Calculation of the chiral Lagrangian coefficients from the underlying theory of QCD: A simple approach

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We calculate the coefficients in the chiral Lagrangian approximately from QCD based on a previous study of deriving the chiral Lagrangian from the first principles of QCD in which the chiral Lagrangian coefficients are defined in terms of certain Green's functions in QCD. We first show that, in the large-N_c limit, the anomaly part contributions to the coefficients are exactly cancelled by certain terms in the normal part contributions, and the final results of the coefficients only concern the remaining normal part contributions depending on QCD interactions. We then do the calculation in a simple approach with the approximations of taking the large-N_c limit, the leading order in dynamical perturbation theory, and the improved ladder approximation; thereby the relevant Green's functions are expressed in terms of the quark self-energy $\Sigma(p^2)$. By solving the Schwinger-Dyson equation for $\Sigma(p^2)$, we obtain the approximate QCD predicted coefficients and quark condensate which are consistent with the experimental values.

I. INTRODUCTION

Because of its nonperturbative nature, studying low energy hadron physics in QCD is a long standing difficult problem. For low lying pseudoscalar mesons, a widely used approach is the theory of the effective chiral Lagrangian based on the consideration of the global symmetry of the system and the momentum expansion without dealing with the nonperturbative dynamics of QCD [1,2]. In the chiral Lagrangian approach, the coefficients in the Lagrangian are all unknown phenomenological parameters which should be determined by experimental inputs. The number of unknown parameters increases rapidly with the increase of the precision in the momentum expansion. Therefore studying the relation between the chiral Lagrangian and the fundamental principles of QCD will not only be theoretically interesting for a deeper understanding of the chiral Lagrangian, but will also be helpful for reducing the number of unknown parameters and increasing the predictive power of the chiral Lagrangian.

In a previous paper, Ref. [3], certain techniques were developed, with which the chiral Lagrangian was formally derived from the first principles of QCD without taking approximations. The chiral Lagrangian coefficients are contributed both by the anomaly part (from the quark functional measure) and the normal part (from the QCD Lagrangian). In Ref. [3], all the chiral Lagrangian coefficients contributed from the normal part of the theory are expressed in terms of certain Green's functions in QCD, which can be regarded as exact QCD definitions of the chiral Lagrangian coefficients. After expanding the effective action in powers of the rotated sources (momentum expansion), the effective

action, up to $O(p^4)$, contributed from the normal part is of the form [3]

$$
\begin{aligned}
S_{\text{eff}}^{(\text{norm})} = \int d^4x \, \text{tr}_f \big[& F_0^2 a_\Omega^2 + F_0^2 B_0 s_\Omega - \mathcal{K}_1 [d_\mu a_\Omega^\mu]^2 - \mathcal{K}_2 (d^\mu a_\Omega^\nu \\
& - d^\nu a_\Omega^\mu)(d_\mu a_{\Omega,\nu} - d_\nu a_{\Omega,\mu}) + \mathcal{K}_3 [a_\Omega^2]^2 \\
& + \mathcal{K}_4 a_\Omega^\mu a_\Omega^\nu a_{\Omega,\mu} a_{\Omega,\nu} + \mathcal{K}_5 a_\Omega^2 \, \text{tr}_f [a_\Omega^2] \\
& + \mathcal{K}_6 a_\Omega^\mu a_\Omega^\nu \, \text{tr}_f [a_{\Omega,\mu} a_{\Omega,\nu}] + \mathcal{K}_7 s_\Omega^2 + \mathcal{K}_8 s_\Omega \, \text{tr}_f [s_\Omega] \\
& + \mathcal{K}_9 p_\Omega^2 + \mathcal{K}_{10} p_\Omega \, \text{tr}_f [p_\Omega] + \mathcal{K}_{11} s_\Omega a_\Omega^2 \\
& + \mathcal{K}_{12} s_\Omega \, \text{tr}_f [a_\Omega^2] - \mathcal{K}_{13} V_\Omega^{\mu\nu} V_{\Omega,\mu\nu} \\
& + i \mathcal{K}_{14} V_\Omega^{\mu\nu} a_{\Omega,\mu} a_{\Omega,\nu} + \mathcal{K}_{15} p_\Omega d_\mu a_\Omega^\mu \big] + O(p^6),
\end{aligned} \tag{1}
$$

where Ω is related to the nonlinearly realized meson field U by $U = \Omega^2$; s_Ω, p_Ω, v_Ω, and a_Ω are, respectively, the external scalar, pseudoscalar, vector, and axial-vector sources rotated by Ω; and the $\mathcal{K}$s are terms with different Lorentz structures in the relevant QCD Green's functions. The obtained expressions for the chiral Lagrangian coefficients are $O(p^2)$:

$$
\begin{aligned}
F_0^2 = \frac{i}{8(N_f^2-1)} \int d^4x \bigg[& \langle [\bar{\psi}^a(0) \gamma^\mu \gamma_5 \psi^b(0)] \\
& \times [\bar{\psi}^b(x) \gamma_\mu \gamma_5 \psi^a(x)] \rangle - \frac{1}{N_f} \langle [\bar{\psi}^a(0) \gamma^\mu \gamma_5 \psi^a(0)] \\
& \times [\bar{\psi}^b(x) \gamma_\mu \gamma_5 \psi^b(x)] \rangle - \langle \bar{\psi}^a(0) \gamma^\mu \gamma_5 \psi^b(0) \rangle \\
& \times \langle \bar{\psi}^b(x) \gamma_\mu \gamma_5 \psi^a(x) \rangle + \frac{1}{N_f} \langle \bar{\psi}^a(0) \gamma^\mu \gamma_5 \psi^a(0) \rangle
\end{aligned}
$$

$$\times \langle \bar{\psi}^b(x) \gamma_\mu \gamma_5 \psi^b(x) \rangle \Bigg],$$

$$F_0^2 B_0 = -\frac{1}{N_f} \langle \bar{\psi}\psi \rangle, \qquad (2)$$

$O(p^4)$:

$$L_1^{(\mathrm{norm})} = \frac{1}{32}\mathcal{K}_4 + \frac{1}{16}\mathcal{K}_5 + \frac{1}{16}\mathcal{K}_{13} - \frac{1}{32}\mathcal{K}_{14},$$

$$L_2^{(\mathrm{norm})} = \frac{1}{16}(\mathcal{K}_4 + \mathcal{K}_6) + \frac{1}{8}\mathcal{K}_{13} - \frac{1}{16}\mathcal{K}_{14},$$

$$L_3^{(\mathrm{norm})} = \frac{1}{16}(\mathcal{K}_3 - 2\mathcal{K}_4 - 6\mathcal{K}_{13} + 3\mathcal{K}_{14}),$$

$$L_4^{(\mathrm{norm})} = \frac{\mathcal{K}_{12}}{16B_0},$$

$$L_5^{(\mathrm{norm})} = \frac{\mathcal{K}_{11}}{16B_0}, \quad L_6^{(\mathrm{norm})} = \frac{\mathcal{K}_8}{16B_0^2},$$

$$L_7^{(\mathrm{norm})} = -\frac{\mathcal{K}_1}{16N_f} - \frac{\mathcal{K}_{10}}{16B_0^2} - \frac{\mathcal{K}_{15}}{16B_0 N_f},$$

$$L_8^{(\mathrm{norm})} = \frac{1}{16}\Bigg[\mathcal{K}_1 + \frac{1}{B_0^2}\mathcal{K}_7 - \frac{1}{B_0^2}\mathcal{K}_9 + \frac{1}{B_0}\mathcal{K}_{15}\Bigg],$$

$$L_9^{(\mathrm{norm})} = \frac{1}{8}(4\mathcal{K}_{13} - \mathcal{K}_{14}), \quad L_{10}^{(\mathrm{norm})} = \frac{1}{2}(\mathcal{K}_2 - \mathcal{K}_{13}),$$

$$H_1^{(\mathrm{norm})} = -\frac{1}{4}(\mathcal{K}_2 + \mathcal{K}_{13}),$$

$$H_2^{(\mathrm{norm})} = \frac{1}{8}\Bigg[-\mathcal{K}_1 + \frac{1}{B_0^2}\mathcal{K}_7 + \frac{1}{B_0^2}\mathcal{K}_9 - \frac{1}{B_0}\mathcal{K}_{15}\Bigg]. \qquad (3)$$

Together with the anomaly part contributions, the complete coefficients are given by

$$L_i = L_i^{(\mathrm{anom})} + L_i^{(\mathrm{norm})}, \quad i = 1, \ldots, 10,$$

$$H_i = H_i^{(\mathrm{anom})} + H_i^{(\mathrm{norm})}, \quad i = 1,2, \qquad (4)$$

where the superscripts (anom) and (norm) denote the anomaly part and normal part contributions, respectively.

In the literature, the anomaly part contributions are usually calculated by means of the heat kernel regularization technique [4]. However, this technique is difficult to implement in the calculation of the normal part contributions which contain complicated functions of the momentum, say the quark self-energy $\Sigma(p^2)$, reflecting nonperturbative QCD dynamics (which are even unspecified in the analytical part of the calculation). In order to treat the anomaly part and the

normal part contributions on equal footing, a certain new regularization technique feasible for the calculations of both parts should be developed. In this paper, we use the generalized Schwinger proper time regularization technique developed in Ref. [5] to regularize the system, which keeps the local chiral symmetry at every step in the calculation, and can be applied to the calculations of both the contributions from the anomaly part and from the normal part. Thus, in this paper, the contributions from the anomaly part and the normal part are calculated by means of the same technique. As the first conclusion of this unified treatment, we show that *the anomaly contributions to the chiral Lagrangian coefficients given in Ref. [4], which are independent of QCD interactions, will actually be cancelled by certain terms in the normal part contributions, and the final expressions for the coefficients concern only the remaining terms from the normal part contributions related to QCD interactions.* It should be so since the coefficients indicate meson interactions which should be residual interactions between quarks and gluons, and thus should depend on QCD interactions. These contributions have not been carefully calculated in the literature. It has been shown in Ref. [3] that in the approximations of large-N_c limit, leading order in dynamical perturbation, and improved ladder approximation, the formula for F_0^2 in Eqs. (2) reduces to the well-known Pagels-Stokar formula [6] in which all dynamical effects from QCD are represented by the quark self-energy $\Sigma(p^2)$ in the formula. In this paper, we take the same approximations to calculate the chiral Lagrangian coefficients (the relevant QCD Green's functions) as an illustration of the main feature of how QCD predicts the chiral Lagrangian coefficients. Similar to the case of the Pagels-Stokar formula, the relevant QCD Green's functions can all be expressed as functions of the quark self-energy $\Sigma(p^2)$. By solving the Schwinger-Dyson equation, we obtain $\Sigma(p^2)$, and thus the approximate QCD predicted values of the coefficients. We shall see that the obtained coefficients $L_1, \ldots, L_{10}$ and quark condensate are consistent with the experimental values. The calculation is checked by the absence of divergences in the large-N_c limit as it should be since the divergent meson-loop contributions are of next-to-the-leading order in the $1/N_c$ expansion. Although the present approximation is rather crude, *it reveals the main feature of QCD predictions for the chiral Lagrangian coefficients.*

This paper is organized as follows: In Sec. II, we calculate the anomaly part contributions to the $O(p^4)$ coefficients using the Schwinger proper time regularization technique, and the results coincide with those in Ref. [4] in the chiral limit. Then, in Sec. III, we apply the same technique to the normal part, and show generally that, in the large-N_c limit, the anomaly part contributions to the chiral Lagrangian coefficients are exactly cancelled by the contributions from a piece in the normal part independent of the quark self-energy, and the contributions from the remaining piece in the normal part depending on the quark self-energy play the real role in the chiral Lagrangian coefficients. Specific approximations in the calculation of the normal part contributions and the formulas for the complete chiral Lagrangian coefficients in terms of the quark self-energy are given in Sec. IV. In Sec. V,

we present the numerical calculations of the quark self-energy and the obtained values of the chiral Lagrangian coefficients. Section VI is a concluding remark.

