low energy parameters are obtained by a renormalization group evolution by running the parameters from the grand unified theory (GUT) scale down to the electroweak scale. Of course one is free to utilize the formulaes derived above for the more general MSSM parameter space. As discussed in Sec. I one can satisfy the EDM constraints in the presence of large phases. This can come about in a variety of ways. As pointed out in Sec. I one possibility is that the internal cancellations can occur which allow for large phases consistent with the EDM constraints. The other possibility is that CP phases appear only in the third generation which suppresses their contributions to the EDMs of the quarks and the leptons

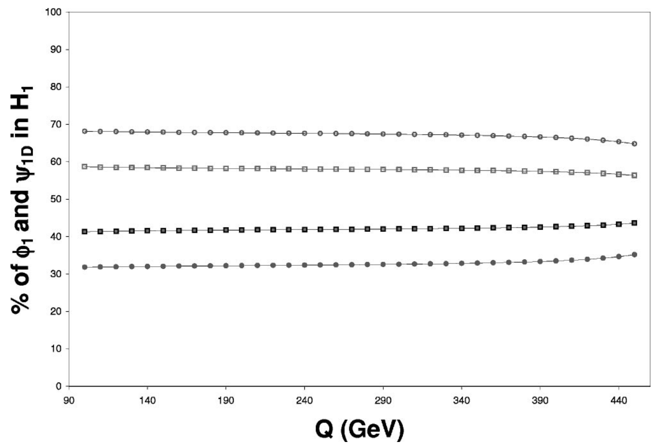

FIG. 1. Plot of the CP even component ϕ_1 of H_1 (upper curves) and the CP odd component ψ_{1D} of H_1 (lower curves) including the top squark, bottom squark, tau slepton, chargino and neutralino sector contributions as a function of the scale Q. The common parameters are $m_A=300$, $\tan\beta=15$, $m_0=100$, $m_{1/2}=500$, $\xi_1=.4$, $\xi_2=.5$, $\alpha_0=.3$, and $|A_0|=1$. The curves with circles are for $\theta_\mu=0.1$ and with squares for $\theta_\mu=0.2$ where all masses are in GeV and all angles are in radians.

in the first two generations to achieve consistency with the experimental constraints. There also exist scenarios which are linear combinations of these two. For the purpose of this analysis we do not revisit the problem of the satisfaction of the EDM constraints. Rather we shall assume that regions of the parameter space exist where such constraints are satisfied and examine the effect of the phases on the Higgs boson masses and mixing. Specifically we are interested in the effects of the neutralino exchange contributions on Δ_{13} and Δ_{23}, and thus, on the mixings of the CP even and the CP sectors.

It was pointed out in Sec. IV that the neutralino, the Z and the neutral Higgs boson exchanges together form a subsector so that in the supersymmetric limit one finds that the one loop correction to the effective potential from this subsector vanishes. This phenomenon is similar to what was also seen in the exchange of the chargino, the W and the charged Higgs boson where the contribution from that sector to the effective potential vanishes in the supersymmetric limit. It was also seen in the analysis of the chargino–W–charged-Higgs-boson exchange that the CP even-odd mixing arising from this sector was roughly Q independent because of the sum of the three separate contributions within this sector. A very similar situation is also realized in the neutralino sector. Here again because of the contributions from the neutralino, the Z and the neutral Higgs boson exchanges their sum contribution to the CP even-odd mixing is roughly scale independent. However, unlike the chargino–W–charged-Higgs-boson exchange where one could demonstrate the above phenomenon analytically, here one has to demonstrate it numerically due to the more analytically complex nature of the results. This is exhibited in Fig. 1 where a plot the percentage of the CP even component ϕ_1 and the CP odd component ψ_{1D} of H_1 as a function of Q is given. The analysis shows an approximate independence in Q of the

