Mass spectra of doubly heavy baryons in the relativistic quark model

D. Ebert
Institut für Physik, Humboldt–Universität zu Berlin, Invalidenstrasse 110, D-10115 Berlin, Germany

R. N. Faustov and V. O. Galkin
Institut für Physik, Humboldt–Universität zu Berlin, Invalidenstrasse 110, D-10115 Berlin, Germany and Russian Academy of Sciences, Scientific Council for Cybernetics, Vavilov Street 40, Moscow 117333, Russia

A. P. Martynenko
Samara State University, Pavlov Street 1, Samara 443011, Russia
(Received 23 January 2002; published 19 July 2002)

Mass spectra of baryons consisting of two heavy (b or c) and one light quark are calculated in the framework of the relativistic quark model. The light-quark–heavy-diquark structure of the baryon is assumed. Under this assumption the ground and excited states of both the diquark and quark-diquark bound system are considered. The quark-diquark potential is constructed. The light quark is treated completely relativistically, while the expansion in the inverse heavy quark mass is used revealing the close similarity with the mass spectra of B and D mesons. We find that the relativistic treatment of the light quark plays an important role. The level inversion of the p-wave excitations of the light quark in doubly heavy baryons is discussed.

DOI: 10.1103/PhysRevD.66.014008 PACS number(s): 14.20.Lq, 12.39.Ki, 14.20.Mr

I. INTRODUCTION

The description of baryons within the constituent quark model is a very important problem in quantum chromodynamics (QCD). Since the baryon is a three-body system [1], its theory is much more complicated compared to the two-body meson system. Up till now it has not even been clear which of the two main QCD models, Y law or Δ law, correctly describes the nonperturbative (long-range) part of the quark interaction in the baryon [2,3]. The popular quark-diquark picture of a baryon is not universal and does not work in all cases [4]. The success of the heavy quark effective theory (HQET) [5] in predicting some properties of the heavy-light $q\bar{Q}$ mesons (B and D) suggests that we apply these methods to heavy-light baryons, too. The simplest baryonic systems of this kind are the so-called doubly heavy baryons (qQQ) [1,6–12]. The two heavy quarks (b or c) compose in this case a bound diquark system in the antitriplet color state which serves as a localized color source. The light quark is orbiting around this heavy source at a distance much larger ($\sim 1/m_q$) than the source size ($\sim 2/m_Q$). Thus the doubly heavy baryons look effectively like a two-body bound system and strongly resemble the heavy-light B and D mesons [2,13]. Then the HQET expansion in the inverse heavy quark mass can be used. The main distinction of the qQQ baryon from the $q\bar{Q}$ meson is that the QQ color source though being almost localized still is a composite system bearing integer spin values $(0,1,\dots)$. Hence it follows that the interaction of the heavy diquark with the light quark is not pointlike but is smeared by the form factor expressed through the overlap of the diquark wave functions. Besides this the diquark excitations contribute to the baryon excited states.

In previous approaches for the calculation of doubly heavy baryon masses the expansion in inverse powers not only of the heavy quark mass (m_Q) but also of the light quark mass (m_q) is carried out. The estimates of the light quark velocity in these baryons show that the light quark is highly relativistic ($v/c\sim 0.7$–0.8). Thus the nonrelativistic approximation is not adequate for the light quark. Here we present a consistent treatment of mass spectra of the doubly heavy baryons in the framework of the relativistic quark model based on the quasipotential wave equation without employing the expansion in $1/m_q$. Thus the light quark is treated fully relativistically. Concerning the heavy diquark (quark) we apply the expansion in $1/M^d_{QQ}(1/m_Q)$. We used a similar approach for the calculation of the mass spectra of B and D mesons [14].

The paper is organized as follows. In Sec. II we describe our relativistic quark model giving special emphasis to the construction of the quark-quark interaction potential in the diquark and the quark-diquark interaction potential in the baryon. In Sec. III we apply our model to the investigation of the heavy diquark properties. The cc and bb diquark mass spectra are calculated. We also determine the diquark interaction vertex with the gluon using the quasipotential approach and calculated diquark wave functions. Thus we take into account the internal structure of the diquark which considerably modifies the quark-diquark potential at small distances and removes fictitious singularities. In Sec. IV we construct the quasipotential of the interaction of a light quark with a heavy diquark. The light quark is treated fully relativistically. We use the expansion in inverse powers of the heavy diquark mass to simplify the construction. First we consider the infinitely heavy diquark limit and then include the $1/M^d_{QQ}$ corrections. In Sec. V we present our predictions for the mass spectra of the ground and excited states of Ξ_{cc}, Ξ_{bb}, Ω_{cc} and Ω_{bb} baryons. We consider both the excitations of the light quark and the heavy diquark. The mixing between excited baryon states with the same total angular momentum and parity is discussed. For Ξ_{cb} and Ω_{cb} baryons, composed from heavy quarks of different flavors we give predictions only for ground states, since the excited states of the cb diquark are unstable under the emission of soft gluons [11]. Moreover, a detailed comparison of our predictions with other approaches is given. We reveal the close similarity of the excitations of the light quark in a doubly heavy baryon and a heavy-light meson. We also test the fulfillment of different relations between mass splittings of doubly heavy baryons with two c or b quarks as well as the relations between splittings in the doubly heavy baryons and heavy-light mesons following from the heavy quark symmetry. Section VI contains our conclusions.