II. ON THE CONTRIBUTIONS FROM THE ANOMALY PART

In order to see the relation between the anomaly part and the normal part contributions to the chiral Lagrangian coefficients, we present here the calculation of the anomaly part contributions by means of the Schwinger proper time regularization. We shall see that the obtained results exactly coincide with those obtained from the heat kernel technique [4]. Our present approach is different from that in Ref. [4] in the sense that the constant constituent quark mass M_Q is not put in by hand as is done in Ref. [4] but is naturally included in the normal part solution through the dynamical quark mass reflecting chiral symmetry breaking. Therefore our result of the anomaly part contribution is to compare with that in Ref. [4] in the chiral limit.

In the Schwinger proper time regularization, the anomaly part does not contribute to the coefficients of the $O(p^2)$ terms in the case corresponding to the result with $M_Q = 0$ in Ref. [4].[1] Therefore we are only going to calculate the anomaly contribution to the coefficients of the $O(p^4)$ terms.

The anomaly term in the path-integral formalism is

$$
\begin{aligned}
S_{\text{eff}}^{(\text{anom})} &\equiv -i \times \text{anomaly terms} \\
&= -iN_c[\text{Tr} \ln(i\slashed{\partial} + J) \\
&\quad - \text{Tr} \ln(i\slashed{\partial} + J_\Omega)] \\
&= iN_c[\text{Tr} \ln(i\slashed{\partial} + J_\Omega) + \Omega\text{-independent term}].
\end{aligned} \tag{5}
$$

The Ω-independent term is independent of the U field, so that it is irrelevant to the chiral Lagrangian coefficients. We shall only evaluate the Ω-dependent term in Eq. (5). To have a unified parametrization, we can parametrize the anomaly contributed effective action similar to that in Eq. (1), i.e.,

$$
\begin{aligned}
S_{\text{eff}}^{(\text{anom})} = \int d^4x \ \text{tr}_f[&-\mathcal{K}_1^{(\text{anom})}[d_\mu a_\Omega^\mu]^2 - \mathcal{K}_2^{(\text{anom})}(d^\mu a_\Omega^\nu - d^\nu a_\Omega^\mu)(d_\mu a_{\Omega,\nu} - d_\nu a_{\Omega,\mu}) + \mathcal{K}_3^{(\text{anom})}[a_\Omega^2]^2 + \mathcal{K}_4^{(\text{anom})} a_\Omega^\mu a_\Omega^\nu a_{\Omega,\mu} a_{\Omega,\nu} \\
&+ \mathcal{K}_5^{(\text{anom})} a_\Omega^2 \ \text{tr}_f[a_\Omega^2] + \mathcal{K}_6^{(\text{anom})} a_\Omega^\mu a_\Omega^\nu \ \text{tr}_f[a_{\Omega,\mu} a_{\Omega,\nu}] + \mathcal{K}_7^{(\text{anom})} s_\Omega^2 + \mathcal{K}_8^{(\text{anom})} s_\Omega \ \text{tr}_f[s_\Omega] + \mathcal{K}_9^{(\text{anom})} p_\Omega^2 \\
&+ \mathcal{K}_{10}^{(\text{anom})} p_\Omega \ \text{tr}_f[p_\Omega] + \mathcal{K}_{11}^{(\text{anom})} s_\Omega a_\Omega^2 + \mathcal{K}_{12}^{(\text{anom})} s_\Omega \ \text{tr}_f[a_\Omega^2] - \mathcal{K}_{13}^{(\text{anom})} V_\Omega^{\mu\nu} V_{\Omega,\mu\nu} + i\mathcal{K}_{14}^{(\text{anom})} V_\Omega^{\mu\nu} a_{\Omega,\mu} a_{\Omega,\nu} \\
&+ \mathcal{K}_{15}^{(\text{anom})} p_\Omega d_\mu a_\Omega^\mu] + O(p^6) + U\text{-independent source terms}.
\end{aligned} \tag{6}
$$

The Ω-dependent term in Eq. (5) suffers from ultraviolet divergence, and we take the Schwinger proper time regularization with an ultraviolet cutoff parameter Λ to regularize it. To apply this regularization, we first work in the Euclidean space-time, and analytically continue the results to the Minkowskian space-time after the evaluation. The main procedure of evaluating the general functional determinant including the quark self-energy Σ is described in Appendix A. In the case of $S_{\text{eff}}^{(\text{anom})}$, there is no Σ-dependence in Eq. (5). However, for regularizing the infrared divergence, we should replace the Σ in Eqs. (A1) and (A3) by an infrared cutoff parameter κ. The momentum integration in Eq. (A3) can be explicitly carried out with a lengthy but elementary calculation. After expanding in powers of the external sources, we can identify the expressions for $\mathcal{K}_1^{(\text{anom})}, \ldots, \mathcal{K}_{15}^{(\text{anom})}$ by comparing with the form of Eq. (6), and we obtain

$$
\mathcal{K}_1^{(\text{anom})} = -\frac{N_c}{24\pi^2},
$$

$$
\mathcal{K}_2^{(\text{anom})} = -\frac{N_c}{48\pi^2} \lim_{\kappa \to 0} \lim_{\Lambda \to \infty} \left(\ln\frac{\kappa^2}{\Lambda^2} + \gamma + 1 \right),
$$

$$
\mathcal{K}_3^{(\text{anom})} = \frac{N_c}{24\pi^2} \lim_{\kappa \to 0} \lim_{\Lambda \to \infty} \left(\ln\frac{\kappa^2}{\Lambda^2} + \gamma + 4 \right),
$$

$$
\mathcal{K}_4^{(\text{anom})} = -\frac{N_c}{24\pi^2} \lim_{\kappa \to 0} \lim_{\Lambda \to \infty} \left(\ln\frac{\kappa^2}{\Lambda^2} + \gamma + 2 \right),
$$

$$
\mathcal{K}_5^{(\text{anom})} = \mathcal{K}_6^{(\text{anom})} = 0,
$$

$$
\mathcal{K}_7^{(\text{anom})} = \mathcal{K}_9^{(\text{anom})} = \frac{N_c}{8\pi^2} \lim_{\Lambda \to \infty} \Lambda^2,
$$

$$
\mathcal{K}_8^{(\text{anom})} = \mathcal{K}_{10}^{(\text{anom})} = \mathcal{K}_{11}^{(\text{anom})} = \mathcal{K}_{12}^{(\text{anom})} = 0,
$$

$$
\mathcal{K}_{13}^{(\text{anom})} = -\frac{N_c}{48\pi^2} \lim_{\kappa \to 0} \lim_{\Lambda \to \infty} \left(\ln\frac{\kappa^2}{\Lambda^2} + \gamma \right),
$$

---

[1] If one takes a momentum cutoff Λ to regularize the divergent integrals as was done in Ref. [4] before putting in the constituent quark mass M_Q, the $O(p^2)$ coefficient F_0^2 will be proportional to Λ^2 (cf. Ref. [4]). As has been pointed out in Ref. [3], this term is exactly cancelled by a corresponding term in the normal part contribution [cf. Eq. (74) in Ref. [3]].

$$\mathcal{K}_{14}^{(\mathrm{anom})} = -\frac{N_c}{12\pi^2}\lim_{\kappa\to0}\lim_{\Lambda\to\infty}\left(\ln\frac{\kappa^2}{\Lambda^2}+\gamma+2\right),$$

$$\mathcal{K}_{15}^{(\mathrm{anom})} = 0. \tag{7}$$

Comparing with the standard form of momentum expansion to identify the $O(p^4)$ chiral Lagrangian coefficients, we obtain the anomaly contribution to these coefficients

$$L_1^{(\mathrm{anom})} = \frac{N_c}{384\pi^2}, \quad L_2^{(\mathrm{anom})} = \frac{N_c}{192\pi^2},$$

$$L_3^{(\mathrm{anom})} = -\frac{N_c}{96\pi^2}, \quad L_4^{(\mathrm{anom})} = L_5^{(\mathrm{anom})} = L_6^{(\mathrm{anom})} = 0,$$

$$L_7^{(\mathrm{anom})} = \frac{N_c}{1152\pi^2}, \quad L_8^{(\mathrm{anom})} = -\frac{N_c}{384\pi^2},$$

$$L_9^{(\mathrm{anom})} = \frac{N_c}{48\pi^2}, \quad L_{10}^{(\mathrm{anom})} = -\frac{N_c}{96\pi^2},$$

$$H_1^{(\mathrm{anom})} = \frac{N_c}{96\pi^2}\lim_{\kappa\to0}\lim_{\Lambda\to\infty}\left(\ln\frac{\kappa^2}{\Lambda^2}+\gamma+\frac{1}{2}\right),$$

$$H_2^{(\mathrm{anom})} = \frac{N_c}{192\pi^2}+\lim_{\Lambda\to\infty}\frac{N_c\Lambda^2}{32\pi^2 B_0^2}. \tag{8}$$

These exactly coincide with the results with $M_Q=0$ in Ref. [4]. Note that the final expressions of the coefficients $L_1,\ldots,L_{10}$ are independent of the infrared cutoff parameter κ and the ultraviolet cutoff Λ although these cutoff parameters appear in $\mathcal{K}_1^{(\mathrm{anom})},\ldots,\mathcal{K}_{15}^{(\mathrm{anom})}$, while H_1 and H_2 depend on the cutoff parameters. This implies that H_1 and H_2 are not measurable quantities. With $N_c=3$, the values of the coefficients are (in units of 10^{-3})

$$L_1=0.79, \quad L_2=1.58, \quad L_3=-3.17,$$

$$L_4=L_5=L_6=0,$$

$$L_7=0.26, \quad L_8=-0.79,$$

$$L_9=6.33, \quad L_{10}=-3.17. \tag{9}$$

These are to be compared with the experimental values (in units of 10^{-3}) [2]

$$L_1=0.9\pm0.3, \quad L_2=1.7\pm0.7, \quad L_3=-4.4\pm2.5,$$

$$L_4=0\pm0.5, \quad L_5=2.2\pm0.5, \quad L_6=0\pm0.3,$$

$$L_7=-0.4\pm0.15, \quad L_8=1.1\pm0.3, \quad L_9=7.4\pm0.7,$$

$$L_{10}=-6.0\pm0.7. \tag{10}$$

The numbers in Eqs. (9) are close to the experimental results of Eqs. (10) except L_7 and L_8 are of wrong signs. This gives people an impression that the coefficients $L_1,\ldots,L_{10}$ might mainly be contributed by the anomaly part, and the normal part might only contribute small corrections [4,7]. However, we note that the results in Eqs. (8) are independent of QCD interactions, i.e., these terms remain unchanged when we switch off the QCD gauge coupling constant α_s. This is somewhat confusing since these coefficients mean meson interactions which should be residual interactions between quarks and gluons. We shall see in the next section that *these terms will actually be completely cancelled by the terms independent of the quark self-energy in the normal part contribution, so that they do not really appear in the final form of the coefficients. What appear in the coefficients are the remaining terms in the normal part contribution which depend on the quark self-energy and hence on the QCD interactions as it should be.* Another feature of the terms in Eqs. (8) indicating that they should be exactly cancelled and should not appear in the final formulas for the coefficients is the divergence of H_1 and H_2 when taking $\Lambda\to\infty$. We know from Ref. [2] that the ultraviolet divergences in the $O(p^4)$ chiral Lagrangian coefficients come merely from the meson-loop corrections with the $O(p^2)$ interactions. In the $1/N_c$ expansion, the meson-loop corrections belonging to $O(1/N_c)$ will not take place in the large-N_c limit. Therefore, in the large-N_c limit, the final expressions for the $O(p^4)$ coefficients should be finite when $\Lambda\to\infty$. Now the ultraviolet divergences in H_1 and H_2 in Eqs. (8) have nothing to do with the meson-loop corrections, so that they should be exactly cancelled by other terms and should not appear in the final expressions for the $O(p^4)$ coefficients.