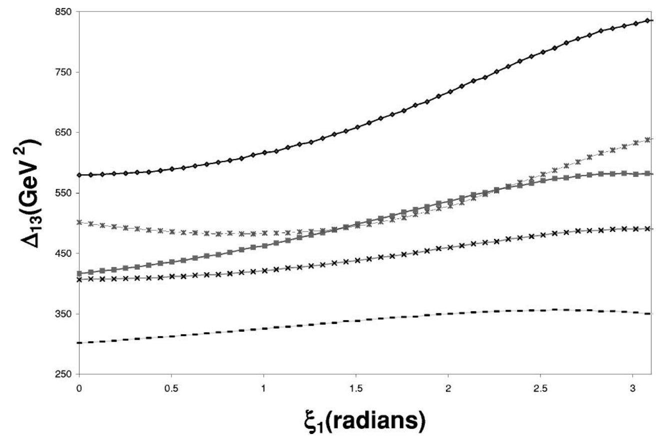

FIG. 2. Plot of Δ_{13} including the top squark, bottom squark, tau slepton, chargino and neutralino sector contributions vs the U(1) gaugino phase ξ_1. The common input for all the curves are $m_0=100$, $m_{1/2}=500$, $M_A=300$, $|A_0|=1$, $\alpha_0=0.3$, $\xi_2=0.5$ and $Q=320$. The five curves correspond to the pairs of $\tan\beta$ and θ_μ values as follows. The curve with $\Delta_{13}=301$ at $\xi_1=0$ corresponds to $\tan\beta=5$, $\theta_\mu=.4$. Similarly the curves with values of $\Delta_{13}=406$ at $\xi_1=0$ correspond to $\tan\beta=6$, $\theta_\mu=.6$, $\Delta_{13}=416$, at $\xi_1=0$ correspond to $\tan\beta=10$, $\theta_\mu=.2$, $\Delta_{13}=501$ at $\xi_1=0$ correspond to $\tan\beta=8$, $\theta_\mu=.8$, and $\Delta_{13}=579$ at $\xi_1=0$ correspond to $\tan\beta=15$, $\theta_\mu=.3$ where all masses are in GeV and all angles are in radians.

CP even-odd mixing. We turn now to a discussion of other aspects of the analysis below.

In Fig. 2 we plot the quantity Δ_{13} as a function of the CP phase of the U(1) gaugino mass ξ_1. The plots exhibited in Fig. 2 contain the top squark, the bottom squark, the tau slepton, the chargino and the neutralino exchange contributions. Among the above exchanges the neutralino exchange contribution is the only one that depends on ξ_1, and thus the variation of Δ_{13} with ξ_1 arises only from this exchange. From Fig. 2 the size of the neutralino exchange contribution can be seen to be fairly substantial. Specifically, the analysis

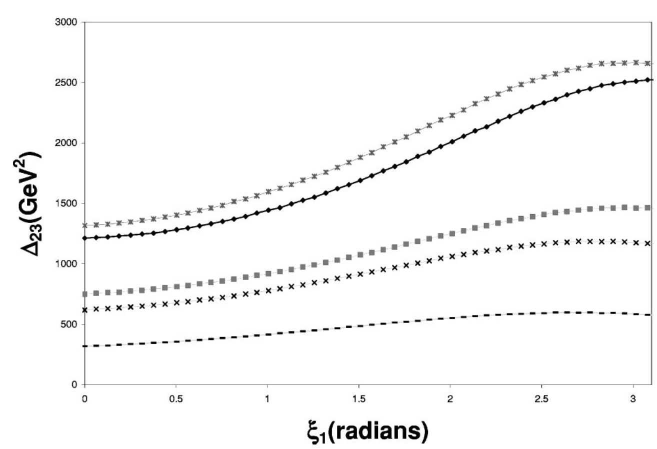

FIG. 3. Plot of Δ_{23} including the top squark, bottom squark, tau slepton, chargino and neutralino sector contributions vs the U(1) gaugino phase ξ_1 for the same input parameters as in Fig. 2. The curves with the same symbols as in Fig. 2 have the same common inputs.