II. RELATIVISTIC QUARK MODEL

In the quasipotential approach and quark-diquark picture of doubly heavy baryons the interaction of two heavy quarks in a diquark and the light quark interaction with a heavy diquark in a baryon are described by the diquark wave function (Ψ_d) of the bound quark-quark state and by the baryon wave function (Ψ_B) of the bound quark-diquark state respectively, which satisfy the quasipotential equation [15] of the Schrödinger type [16]

$$\left(\frac{b^2(M)}{2\mu_R}-\frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_{d,B}(\mathbf{p})=\int\frac{d^3q}{(2\pi)^3}V(\mathbf{p},\mathbf{q};M)\Psi_{d,B}(\mathbf{q}), \tag{1}$$

where the relativistic reduced mass is

$$\mu_R=\frac{E_1E_2}{E_1+E_2}=\frac{M^4-(m_1^2-m_2^2)^2}{4M^3}, \tag{2}$$

and E_1,E_2 are given by

$$E_1=\frac{M^2-m_2^2+m_1^2}{2M},\quad E_2=\frac{M^2-m_1^2+m_2^2}{2M}; \tag{3}$$

here $M=E_1+E_2$ is the bound state mass (diquark or baryon), $m_{1,2}$ are the masses of heavy quarks (Q_1 and Q_2) which form the diquark or of the heavy diquark (d) and light quark (q) which form the doubly heavy baryon (B), and $\mathbf{p}$ is their relative momentum. In the center of mass system the relative momentum squared on mass shell reads

$$b^2(M)=\frac{[M^2-(m_1+m_2)^2][M^2-(m_1-m_2)^2]}{4M^2}. \tag{4}$$

The kernel $V(\mathbf{p},\mathbf{q};M)$ in Eq. (1) is the quasipotential operator of the quark-quark or quark-diquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive energy states. In the following analysis we closely follow the similar construction of the quark-antiquark interaction in mesons which were extensively studied in our relativistic quark model [14,17]. For the quark-quark interaction in a diquark we use the relation $V_{QQ}=V_{Q\bar{Q}}/2$ arising under the assumption about the octet structure of the interaction from the difference of the QQ and $Q\bar{Q}$ color states. An important role in this construction is played by the Lorentz structure of the confining interaction. In our analysis of mesons while constructing the quasipotential of quark-antiquark interaction we adopted that the effective interaction is the sum of the usual one-gluon exchange term with the mixture of long-range vector and scalar linear confining potentials, where the vector confining potential contains the Pauli terms. We use the same conventions for the construction of the quark-quark and quark-diquark interactions in the baryon. The quasipotential is then defined [10,17] as follows.

(a) For the quark-quark (QQ) interaction

$$V(\mathbf{p},\mathbf{q};M)=\bar{u}_1(p)\bar{u}_2(-p)\mathcal{V}(\mathbf{p},\mathbf{q};M)u_1(q)u_2(-q), \tag{5}$$

with

$$\mathcal{V}(\mathbf{p},\mathbf{q};M)=\frac{2}{3}\alpha_SD_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu+\frac{1}{2}V_{\rm conf}^V(\mathbf{k})\Gamma_1^\mu(\mathbf{k})\Gamma_{2;\mu}(-\mathbf{k})+\frac{1}{2}V_{\rm conf}^S(\mathbf{k}).$$

(b) For quark-diquark (qd) interaction

$$\begin{aligned}V(\mathbf{p},\mathbf{q};M)=&\frac{\langle d(P)|J_\mu|d(Q)\rangle}{2\sqrt{E_d(p)E_d(q)}}\bar{u}_q(p)\frac{4}{3}\alpha_SD_{\mu\nu}(\mathbf{k})\gamma^\nu u_q(q)\\&+\psi_d^*(P)\bar{u}_q(p)J_{d;\mu}\Gamma_q^\mu(\mathbf{k})V_{\rm conf}^V(\mathbf{k})u_q(q)\psi_d(Q)\\&+\psi_d^*(P)\bar{u}_q(p)V_{\rm conf}^S(\mathbf{k})u_q(q)\psi_d(Q),\end{aligned} \tag{6}$$

where α_S is the QCD coupling constant, the color factor is equal to 2/3 for quark-quark and 4/3 for quark-diquark interactions, and $\langle d(P)|J_\mu|d(Q)\rangle$ is the vertex of the diquark-gluon interaction which is discussed in detail below [$P=(E_d,-\mathbf{p})$, $Q=(E_d,-\mathbf{q})$, $E_d=(M^2-m_q^2+M_d^2)/(2M)$]. $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge

$$\begin{aligned}&D^{00}(\mathbf{k})=-\frac{4\pi}{\mathbf{k}^2},\quad D^{ij}(\mathbf{k})=-\frac{4\pi}{k^2}\left(\delta^{ij}-\frac{k^ik^j}{\mathbf{k}^2}\right),\\&D^{0i}=D^{i0}=0,\end{aligned} \tag{7}$$

and $\mathbf{k}=\mathbf{p}-\mathbf{q}$; γ_μ and $u(p)$ are the Dirac matrices and spinors

$$u^\lambda(p)=\sqrt{\frac{\epsilon(p)+m}{2\epsilon(p)}}\begin{pmatrix}1\\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{p}}{\epsilon(p)+m}\end{pmatrix}\chi^\lambda, \tag{8}$$

with $\epsilon(p)=\sqrt{\mathbf{p}^2+m^2}$.

The diquark state in the confining part of the quark-diquark quasipotential (6) is described by the wave functions

$$\psi_d(p)=\begin{cases}1 & \text{for scalar diquark,}\\ \varepsilon_d(p) & \text{for (axial) vector diquark,}\end{cases} \tag{9}$$

where the four vector

$$\varepsilon_d(p)=\left(\frac{(\boldsymbol{\varepsilon}_d\cdot\mathbf{p})}{M_d},\boldsymbol{\varepsilon}_d+\frac{(\boldsymbol{\varepsilon}_d\cdot\mathbf{p})\mathbf{p}}{M_d(E_d(p)+M_d)}\right),$$
$$\varepsilon_d(p)\cdot p=0, \tag{10}$$

is the polarization vector of the (axial) vector diquark with momentum $\mathbf{p}$, $E_d(p)=\sqrt{\mathbf{p}^2+M_d^2}$, and $\varepsilon_d(0)=(0,\boldsymbol{\varepsilon}_d)$ is the polarization vector in the diquark rest frame. The effective long-range vector vertex of the diquark can be presented in the form