III. ON THE CONTRIBUTIONS FROM THE NORMAL PART

In this section we use the same regularization technique as in Sec. II to calculate the normal part contributions to the chiral Lagrangian coefficients. We start from the effective action $S_{\mathrm{eff}}^{(\mathrm{norm})}$ given in Ref. [3],

$$e^{iS_{\mathrm{eff}}^{(\mathrm{nrom})}} = \int \mathcal{D}\Xi\, e^{i\tilde{\Gamma}[1,J_\Omega,\Xi,\Phi_\Omega,\Pi_\Omega]}$$

$$= \int \mathcal{D}\Xi\mathcal{D}\Phi_\Omega\exp\left\{i\Gamma_0[J_\Omega,\Phi_\Omega,\Pi_{\Omega c}]+i\Gamma_I[\Phi_\Omega]\right.$$

$$+iN_c\int d^4x\,\mathrm{tr}_{lf}\left[\Xi(x)\left(-i\sin\frac{\vartheta(x)}{N_f}\right.\right.$$

$$\left.\left.\left.+\gamma_5\cos\frac{\vartheta(x)}{N_f}\right)\Phi_\Omega^T(x,x)\right]\right\} \tag{11}$$

which satisfies a useful relation [3]

$$\left.\frac{dS_{\mathrm{eff}}^{(\mathrm{norm})}}{dJ_\Omega^{\sigma\rho}(x)}\right|_{U\,\mathrm{fix,\,anomaly\,ignored}} = N_c\overline{\Phi_{\Omega c}^{\sigma\rho}(x,x)}. \tag{12}$$

The symbols are defined in Ref. [3].

In the large-N_c limit, the integrations in Eq. (11) can be carried out by the saddle point approximation with the saddle point equations

$$\Phi_{\Omega c}^{(a\eta)(b\zeta)}(x,y) = -i[(i\partial\!\!\!/+J_\Omega-\Pi_{\Omega c})^{-1}]^{(b\zeta)(a\eta)}(y,x),$$ (13)

$$\Pi_{\Omega c}^{\sigma\rho}(x,y) = -\tilde{\tilde{\Xi}}^{\sigma\rho}(x)\delta^4(x-y)$$

$$-\sum_{n=1}^{\infty}\int d^4x_1\cdots d^4x_n d^4x'_1\cdots d^4x'_n$$

$$\times\frac{(-i)^{n+1}(N_cg_s^2)^n}{n!}$$

$$\times\bar{G}_{\rho\rho_1\cdots\rho_n}^{\sigma\sigma_1\cdots\sigma_n}(x,y,x_1,x'_1,\ldots,x_n,x'_n)$$

$$\Phi_{\Omega c}^{\sigma_1\rho_1}(x_1,x'_1)\cdots\Phi_{\Omega c}^{\sigma_n\rho_n}(x_n,x'_n),$$ (14)

$$\text{tr}_l\left[\left(-i\sin\frac{\vartheta(x)}{N_f}+\gamma_5\cos\frac{\vartheta(x)}{N_f}\right)\Phi_{\Omega c}^T(x,x)\right]=0,$$ (15)

where $\tilde{\tilde{\Xi}}$ is a short notation for the following quantity:

$$\tilde{\tilde{\Xi}}^{\sigma\rho}(x)\equiv\frac{\partial}{\partial\Phi_{\Omega c}^{\sigma\rho}(x,x)}\int d^4y\,\text{tr}_{lf}\left\{\Xi_c(y)\left[-i\sin\frac{\vartheta_c(y)}{N_f}\right.\right.$$

$$\left.\left.+\gamma_5\cos\frac{\vartheta_c(y)}{N_f}\right]\Phi_{\Omega c}^T(y,y)\right\}\Bigg|_{\Xi_c\text{ fixed}}.$$ (16)

Then the obtained $S_{\text{eff}}^{(\text{norm})}$ in this approximation is

$$S_{\text{eff}}^{(\text{norm})}=\tilde{\Gamma}[1,J_\Omega,\Xi_c,\Phi_{\Omega c},\Phi_{\Omega c}]=-iN_c\,\text{Tr}\ln[i\partial\!\!\!/+J_\Omega$$

$$-\Pi_{\Omega c}]+N_c\int d^4x d^4x'\,\Phi_{\Omega c}^{\sigma\rho}(x,x')\Pi_{\Omega c}^{\sigma\rho}(x,x')$$

$$+N_c\sum_{n=2}^{\infty}\int d^4x_1\cdots d^4x'_n\frac{(-i)^n(N_cg_s^2)^{n-1}}{n!}$$

$$\times\bar{G}_{\rho_1\cdots\rho_n}^{\sigma_1\cdots\sigma_n}(x_1,x'_1,\ldots,x_n,x'_n)$$

$$\times\Phi_{\Omega c}^{\sigma_1\rho_1}(x_1,x'_1)\cdots\Phi_{\Omega c}^{\sigma_n\rho_n}(x_n,x'_n)$$

$$+iN_c\int d^4x\,\text{tr}_{lf}\left\{\Xi_c(x)\left[-i\sin\frac{\vartheta_c(x)}{N_f}\right.\right.$$

$$\left.\left.+\gamma_5\cos\frac{\vartheta_c(x)}{N_f}\right]\Phi_{\Omega,c}^T(x,x)\right\},$$ (17)

in which the $O(1/N_c)$ term Γ_I is neglected. Note that the last term in Eq. (17) actually vanishes due to Eq. (15). We keep it here for showing the relation between the effective action $S_{\text{eff}}^{(\text{norm})}$ and its stationary conditions Eqs. (13)–(15).

In the large-N_c limit, $\overline{\Phi_{\Omega c}}=\Phi_{\Omega c}$ on the right-hand side of Eq. (12). The left-hand side of Eq. (12) can be carried out from Eq. (17) using Eqs. (13)–(15). Then the explicit form of Eq. (12) in this approximation is

$$-i[(i\partial\!\!\!/+J_\Omega-\Pi_{\Omega c})^{-1}]^{\rho\sigma}(x,x)=\Phi_{\Omega c}^{\sigma\rho}(x,x).$$ (18)

We see that $\Pi_{\Omega c}$ and $\Phi_{\Omega c}$ play the roles of the quark self-energy and the quark propagator, respectively, in the case with $J_\Omega\neq 0$.

Now we decompose $S_{\text{eff}}^{(\text{norm})}$ into a part *independent of* $\Pi_{\Omega c}$ and a part *depending on* $\Pi_{\Omega c}$. The part independent of $\Pi_{\Omega c}$ can be extracted from $S_{\text{eff}}^{(\text{norm})}$ by setting $\Pi_{\Omega c}=0$, i.e.,

$$S_{\text{eff}}^{(\text{norm},\Pi_{\Omega c}=0)}=-iN_c\,\text{Tr}\ln[i\partial\!\!\!/+J_\Omega]$$

$$+N_c\left[\sum_{n=2}^{\infty}\int d^4x_1\cdots d^4x'_n\frac{(-i)^n(N_cg_s^2)^{n-1}}{n!}\right.$$

$$\times\bar{G}_{\rho_1\cdots\rho_n}^{\sigma_1\cdots\sigma_n}(x_1,x'_1,\ldots,x_n,x'_n)$$

$$\left.\times\Phi_{\Omega c}^{\sigma_1\rho_1}(x_1,x'_1)\cdots\Phi_{\Omega c}^{\sigma_n\rho_n}(x_n,x'_n)\right]_{\Pi_{\Omega c}=0}.$$ (19)

Here we have ignored the last term in Eq. (17) which actually vanishes due to Eq. (15). We show in Appendix B that the last term in Eq. (19) is actually Ω independent. Therefore, Eq. (19) can be written as

$$S_{\text{eff}}^{(\text{norm},\Pi_{\Omega c}=0)}=-iN_c[\text{Tr}\ln(i\partial\!\!\!/+J_\Omega)$$

$$+\Omega\text{-independent terms}].$$ (20)

Comparing the J_Ω-dependent terms in Eqs. (5) and (20), we see that they are of the same form but with an opposite sign. Thus *their contributions to the chiral Lagrangian coefficients exactly cancel each other to all orders in the momentum expansion.* The cancellation in the case of the $O(p^2)$ coefficient F_0^2 has been described in footnote 1 in Sec. II. For the $O(p^4)$ coefficients, we have

$$\mathcal{K}_i^{(\text{anom})}+\mathcal{K}_i^{(\Pi_{\Omega c}=0)}=0,\quad i=1,\ldots,16$$

$$L_i^{(\text{anom})}+L_i^{(\text{norm},\Pi_{\Omega c}=0)}=0,\quad i=1,\ldots,10,$$

$$H_i^{(\text{anom})}+H_i^{(\text{norm},\Pi_{\Omega c}=0)}=0,\quad i=1,2.$$ (21)

Thus, in the large-N_c limit, the anomaly part contributed chiral Lagrangian coefficients in Eqs. (8) do not really appear in the final results of the chiral Lagrangian coefficients although their values Eqs. (9) are close to the experimental values. *The chiral Lagrangian coefficients are actually contributed from the $\Pi_{\Omega c}\neq 0$ part of $S_{\text{eff}}^{(\text{norm})}$*,

$$S_{\text{eff}}^{(\text{norm},\Pi_{\Omega c}\neq 0)}\equiv S_{\text{eff}}^{(\text{norm})}-S_{\text{eff}}^{(\text{norm},\Pi_{\Omega c}=0)}$$ (22)

which leads to the $\Pi_{\Omega c}\neq 0$ part of $\mathcal{K}_1^{(\text{norm})},\ldots,\mathcal{K}_{15}^{(\text{norm})}$,

$$\mathcal{K}_i^{(\text{norm},\Pi_{\Omega c}\neq 0)}=\mathcal{K}_i^{(\text{norm})}-\mathcal{K}_i^{(\text{norm},\Pi_{\Omega c}=0)},\quad i=1,\ldots,15.$$ (23)

This is our first new conclusion in this study.