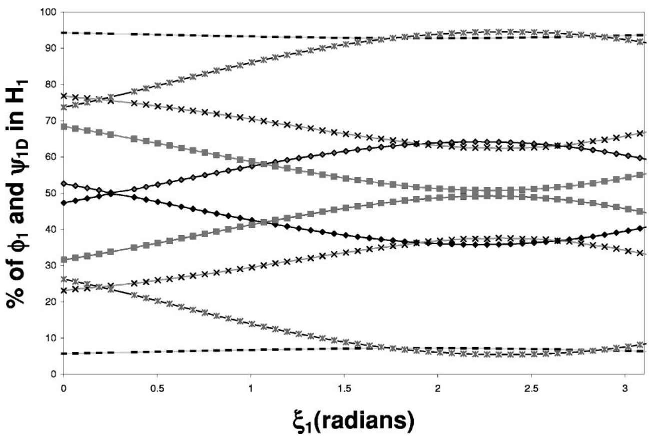

FIG. 4. Plot of the CP even component ϕ_1 of H_1 (upper curves) and the CP odd component ψ_{1D} of H_1 (lower curves) including the top squark, bottom squark, tau slepton, chargino and neutralino sector contributions as a function of the U(1) gaugino phase ξ_1 for the same inputs as in Fig. 2. The curves with the same symbols as in Fig. 2 have the same common inputs.

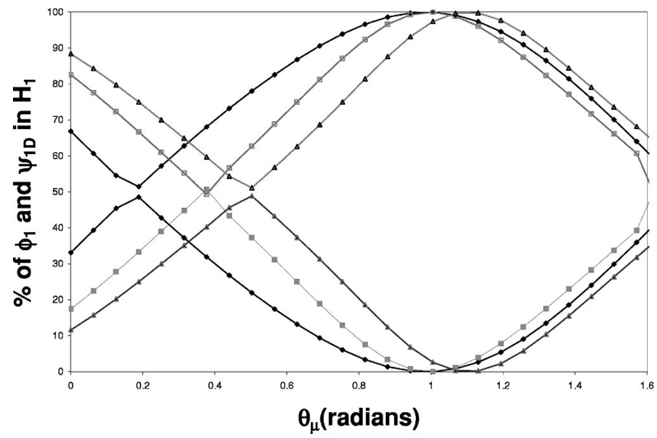

FIG. 6. Plot of the CP even component ϕ_1 of H_1 (upper curves) and the CP odd component ψ_{1D} of H_1 (lower curves) including the top squark, bottom squark, tau slepton, chargino and neutralino sector contributions as a function of θ_μ. The common parameters are $m_A=300$, $Q=320$, $m_0=100$, $m_{1/2}=500$, $\xi_2=.5$, $\alpha_0=.3$, and $|A_0|=1$. For curves with diamonds $\tan\beta=15$, $\xi_1=1.5$, for squares $\tan\beta=8$, $\xi_1=1.5$, and for triangles $\tan\beta=8$, $\xi_1=0.5$ where all masses are in GeV and all angles are in radians.

of Fig. 2 shows that the neutralino exchange contribution is comparable to the effects from the stop and chargino exchanges. A plot of Δ_{23} vs ξ_1 is given Fig. 3. As in Fig. 2 one finds that Δ_{23} is quite sensitive to the CP violating phase ξ_1. As in Fig. 2 here again the neutralino exchange contribution is comparable to the top squark and the chargino exchange contribution. An analysis of the percentage of the CP even component ϕ_1 of H_1 (upper curves) and of the percentage of the CP odd component ψ_{1D} of H_1 (lower curves) arising from the exchange of the top squark, the bottom squark, the tau slepton, the chargino and the neutralino sector contributions as a function of ξ_1 is given in Fig. 4. As expected from the analysis of Fig. 2 and Fig. 3 one finds that there is a significant mixing between the CP even and the CP odd components of H_1. Further, as also expected from the analysis of Figs. 2 and 3, the CP even and CP odd components of H_1 show a reasonably strong dependence on ξ_1.

