Production of singlet P-wave $c\bar{c}$ and $b\bar{b}$ states

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No spin-singlet $b\bar{b}$ quarkonium state has yet been observed. In this paper we discuss the production of the singlet P-wave $b\bar{b}$ and $c\bar{c}$ $^1P_1$ states h_b and h_c. We consider two possibilities. In the first the $^1P_1$ states are produced via the electromagnetic cascades $\Upsilon(3S)\to\eta_b(2S)+\gamma\to h_b+\gamma\gamma\to\eta_b+\gamma\gamma\gamma$ and $\psi'\to\eta_c'+\gamma\to h_c+\gamma\gamma\to\eta_c+\gamma\gamma\gamma$. A more promising process consists of single pion transition to the $^1P_1$ state followed by the radiative transition to the $1\,^1S_0$ state: $\Upsilon(3S)\to h_b+\pi^0\to\eta_b+\pi^0+\gamma$ and $\psi'\to h_c+\pi^0\to\eta_c+\pi^0+\gamma$. For a million $\Upsilon(3S)$ or ψ''s produced we expect these processes to produce several hundred events.

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The study of bound states of heavy quarks has provided important tests of quantum chromodynamics (QCD) [1]. The heavy quarkonium $c\bar{c}$ and $b\bar{b}$ resonances have a rich spectroscopy with numerous narrow S-, P-, and D-wave levels below the production threshold of open charm and b-flavored mesons. The spin-triplet S-wave states $\psi(nS)$ and $\Upsilon(nS)$, with $J^{PC}=1^{--}$, are readily produced by virtual photons in e^+e^- or hadronic interactions, and then undergo electric dipole ($E1$) transition to the spin-triplet P-wave levels. Previous studies have discussed the production of the spin-triplet D-wave $b\bar{b}$ states [2,3] and there has been some discussion of how one might produce the $1\,^1P_1c\bar{c}$ state [4–9]. Up to now, the only observed heavy quarkonium spin-singlet state has been the $\eta_c(1^1S_0)$, but the Belle Collaboration [10] has just announced the discovery of the $\eta_c'(2^1S_0)$ in B decays at a mass of $(3654\pm6\pm8)$ MeV/c^2. There have also been a few measurements suggesting the $1^1P_1(c\bar{c})$ state in $\bar{p}p$ annihilation experiments [11–13] but these results have yet to be confirmed. No $b\bar{b}$ spin-singlet states have yet been seen.

The mass predictions for the singlet states are an important test of QCD motivated potential models [14–23] and the applicability of perturbative quantum chromodynamics to the heavy quarkonia $c\bar{c}$ and $b\bar{b}$ systems [24–27], as well as the more recent nonrelativistic QCD (NRQCD) [28] approach. For QCD-motivated potential models the triplet-singlet splittings test the Lorentz nature of the confining potential with different combinations of Lorentz scalar, vector, etc., giving rise to different orderings of the triplet-singlet splittings in the heavy quarkonium P-wave mesons. Furthermore, the observation of $c\bar{c}$ and $b\bar{b}$ states and the measurement of their masses is an important validation of lattice QCD calculations [29–34], which will lead to greater confidence in their application in extracting electroweak quantities from hadronic processes. Under the assumption of a Fermi-Breit Hamiltonian and only vector-like and scalar-like components in the central potential, Stubbe and Martin [35] predicted that the $n\,^1P_1$ mass lies no lower than the spin-averaged ^3P_J masses (weighted with the factors $2J+1$), denoted by $n\,^3P_{\rm cog}$. Violation of these bounds would indicate a significant underestimate by [35] of relativistic effects.

In Table I we summarize some predictions for hyperfine mass splittings for P-wave $c\bar{c}$ and $b\bar{b}$ levels. The wide variation in the predicted splittings demonstrates the need for experimental tests of the various calculational approaches.