The final chiral Lagrangian coefficients are then

$$L_i = L_i^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)}, \quad i = 1, \ldots, 10,$$

$$H_i = H_i^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)}, \quad i = 1, 2, \tag{24}$$

and

$$L_1 = \frac{1}{32} \mathcal{K}_4^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)} + \frac{1}{16} \mathcal{K}_5^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)} + \frac{1}{16} \mathcal{K}_{13}^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)}$$
$$- \frac{1}{32} \mathcal{K}_{14}^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)},$$

$$L_2 = \frac{1}{16} (\mathcal{K}_4^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)} + \mathcal{K}_6^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)}) + \frac{1}{8} \mathcal{K}_{13}^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)}$$
$$- \frac{1}{16} \mathcal{K}_{14}^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)},$$

$$L_3 = \frac{1}{16} (\mathcal{K}_3^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)} - 2 \mathcal{K}_4^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)} - 6 \mathcal{K}_{13}^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)}$$
$$+ 3 \mathcal{K}_{14}^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)}),$$

$$L_4 = \frac{\mathcal{K}_{12}^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)}}{16 B_0}, \quad L_5 = \frac{\mathcal{K}_{11}^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)}}{16 B_0},$$

$$L_6 = \frac{\mathcal{K}_8^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)}}{16 B_0^2},$$

$$L_7 = -\frac{\mathcal{K}_1^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)}}{16 N_f} - \frac{\mathcal{K}_{10}^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)}}{16 B_0^2} - \frac{\mathcal{K}_{15}^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)}}{16 B_0 N_f},$$

$$L_8 = \frac{1}{16} \left[\mathcal{K}_1^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)} + \frac{1}{B_0^2} \mathcal{K}_7^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)} \right.$$
$$\left. - \frac{1}{B_0^2} \mathcal{K}_9^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)} + \frac{1}{B_0} \mathcal{K}_{15}^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)} \right],$$

$$L_9 = \frac{1}{8} (4 \mathcal{K}_{13}^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)} - \mathcal{K}_{14}^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)}),$$

$$L_{10} = \frac{1}{2} (\mathcal{K}_2^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)} - \mathcal{K}_{13}^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)}),$$

$$H_1 = -\frac{1}{4} (\mathcal{K}_2^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)} + \mathcal{K}_{13}^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)}),$$

$$H_2 = \frac{1}{8} \left[-\mathcal{K}_1^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)} + \frac{1}{B_0^2} \mathcal{K}_7^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)} \right.$$
$$\left. + \frac{1}{B_0^2} \mathcal{K}_9^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)} - \frac{1}{B_0} \mathcal{K}_{15}^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)} \right]. \tag{25}$$

Since $\Pi_{\Omega c}|_{g_s = 0} = 0$, these $O(p^4)$ chiral Lagrangian coefficients will vanish if we switch off the QCD coupling constant g_s as it should be.

IV. CALCULATION OF THE CHIRAL LAGRANGIAN COEFFICIENTS

We see that to calculate the chiral Lagrangian coefficients from $S_{\mathrm{eff}}^{(\mathrm{norm}, \Pi_{\Omega c} \neq 0)}$, we should mainly deal with $S_{\mathrm{eff}}^{(\mathrm{norm})}$ given in Eq. (17) which has never been carefully calculated in the literature. Ignoring the vanishing last term in Eq. (17), there are still rather complicated terms in it. For example, the third term includes various ranks of gluon Green's functions which concern very complicated calculations of QCD dynamics. As the first time of doing this kind of calculation, we shall take futher approximations to simplify the evaluation of $S_{\mathrm{eff}}^{(\mathrm{norm})}$. We know that the pion decay constant f_π has been studied from QCD in Ref. [6] by taking the approximation of keeping only the leading order in dynamical perturbation, i.e., taking into account only the QCD interaction in the Schwinger-Dyson equation leading to the nonperturbative solution of chiral symmetry breaking, and neglecting other QCD corrections in positive powers of g_s (perturbative). This approximation leads to the widely used Pagels-Stokar formula which is reasonable though not perfect. In the large-N_c limit, f_π is just the $O(p^2)$ chiral Lagrangian coefficient F_0 given in Eqs. (2). So, as in Ref. [6], we take the approximation of keeping only the leading order in dynamical perturbation to calculate the chiral Lagrangian coefficients from $S_{\mathrm{eff}}^{(\mathrm{norm})}$. In this spirit, we neglect the complicated third term in Eq. (17) which contains only positive powers of g_s. Furthermore, we see from Eq. (14) that the second term in Eq. (17) is of the same order as the third term, so that we neglect the second term in Eq. (17) as well. With this approximation, $S_{\mathrm{eff}}^{(\mathrm{norm})}$ is simplified as

$$S_{\mathrm{eff}}^{(\mathrm{norm})} = -i N_c \, \mathrm{Tr} \ln[i \slashed{\partial} + J_\Omega - \Pi_{\Omega c}]. \tag{26}$$

Now the concerned QCD dynamics resides in the $\Pi_{\Omega c}$ term which is related to the quark self-energy [cf. Eq. (18)]. We expect such a simple approximation may also lead to reasonable results of the $O(p^4)$ chiral Lagrangian coefficients since reasonable values of the $O(p^4)$ chiral Lagrangian coefficients have been obtained in a model by Holdom [8] considering only the quark self-energy contribution with certain phenomenological ansatz. We shall see in Sec. V that our obtained $O(p^4)$ chiral Lagrangian coefficients are indeed reasonable. Although this approximation is crude, it provides a simple illustration of the main feature of how QCD predicts the chiral Lagrangian coefficients. Further improved study beyond this simple approximation taking into account the second and third terms in Eq. (17) is of course needed. That will be presented in a later paper. Now we need to calculate $\Pi_{\Omega c}^{\sigma \rho}(x, y)$ and carry out the explicit expression for $S_{\mathrm{eff}}^{(\mathrm{norm})}$ in Eq. (26).

We have noticed that $\Pi_{\Omega c}^{\sigma \rho}$ is related to the quark self-energy. If we find out the relation between $\Pi_{\Omega c}^{\sigma \rho}(x, y)$ and the conventional quark self-energy $\Sigma(-p^2)$, then we can obtain

$\Sigma(-p^2)$ by solving the well-known Schwinger-Dyson equation. As in the literature, we shall write down the Schwinger-Dyson equation in the Landau gauge which is stable against the gauge parameter. In the same approximation of taking the leading order in dynamical perturbation theory and with the improved ladder approximation, the Schwinger-Dyson equation in the Euclidean space-time reads [9–11]

$$\Sigma(p^2) - \frac{3N_c}{2} \int \frac{d^4q}{4\pi^3} \frac{\alpha_s[(p-q)]}{(p-q)^2} \frac{\Sigma(q^2)}{q^2 + \Sigma^2(q^2)} = 0. \tag{27}$$

This equation can be solved numerically and the details will be presented in Sec. V. Naively, we may expect that $\Pi_{\Omega c}^{\sigma\rho}(x,y) = \delta^{\sigma\rho}\Sigma(\partial_x^2)\delta^4(x-y)$. But this is not correct. Under a local chiral transformation $h(x)$ (hidden symmetry transformation [3]), $\Pi_{\Omega c}^{\sigma\rho}$ transforms as

$$\Pi_{\Omega c}(x,y) \to \Pi'_{\Omega c}(x,y) = h^\dagger(x)\Pi_{\Omega c}(x,y)h(y), \tag{28}$$

while $\delta^{\sigma\rho}\Sigma(\partial_x^2)\delta^4(x-y)$ does not transform like this. The correct relation can be found by replacing the ordinary derivative ∂_x^μ by the covariant derivative

$$\bar{\nabla}_x^\mu = \partial_x^\mu - iv_\Omega^\mu(x). \tag{29}$$

Since the external source $v_\Omega^\mu(x)$ transforms as

$$v_\Omega^\mu(x) \to v_\Omega^{\mu'}(x) = h^\dagger(x)v_\Omega^\mu(x)h(x) + ih^\dagger(x)[\partial^\mu h(x)], \tag{30}$$

the covariant derivative $\bar{\nabla}_x^\mu$ transforms as

$$\bar{\nabla}_x^\mu \to \bar{\nabla}_x^{\mu'} = h^\dagger(x)\bar{\nabla}_x^\mu h(x). \tag{31}$$

Thus the correct identification is

$$\Pi_{\Omega c}^{\sigma\rho}(x,y) = [\Sigma(\bar{\nabla}_x^2)]^{\sigma\rho}\delta^4(x-y). \tag{32}$$

Then the effective action (26) can be written as

$$S_{\text{eff}}^{(\text{norm})} = -iN_c \operatorname{Tr}\ln[i\partial\!\!\!/ + J_\Omega - \Sigma(\bar{\nabla}^2)]. \tag{33}$$

Next we evaluate the effective action (33) using the Schwinger proper time regularization as before (cf. Appendix A) to obtain the expressions for the chiral Lagrangian coefficients. This is not trivial since usually this regularization scheme is used in the case that Σ is a constant, and thus the momentum integration can be explicitly carried out to check the local gauge invariance of the result. Now we leave $\Sigma(\nabla^2)$ as an unspecified function in Eq. (33), so that the momentum integration cannot be carried out explicitly. Organizing terms to guarantee local chiral invariance is rather tedious and the details are given in Ref. [5]. Our obtained expressions in the Minkowskian space-time are

$$F_0^2 B_0 = 4 \int d\tilde{p}\,\Sigma_p X_p, \tag{34}$$

$$F_0^2 = 2 \int d\tilde{p}\left[(-2\Sigma_p^2 - p^2\Sigma_p\Sigma_p')X_p^2 + (2\Sigma_p^2 + p^2\Sigma_p\Sigma_p')\frac{X_p}{\Lambda^2}\right], \tag{35}$$

$$\mathcal{K}_1^{(\text{norm})} = 2 \int d\tilde{p}\left[-2A_p X_p^3 + 2A_p\frac{X_p^2}{\Lambda^2} - A_p\frac{X_p}{\Lambda^4} + \frac{p^2}{2}\Sigma_p'^2\frac{X_p}{\Lambda^2}\right.$$
$$\left. - \frac{p^2}{2}\Sigma_p'^2 X_p^2\right],$$

$$\mathcal{K}_2^{(\text{norm})} = \int d\tilde{p}\left[-2B_p X_p^3 + 2B_p\frac{X_p^2}{\Lambda^2} - B_p\frac{X_p}{\Lambda^4} + \frac{p^2}{2}\Sigma_p'^2\frac{X_p}{\Lambda^2}\right.$$
$$\left. - \frac{p^2}{2}\Sigma_p'^2 X_p^2\right],$$

$$\mathcal{K}_3^{(\text{norm})} = 2 \int d\tilde{p}\left[\left(\frac{4\Sigma_p^4}{3} - \frac{2p^2\Sigma_p^2}{3} + \frac{p^4}{18}\right)\left(6X_p^4 - \frac{6X_p^3}{\Lambda^2}\right.\right.$$
$$\left.+ \frac{3X_p^2}{\Lambda^4} - \frac{X_p}{\Lambda^6}\right) + \left(-4\Sigma_p^2 + \frac{p^2}{2}\right)\left(-2X_p^3 + \frac{2X_p^2}{\Lambda^2}\right.$$
$$\left.\left.- \frac{X_p}{\Lambda^4}\right) - \frac{X_p}{\Lambda^2} + X_p^2\right],$$