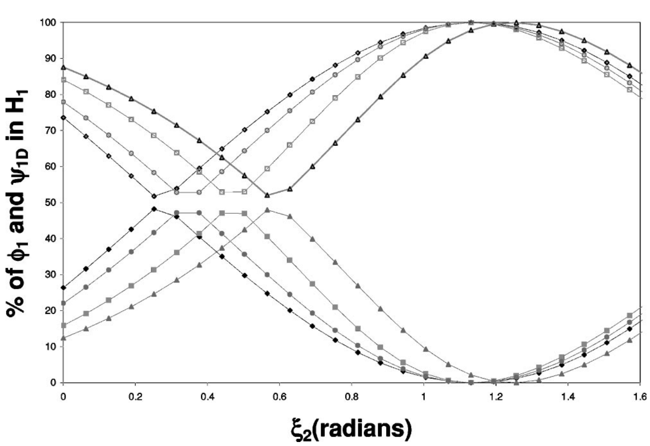

FIG. 5. Plot of the CP even component ϕ_1 of H_1 (upper curves) and the CP odd component ψ_{1D} of H_1 (lower curves) including the top squark, bottom squark, tau slepton, chargino and neutralino sector contributions as a function of the ξ_2. The common parameters are $m_A=300$, $Q=320$, $m_0=100$, $m_{1/2}=500$, $\alpha_0=.3$, $|A_0|=1$, and $\theta_\mu=.4$. For the curves with diamonds $\tan\beta=15$, $\xi_1=1.5$, for squares $\tan\beta=8$, $\xi_1=1.5$, for triangles $\tan\beta=8$, $\xi_1=0.5$, and for circles $\tan\beta=10$, $\xi_1=1.5$ where all masses are in GeV and all angles are in radians.

An analysis of the CP even and CP odd mixing in H_1 as a function of the SU(2) gaugino phase is given in Fig. 5. Unlike Figs. 2–4, where the entire ξ_1 dependence arose from the neutralino exchange contribution here the ξ_2 dependence of the CP even and CP odd components of H_1 arises from two sources, i.e., from the chargino and the neutralino exchange contributions. Because of this the dependence of the CP even and CP odd components on ξ_2 is much stronger than on ξ_1 as may be seen by comparing the plots of Figs. 2–4 with the plots of Fig. 5. In Fig. 6 a plot of the percentage of the CP even component ϕ_1 of H_1 (upper sets) and the CP odd component ψ_{1D} of H_1 (lower sets) arising from the exchange of the top squark, the bottom squark, the tau slepton, the chargino and the neutralino sector contributions is given as a function of θ_μ. In this case we find that the dependence of the CP even and the CP odd components on θ_μ is also very strong. Indeed in this case the mixings between the CP even and the CP odd states can be maximal depending on the value of θ_μ. The strong dependence on θ_μ can be understood as due to the fact that all contributions, i.e., the top squark, the bottom squark, the tau slepton, the chargino, and the neutralino contributions, depend on θ_μ. This in contrast to the dependence on ξ_1 which arises only from the neutralino exchange.

Finally, in Fig. 7 we give an analysis of the percentage of the CP even component ϕ_1 of H_1 (upper sets) and the CP odd component ψ_{1D} of H_1 (lower sets) arising from the exchange of the top squark, the bottom squark, the tau squark, the chargino and the neutralino sector contributions as a function of $\tan\beta$. We find that the CP even and the CP odd mixings show a strong dependence on $\tan\beta$. A similar strong