There are two possibilities for producing spin-singlet states. In the first, the system undergoes a magnetic dipole ($M1$) transition from a spin-triplet state to a spin-singlet state. The predictions for $M1$ transitions from the $\Upsilon(n\,^3S_1)$ levels to the $\eta_b(n'\,^1S_0)$ states, for both favored $M1$ transitions and hindered $M1$ transitions with changes of the principal quantum number, have been reviewed in Ref. [36]. The second route begins with a hadronic transition, from a n^3S_1 state to a $^1P_1$ state, emitting one or more pions, followed by the electromagnetic decay of the $^1P_1$ state.

In this paper we examine the production of the spin-singlet P-wave $c\bar{c}$ and $b\bar{b}$ states. We examined the decay chains that start with the $M1$ transition from the ψ' to the η_c' in the $c\bar{c}$ system and from the $\Upsilon(3S)$ to either the $\eta_b(3S)$ or $\eta_b(2S)$ state in the $b\bar{b}$ system. In both cases the $M1$ transition is followed by an $E1$ transition to the spin-singlet $2P$ or $1P$ state. This is in turn followed by a second $E1$ transition to a $n\,^1S_0$ state. In addition, the $2\,^1P_1$ $b\bar{b}$ state can undergo an $E1$ transition to the $1\,^1D_2$ state. However, with the current CLEO data set, the only decay chain which has any hope of being seen in the $b\bar{b}$ system is $\Upsilon(3S)\to\eta_b(2S)\gamma\to h_b(1P)\gamma\gamma$. We therefore only present results relevant to this set of decays.

The decay chains originating with the hadronic transitions are more promising. We therefore include estimates of branching ratios for chains originating with the direct hadronic transition $\Upsilon(3S)\to h_b(^1P_1)+\pi^0$ discussed by Voloshin [37] followed by the radiative decay $h_b(^1P_1)$

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TABLE I. Predictions for hyperfine splittings $M(n\,^3P_{\rm cog}) - M(n\,^1P_1)$ for $c\bar{c}$ and $b\bar{b}$ levels.

Reference	Approach	$n=1$ $c\bar{c}$ (MeV)	$n=1$ $b\bar{b}$ (MeV)	$n=2$ $b\bar{b}$ (MeV)
GI85 [14]	a	8	2	2
MR83 [15]	b	0	0	1
LPR92 [16]	c	4	2	1
OS82 [17]	d	10	3	3
MB83 [18]	e	−5	−2	−2
GRR86 [19]	f	−2	−1	−1
IO87 [20]	g	24.1±2.5	3.73±0.1	3.51±0.02
GOS84 $\eta_s=1$ [21]	h	6	3	2
GOS84 $\eta_s=0$ [21]	h	17	8	6
PJF92 [22]	i	−20.3±3.7	−2.5±1.6	−3.7±0.8
HOOS92 [23]	j	−0.7±0.2	−0.18±0.03	−0.15±0.03
PTN86 [25]	j	−3.6	−0.4	−0.3
PT88 [26]	j	−1.4	−0.5	−0.4
SESAM98 [31]	k	–	~ -1	–
CP-PACS00 [33]	l	1.7–4.0	1.6–5.0	–

[a]Potential model with smeared short range hyperfine interaction. (The splittings are based on masses rounded to 1 MeV, not the results rounded to 10 MeV as given in Ref. [14].)
[b]Potential model with long range longitudinal color electric field.
[c]Potential model with PQCD corrections to the short distance piece.
[d]Potential model with smeared hyperfine interaction.
[e]Potential model with smeared hyperfine interaction and relativistic corrections.
[f]Potential model includes 1-loop QCD corrections.
[g]Potential model with short distance from the 2-loop PQCD calculation. Results shown are for $\Lambda_{\overline{MS}} = 200$ MeV.
[h]Potential model with confining potential with both Lorentz scalar and vector. η_s gives the fraction of the confining potential that is pure Lorentz scalar versus Lorentz vector.
[i]Potential model. The solution is for the Richardson potential and $m_c = 1.49 \pm 0.1$ GeV. Other solutions given in Ref. [22] are consistent with this result within errors.
[j]PQCD.
[k]Unquenched nonrelativistic lattice QCD.
[l]Lattice QCD; the result is dependent on the value used for β and m_Q.