$$J_{d;\mu}=\begin{cases}\dfrac{(P+Q)_\mu}{2\sqrt{E_d(p)E_d(q)}} & \text{for scalar diquark,}\\[2ex] \dfrac{(P+Q)_\mu}{2\sqrt{E_d(p)E_d(q)}}-\dfrac{i\mu_d}{2M_d}\Sigma_\mu^\nu\tilde{k}_\nu & \text{for (axial) vector diquark,}\end{cases} \tag{11}$$

where $\tilde{k}=(0,\mathbf{k})$ and we neglected the contribution of the chromoquadrupole moment of the (axial) vector diquark, which is suppressed by an additional power of k/M_d. Here the antisymmetric tensor

$$(\Sigma_{\rho\sigma})_\mu^\nu=-i(g_{\mu\rho}\delta_\sigma^\nu-g_{\mu\sigma}\delta_\rho^\nu) \tag{12}$$

and the (axial) vector diquark spin $\mathbf{S}_d$ is given by $(S_{d;k})_{il}=-i\varepsilon_{kil}$. We choose the total chromomagnetic moment of the (axial) vector diquark $\mu_d=2$ [10,18].

The effective long-range vector vertex of the quark is defined by [14,17]

$$\Gamma_\mu(\mathbf{k})=\gamma_\mu+\frac{i\kappa}{2m}\sigma_{\mu\nu}\tilde{k}^\nu,\quad \tilde{k}=(0,\mathbf{k}), \tag{13}$$

where κ is the Pauli interaction constant characterizing the anomalous chromomagnetic moment of quarks. In the configuration space the vector and scalar confining potentials in the nonrelativistic limit reduce to

$$V_{\rm conf}^V(r)=(1-\varepsilon)V_{\rm conf}(r),$$
$$V_{\rm conf}^S(r)=\varepsilon V_{\rm conf}(r), \tag{14}$$

with

$$V_{\rm conf}(r)=V_{\rm conf}^S(r)+V_{\rm conf}^V(r)=Ar+B, \tag{15}$$

where ε is the mixing coefficient.

All the parameters of our model like quark masses, parameters of linear confining potential A and B, mixing coefficient ε and anomalous chromomagnetic quark moment κ were fixed from the analysis of heavy quarkonium masses [17] and radiative decays [19]. The quark masses $m_b=4.88$ GeV, $m_c=1.55$ GeV, $m_s=0.50$ GeV, $m_{u,d}=0.33$ GeV and the parameters of the linear potential $A=0.18$ GeV^2 and $B=-0.30$ GeV have standard values of quark models. The value of the mixing coefficient of vector and scalar confining potentials $\varepsilon=-1$ has been determined from the consideration of the heavy quark expansion [20] and meson radiative decays [19]. Finally, the universal Pauli interaction constant $\kappa=-1$ has been fixed from the analysis of the fine splitting of heavy quarkonia ^3P_J states [17] and also from the heavy quark expansion [20]. Note that the long-range magnetic contribution to the potential in our model is proportional to $(1+\kappa)$ and thus vanishes for the chosen value of $\kappa=-1$.

III. DIQUARKS IN THE RELATIVISTIC QUARK MODEL

The quark-quark interaction in the diquark consists of the sum of the spin-independent and spin-dependent parts

$$V_{QQ}(r)=V_{QQ}^{\rm SI}(r)+V_{QQ}^{\rm SD}(r). \tag{16}$$

The spin-independent part with the account of v^2/c^2 corrections including retardation effects [21] is given by

$$\begin{aligned}V_{QQ}^{\rm SI}(r)=&-\frac{2}{3}\frac{\alpha_s(\mu^2)}{r}+\frac{1}{2}(Ar+B)+\frac{1}{8}\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}\right)\\&\times\Delta\left[-\frac{2}{3}\frac{\alpha_s(\mu^2)}{r}+\frac{1}{2}(1-\varepsilon)(1+2\kappa)Ar\right]\\&+\frac{1}{2m_1m_2}\left\{-\frac{2}{3}\frac{\alpha_s}{r}\left[\mathbf{p}^2+\frac{(\mathbf{p}\cdot\mathbf{r})^2}{r^2}\right]\right\}_W\\&+\frac{1}{2}\left[\frac{1-\varepsilon}{2m_1m_2}-\frac{\varepsilon}{4}\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}\right)\right]\\&\times\left\{Ar\left[\mathbf{p}^2-\frac{(\mathbf{p}\cdot\mathbf{r})^2}{r^2}\right]\right\}_W+\frac{1}{2}\left[\frac{1-\varepsilon}{2m_1m_2}\right.\\&\left.-\frac{\varepsilon}{4}\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}\right)\right]B\mathbf{p}^2,\end{aligned} \tag{17}$$

where $\{\cdots\}_W$ denotes the Weyl ordering of operators and

$$\alpha_s(\mu^2)=\frac{4\pi}{\beta_0\ln(\mu^2/\Lambda^2)},\qquad \beta_0=11-\frac{2}{3}n_f, \tag{18}$$

with μ fixed to be equal to the reduced mass, n_f is a number of flavors and $\Lambda=85$ MeV.