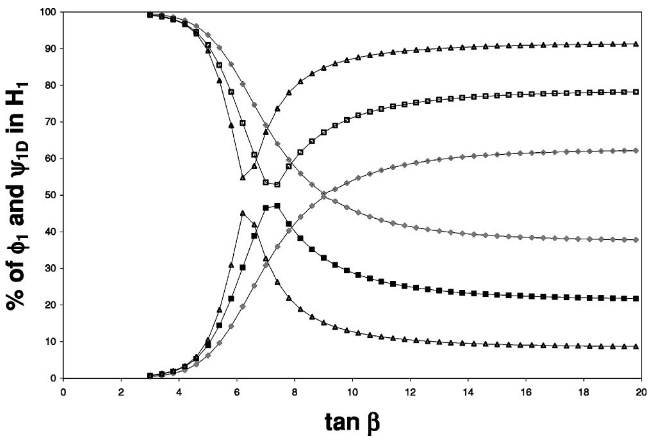

FIG. 7. Plot of the CP even component ϕ_1 of H_1 (upper curves) and the CP odd component ψ_{1D} of H_1 (lower curves) including the top squark, bottom squark, tau slepton, chargino and neutralino sector contributions as a function of $\tan\beta$. The common input parameters for the curves are $m_A=300$, $Q=320$, $m_0=100$, $m_{1/2}=500$, $\xi_1=.5$, $\xi_2=.5$, $\alpha_0=.3$, and $|A_0|=1$. For the curves with diamonds, $\theta_\mu=.4$, for squares $\theta_\mu=.6$, and for triangles $\theta_\mu=.8$ where all masses are in GeV and all angles are in radians.

dependence on $\tan\beta$ was seen also in previous analyses [16]. We note that the inclusion of the neutralino contribution further sharpens the $\tan\beta$ dependence and one finds that the CP even (odd) component can vary from 100% (0%) to less than 60% (more than 40%) as $\tan\beta$ is varied. This sharper behavior of the amplitudes with $\tan\beta$ arises from the additional contributions from the neutralino, the neutral Higgs boson and the Z boson exchanges. An analysis similar to the above can be carried out for the case of the H_2 and H_3 fields. In the analysis of chargino exchange contributions it wasfound that the CP odd component of H_2 is rather small while the analysis of H_3 parallels the analysis of H_1 with the only difference that the roles of the CP even and the CP odd components is reversed. Much the same situation occurs in this case and thus we omit the detailed discussion of these states.

VI. CONCLUSIONS

In this paper we have developed a calculus for the derivatives of the eigenvalues of the neutralino mass matrix with respect to the background fields which are in general dependent on CP violating phases. The calculus allows one to deduce the derivatives of the eigenvalues of the neutralino mass matrix analytically even though the eigenvalues themselves cannot be gotten analytically in a compact form. We use this calculus to obtain analytical results for the neutralino–Z–neutral-Higgs-boson exchange contribution to the masses and mixings in the CP-even–CP-odd–neutral-Higgs-boson sector. The above computation along with the top-squark–top, the bottom-squark–bottom, the tau–tau-slepton and the chargino–W–charged-Higgs-boson exchange contribution computed previously provide us with a complete one loop contribution to the Higgs boson mass matrix with the inclusion of CP phases. This full one loop result was then used to discuss the phenomenon of CP violation in the neutral Higgs sector. The numerical analysis shows that the mixings between the CP even and the CP odd sectors are significantly affected by the neutralino exchange contribution. The mixing of the CP even and the CP odd Higgs sector has many important consequences [15,16,18]. Thus one consequence is that CP even-odd mixing affects the couplings of the Higgs bosons with quarks and leptons and this effect can be discerned in Higgs searches in collider experiments. Another important implication is that the CP even-odd mixing will affect the relic density analysis and thus modify the parameter space allowed by the relic density constraints. Further, since the couplings of the quark and leptons with the Higgs bosons are affected due to the CP even-odd mixing there will also be an effect of these mixings on detection rates in the direct searches for dark matter. It would be interesting to carry out an analysis of these phenomena.