$\rightarrow \eta_b(1S) + \gamma$ and the analogous transitions in the charmonium system $\psi'(2S) \rightarrow h_c(^1P_1) + \pi^0 \rightarrow \eta_c(1S)\gamma\pi^0$. Kuang and Yan [38] have also considered the related spin-flip transition $\Upsilon(3S) \rightarrow h_b(^1P_1) + \pi\pi$ which may provide an additional path to the h_b.

Searches for the $^1P_1$ states have taken on renewed interest because of the current data-taking runs of the CLEO Collaboration at the Cornell Electron Storage Ring (CESR), which are expected to significantly increase their sample of data at the $\Upsilon(3S)$ resonance, and the proposed CLEO-c project which will study physics in the charmonium system.

We begin with the $b\bar{b}$ mesons and decay chains involving only radiative transitions. To estimate the number of events expected from these decay chains we need to estimate the radiative partial decay widths between states and the hadronic partial widths of the appropriate $^1S_0$ and $^1P_1$ states.

The $M1$ transitions from the $\Upsilon(3S)$ to the $\eta_b(3S)$ and $\eta_b(2S)$ were studied in detail in Ref. [36], which we will use in what follows. The $E1$ transitions are straightforward to work out [2] and in the nonrelativistic limit are given by

$$\Gamma(^1S_0 \rightarrow {}^1P_1 + \gamma) = \frac{4}{3}\alpha e_Q^2 \omega^3 |\langle ^1P_1 | r | ^1S_0 \rangle|^2 \qquad (1)$$

$$\Gamma(^1P_1 \rightarrow {}^1S_0 + \gamma) = \frac{4}{9}\alpha e_Q^2 \omega^3 |\langle ^1S_0 | r | ^1P_1 \rangle|^2 \qquad (2)$$

$$\Gamma(^1P_1 \rightarrow {}^1D_2 + \gamma) = \frac{8}{9}\alpha e_Q^2 \omega^3 |\langle ^1D_2 | r | ^1P_1 \rangle|^2 \qquad (3)$$

where $\alpha = 1/137.036$ is the fine-structure constant, e_Q is the quark charge in units of $|e|$ ($-1/3$ for $Q=b$), and ω is the photon's energy. The photon energies, overlap integrals, and partial widths for the $E1$ transitions between $^1P_1$ and $^1S_0$ levels are given in Table II and summarized in Fig. 1, along with the relevant $M1$ transitions. The $n\,^1S_0$ masses were obtained by subtracting the predictions of Ref. [14] for the $n^3S_1 - n^1S_0$ splittings from the measured $n\,^3S_1$ masses, while the $n\,^1P_1$ masses were obtained by subtracting the predictions for the $n\,^3P_{\rm cog} - n\,^1P_1$ splittings of Ref. [14] from measured $n\,^3P_{\rm cog}$ values. The overlap integrals, $\langle r \rangle$

TABLE II. Radiative electric dipole transitions involving $h_b(1^1P_1)$ and $h_b'(2^1P_1)$ $b\bar{b}$ states. The details of the calculation are given in the text.