The spin-dependent part of the quark-quark potential can be presented in our model [17] as follows:

$$\begin{aligned}V_{QQ}^{\rm SD}(r)=&a\mathbf{L}\cdot\tilde{\mathbf{S}}+b\left[\frac{3}{r^2}(\mathbf{S}_1\cdot\mathbf{r})(\mathbf{S}_2\cdot\mathbf{r})-(\mathbf{S}_1\cdot\mathbf{S}_2)\right]\\&+c\mathbf{S}_1\cdot\mathbf{S}_2+d\mathbf{L}\cdot(\mathbf{S}_1-\mathbf{S}_2),\end{aligned} \tag{19}$$

$$\begin{aligned}a=\frac{1}{m_1m_2}\Bigg\{&\left(1+\frac{m_1^2+m_2^2}{4m_1m_2}\right)\frac{2}{3}\frac{\alpha_s(\mu^2)}{r^3}-\frac{1}{2}\frac{m_1^2+m_2^2}{4m_1m_2}\frac{A}{r}\\&+\frac{1}{2}(1+\kappa)\frac{(m_1+m_2)^2}{2m_1m_2}(1-\varepsilon)\frac{A}{r}\Bigg\},\end{aligned}$$

$$b=\frac{1}{3m_1m_2}\left\{\frac{2\alpha_s(\mu^2)}{r^3}+\frac{1}{2}(1+\kappa)^2(1-\varepsilon)\frac{A}{r}\right\},$$

$$c=\frac{2}{3m_1m_2}\left\{\frac{8\pi\alpha_s(\mu^2)}{3}\delta^3(r)+\frac{1}{2}(1+\kappa)^2(1-\varepsilon)\frac{A}{r}\right\},$$

$$d=\frac{1}{m_1m_2}\left\{\frac{m_2^2-m_1^2}{4m_1m_2}\left[\frac{2}{3}\frac{\alpha_s(\mu^2)}{r^3}-\frac{1}{2}\frac{A}{r}\right]\right.$$
$$\left.+\frac{1}{2}(1+\kappa)\frac{m_2^2-m_1^2}{2m_1m_2}(1-\varepsilon)\frac{A}{r}\right\}, \tag{20}$$

where $\mathbf{L}$ is the orbital momentum and $\mathbf{S}_{1,2}$, $\tilde{\mathbf{S}}=\mathbf{S}_1+\mathbf{S}_2$ are the spin momenta.

Now we can calculate the mass spectra of heavy diquarks with the account of all relativistic corrections (including retardation effects) of order v^2/c^2. For this purpose we substitute the quasipotential which is a sum of the spin-independent (17) and spin-dependent (19) parts into the quasipotential equation (1). Then we multiply the resulting expression from the left by the quasipotential wave function of a bound state and integrate with respect to the relative momentum. Taking into account the accuracy of the calculations, we can use for the resulting matrix elements the wave functions of Eq. (1) with the static potential

$$V_{QQ}^{\rm NR}(r)=-\frac{2}{3}\frac{\alpha_s(\mu^2)}{r}+\frac{1}{2}(Ar+B). \tag{21}$$

As a result we obtain the mass formula

$$\frac{b^2(M)}{2\mu_R}=W+\langle a\rangle\langle\mathbf{L}\cdot\tilde{\mathbf{S}}\rangle+\langle b\rangle\left\langle\left[\frac{3}{r^2}(\mathbf{S}_1\cdot\mathbf{r})(\mathbf{S}_2\cdot\mathbf{r})\right.\right.$$
$$\left.\left.-(\mathbf{S}_1\cdot\mathbf{S}_2)\right]\right\rangle+\langle c\rangle\langle\mathbf{S}_1\cdot\mathbf{S}_2\rangle+\langle d\rangle\langle\mathbf{L}\cdot(\mathbf{S}_1-\mathbf{S}_2)\rangle, \tag{22}$$

where

$$W=\langle V_{QQ}^{\rm SI}\rangle+\frac{\langle\mathbf{p}^2\rangle}{2\mu_R},$$

$$\langle\mathbf{L}\cdot\tilde{\mathbf{S}}\rangle=\frac{1}{2}[\tilde{J}(\tilde{J}+1)-L(L+1)-\tilde{S}(\tilde{S}+1)],$$

$$\left\langle\left[\frac{3}{r^2}(\mathbf{S}_1\cdot\mathbf{r})(\mathbf{S}_2\cdot\mathbf{r})-(\mathbf{S}_1\cdot\mathbf{S}_2)\right]\right\rangle$$
$$=-\frac{6(\langle\mathbf{L}\cdot\tilde{\mathbf{S}}\rangle)^2+3\langle\mathbf{L}\cdot\tilde{\mathbf{S}}\rangle-2\tilde{S}(\tilde{S}+1)L(L+1)}{2(2L-1)(2L+3)},$$

$$\langle\mathbf{S}_1\cdot\mathbf{S}_2\rangle=\frac{1}{2}\left(\tilde{S}(\tilde{S}+1)-\frac{3}{2}\right),\quad \tilde{\mathbf{S}}=\mathbf{S}_1+\mathbf{S}_2,$$

TABLE I. Mass spectrum and mean squared radii of the cc diquark.

State	Mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)	State	Mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)
1^3S_1	3.226	0.56	1^1P_1	3.460	0.82
2^3S_1	3.535	1.02	2^1P_1	3.712	1.22
3^3S_1	3.782	1.37	3^1P_1	3.928	1.54

and $\langle a\rangle,\langle b\rangle,\langle c\rangle,\langle d\rangle$ are the appropriate averages over radial wave functions of Eq. (20). We use the notations for the heavy diquark classification: $n^{2\tilde{S}+1}L_{\tilde{J}}$, where $n=1,2,\ldots$ is a radial quantum number, L is the angular momentum, $\tilde{S}=0,1$ is the total spin of two heavy quarks, and $\tilde{J}=L-\tilde{S},L,L+\tilde{S}$ is the total angular momentum ($\tilde{\mathbf{J}}=\mathbf{L}+\tilde{\mathbf{S}}$), which is considered as the spin of diquark ($\mathbf{S}_d$) in the following section. The first term on the right-hand side of the mass formula (22) contains all spin-independent contributions, the second and the last terms describe the spin-orbit interaction, the third term is responsible for the tensor interaction, while the fourth term gives the spin-spin interaction. The results of our numerical calculations of the mass spectra of cc and bb diquarks are presented in Tables I and II. The mass of the ground state of the bc diquark in the axial vector ($1\,^3S_1$) state is

$$M_{bc}^A=6.526\ \text{GeV}$$

and in the scalar ($1\,^1S_1$) state is

$$M_{bc}^S=6.519\ \text{GeV}.$$

In order to determine the diquark interaction with the gluon field, which takes into account the diquark structure, it is necessary to calculate the corresponding matrix element of the quark current between diquark states. This diagonal matrix element can be parametrized by the following set of elastic form factors.