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APPENDIX A: NEUTRALINO EIGENVALUES AND DERIVATIVES

The characteristic equation for the square of the neutralino mass is $F(\lambda)=\mathrm{Det}(M^{\dagger}_{\chi^0}\, M_{\chi^0}-\lambda I)=0$ where λ represents the square of the neutralino mass eigenvalues. It can be expanded as

$$F(\lambda)=\lambda^4+a\lambda^3+b\lambda^2+c\lambda+d=0. \tag{A1}$$

In the above a, b, c and d are computed using Eq. (14). The computation of the coefficients is done to leading and to next to the leading order in an expansion in M_Z^2/M_S^2 where M_S stands for the soft SUSY parameters. Thus, e.g., a is expanded to $O(M_S^2)$ and $O(M_Z^2)$ orders (it is actually exact when expanded to this order), b is expanded to $O(M_S^4)$ and $O(M_S^2M_Z^2)$ orders, etc. The analysis for a, b and c [d does not enter in Eqs. (23)–(29) and is not exhibited] gives

$$a=-[|\tilde{m}_1|^2+|\tilde{m}_2|^2+2|\mu|^2+2M_Z^2] \tag{A2}$$

$$\begin{aligned}b=&|\tilde{m}_1|^2|\tilde{m}_2|^2+|\mu|^4+2|\mu|^2(|\tilde{m}_1|^2+|\tilde{m}_2|^2)\\&+M_Z^2[|m_1|^2+|m_2|^2+2|\mu|^2\\&+(|\tilde{m}_1|^2-|\tilde{m}_2|^2)\cos 2\theta_W\\&-4\cos\beta\sin\beta C_W^2|\tilde{m}_2||\mu|\cos\gamma_2\\&-4\cos\beta\sin\beta S_W^2|\tilde{m}_1||\mu|\cos\gamma_1]\end{aligned} \tag{A3}$$

where $C_W^2 = g_2^2/(g_1^2+g_2^2)$ and $S_W^2 = g_1^2/(g_1^2+g_2^2)$

$$c = -2|\mu|^2|\tilde{m}_1|^2|\tilde{m}_2|^2 - |\mu|^4(|\tilde{m}_1|^2+|\tilde{m}_2|^2) + 4M_Z^2 \sin\beta\cos\beta|\mu|[(|\tilde{m}_2|^2+|\mu|^2)S_W^2|\tilde{m}_1|\cos\gamma_1 + (|\tilde{m}_1|^2+|\mu|^2)C_W^2|\tilde{m}_2|\cos\gamma_2]. \quad \text{(A4)}$$

The derivatives $\partial\lambda_i/\partial\Phi_\alpha$ can be obtained explicitly as follows:

$$\frac{\partial\lambda_i}{\partial\Phi_\alpha} = -\left.\frac{a_\alpha\lambda^3+b_\alpha\lambda^2+c_\alpha\lambda+d_\alpha}{4\lambda^3+3a\lambda^2+2b\lambda+c}\right|_{\lambda=\lambda_i}. \quad \text{(A5)}$$

The second derivatives are given by

$$\frac{\partial^2\lambda_i}{\partial\Phi_\alpha\partial\Phi_\beta} = \Bigg[-\frac{(a_\alpha\lambda^3+b_\alpha\lambda^2+c_\alpha\lambda+d_\alpha)}{(4\lambda^3+3a\lambda^2+2b\lambda+c)^3}(a_\beta\lambda^3+b_\beta\lambda^2 + c_\beta\lambda+d_\beta)(12\lambda^2+6a\lambda+2b) + \frac{(a_\alpha\lambda^3+b_\alpha\lambda^2+c_\alpha\lambda+d_\alpha)}{(4\lambda^3+3a\lambda^2+2b\lambda+c)^2}(3a_\beta\lambda^2+2b_\beta\lambda + c_\beta) + \frac{(a_\beta\lambda^3+b_\beta\lambda^2+c_\beta\lambda+d_\beta)}{(4\lambda^3+3a\lambda^2+2b\lambda+c)^2} \times(3a_\alpha\lambda^2+2b_\alpha\lambda+c_\alpha) - \frac{(a_{\alpha\beta}\lambda^3+b_{\alpha\beta}\lambda^2+c_{\alpha\beta}\lambda+d_{\alpha\beta})}{(4\lambda^3+3a\lambda^2+2b\lambda+c)}\Bigg]_{\lambda=\lambda_i} \quad \text{(A6)}$$

where

$$a_\alpha = \frac{\partial a}{\partial\Phi_\alpha}, \quad a_{\alpha\beta} = \frac{\partial^2 a}{\partial\Phi_\alpha\partial\Phi_\beta}. \quad \text{(A7)}$$