Transition	M_i (MeV)	M_f (MeV)	ω (MeV)	$\langle r\rangle$ (GeV^{-1})	Γ (keV)
$3\,^1S_0\to2\,^1P_1$	10337	10258	78.7	-2.46	3.2
$3\,^1S_0\to1\,^1P_1$	10337	9898	430	0.126	1.4
$2\,^1P_1\to1\,^1D_2$	10258	10148	109	-1.69	2.7
$2\,^1P_1\to2\,^1S_0$	10258	9996	259	1.57	15.4
$2\,^1P_1\to1\,^1S_0$	10258	9397	825	0.222	10.0
$2\,^1S_0\to1\,^1P_1$	9996	9898	97.5	-1.53	2.3
$1\,^1P_1\to1\,^1S_0$	9898	9397	488	0.940	37

$\equiv\langle^1L'_{L'}|r|^1L_L\rangle$, were evaluated using the wave functions of Ref. [14]. We found that the relativistic effects considered in Ref. [14] reduce the partial widths by a few percent at most. Somewhat larger matrix elements were obtained in an inverse-scattering approach [2], except for the highly suppressed $3S\to1P$ transition, whose matrix element is very sensitive to details of wave functions [39].

To estimate the number of events in a particular decay chain requires branching fractions which depend on knowing all important partial decay widths. Inclusive strong decays to gluon and quark final states generally make large contributions to the total width and have been studied extensively [40–45]. The relevant theoretical expressions, including leading-order QCD corrections [41], are summarized in Ref. [44]:

$$\Gamma(^1S_0\to gg)=\frac{8\pi\alpha_s^2}{3m_Q^2}|\psi(0)|^2 \tag{4}$$

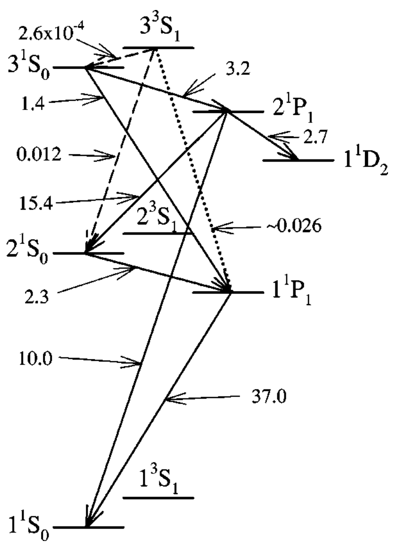

FIG. 1. Radiative transitions in the $b\bar{b}$ system. The dashed lines represent $M1$ transitions, the solid lines $E1$ transitions and the dotted lines single π^0 emission. The transitions are labeled with their partial widths given in keV.

with a multiplicative correction factor of $[1+4.4(\alpha_s/\pi)]$ for $b\bar{b}$ and $[1+4.8(\alpha_s/\pi)]$ for $c\bar{c}$,

$$\Gamma(^1P_1\to ggg)=\frac{20\alpha_s^3}{9\pi m_Q^4}|R_P'(0)|^2\ln(m_Q\langle r\rangle) \tag{5}$$

and

$$\Gamma(^1P_1\to gg+\gamma)=\frac{36}{5}e_q^2\frac{\alpha}{\alpha_s}\Gamma(^1P_1\to ggg) \tag{6}$$

where we also include the decay $^1P_1\to gg+\gamma$.

Considerable uncertainties arise in these expressions from the model dependence of the wave functions and possible relativistic contributions [14]. In addition, the logarithm in the decay $\Gamma(^1P_1\to ggg)$ is a measure of the virtuality of the quark emitting the gluon. Different choices have been proposed for its argument, introducing further uncertainty. Rather than evaluating these expressions in a specific potential model, we can obtain less model-dependent estimates of strong decays by relating ratios of theoretical predictions, in which much of the theoretical uncertainties factor out, to experimentally measured widths. Although we expect the wave function at the origin to be slightly larger for the singlet state than the triplet state, we expect this difference to be much smaller than the uncertainties mentioned above.