(a) Scalar diquark (S)

$$\langle S(P)|J_\mu|S(Q)\rangle=h_+(k^2)(P+Q)_\mu. \tag{23}$$

(b) (Axial) vector diquark (V)

TABLE II. Mass spectrum and mean squared radii of bb diquark.

State	Mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)	State	Mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)
1^3S_1	9.778	0.37	1^1P_1	9.944	0.57
2^3S_1	10.015	0.71	2^1P_1	10.132	0.87
3^3S_1	10.196	0.98	3^1P_1	10.305	1.12
4^3S_1	10.369	1.22	4^1P_1	10.453	1.34

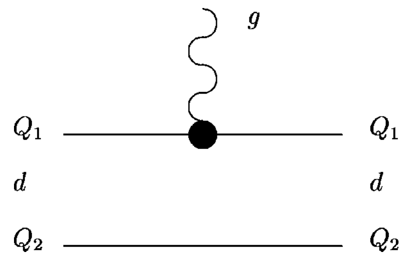

FIG. 1. Lowest order vertex function $\Gamma^{(1)}$ corresponding to Eq. (26). The gluon interaction only with one heavy quark is shown.

$$\begin{aligned}\langle V(P)|J_\mu|V(Q)\rangle = & -[\varepsilon_d^*(P)\cdot\varepsilon_d(Q)]h_1(k^2)(P+Q)_\mu \\ & +h_2(k^2)\{[\varepsilon_d^*(P)\cdot Q]\varepsilon_{d;\mu}(Q) \\ & +[\varepsilon_d(Q)\cdot P]\varepsilon_{d;\mu}^*(P)\}+h_3(k^2)\frac{1}{M_V^2} \\ & \times[\varepsilon_d^*(P)\cdot Q][\varepsilon_d(Q)\cdot P](P+Q)_\mu,\end{aligned} \tag{24}$$

where $k=P-Q$ and $\varepsilon_d(P)$ is the polarization vector of the (axial) vector diquark (10).

In the quasipotential approach, the matrix element of the quark current $J_\mu=\bar{Q}\gamma^\mu Q$ between the diquark states (d) has the form [22]

$$\langle d(P)|J_\mu(0)|d(Q)\rangle=\int\frac{d^3p\,d^3q}{(2\pi)^6}\bar{\Psi}_P^d(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi_Q^d(\mathbf{q}), \tag{25}$$

where $\Gamma_\mu(\mathbf{p},\mathbf{q})$ is the two-particle vertex function and Ψ_P^d are the diquark wave functions projected onto the positive energy states of quarks and boosted to the moving reference frame with momentum P. The leading contribution to Γ comes from Fig. 1. Other nonleading terms are the consequence of the projection onto the positive-energy states and give contributions only at the order of $1/m_Q^2$ for diquarks composed from two heavy quarks and can be neglected. Thus we will limit our analysis only to the contribution coming from Fig. 1. The corresponding vertex function is given by

$$\Gamma_\mu^{(1)}(\mathbf{p},\mathbf{q})=\bar{u}_{Q_1}(p_1)\gamma^\mu u_{Q_1}(q_1)(2\pi)^3\delta(\mathbf{p}_2-\mathbf{q}_2)+(1\leftrightarrow 2), \tag{26}$$

where [22]

$$p_{1,2}=\epsilon_{1,2}(p)\frac{P}{M_d}\pm\sum_{i=1}^{3}n^{(i)}(P)p^i,$$

$$q_{1,2}=\epsilon_{1,2}(q)\frac{Q}{M_d}\pm\sum_{i=1}^{3}n^{(i)}(Q)q^i,$$

and $n^{(i)}$ are three four-vectors defined by

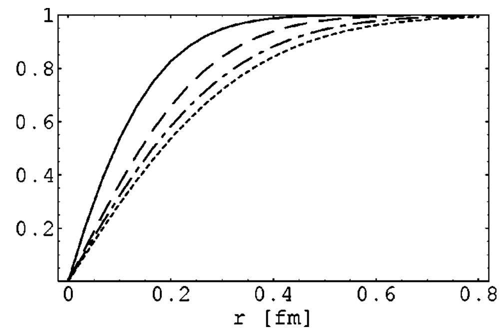

FIG. 2. The form factors $F(r)$ for the cc diquark. The solid curve is for the $1S$ state, the dashed curve for the $1P$ state, the dashed-dotted curve for the $2S$ state, and the dotted curve for the $2P$ state.

$$n^{(i)\mu}(P)=\left\{\frac{P^i}{M},\delta_{ij}+\frac{P^iP^j}{M(E+M)}\right\},$$

$$E=\sqrt{\mathbf{P}^2+M^2}.$$

We substitute the vertex function $\Gamma^{(1)}$ given by Eq. (26) in the matrix element (25) and expand it in $1/m_Q$ up to the leading order. Comparing the resulting expressions with the form factor decompositions (23) and (24) we find

$$\begin{aligned}& h_+(k^2)=h_1(k^2)=h_2(k^2)=2F(\mathbf{k}^2), \\ & h_3(k^2)=0, \\ & F(\mathbf{k}^2)=\frac{\sqrt{E_dM_d}}{E_d+M_d}\left[\int\frac{d^3p}{(2\pi)^3}\bar{\Psi}_d\left(\mathbf{p}+\frac{2m_2}{E_d+M_d}\mathbf{k}\right)\right. \\ & \qquad\left.\times\Psi_d(\mathbf{p})+(1\leftrightarrow 2)\right],\end{aligned} \tag{27}$$