$$\mathcal{K}_4^{(\text{norm})} = \int d\tilde{p}\left[\left(\frac{-4\Sigma_p^4}{3} + \frac{2p^2\Sigma_p^2}{3} + \frac{p^4}{18}\right)\left(6X_p^4 - \frac{6X_p^3}{\Lambda^2}\right.\right.$$
$$\left.+ \frac{3X_p^2}{\Lambda^4} - \frac{X_p}{\Lambda^6}\right) + 4\Sigma_p^2\left(-2X_p^3 + \frac{2X_p^2}{\Lambda^2} - \frac{X_p}{\Lambda^4}\right)$$
$$\left.+ \frac{X_p}{\Lambda^2} - X_p^2\right],$$

$$\mathcal{K}_5^{(\text{norm})} = \mathcal{K}_6^{(\text{norm})} = 0,$$

$$\mathcal{K}_7^{(\text{norm})} = 2 \int d\tilde{p}\left[(3\Sigma_p^2 + 2p^2\Sigma_p\Sigma_p')X_p^2\right.$$
$$\left.+ [-2\Sigma_p^2 - p^2(1 + 2\Sigma_p\Sigma_p')]\frac{X_p}{\Lambda^2}\right],$$

$$\mathcal{K}_8^{(\text{norm})} = 0,$$

$$\mathcal{K}_9^{(\text{norm})} = 2 \int d\tilde{p}\left[(\Sigma_p^2 + 2p^2\Sigma_p\Sigma_p')X_p^2\right.$$
$$\left.- p^2(1 + 2\Sigma_p\Sigma_p')\frac{X_p}{\Lambda^2}\right],$$

$$\mathcal{K}_{10}^{(\mathrm{norm})}=0,$$

$$\mathcal{K}_{11}^{(\mathrm{norm})}=4\int d\tilde{p}\left[(-4\Sigma_p^3+p^2\Sigma_p)X_p^3+(4\Sigma_p^3-p^2\Sigma_p)\frac{X_p^2}{\Lambda^2}\right.$$
$$\left.-\left(2\Sigma_p^3-\frac{1}{2}p^2\Sigma_p\right)\frac{X_p}{\Lambda^4}+3\Sigma_p\frac{X_p}{\Lambda^2}-3\Sigma_pX_p^2\right],$$

$$\mathcal{K}_{12}^{(\mathrm{norm})}=0,$$

$$\mathcal{K}_{13}^{(\mathrm{norm})}=\int d\tilde{p}\left[\left(\frac{1}{3}p^2\Sigma_p'\Sigma_p''+\frac{1}{3}\Sigma_p\Sigma_p''\right)X_p+(C_p-D_p)\frac{X_p}{\Lambda^2}\right.$$
$$\left.-(C_p-D_p)X_p^2-2E_pX_p^3+2E_p\frac{X_p^2}{\Lambda^2}-E_p\frac{X_p}{\Lambda^4}\right],$$

$$\mathcal{K}_{14}^{(\mathrm{norm})}=-4\int d\tilde{p}\left[-2F_pX_p^3+2F_p\frac{X_p^2}{\Lambda^2}-F_p\frac{X_p}{\Lambda^4}\right.$$
$$\left.+\frac{p^2}{2}\Sigma_p'^2\frac{X_p}{\Lambda^2}-\frac{p^2}{2}\Sigma_p'^2X_p^2\right],$$

$$\mathcal{K}_{15}^{(\mathrm{norm})}=-4\int d\tilde{p}\left[-\left(\Sigma_p+\frac{1}{2}p^2\Sigma_p'\right)\frac{X_p}{\Lambda^2}\right.$$
$$\left.+\left(\Sigma_p+\frac{1}{2}p^2\Sigma_p'\right)X_p^2\right],\tag{36}$$

in which the short notations (in the Minkowskian space-time) are

$$\Sigma_p\equiv\Sigma(-p^2),\tag{37}$$

$$\int d\tilde{p}\equiv iN_c\int\frac{d^4p}{(2\pi)^4}e^{(p^2-\Sigma_p^2)/\Lambda^2},$$

$$X_p\equiv\frac{1}{p^2-\Sigma_p^2},$$

$$A_p=-\frac{2}{3}p^2\Sigma_p\Sigma_p'(-1-2\Sigma_p\Sigma_p')$$
$$-\frac{1}{3}\Sigma_p^2(-1-2\Sigma_p\Sigma_p')+\frac{1}{3}p^2\Sigma_p^2(-\Sigma_p'^2-\Sigma_p\Sigma_p'')$$
$$-\frac{1}{6}p^4(-\Sigma_p'^2-\Sigma_p\Sigma_p''),$$

$$B_p=-\frac{2}{3}p^2\Sigma_p\Sigma_p'(-1-2\Sigma_p\Sigma_p')$$
$$-\frac{1}{3}\Sigma_p^2(-1-2\Sigma_p\Sigma_p')+\frac{1}{3}p^2\Sigma_p^2(-\Sigma_p'^2-\Sigma_p\Sigma_p'')$$
$$-\frac{1}{18}p^4(-\Sigma_p'^2-\Sigma_p\Sigma_p'')-\frac{1}{6}p^2(-1-2\Sigma_p\Sigma_p'),$$

$$C_p=\frac{1}{3}-\frac{1}{3}\Sigma_p\Sigma_p'-\frac{1}{2}p^2\Sigma_p'^2,$$

$$D_p=\frac{1}{2}p^2\Sigma_p'^2-\frac{1}{6}p^2\Sigma_p\Sigma_p''(-1-2\Sigma_p\Sigma_p')$$
$$-\frac{2}{9}p^4\Sigma_p'\Sigma_p''(-1-2\Sigma_p\Sigma_p')$$
$$-\frac{2}{9}p^4\Sigma_p'^2(-\Sigma_p'^2-\Sigma_p\Sigma_p'')$$
$$-\frac{1}{3}p^2\Sigma_p\Sigma_p'(-\Sigma_p'^2-\Sigma_p\Sigma_p''),$$

$$E_p=-\frac{1}{6}p^2\Sigma_p\Sigma_p'(-1-2\Sigma_p\Sigma_p')^2$$
$$-\frac{1}{9}p^4\Sigma_p'^2(-1-2\Sigma_p\Sigma_p')^2,$$

$$F_p=-\frac{4}{3}p^2\Sigma_p\Sigma_p'+\frac{4}{3}p^2(\Sigma_p\Sigma_p')^2-\frac{2}{3}\Sigma_p^2+\frac{2}{3}\Sigma_p^3\Sigma_p'$$
$$+\frac{1}{3}p^2\Sigma_p^2(-\Sigma_p'^2-\Sigma_p\Sigma_p'')-\frac{1}{9}p^4(-\Sigma_p'^2-\Sigma_p\Sigma_p'')$$
$$-\frac{1}{3}p^2(-1-2\Sigma_p\Sigma_p')-\frac{1}{2}p^2.\tag{38}$$

For the coefficient F_0^2, Eq. (35) is just the well-known Pagels-Stokar formula [6] when taking the regularization cutoff parameter $\Lambda\to\infty$.

It is easy to check that these $\mathcal{K}_i^{(\mathrm{norm})}$ ($i=1,\ldots,15$) do contain the $\Pi_{\Omega c}$-independent (Σ_p-independent) piece which exactly cancel the anomaly contributions in Eqs. (7) mentioned in Sec. III. This can be done by taking a constant Σ_p to carry out the momentum integrations, and picking up the Σ_p-independent terms which are just the $\Pi_{\Omega c}$-independent terms mentioned in Sec. III. Subtracting these $\Pi_{\Omega c}$-independent terms from the obtained $\mathcal{K}_i^{(\mathrm{norm})}$ in Eqs. (36), we get the desired $\mathcal{K}_i^{(\mathrm{norm},\Pi_{\Omega c}\neq0)}$ in Eq. (23), which is needed in the final expressions for the chiral Lagrangian coefficients in Eqs. (25).

We can also check that the regularization cutoff Λ does not appear in $\mathcal{K}_i^{(\mathrm{norm},\Pi_{\Omega c}\neq0)}$, so that the obtained chiral Lagrangian coefficients $L_1,\ldots,L_{10}$ are all finite as it should be since there is no divergence in the large-N_c limit [the divergent meson-loop corrections are of $O(1/N_c)$].
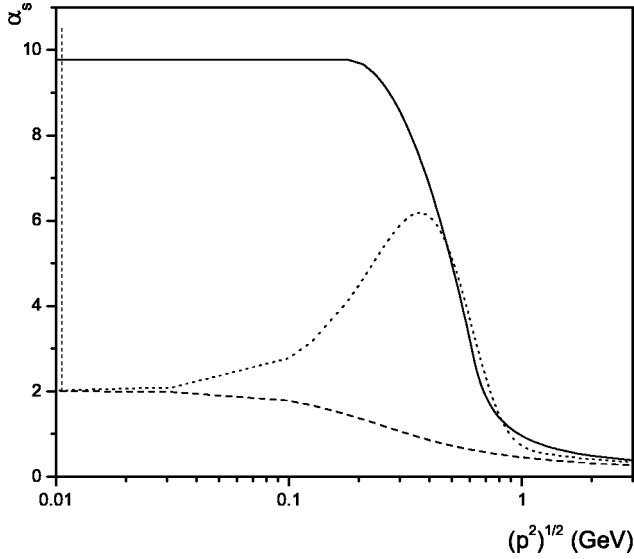
FIG. 1. $\alpha_s(p)$ for Model A [Eq. (40)], Model B [Eq. (41)], and Model C [Eq. (42)]. The solid, dashed, and dotted lines are for Models A, B, and C, respectively.
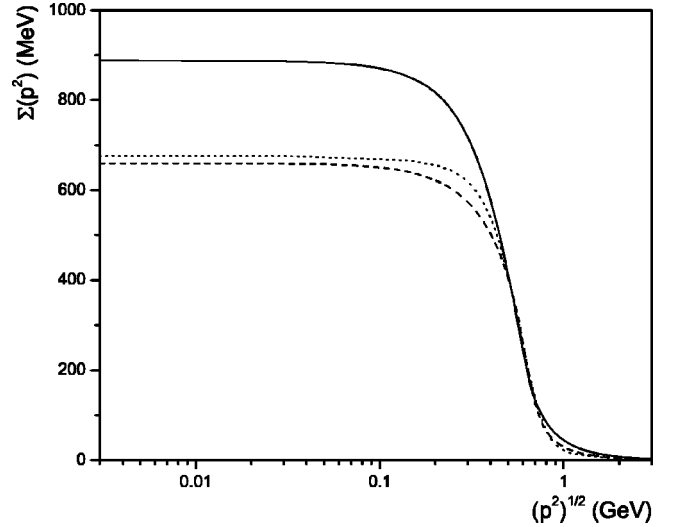

FIG. 2. The obtained $\Sigma(p^2)$ from the Schwinger-Dyson equation (27) with Model A [Eq. (40)], Model B [Eq. (41)], and Model C [Eq. (42)] for the running coupling constant α_s. The solid, dashed, and dotted lines are for Models A, B, and C, respectively.