To make our estimates, we will need in addition to Eqs. (4)–(6), the following expressions [44]:

$$\Gamma(^3S_1\to ggg)=\frac{40(\pi^2-9)\alpha_s^3}{81m_Q^2}|\psi(0)|^2 \tag{7}$$

with a multiplicative correction factor of $[1-4.9(\alpha_s/\pi)]$ for $b\bar{b}$ and $[1-3.7(\alpha_s/\pi)]$ for $c\bar{c}$,

$$\Gamma(^3S_1\to\gamma+gg)=\frac{32(\pi^2-9)e_Q^2\alpha\alpha_s^2}{9m_Q^2}|\psi(0)|^2 \tag{8}$$

with a multiplicative correction factor of $[1-7.4(\alpha_s/\pi)]$ for $b\bar{b}$ and $[1-6.7(\alpha_s/\pi)]$ for $c\bar{c}$, and

$$\Gamma(^3P_1\to q\bar{q}+g)=\frac{8\alpha_s^3n_f}{9\pi m_Q^4}|R_P'(0)|^2\ln(m_Q\langle r\rangle), \tag{9}$$

where the QCD correction factor for the last expression is not known. Taking account of decays of the $2\,^3S_1$ and $3\,^3S_1$ states to $\pi\pi\Upsilon(nS)$, lepton pairs, and $\chi_b(nP)\gamma$, as quoted in Ref. [46], we find total branching ratios to non-glue final states, and assume glue to constitute the remainder. Using branching ratios and total widths quoted in Ref. [46], we then arrive at the estimates summarized in Table III for $\Gamma[\Upsilon(nS)\to\text{glue}]\equiv\Gamma[\Upsilon(nS)\to\text{hadrons}]+\Gamma[\Upsilon(nS)\to\gamma+\text{hadrons}]$.

TABLE III. Ingredients in estimates of $\Gamma[\Upsilon(nS)\to\text{glue}]$.

$\Upsilon(nS)$ state	$\mathcal{B}$(non-glue) (%)	$\mathcal{B}$(glue) (%)	Γ(tot) (keV)	Γ(glue) (keV)
$\Upsilon(2S)$	49.1 ± 1.5	50.9 ± 1.5	44 ± 7	22.4 ± 3.6
$\Upsilon(3S)$	44.9 ± 1.4	55.1 ± 1.4	26.3 ± 3.5	14.5 ± 2.0

Using Eqs. (7) and (8), $\alpha_s(\Upsilon_{2S})=0.181$, and $\alpha_s(\Upsilon_{3S})=0.180$ we find $\Gamma[\Upsilon(2S)\to\text{hadrons})]=21.8\pm3.5$ keV and $\Gamma[\Upsilon(3S)\to\text{hadrons})]=14.1\pm2.0$ keV. The ratio of the widths from Eqs. (4) and (7):

$$\Gamma(^1S_0\to gg)=\frac{27\pi}{5(\pi^2-9)}\frac{1}{\alpha_s}\frac{\left(1+4.4\frac{\alpha_s}{\pi}\right)}{\left(1-4.9\frac{\alpha_s}{\pi}\right)}\times\Gamma(^3S_1\to ggg) \tag{10}$$

results in $\Gamma[\eta_b(2S)\to\text{hadrons})]=4.1\pm0.7$ MeV and $\Gamma[\eta_b(3S)\to\text{hadrons})]=2.7\pm0.4$ MeV.

We follow the same procedure to estimate the hadronic width for the $^1P_1$ states, although in this case we need to make use of a theoretical estimate for the partial width $^3P_1\to{}^3S_1\gamma$. Here we have [44]

$$\Gamma(^1P_1\to\text{hadrons})=\frac{5}{2n_f}\times\Gamma(^3P_1\to\text{hadrons}) \tag{11}$$

where n_f is the number of light quark flavors in the final state which we will take to be 3, ignoring the kinematically suppressed charm-anticharm channel. This results in a conservative upper limit for $\Gamma(^1P_1\to\text{hadrons})$ and hence a lower limit for the branching ratio of this state to $\gamma+\eta_b$. As mentioned, the QCD corrections to these widths are not known. The large uncertainties arising from the wave function and logarithms in Eqs. (5) and (9) cancel out.