where $\Psi_d\equiv\Psi_0^d$ are the diquark wave functions at rest. We calculated corresponding form factors $F(r)/r$ which are the Fourier transforms of $F(\mathbf{k}^2)/\mathbf{k}^2$ using the diquark wave functions found by numerically solving the quasipotential equation. In Fig. 2 the functions $F(r)$ for the cc diquark in $(1S,1P,2S,2P)$ states are shown as an example. We see that the slope of $F(r)$ decreases with the increase of the diquark excitation. Our estimates show that this form factor can be approximated with a high accuracy by the expression

$$F(r)=1-e^{-\xi r-\zeta r^2}, \tag{28}$$

which agrees with previously used approximations [23]. The values of parameters ξ and ζ for different cc and bb diquark states are given in Tables III and IV. As we see, the functions $F(r)$ vanish in the limit $r\to 0$ and become unity for large values of r. Such a behavior can be easily understood intuitively. At large distances a diquark can be well approximated by a pointlike object and its internal structure cannot be resolved. When the distance to the diquark decreases the internal structure plays a more important role. As the distance approaches zero, the interaction weakens and turns to zero for $r=0$ since this point coincides with the center of

TABLE III. Parameters ξ and ζ for ground and excited states of cc diquark.

State	ξ (GeV)	ζ (GeV^2)	State	ξ (GeV)	ζ (GeV^2)
$1S$	1.30	0.42	$1P$	0.74	0.315
$2S$	0.67	0.19	$2P$	0.60	0.155
$3S$	0.57	0.12	$3P$	0.55	0.075

TABLE IV. Parameters ξ and ζ for ground and excited states of bb diquark.

State	ξ (GeV)	ζ (GeV^2)	State	ξ (GeV)	ζ (GeV^2)
$1S$	1.30	1.60	$1P$	0.90	0.59
$2S$	0.85	0.31	$2P$	0.65	0.215
$3S$	0.66	0.155	$3P$	0.58	0.120
$4S$	0.56	0.09	$4P$	0.51	0.085

gravity of the two heavy quarks forming the diquark. Thus the function $F(r)$ gives an important contribution to the short-range part of the interaction of the light quark with the heavy diquark in the baryon and can be neglected for the long-range (confining) interaction. It is important to note that the inclusion of such a function removes a fictitious singularity $1/r^3$ at the origin arising from the one-gluon exchange part of the quark-diquark potential when the expansion in inverse powers of the heavy quark is used.

IV. QUASIPOTENTIAL OF THE INTERACTION OF A LIGHT QUARK WITH A HEAVY DIQUARK

The expression for the quasipotential (6) can, in principle, be used for arbitrary quark and diquark masses. The substitution of the Dirac spinors (8) and diquark form factors (23) and (24) into Eq. (6) results in an extremely nonlocal potential in the configuration space. Clearly, it is very hard to deal with such potentials without any simplifying expansion. Fortunately, in the case of the heavy-diquark–light-quark picture of the baryon, one can carry out (following HQET) the expansion in inverse powers of the heavy diquark mass M_d. The leading terms then follow in the limit $M_d\to\infty$.

A. Infinitely heavy diquark limit

In the limit $M_d\to\infty$ the heavy diquark vertices (23) and (24) have only the zeroth component, and the diquark mass and spin decouple from the consideration. As a result we get in this limit the quasipotential for the light quark similar to the one in the heavy-light meson in the limit of an infinitely heavy antiquark [14]. The only difference consists in the extra factor $F(k^2)$, defined in Eq. (27), in the one-gluon exchange part which accounts for the heavy diquark structure. The quasipotential in this limit is given by

$$V(\mathbf{p},\mathbf{q};M)=\bar{u}_q(p)\left\{-\frac{4}{3}\alpha_s F(k^2)\frac{4\pi}{\mathbf{k}^2}\gamma_q^0+V^V_{\rm conf}(\mathbf{k})\left[\gamma_q^0+\frac{\kappa}{2m_q}\gamma_q^0(\boldsymbol{\gamma}\mathbf{k})\right]+V^S_{\rm conf}(\mathbf{k})\right\}u_q(q). \tag{29}$$

The resulting interaction is still nonlocal in configuration space. However, taking into account that doubly heavy baryons are weakly bound, we can replace $\epsilon_q(p)\to E_q=(M^2-M_d^2+m_q^2)/(2M)$ in the Dirac spinors (8) [14]. Such a simplifying substitution is widely used in quantum electrodynamics [24–26] and introduces only minor corrections of order of the ratio of the binding energy $\langle V\rangle$ to E_q. This substitution makes the Fourier transformation of the potential (29) local. In contrast with the heavy-light meson case no special consideration of the one-gluon exchange term is necessary, since the presence of the diquark structure described by an extra function $F(k^2)$ in Eq. (29) removes fictitious $1/r^3$ singularity at the origin in configuration space.

The resulting local quark-diquark potential for $M_d\to\infty$ can be presented in configuration space in the following form:

$$\begin{aligned}V_{M_d\to\infty}(r)=\frac{E_q+m_q}{2E_q}\Bigg[&V_{\rm Coul}(r)+V_{\rm conf}(r)+\frac{1}{(E_q+m_q)^2}\\&\times\Bigg\{\mathbf{p}[V_{\rm Coul}(r)+V^V_{\rm conf}(r)-V^S_{\rm conf}(r)]\mathbf{p}\\&-\frac{E_q+m_q}{2m_q}\Delta V^V_{\rm conf}(r)[1-(1+\kappa)]\\&+\frac{2}{r}\Bigg(V'_{\rm Coul}(r)-V'^S_{\rm conf}(r)-V'^V_{\rm conf}(r)\\&\times\left[\frac{E_q}{m_q}-2(1+\kappa)\frac{E_q+m_q}{2m_q}\right]\Bigg)\mathbf{l}\cdot\mathbf{S}_q\Bigg\}\Bigg],\end{aligned} \tag{30}$$

where $V_{\rm Coul}(r)=-(4/3)\alpha_s F(r)/r$ is the smeared Coulomb potential. The prime denotes differentiation with respect to r, $\mathbf{l}$ is the orbital momentum, and $\mathbf{S}_q$ is the spin operator of the light quark. Note that the last term in Eq. (30) is of the same order as the first two terms and thus cannot be treated perturbatively. It is important to note that the quark-diquark potential $V_{M_d\to\infty}(r)$ almost coincides with the quark-antiquark potential in heavy-light (B and D) mesons for $m_Q\to\infty$ [14]. The only difference is the presence of the extra factor $F(r)$ in $V_{\rm Coul}(r)$ which accounts for the internal structure of the diquark. This is the consequence of the heavy quark (diquark) limit in which its spin and mass decouple from the consideration.