APPENDIX B: LIST OF PARAMETERS

The explicit evaluation of the coefficients a_1, b_1, c_1, d_1 is given below

$$a_1 = -(g_1^2+g_2^2)v_1$$

$$b_1 = -g_2^2|\mu||\tilde{m}_2|v_2\cos\gamma_2 - g_1^2|\mu||\tilde{m}_1|v_2\cos\gamma_1 + v_1[|\tilde{m}_1|^2g_2^2+|\tilde{m}_2|^2g_1^2+(g_1^2+g_2^2)|\mu|^2] \quad \text{(B1)}$$

$$c_1 = g_2^2(|\tilde{m}_1|^2+|\mu|^2)|\mu||\tilde{m}_2|v_2\cos\gamma_2 + g_1^2(|\tilde{m}_2|^2 + |\mu|^2)|\mu||\tilde{m}_1|v_2\cos\gamma_1 - g_2^2|\mu|^2|\tilde{m}_1|^2v_1 - g_1^2|\mu|^2|\tilde{m}_2|^2v_1 \quad \text{(B2)}$$

$$d_1 = -g_2^2|\mu|^3|\tilde{m}_1|^2|\tilde{m}_2|v_2\cos\gamma_2 - g_1^2|\mu|^3|\tilde{m}_2|^2|\tilde{m}_1|v_2\cos\gamma_1. \quad \text{(B3)}$$

The coefficients a_2, b_2, c_2, d_2 can be obtained from a_1, b_1, c_1, d_1 with the following interchanges:

$$a_2 = a_1(v_1 \longleftrightarrow v_2), \quad b_2 = b_1(v_1 \longleftrightarrow v_2), \quad c_2 = c_1(v_1 \longleftrightarrow v_2), \quad d_2 = d_1(v_1 \longleftrightarrow v_2). \quad \text{(B4)}$$

The coefficients a_3, b_3, c_3, d_3 are given as follows:

$$a_3 = 0$$

$$b_3 = -g_2^2|\tilde{m}_2||\mu|v_2\sin\gamma_2 - g_1^2|\tilde{m}_1||\mu|v_2\sin\gamma_1 \quad \text{(B5)}$$

$$c_3 = g_2^2(|\tilde{m}_1^2|^2+|\mu|^2)|\tilde{m}_2||\mu|v_2\sin\gamma_2 + g_1^2(|\tilde{m}_2^2|^2+|\mu|^2)|\tilde{m}_1||\mu|v_2\sin\gamma_1 \quad \text{(B6)}$$

$$d_3 = -g_2^2|\tilde{m}_1|^2|\mu|^3|\tilde{m}_2|v_2\sin\gamma_2 - g_1^2|\tilde{m}_2|^2|\mu|^3|\tilde{m}_1|v_2\sin\gamma_1. \quad \text{(B7)}$$

The coefficients a_3', b_3', c_3', d_3' can be obtained from a_3, b_3, c_3, d_3 with the following interchanges:

$$a_3' = a_3(v_1 \longleftrightarrow v_2), \quad b_3' = b_3(v_1 \longleftrightarrow v_2), \quad c_3' = c_3(v_1 \longleftrightarrow v_2), \quad d_3' = d_3(v_1 \longleftrightarrow v_2). \quad \text{(B8)}$$

A_0 and B_0 are given by

$$A_0 = 2(g_1^2+g_2^2)v_1(M_Z^2-M_{A^0}^2)\cos 2\beta + (g_1^2+g_2^2)v_2(M_Z^2 + M_{A^0}^2)\sin 2\beta \quad \text{(B9)}$$

$$B_0 = -2(g_1^2+g_2^2)v_2(M_Z^2-M_{A^0}^2)\cos 2\beta + (g_1^2+g_2^2)v_1(M_Z^2 + M_{A^0}^2)\sin 2\beta. \quad \text{(B10)}$$

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