The only branching ratios quoted in Ref. [46] for the n^3P_1 states are for decays to $n'^3S_1\gamma$. Using quark model predictions for the radiative transitions and assuming that hadronic decays dominate the remainder of the total widths, we can estimate the hadronic partial widths of these states. The results are summarized in Table IV.

TABLE V. Partial widths and branching ratios for spin-singlet $b\bar{b}$ states. The details of the calculation are given in the text.

Initial state	Final state	Width (keV)	$\mathcal{B}$ (%)
$3\,^1S_0$	$2\,^1P_1\gamma$	3.2	0.12
	$1\,^1P_1\gamma$	1.4	0.05
	gg	2700	99.8
$2\,^1P_1$	$2\,^1S_0\gamma$	15.4	19.3
	$1\,^1S_0\gamma$	10.0	12.5
	$1\,^1D_2\gamma$	2.7	3.4
	ggg	50.2[a]	62.8
	γgg	1.6	2.0
$2\,^1S_0$	$1\,^1P_1\gamma$	2.3	0.057
	gg	4100	99.9
$1\,^1P_1$	$1\,^1S_0\gamma$	37.0	41.4
	ggg	50.8[a]	56.8
	γgg	1.6	1.8

[a]Based on the partial width for $^3P_1\to{}^3S_1\gamma$ of Ref. [2] in Table IV.

The branching ratios obtained by combining the partial widths given in Tables II and IV are summarized in Table V.

To study the singlet P-wave $b\bar{b}$ states we considered the two-photon inclusive transitions $3\,^3S_1\xrightarrow{\gamma}3\,^1S_0\xrightarrow{\gamma}2\,^1P_1$ or $\xrightarrow{\gamma}1\,^1P_1$ and $3\,^3S_1\xrightarrow{\gamma}2\,^1S_0\xrightarrow{\gamma}1\,^1P_1$. In all cases the $^1P_1$ states can undergo further $E1$ radiative transitions to $^1S_0$ states. It may be that this last photon provides a useful tag to distinguish the cascade of interest from other possible decays involving triplet P and D-wave $b\bar{b}$ states. We use the branching ratios predicted in Ref. [36] for the initial $M1$ transitions, $\mathcal{B}(\Upsilon(3S)\to\eta_b(3S)+\gamma)=0.10\times10^{-4}$ and $\mathcal{B}(\Upsilon(3S)\to\eta_b(2S)+\gamma)=4.7\times10^{-4}$, which correspond to the GI mass splittings and wave functions [14] and where the latter result takes into account relativistic corrections. Combined with the branching ratios for the subsequent $E1$ transitions given in Table V the only decay chain that might yield enough events to be observed is $3\,^3S_1\xrightarrow{\gamma}2\,^1S_0\xrightarrow{\gamma}1\,^1P_1$ which yields roughly 0.3 events per million $\Upsilon(3S)$ states produced.

A more promising approach is the decay chain $\Upsilon(3S)\to{}^1P_1\pi^0$ followed by the $E1$ radiative transition $^1P_1\to{}^1S_0\gamma$. Voloshin estimates $\mathcal{B}(\Upsilon(3S)\to1\,^1P_1+\pi^0)=0.10$

TABLE IV. Partial widths of $^3P_1$ and $^1P_1$ states. The details of the calculation are given in the text.

$^3P_1$ state	$\Sigma_n\mathcal{B}(^3P_1\to n^3S_1\gamma)$[a] (%)	$\Gamma(^3P_1\to{}^3S_1\gamma)$ (keV)	$\Gamma(\to\text{hadrons})$ $^3P_1$ (keV)	$^1P_1$ (keV)
$1\,^3P_1(b\bar{b})$	35 ± 8	32.8[b]	60.9	50.8
		28.9[c]	53.7	44.7
$2\,^3P_1(b\bar{b})$	29.5 ± 4.2	25.2[b]	60.2	50.2
		16.8[c]	40.1	33.4

[a]Particle Data Group [46].
[b]Kwong and Rosner [2].
[c]Godfrey and Isgur [14].