V. NUMERICAL CALCULATIONS

The last step in the calculation is to solve the Schwinger-Dyson equation (27) numerically to obtain $\Sigma(p^2)$. In the integrand in the Schwinger-Dyson equation (27), there is still the QCD running coupling constant $\alpha_s(p-q)$ unspecified. The high momentum behavior of α_s is well known. The one-loop level formula is

$$\alpha_s(p) \xrightarrow{p^2 \to \infty} \frac{12\pi}{(33-2N_f)} \frac{1}{\ln(p^2/\Lambda_{\text{QCD}}^2)}. \qquad (39)$$

The low momentum behavior of $\alpha_s(p)$ is not known yet due to the ignorance of nonperturbative QCD. Inevitably, we have to take a certain QCD motivated model for it as in the literature. We shall take the following Model A from Ref. [9], and Model B and Model C from Ref. [10] as examples to do the calculation. They are

$$\text{A:} \quad \alpha_s(p) = 7\frac{12\pi}{(33-2N_f)}, \quad \text{for } \ln(p^2/\Lambda_{\text{QCD}}^2) \leqslant -2;$$

$$= \left\{ 7 - \frac{4}{5}[2 + \ln(p^2/\Lambda_{\text{QCD}}^2)]^2 \right\} \frac{12\pi}{(33-2N_f)},$$

$$\text{for } -2 \leqslant \ln(p^2/\Lambda_{\text{QCD}}^2) \leqslant 0.5;$$

$$= \frac{1}{\ln(p^2/\Lambda_{\text{QCD}}^2)} \frac{12\pi}{(33-2N_f)},$$

$$\text{for } 0.5 \leqslant \ln(p^2/\Lambda_{\text{QCD}}^2). \qquad (40)$$

$$\text{B:} \quad \alpha_s(p) = 4\pi^3 \eta^2 p^2 \delta^4(p) + \frac{12\pi}{(33-2N_f)} \frac{1}{\ln(2+p^2/\Lambda_{\text{QCD}}^2)}; \qquad (41)$$

$$\text{C:} \quad \alpha_s(p) = \frac{4\pi^3}{\mu^2} p^2 e^{-p^2/p_0^2} + \frac{12\pi}{(33-2N_f)} \frac{1}{\ln(2+p^2/\Lambda_{\text{QCD}}^2)}. \qquad (42)$$

They all have the asymptotic behavior (39). In Eq. (40), there is only one parameter Λ_{QCD}, while in Eqs. (41) and (42), in addition to Λ_{QCD}, there are extra parameters η, μ, and p_0, respectively. We shall determine the parameters in the following way. In the present approach, there are no meson-loop corrections. Thus we should identify $F_0 = f_\pi = 93$ MeV [2], and F_0 is given by the Pagels-Stokar formula. Changing the parameters will cause a change in $\Sigma(p^2)$, and thus a change in F_0. We take $F_0 = 93$ MeV as a requirement to determine the parameters. In the case of Model A, the determined Λ_{QCD} is $\Lambda_{\text{QCD}} = 484$ MeV [9]. In the cases of Model B and Model C, there are extra parameters. We take the original values $p_0 = 380$ MeV and $\Lambda_{\text{QCD}} = 230$ MeV as in Ref. [10], and determine η and μ in the above way. The determined values are $\eta = 290$ MeV and $\mu = 1160$ MeV.[2] The running coupling constant $\alpha_s(p)$ in the three cases are plotted in Fig. 1. We see that they are different mainly in the low momentum region.

To solve the Schwinger-Dyson equation (27), we further take the usual approximation $\alpha_s(p-q) \approx \theta(p^2-q^2)\alpha_s(p^2) + \theta(q^2-p^2)\alpha_s(q^2)$ [12] with which the angular integration can be easily carried out, and the integral equation can be

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[2]The original values of η and μ in Ref. [10] are $\eta = 920$ MeV, $\mu = 600$ MeV which are different from ours. The reason is that in Ref. [10] the number of quark flavors is taken as $N_f = 6$ rather than $N_f = 3$, and the formula for f_π is more complicated than the Pagels-Stokar formula.

TABLE I. The obtained values of the $O(p^4)$ coefficients $L_1, \ldots, L_{10}$ for Model A [Eq. (40)], Model B [Eq. (41)], and Model C [Eq. (42)] with $\Lambda, \bar{\Lambda} \to \infty$ together with the experimental values [Eq. (10)] for comparison. Λ_{QCD} is in MeV, and the coefficients are in units of 10^{-3}.

	Λ_{QCD}	L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	L_9	L_{10}
A:	484	1.10	2.20	-7.82	0	1.62	0	-0.70	1.75	5.07	-7.06
B:	230	0.921	1.84	-6.73	0	1.43	0	-0.673	1.64	3.80	-6.22
C:	230	0.948	1.90	-6.90	0	1.29	0	-0.632	1.56	3.95	-6.21
Expt:		0.9 ± 0.3	1.7 ± 0.7	-4.4 ± 2.5	0 ± 0.5	2.2 ± 0.5	0 ± 0.3	-0.4 ± 0.15	1.1 ± 0.3	7.4 ± 0.7	-6.0 ± 0.7

converted into the following differential equation[3]

$$\frac{d}{dp^2}\frac{\Sigma'(p^2)}{\left(\frac{\alpha_s(p)}{p^2}\right)'} - \frac{3N_c}{8\pi}p^2\frac{\Sigma(p^2)}{p^2+\Sigma^2(p^2)} = 0, \quad (43)$$

with boundary conditions:

$$\Sigma(\bar{\Lambda}^2) - \frac{3N_c}{8\pi}\frac{\alpha(\bar{\Lambda}^2)}{\bar{\Lambda}^2}\int_0^{\bar{\Lambda}^2}dq^2\frac{q^2\Sigma(q^2)}{q^2+\Sigma^2(q^2)} = 0, \quad (44)$$

$$\Sigma'(0) + \frac{\frac{3N_c}{16\pi}\alpha_s(0)}{\Sigma(0)} = 0, \quad (45)$$

where $\bar{\Lambda}$ is a momentum cutoff regularizing the integral. We shall eventually take $\bar{\Lambda} \to \infty$.

We know that the asymptotic behavior of $\Sigma(p^2)$ reflecting chiral symmetry breaking is

$$\Sigma(p^2) \xrightarrow{p^2 \to \infty} \frac{\ln^{\gamma-1}(p^2/\Lambda_{QCD}^2)}{p^2}, \quad (46)$$

where $\gamma \equiv (9N_c)/[2(33-2N_f)]$. We have found the numerical solution of Eqs. (43)–(45) satisfying this asymptotic behavior. The obtained solution with $\bar{\Lambda} \to \infty$ (a large enough number which can be regarded as infinity) in the three cases are plotted in Fig. 2. Again they are different mainly in the low momentum region.

With the obtained $\Sigma(p^2)$, we can calculate the $O(p^4)$ chiral Lagrangian coefficients from Eqs. (25), (23), and (36). The obtained values of $L_1, \ldots, L_{10}$ are listed in Table I together with the experimental values [2] for comparison. Note that there is no running of $L_1, \ldots, L_{10}$ in this simple approach since the meson-loop effects causing the running of

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[3]In the case of Model B, there is a term containing $\delta^4(p)$ which is not a function of p^2, and the integration can be directly carried out. Therefore, in this case, the differential equation and boundary conditions are different from Eqs. (43)–(45). They are $d/dp^2 d/dp^2 \Sigma(p^2)[1-3N_c/2/p^2+\Sigma^2(p^2)]/d/dp^2\beta/$ $p^2 \ln(2 +p^2/\Lambda_{QCD}^2) - 3N_c/8\pi p^2\Sigma(p^2)/p^2 +\Sigma^2(p^2)=0$, $\Sigma(\bar{\Lambda}^2)[1-3 N_c/2/\bar{\Lambda}^2+\Sigma(\bar{\Lambda}^2)] - 3N_c/8\pi\alpha_s(\bar{\Lambda}^2)/\bar{\Lambda}^2\int_0^{\bar{\Lambda}^2}dq^2q^2\Sigma(q^2)/q^2 + \Sigma (q^2)=0$, and $[d/dp^2\{\Sigma(p^2)[1-3N_c/2/p^2+\Sigma(p^2)]\}]_{p^2=0}$ $+3N_c/16\pi\alpha_s(0)/\Sigma(0)=0$, respectively.

$L_1, \ldots, L_{10}$ [2] are of the order of $1/N_c$, and are neglected in this approach. Thus the predicted numbers of $L_1, \ldots, L_{10}$ can be directly compared with the experimental values. We see from Table I that:

(i) these coefficients are not so sensitive to the forms of $\alpha_s(p)$;

(ii) all the obtained $L_1, \ldots, L_{10}$ are of the right orders of magnitude and the right signs;

(iii) L_1, L_2, L_4, L_6, and L_{10} are consistent with the experiments at the 1σ level;

(iv) L_3, L_5, L_7, and L_8 are consistent with the experiments at the 2σ level; and

(v) only L_9 deviates from the experimental value by $(3–4)\sigma$.

Considering the large theoretical uncertainty in this simple approach, *the obtained $L_1, \ldots, L_{10}$ are consistent with the experiments*. We see that the nonperturbative quark self-energy plays an important role in QCD contributions to the chiral Lagrangian coefficients. This supports the phenomenological model of Holdom [8].

In addition to $L_1, \ldots, L_{10}$, we can also calculate the quark condensate $\langle\bar{\psi}\psi\rangle$ from the $O(p^2)$ coefficient $F_0^2 B_0$ in Eq. (34). In the simple approach in this paper, the relation between $\langle\bar{\psi}\psi\rangle$ and $F_0^2 B_0$ is [3]

$$\langle\bar{\psi}\psi\rangle = -N_f F_0^2 B_0. \quad (47)$$

We know that, in this simple approach, $F_0 = f_\pi = 93$ MeV is finite. But $F_0^2 B_0$ in Eq. (34) is divergent,

$$F_0^2 B_0(\Lambda^2, \Lambda_{QCD}^2) \propto \ln^\gamma\left(\frac{\Lambda^2}{\Lambda_{QCD}^2}\right), \quad (48)$$

so that it needs to be renormalized. We take a simple renormalization scheme by taking the counter term as $F_0^2 B_0(\Lambda^2, \mu^2)$, in which μ is the renormalization scale.[4] Thus the renormalized quantity

$$F_0^2 B_{0r} \propto \ln^\gamma\left(\frac{\mu^2}{\Lambda_{QCD}^2}\right). \quad (49)$$

Then the renormalized (47) is

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[4]This corresponds to the modified minimal subtraction ($\overline{MS}$) scheme [13].

TABLE II. The same as in Table I but with $\Lambda = 1$ GeV instead of $\Lambda \to \infty$.

	L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	L_9	L_{10}	F_0
A:	0.403	0.805	-3.47	0	1.47	0	-0.792	1.83	2.28	-4.08	88.7
B:	0.281	0.563	-2.71	0	1.44	0	-0.836	1.83	1.46	-3.69	89.6
C:	0.304	0.608	-2.86	0	1.43	0	-0.855	1.87	1.56	-3.64	89.4

$$\langle \bar{\psi}\psi \rangle_r = -N_f F_0^2 B_{0r}. \tag{50}$$

We take the renormalization scale to be $\mu = 1$ GeV to define the quark condensate. The obtained values of $\langle \bar{\psi}\psi \rangle_r$ for the three forms of $\alpha_s(p)$ are

$$\text{A:} \quad \langle \bar{\psi}\psi \rangle_r = -(296 \text{ MeV})^3,$$

$$\text{B:} \quad \langle \bar{\psi}\psi \rangle_r = -(296 \text{ MeV})^3, \tag{51}$$

$$\text{C:} \quad \langle \bar{\psi}\psi \rangle_r = -(301 \text{ MeV})^3.$$

These are to be compared with the experimentally determined value $\langle \bar{\psi}\psi \rangle_{\text{expt}} = -(250 \text{ MeV})^3$ from the QCD sum rule at the scale of the typical hadronic mass [14]. Considering the large theoretical uncertainty in this calculation, the predicted quark condensate is also consistent with the experiment.