In the infinitely heavy diquark limit the quasipotential equation (1) in configuration space becomes

$$\left(\frac{E_q^2-m_q^2}{2E_q}-\frac{\mathbf{p}^2}{2E_q}\right)\Psi_B(r)=V_{M_d\to\infty}(r)\Psi_B(r), \tag{31}$$

and the mass of the baryon is given by $M=M_d+E_q$.

TABLE V. The values of E_q in the limit $M_d \to \infty$ (in GeV).

Baryon State	ccq E_q	ccs E_s	bbq E_q	bbs E_s
$1S1s(1/2)$	0.491	0.638	0.492	0.641
$1S1p(3/2)$	0.788	0.906	0.785	0.904
$1S1p(1/2)$	0.877	0.968	0.880	0.969
$1S2s(1/2)$	0.987	1.080	0.993	1.084
$1P1s(1/2)$	0.484	0.633	0.489	0.636
$1P1p(3/2)$	0.793	0.909	0.789	0.906
$1P1p(1/2)$	0.873	0.965	0.876	0.967
$1P2s(1/2)$	0.980	1.075	0.984	1.078
$2S1s(1/2)$	0.481	0.631	0.486	0.634
$2S1p(3/2)$	0.794	0.909	0.791	0.908
$2S1p(1/2)$	0.871	0.963	0.874	0.965
$2S2s(1/2)$	0.979	1.074	0.982	1.076
$2P1s(1/2)$	0.479	0.630	0.481	0.631
$3S1s(1/2)$	0.478	0.630	0.480	0.630

Solving Eq. (31) numerically we get the eigenvalues E_q and the baryon wave functions Ψ_B. The obtained results are presented in Table V. We use the notation $n_d L n_q l(j)$ for the classification of baryon states in the infinitely heavy diquark limit. Here we first give the radial quantum number n_d and the angular momentum L of the heavy diquark. Then the radial quantum number n_q, the angular momentum l and the value j of the total angular momentum ($\mathbf{j}=\mathbf{l}+\mathbf{S}_q$) of the light quark are shown. We see that the heavy diquark spin and mass decouple in the limit $M_d \to \infty$, and thus we get the number of degenerated states in accord with the heavy quark symmetry prediction. This symmetry predicts also almost an equality of corresponding light-quark energies E_q for bbq and ccq baryons and their nearness in the same limit to the light-quark energies E_q of B and D mesons [14]. The small deviations of the baryon energies from values of the meson energies are connected with the different forms of the singularity smearing at $r=0$ in the baryon and meson cases.

B. $1/M_d$ corrections

The heavy quark symmetry degeneracy of states is broken by $1/M_d$ corrections. The corrections of order $1/M_d$ to the potential (30) arise from the spatial components of the heavy diquark vertex. Other contributions at first order in $1/M_d$ come from the one-gluon-exchange potential and the vector confining potential, while the scalar potential gives no contribution at first order. The resulting $1/M_d$ correction to the quark-diquark potential (30) is given by the following expression:

(a) Scalar diquark:

$$\delta V_{1/M_d}(r)=\frac{1}{E_q M_d}\left\{\mathbf{p}[V_{\rm Coul}(r)+V^V_{\rm conf}(r)]\mathbf{p}+V'_{\rm Coul}(r)\frac{\mathbf{l}^2}{2r}-\frac{1}{4}\Delta V^V_{\rm conf}(r)+\left[\frac{1}{r}V'_{\rm Coul}(r)+\frac{(1+\kappa)}{r}V'^V_{\rm conf}(r)\right]\mathbf{l}\cdot\mathbf{S}_q\right\}. \tag{32}$$

(b) (Axial) vector diquark:

$$\delta V_{1/M_d}(r)=\frac{1}{E_q M_d}\Bigg\{\mathbf{p}[V_{\rm Coul}(r)+V^V_{\rm conf}(r)]\mathbf{p}+V'_{\rm Coul}(r)\frac{\mathbf{l}^2}{2r}-\frac{1}{4}\Delta V^V_{\rm conf}(r)+\left[\frac{1}{r}V'_{\rm Coul}(r)+\frac{(1+\kappa)}{r}V'^V_{\rm conf}(r)\right]\mathbf{l}\cdot\mathbf{S}_q$$
$$+\frac{1}{2}\left[\frac{1}{r}V'_{\rm Coul}(r)+\frac{(1+\kappa)}{r}V'^V_{\rm conf}(r)\right]\mathbf{l}\cdot\mathbf{S}_d+\frac{1}{3}\left(\frac{1}{r}V'_{\rm Coul}(r)-V''_{\rm Coul}(r)+(1+\kappa)\left[\frac{1}{r}V'^V_{\rm conf}(r)-V''^V_{\rm conf}(r)\right]\right)$$
$$\times\left[-\mathbf{S}_q\cdot\mathbf{S}_d+\frac{3}{r^2}(\mathbf{S}_q\cdot\mathbf{r})(\mathbf{S}_d\cdot\mathbf{r})\right]+\frac{2}{3}[\Delta V_{\rm Coul}(r)+(1+\kappa)\Delta V^V_{\rm conf}(r)]\mathbf{S}_d\cdot\mathbf{S}_q\Bigg\}, \tag{33}$$

where $\mathbf{S}=\mathbf{S}_q+\mathbf{S}_d$ is the total spin, and $\mathbf{S}_d$ is the diquark spin (which is equal to the total angular momentum $\tilde{\mathbf{J}}$ of two heavy quarks forming the diquark). The first three terms in Eq. (33) represent spin-independent corrections, the fourth and the fifth terms are responsible for the spin-orbit interaction, the sixth one is the tensor interaction, and the last one is the spin-spin interaction. It is necessary to note that the confining vector interaction gives a contribution to the spin-dependent part which is proportional to $(1+\kappa)$. Thus it vanishes for the chosen value of $\kappa=-1$, while the confining vector contribution to the spin-independent part is nonzero.