TABLE VI. Partial widths and branching ratios for spin-singlet $c\bar{c}$ states. In column 5, $\mathcal{O}$ represents the operator relevant to the particular electromagnetic transition; $\mathcal{O}=r(\mathrm{GeV}^{-1})$ for $E1$ transitions and $\mathcal{O}=j_0(kr/2)$ for $M1$ transitions. The details of the calculation are given in the text.

Initial state	Final state	M_i (MeV)	M_f (MeV)	ω (MeV)	$\langle f\lvert\mathcal{O}\rvert i\rangle$	Width (keV)	$\mathcal{B}$ (%)
$2\,^3S_1$	$2\,^1S_0\gamma$	3686	3654	31.8	0.982	0.051	0.018
	$1\,^1S_0\gamma$	3686	2980	638	0.151	9.7	3.5
$2\,^1S_0$	$1\,^1P_1\gamma$	3654	3517	134.4	-2.21	51.3	0.69
	$1\,^3S_1\gamma$	3654	3097	515	-0.0973	6.3	0.084
	gg					7400	99.2
$1\,^1P_1$	$1\,^1S_0\gamma$	3517	2980	496	1.42	354	37.7
	ggg					533	56.8
	γgg					52	5.5

$\times 10^{-2}$ [37]. Thus, $\mathcal{B}[\Upsilon(3S)\to 1\,^1P_1+\pi^0\to 1\,^1S_0\gamma]\simeq 4\times 10^{-4}$, which would yield $\simeq 400$ events per million $\Upsilon(3S)$ produced. This signature should be easily seen by the CLEO detector, which has excellent photon detection capabilities. Since the recoil of the 1^1P_1 state is relatively small, the 488 MeV photon from the $1^1P_1\to 1^1S_0$ decay (suitably Doppler-shifted by up to ± 20 MeV) should provide a useful tag. Kuang and Yan predict [38] the partial width for the hadronic transition $\Upsilon(3S)\to h_b(1^1P_1)+\pi\pi$ to be 0.1–0.2 keV, giving a branching ratio of $\sim(3.8\text{–}7.6)\times 10^{-3}$. This is substantially higher than the value for $\mathcal{B}(\Upsilon(3S)\to 1^1P_1+\pi^0)$ quoted above so it could provide an alternative path to the h_b. However, Voloshin [37] does not obtain such a favorable branching ratio for this process, finding instead $<10^{-4}$.

We now turn to the charmonium system. The search for the h_c was discussed recently by Kuang [5] so we will be brief in our analysis, emphasizing aspects that are different from Kuang [5]. As in the case of $b\bar{b}$ there are two routes to the h_c. The first is the decay chain $\psi'\to\eta_c'\gamma\to h_c\gamma$ and the second is through the hadronic transition $\psi'\to h_c\pi^0$.

For the first case we need the various radiative widths. The expression for the $E1$ width is given by Eq. (1), while the rates for magnetic dipole transitions are given in the nonrelativistic approximation by

$$\Gamma(^3S_1\to {}^1S_0+\gamma)=\frac{4\alpha e_Q^2}{3m_Q^2}\,\omega^3\lvert\langle f\lvert j_0(kr/2)\rvert i\rangle\rvert^2 \qquad (12)$$

$$\Gamma(^1S_0\to {}^3S_1+\gamma)=\frac{4\alpha e_Q^2}{m_Q^2}\,\omega^3\lvert\langle f\lvert j_0(kr/2)\rvert i\rangle\rvert^2 \qquad (13)$$