The above results show that *the present simple approach does reveal the main feature of the QCD predictions for the chiral Lagrangian coefficients* although the approximations in this approach are rather crude. Of course, further improvements of the approximations beyond this simple approach are needed. This kind of study is in progress.

Finally, we would like to mention that, in our calculation, we have taken the ultraviolet cutoff parameters $\Lambda, \bar{\Lambda} \to \infty$, i.e., we have taken account of the QCD contributions in the whole momentum range. Note that this has nothing to do with the validity range of the chiral Lagrangian determined by the range in which the expansion in the meson momentum makes sense, i.e., up to $\Lambda_\chi \approx 4\pi f_\pi$. To see the role of the QCD contributions from the high momentum region, say above 1 GeV, we have made a check by doing the calculations with the same $\Sigma(p^2)$ but taking $\Lambda = 1$ GeV instead of $\Lambda \to \infty$. The results are listed in Table II. Comparing the nonvanishing results in Table II with the corresponding $\Lambda, \bar{\Lambda} \to \infty$ results in Table I, we see that this change of Λ does not cause much difference in L_5, L_7, L_8, and F_0, while it causes L_1, L_2, L_3, L_9, and L_{10} to reduce by at least a factor of 2. Therefore, we see that L_5, L_7, L_8, and F_0 are mainly contributed by the QCD dynamics in the low momentum region, while high momentum region contributions to L_1, L_2, L_3, L_9, and L_{10} are not negligible.

VI. CONCLUSIONS

In this paper, we have calculated the coefficients in the Gasser-Leutwyler Lagrangian from the underlying theory of QCD in a simple approach with the approximations of taking the large-N_c limit, the leading order in dynamical perturba-

tion theory, and the improved ladder approximation based on the QCD formulas given in Ref. [3] to illustrate the main feature of how QCD predicts the chiral Lagrangian coefficients. In the calculation, we use the same regularization technique, the generalized Schwinger proper time regularization, in the calculations of the contributions from both the anomaly part and the normal part, so that the relation between the contributions from the two parts can be clearly seen.

We first take the large-N_c limit to evaluate the effective action in QCD. Our first conclusion in this study is that, *in the large-N_c limit, to all orders in momentum expansion, the anomaly part contributions to the chiral Lagrangian coefficients [cf. Eqs. (8)] given in the literature [4] from the effective action $S_{\text{eff}}^{(\text{anom})}$ [cf. Eq. (5)] are exactly cancelled by the contributions from the piece of the effective action $S_{\text{eff}}^{(\text{norm},\Pi_{\Omega c}=0)}$ [cf. Eq. (19)] in the normal part contributions, so that the chiral Lagrangian coefficients are eventually contributed by the remaining piece of the normal part effective action $S_{\text{eff}}^{(\text{norm},\Pi_{\Omega c} \neq 0)}$ [cf. Eq. (22)].* The final QCD expressions for the $O(p^4)$ coefficients are given in Eqs. (25).

To simplify $S_{\text{eff}}^{(\text{norm},\Pi_{\Omega c} \neq 0)}$, we further make the approximation of taking the leading order in dynamical perturbation theory. Then $S_{\text{eff}}^{(\text{norm},\Pi_{\Omega c} \neq 0)}$ is reduced to the simple form in Eq. (33), and all the chiral Lagrangian coefficients are approximately expressed in terms of the quark self-energy $\Sigma(p^2)$ shown in Eqs. (34)–(36). To solve the Schwinger-Dyson equation for $\Sigma(p^2)$, we further take the improved ladder approximation. Lacking the knowledge about the running coupling constant $\alpha_s(p)$ in the nonperturbative region, we take certain models for it from the literature [9,10] [cf. Eqs. (40)–(42)], and we further take the usual approximation $\alpha_s(p-q) \approx \theta(p^2-q^2)\alpha_s(p^2) + \theta(q^2-p^2)\alpha_s(q^2)$ to simplify the calculation. The quark self-energy reflecting chiral symmetry breaking is obtained by solving the simplified Schwinger-Dyson equation numerically. The obtained results of the $O(p^4)$ coefficients are listed in Table I. Compared with the experimental values of $L_1, \ldots, L_{10}$ [2], *the agreement of L_1, L_2, L_4, L_6, and L_{10} is of the level of 1σ, and that of L_3, L_5, L_7, and L_8 is of the level of 2σ. Only L_9 deviates from the experimental value by $(3-4)\sigma$.* Considering the large theoretical uncertainty in this simple approach, *all the obtained coefficients $L_1, \ldots, L_{10}$ are consistent with the experiments.* We have also calculated the renormalized quark condensate $\langle \bar{\psi}\psi \rangle_r$ from the obtained $O(p^2)$ coefficient [cf. Eq. (51)] *which is also consistent with the experiment.*

Although the approximations in this simple approach are rather crude, the above results show that *this simple approach does reveal the main feature of QCD predictions for*

the chiral Lagrangian coefficients. For studying physics not requiring high precision, this simple approach may already be useful. Of course further improvements of the approximations beyond this simple approach (reflecting more about QCD dynamics) are needed. This kind of study is in progress and will be presented in another paper.

The approach can also be applied to electroweak theories to study how the coefficients in the electroweak chiral Lagrangian are predicted by various kinds of underlying gauge theories of the electroweak symmetry breaking mechanism. This kind of study is also in progress, and will be presented in separate papers.

$$\text{Re} \ln \text{Det}[D + \Sigma(-\bar{\nabla}^2)] = \frac{1}{2} \text{Tr} \ln[[D^\dagger + \Sigma(-\bar{\nabla}^2)][D + \Sigma(-\bar{\nabla}^2)]]$$

$$= -\frac{1}{2} \lim_{\Lambda \to \infty} \int_{\frac{1}{\Lambda^2}}^{\infty} \frac{d\tau}{\tau} \text{Tr} \, e^{-\tau[\bar{E} - \nabla^2 + \Sigma^2(-\bar{\nabla}^2) + \hat{I}_\Omega \Sigma(-\bar{\nabla}^2) + \Sigma(-\bar{\nabla}^2)\tilde{I}_\Omega - d\!\!\!/\Sigma(-\bar{\nabla}^2)]} \tag{A1}$$

where

$$D \equiv \bar{\nabla}\!\!\!/ - s_\Omega + i p_\Omega \gamma_5,$$

$$\nabla_\mu \equiv \partial_\mu - i v_{\Omega\mu} - i a_{\Omega\mu} \gamma_5 = -\nabla_\mu^\dagger,$$

$$\bar{\nabla}^\mu \equiv \partial^\mu - i v_\Omega^\mu(x),$$

$$\bar{E} - \nabla^2 + \Sigma^2(-\bar{\nabla}^2) + \hat{I}_\Omega \Sigma(-\bar{\nabla}^2) + \Sigma(-\bar{\nabla}^2)\tilde{I}_\Omega - d\!\!\!/\Sigma(-\bar{\nabla}^2)$$

$$= [D^\dagger + \Sigma(-\bar{\nabla}^2)][D + \Sigma(-\bar{\nabla}^2)],$$

$$\hat{I}_\Omega = -i d\!\!\!/_\Omega \gamma_5 - s_\Omega - i p_\Omega \gamma_5,$$

$$\tilde{I}_\Omega = -i d\!\!\!/_\Omega \gamma_5 - s_\Omega + i p_\Omega \gamma_5,$$

$$[d\!\!\!/\Sigma(-\bar{\nabla}^2)] \equiv \gamma^\mu[d_\mu \Sigma(-\bar{\nabla}^2)] = \gamma^\mu(\partial_\mu \Sigma(-\bar{\nabla}^2)$$

$$- i[v_{\Omega\mu}, \Sigma(-\bar{\nabla}^2)]). \tag{A2}$$

The matrix element in Eq. (A1) can be evaluated in the momentum representation

APPENDIX A: FUNCTIONAL DETERMINANT CONTAINING QUARK SELF-ENERGY

In this appendix, we take the Schwinger proper time regulation to regularize the one-loop functional determinant in which the quark self-energy $\Sigma(\bar{\nabla}^2)$ reflecting chiral symmetry breaking takes place.

For convenience, the evaluation is done in the Euclidean space-time, and will be analytically continued to the Minkowskian space-time after the evaluation. The functional determinant is complex. The imaginary part is just the Wess-Zumino-Witten term, and its expression in terms of Σ has already been given in Ref. [15] which exactly coincides Witten's result [16]. The phenomenology of the Wess-Zumino-Witten term is well-known and is not related to the main purpose of this paper. So we shall ignore the imaginary part here and concentrate on the evaluation of the following real part of the functional determinant

$$\langle x | e^{-\tau[\bar{E} - \nabla^2 + \Sigma^2(-\bar{\nabla}^2) + \hat{I}_\Omega \Sigma(-\bar{\nabla}^2) + \Sigma(-\bar{\nabla}^2)\tilde{I}_\Omega - d\!\!\!/\Sigma(-\bar{\nabla}^2)]} | x \rangle$$

$$= \int \frac{d^4 p}{(2\pi)^4} \exp\{-\tau[\bar{E}(x) - \nabla_x^2 - 2ip \cdot \nabla_x + p^2 + \Sigma^2(-\bar{\nabla}_x^2$$

$$- 2ip \cdot \bar{\nabla}_x + p^2) + \hat{I}_\Omega \Sigma(-\bar{\nabla}_x^2 - 2ip \cdot \bar{\nabla}_x + p^2) + \Sigma(-\bar{\nabla}_x^2$$

$$- 2ip \cdot \bar{\nabla}_x + p^2)\tilde{I}_\Omega - d\!\!\!/\Sigma(-\bar{\nabla}_x^2 - 2ip \cdot \bar{\nabla}_x + p^2)]\}. \tag{A3}$$

Then after lengthy but elementary calculations and expanding in powers of the external sources, we can identify the expressions for F_0^2, $F_0^2 B_0$, $\mathcal{K}_1^{\text{(norm)}}, \ldots, \mathcal{K}_{15}^{\text{(norm)}}$ by comparing with the form of Eqs. (1), and the obtained results in the Minkowskian space-time are just those given in Eqs. (35), (34), and (36) in the text. The details are given in Ref. [5].

For the evaluation of the effective action $S_{\text{eff}}^{\text{(anom)}}$ in Eq. (5) in the Minkowskian space-time, we note that there is no $\Sigma(-\bar{\nabla}^2)$ term in Eq. (A1), but we still have to replace $\Sigma(-\bar{\nabla}^2)$ by an infrared cutoff parameter κ in Eq. (A1) to regularize the infrared divergence. Then the momentum integration can be explicitly carried out, and we obtain the results in Eqs. (7) in the text.