where we take $m_c=1.628$ GeV. The results, using the wave functions and 1^1P_1 mass of Ref. [14], are summarized in Table VI. To calculate $\Gamma(\psi'\to\eta_c'\gamma)$, we took $M(\eta_c')=3654$ MeV, the central value quoted in Ref. [10]. Note that the widths for the hindered $M1$ transitions are very sensitive to the wave functions. The hadronic widths for the η_c' and h_c given in Table VI were obtained using the same procedure used for the $b\bar{b}$ hadronic widths: We relate theoretical expressions for ratios of the widths to a known measured width and take $\alpha_s(\psi')=0.236$. [In contrast with the $b\bar{b}$ system, the total width of the 1^3P_1 $c\bar{c}$ meson is known [46]: $\Gamma_{\mathrm{tot}}(\chi_{c1})=0.88\pm 0.14$ MeV.] The predicted result for $h_c\to$hadrons is consistent with the NRQCD result obtained by Bodwin, Braaten and Lepage [47]. Combining these results we find that $\mathcal{B}(\psi'\to\eta_c'\gamma)\times\mathcal{B}(\eta_c'\to h_c\gamma)\sim 10^{-6}$, which would yield a modest number of h_c mesons at best.

As in the case of the h_b, a more promising avenue is the single pion transition $\psi'\to h_c\pi^0$ followed by the radiative transition $h_c\to\eta_c\gamma$, where the photon is expected to have an energy very close to 496 MeV and can be used to tag the event. Using the branching ratio of $\mathcal{B}(\psi'\to h_c\pi^0)=0.1\%$ predicted by Voloshin [37] (see also Refs. [7,38]) and the branching ratio given for $h_c\to\eta_c\gamma$ in Table VI, we obtain $\mathcal{B}(\psi'\to h_c\pi^0)\times\mathcal{B}(h_c\to\eta_c\gamma)=3.8\times 10^{-4}$, which is substantially larger than the decay chain proceeding only via radiative transitions. In his recent paper Kuang [5] finds h_c production to be sensitive to $^3S_1$-$^3D_1$ mixing, so that a measurement of $\mathcal{B}(\psi'\to h_c\pi^0)$ would be a useful test of detailed mixing schemes between the ψ' and the $\psi(3770)\equiv\psi''$, some of which are discussed in Ref. [48,49].

Another promising approach for the detection of the h_c has recently been proposed by Suzuki [4]. He suggests looking for the h_c by measuring the final state $\gamma\eta_c$ of the cascade $B\to h_cK/K^*\to\gamma\eta_cK/K^*$. This channel is especially timely given the announcement by the Belle Collaboration of the discovery of the $\eta_c'(2^1S_0)$ in B decays [10] and, previously, the observation of the related decay, $B\to\chi_0K$ [50].

In the case of the S-wave ($^1S_0$) states, one should also bear in mind that $\gamma\gamma$ collisions have been used to observed the η_c in several experiments (see [46]). One candidate for $\gamma\gamma\to\eta_b$ with mass $9.30\pm 0.02\pm 0.02$ GeV/c^2 (consistent, however, with background) has been reported by the ALEPH Collaboration [51].

To conclude, we have explored different means of looking for the $^1P_1$ states in heavy quarkonium. In both the $b\bar{b}$ and $c\bar{c}$ systems the 1^1P_1 state can be reached via the chain $^3S_1\to {}^1S_0+\gamma\to 1^1P_1+\gamma\gamma$. However, in both systems one only expects of the order of a few events per million $\Upsilon(3S)$ or ψ' produced. In both systems, a more promising avenue is the transition $^3S_1\to {}^1P_1+\pi^0$ followed by the $E1$ radiative transition to the 1^1S_0 state which would yield several hundred events per million $\Upsilon(3S)$ or ψ''s produced. The alternative suggestion [38] of searching for the transitions $^3S_1\to {}^1P_1+\pi\pi$ also is worth pursuing.

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