APPENDIX B: Ω-INDEPENDENCE OF THE LAST TERM IN EQ. (19)

Here we show that the last term in Eq. (19),

$$
S_{\text{eff}}^{(\bar{G})} \equiv N_c \sum_{n=2}^{\infty} \int d^4x_1 \cdots d^4x_n' \frac{(-i)^n (N_c g_s^2)^{n-1}}{n!}
$$

$$
\times \bar{G}_{\rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x_1', \ldots, x_n, x_n')
$$

$$
\times \Phi_{\Omega c}^{\sigma_1 \rho_1}(x_1, x_1') \cdots \Phi_{\Omega c}^{\sigma_n \rho_n}(x_n, x_n')|_{\Pi_{\Omega c}=0}, \quad \text{(B1)}
$$

with the saddle point equation [cf. Eq. (13)]

$$
\Phi_{\Omega c}^{\sigma \rho}(x, y)|_{\Pi_{\Omega c}=0} = -i[(i\slashed{\partial} + J_\Omega)^{-1}]^{\rho \sigma}(y, x) \quad \text{(B2)}
$$

is Ω-independent. The Ω-rotated quantities in Eqs. (B1) and (B2) are defined by [3]

$$
J_\Omega(x) = [\Omega(x)P_R + \Omega^\dagger(x)P_L] [J(x) + i\slashed{\partial}]
$$

$$
\times [\Omega(x)P_R + \Omega^\dagger(x)P_L],
$$

$$
\Phi_\Omega^T(x, y) = [\Omega^\dagger(x)P_R + \Omega(x)P_L]\Phi^T(x, y)
$$

$$
\times [\Omega^\dagger(y)P_R + \Omega(y)P_L], \quad \text{(B3)}
$$

and the $2n$-point Green's function

$$
\bar{G}_{\rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x_1', \ldots, x_n, x_n')
$$

is defined by [3]

$$
G_{\mu_1 \cdots \mu_n}^{i_1 \cdots i_n}(x_1, \ldots, x_n) \left[\bar{\psi}_{\alpha_1}^{a_1}(x_1) \left(\frac{\lambda_{i_1}}{2} \right)_{\alpha_1 \beta_1} \gamma^{\mu_1} \psi_{\beta_1}^{a_1}(x_1) \right] \cdots \left[\bar{\psi}_{\alpha_n}^{a_n}(x_n) \left(\frac{\lambda_{i_n}}{2} \right)_{\alpha_n \beta_n} \gamma^{\mu_n} \psi_{\beta_n}^{a_n}(x_n) \right]
$$

$$
= \int d^4x_1' \cdots d^4x_n' g_s^{n-2} \bar{G}_{\rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x_1', \cdots, x_n, x_n') \bar{\psi}_{\alpha_1}^{\sigma_1}(x_1) \psi_{\alpha_1}^{\rho_1}(x_1') \cdots \bar{\psi}_{\alpha_n}^{\sigma_n}(x_n) \psi_{\alpha_n}^{\rho_n}(x_n'). \quad \text{(B4)}
$$

First we see from Eqs. (B3) that Eq. (B2) can be written as

$$
\Phi_{\Omega c}^{\sigma \rho}(x, y)|_{\Pi_{\Omega c}=0} = (\Phi^T)_{\Omega c}^{\rho \sigma}(y, x)|_{\Pi_{\Omega c}=0} = [[\Omega^\dagger(y)P_R + \Omega(y)P_L]\Phi_c^T(y, x)[\Omega^\dagger(x)P_R + \Omega(x)P_L]]^{\rho \sigma}|_{\Pi_c=0}
$$

$$
= -i[[\Omega^\dagger(y)P_R + \Omega(y)P_L][(i\slashed{\partial} + J)^{-1}](y, x)[\Omega^\dagger(x)P_R + \Omega(x)P_L]]^{\rho \sigma}
$$

$$
= -i[\Omega^\dagger(y)P_R + \Omega(y)P_L]^{\rho \rho'}[(i\slashed{\partial} + J)^{-1}]^{\rho' \sigma'}(y, x)[\Omega^\dagger(x)P_R + \Omega(x)P_L]^{\sigma' \sigma}
$$

$$
= -i[\gamma_0 V_\Omega \gamma_0]^{\dagger \rho \rho'}(y)[(i\slashed{\partial} + J)^{-1}]^{\rho' \sigma'}(y, x) V_\Omega^{\sigma' \sigma}(x), \quad \text{(B5)}
$$

in which

$$
V_\Omega(x) \equiv \Omega^\dagger(x)P_R + \Omega(x)P_L \quad \text{(B6)}
$$

satisfies

$$
\gamma_0 V_\Omega^\dagger(x) \gamma_0 \gamma_\mu = \gamma_\mu V_\Omega^\dagger(x), \quad \text{(B7)}
$$

and

$$
\Phi_c^T(y, x)|_{\Pi_c=0} = -i[(i\slashed{\partial} + J)^{-1}](y, x) \quad \text{(B8)}
$$

is Ω-independent.

With the expression (B5) for $\Phi_{\Omega c}^{\sigma \rho}(x, y)|_{\Pi_{\Omega c}=0}$, Eq. (B1) becomes

$$S_{\text{eff}}^{(\bar{G})} = N_c \sum_{n=2}^{\infty} \int d^4x_1 \cdots d^4x_n' \frac{(-i)^n (N_c g_s^2)^{n-1}}{n!} \bar{G}_{\rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x_1', \ldots, x_n, x_n') \Phi_{\Omega c}^{\sigma_1 \rho_1}(x_1, x_1') \cdots \Phi_{\Omega c}^{\sigma_n \rho_n}(x_n, x_n') \big|_{\Pi_{\Omega c} = 0}$$

$$= N_c \sum_{n=2}^{\infty} \int d^4x_1 \cdots d^4x_n' \frac{(-i)^n (N_c g_s^2)^{n-1}}{n!} \bar{G}_{\rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x_1', \ldots, x_n, x_n')$$

$$\times (\gamma_0 V_\Omega^\dagger \gamma_0)^{\sigma_1 \sigma_1'}(x_1) \cdots (\gamma_0 V_\Omega^\dagger \gamma_0)^{\sigma_n \sigma_n'}(x_n) V_\Omega^{\rho_1' \rho_1}(x_1') \cdots V_\Omega^{\rho_n' \rho_n}(x_n') \Phi_c^{\sigma_1 \rho_1}(x_1, x_1') \cdots \Phi_c^{\sigma_n \rho_n}(x_n, x_n') \bigg|_{\Pi_c = 0}$$

$$= N_c \sum_{n=2}^{\infty} \int d^4x_1 \cdots d^4x_n' \frac{(-i)^n (N_c g_s^2)^{n-1}}{n!} \bar{G}_{V_\Omega, \rho_1' \cdots \rho_n'}^{\sigma_1' \cdots \sigma_n'}(x_1, x_1', \ldots, x_n, x_n') \Phi_c^{\sigma_1' \rho_1'}(x_1, x_1') \cdots \Phi_c^{\sigma_n' \rho_n'}(x_n, x_n') \bigg|_{\Pi_c = 0},$$

$$\tag{B9}$$

where $\bar{G}_{V_\Omega, \rho_1' \cdots \rho_n'}^{\sigma_1' \cdots \sigma_n'}(x_1, x_1', \ldots, x_n, x_n')$ is

$$\bar{G}_{V_\Omega, \rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x_1', \ldots, x_n, x_n') \equiv (\gamma_0 V_\Omega^\dagger \gamma_0)^{\sigma_1 \sigma_1'}(x_1) \cdots (\gamma_0 V_\Omega^\dagger \gamma_0)^{\sigma_n \sigma_n'}(x_n)$$

$$\times \bar{G}_{\rho_1' \cdots \rho_n'}^{\sigma_1' \cdots \sigma_n'}(x_1, x_1', \ldots, x_n, x_n') V_\Omega^{\rho_1' \rho_1}(x_1') \cdots V_\Omega^{\rho_n' \rho_n}(x_n'). \tag{B10}$$

Next, we look at this transformed Green's function $\bar{G}_{V_\Omega, \rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x_1', \ldots, x_n, x_n')$. From the definition (B4) and the property (B7) we have

$$\int d^4x_1' \cdots d^4x_n' g_s^{n-2} \bar{G}_{V_\Omega, \rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x_1', \ldots, x_n, x_n') \bar{\psi}_{\alpha_1}^{\sigma_1}(x_1) \psi_{\alpha_1}^{\rho_1}(x_1') \cdots \bar{\psi}_{\alpha_n}^{\sigma_n}(x_n) \psi_{\alpha_n}^{\rho_n}(x_n')$$

$$= \int d^4x_1' \cdots d^4x_n' g_s^{n-2} \bar{G}_{\rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x_1', \ldots, x_n, x_n') (\bar{\psi} \gamma_0 V_\Omega^\dagger \gamma_0)_{\alpha_1}^{\sigma_1}(x_1) (V_\Omega \psi)_{\alpha_1}^{\rho_1}(x_1') \cdots (\bar{\psi} \gamma_0 V_\Omega^\dagger \gamma_0)_{\alpha_n}^{\sigma_n}(x_n) (V_\Omega \psi)_{\alpha_n}^{\rho_n}(x_n')$$

$$= G_{\mu_1 \cdots \mu_n}^{i_1 \cdots i_n}(x_1, \ldots, x_n) \left[(\bar{\psi} \gamma_0 V_\Omega^\dagger \gamma_0)_{\alpha_1}^{a_1}(x_1) \left(\frac{\lambda_{i_1}}{2}\right)_{\alpha_1 \beta_1} \gamma^{\mu_1} (V_\Omega \psi)_{\beta_1}^{a_1}(x_1) \right] \cdots \left[(\bar{\psi} \gamma_0 V_\Omega^\dagger \gamma_0)_{\alpha_n}^{a_n}(x_n) \right.$$

$$\times \left. \left(\frac{\lambda_{i_n}}{2}\right)_{\alpha_n \beta_n} \gamma^{\mu_n} (V_\Omega \psi)_{\beta_n}^{a_n}(x_n) \right]$$

$$= G_{\mu_1 \cdots \mu_n}^{i_1 \cdots i_n}(x_1, \ldots, x_n) \left[\bar{\psi}_{\alpha_1}^{a_1}(x_1) \left(\frac{\lambda_{i_1}}{2}\right)_{\alpha_1 \beta_1} \gamma^{\mu_1} \psi_{\beta_1}^{a_1}(x_1) \right] \cdots \left[\bar{\psi}_{\alpha_n}^{a_n}(x_n) \left(\frac{\lambda_{i_n}}{2}\right)_{\alpha_n \beta_n} \gamma^{\mu_n} \psi_{\beta_n}^{a_n}(x_n) \right]$$

$$= \int d^4x_1' \cdots d^4x_n' g_s^{n-2} \bar{G}_{\rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x_1', \ldots, x_n, x_n') \bar{\psi}_{\alpha_1}^{\sigma_1}(x_1) \psi_{\alpha_1}^{\rho_1}(x_1') \cdots \bar{\psi}_{\alpha_n}^{\sigma_n}(x_n) \psi_{\alpha_n}^{\rho_n}(x_n'),$$

i.e.,

$$\bar{G}_{V_\Omega, \rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x_1', \ldots, x_n, x_n') = \bar{G}_{\rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x_1', \ldots, x_n, x_n'). \tag{B11}$$

Thus the transformed Green's function $\bar{G}_{V_\Omega \rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}$ in Eq. (B9) can be replaced by $\bar{G}_{\rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}$, and Eq. (B9) becomes

$$S_{\text{eff}}^{(\bar{G})} = N_c \sum_{n=2}^{\infty} \int d^4x_1 \cdots d^4x_n' \frac{(-i)^n (N_c g_s^2)^{n-1}}{n!} \bar{G}_{\rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x_1', \ldots, x_n, x_n') \Phi_c^{\sigma_1 \rho_1}(x_1, x_1') \cdots \Phi_c^{\sigma_n \rho_n}(x_n, x_n') \bigg|_{\Pi_c = 0}$$

$$\tag{B12}$$

which is independent of Ω.

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