In order to estimate the matrix elements of spin-dependent terms in the $1/M_d$ corrections to the quark-diquark potential (32) and (33) as well as different mixings of baryon states, it is convenient to use the following relations:

$$|J;j\rangle=\sum_S(-1)^{J+l+S_d+S_q}\sqrt{(2S+1)(2j+1)}\times\begin{Bmatrix} S_d & S_q & S\\ l & J & j\end{Bmatrix}|J,S\rangle \tag{34}$$

and

$$|J;j\rangle=\sum_{J_d}(-1)^{J+l+S_d+S_q}\sqrt{(2J_d+1)(2j+1)}$$
$$\times\begin{Bmatrix} S_d & l & J_d \\ S_q & J & j \end{Bmatrix}|J,J_d\rangle, \tag{35}$$

where $\mathbf{J}=\mathbf{j}+\mathbf{S}_d$ is the baryon total angular momentum, $\mathbf{j}=\mathbf{l}+\mathbf{S}_q$ is the light quark total angular momentum, $\mathbf{S}=\mathbf{S}_q+\mathbf{S}_d$ is the baryon total spin, and $\mathbf{J}_d=\mathbf{L}+\mathbf{S}_d$.

V. RESULTS AND DISCUSSION

For the description of the quantum numbers of baryons we use the notations $(n_dLn_ql)J^P$, where we first show the radial quantum number of the diquark ($n_d=1,2,3\ldots$) and its orbital momentum by a capital letter ($L=S,P,D\ldots$), then the radial quantum number of the light quark ($n_q=1,2,3\ldots$) and its orbital momentum by a lowercase letter ($l=s,p,d\ldots$), and at the end the total angular momentum J and parity P of the baryon.

The presence of the spin-orbit interaction proportional to $\mathbf{l}\cdot\mathbf{S}_d$ and of the tensor interaction in the quark-diquark potential at $1/M_d$ order (33) results in a mixing of states which have the same total angular momentum J and parity but different light quark total momentum j. For example, the baryon states with diquark in the ground state and light quark in the p-wave $(1S1p)$ for $J=1/2$ or 3/2 have different values of the light quark angular momentum $j=1/2$ and 3/2, which mix between themselves. In the case of the ccq baryon we have the mixing matrix for $J=1/2$,

$$\begin{pmatrix} -55.6 & -7.3 \\ -8.5 & -37.9 \end{pmatrix} \text{ MeV}, \tag{36}$$

with the following eigenvectors:

$$|(1S2p)1/2'^-\rangle=-0.334|j=3/2\rangle+0.943|j=1/2\rangle,$$
$$|(1S2p)1/2^-\rangle=0.925|j=3/2\rangle+0.380|j=1/2\rangle. \tag{37}$$

For the ccq baryon with $J=3/2$ the mixing matrix is given by

$$\begin{pmatrix} -23.0 & 18.1 \\ 21.3 & 18.9 \end{pmatrix} \text{ MeV}, \tag{38}$$

and the eigenvectors are equal to

$$|(1S2p)3/2'^-\rangle=0.343|j=3/2\rangle+0.939|j=1/2\rangle,$$
$$|(1S2p)3/2^-\rangle=0.919|j=3/2\rangle-0.394|j=1/2\rangle. \tag{39}$$

For the bbq baryon we get the mixing matrix for $J=1/2$

$$\begin{pmatrix} -18.0 & -2.4 \\ -2.8 & -12.6 \end{pmatrix} \text{ MeV}, \tag{40}$$

which has eigenvalues

$$|(1S2p)1/2'^-\rangle=0.349|j=3/2\rangle+0.937|j=1/2\rangle,$$
$$|(1S2p)1/2^-\rangle=0.915|j=3/2\rangle+0.402|j=1/2\rangle, \tag{41}$$

and for $J=3/2$ the mixing matrix is

$$\begin{pmatrix} -7.4 & 5.9 \\ 7.1 & 6.3 \end{pmatrix} \text{ MeV}, \tag{42}$$

so that

$$|(1S2p)3/2'^-\rangle=0.341|j=3/2\rangle+0.940|j=1/2\rangle,$$
$$|(1S2p)3/2^-\rangle=0.917|j=3/2\rangle+0.400|j=1/2\rangle. \tag{43}$$

The quasipotential with $1/m_Q$ corrections is given by the sum of $V_{m_Q\to\infty}(r)$ from Eq. (30) and $\delta V_{1/m_Q}(r)$ from Eqs. (32) and (33). By substituting it in the quasipotential equation (1) and treating the $1/m_Q$ correction term $\delta V_{1/m_Q}(r)$

TABLE VI. Mass spectrum of Ξ_{cc} baryons (in GeV).

State	Mass		State	Mass	
$(n_dLn_ql)J^P$	Our	[11]	$(n_dLn_ql)J^P$	Our	[11]
$(1S1s)1/2^+$	3.620	3.478	$(1P1s)1/2^-$	3.838	3.702
$(1S1s)3/2^+$	3.727	3.61	$(1P1s)3/2^-$	3.959	3.834
$(1S1p)1/2^-$	4.053	3.927	$(2S1s)1/2^+$	3.910	3.812
$(1S1p)3/2^-$	4.101	4.039	$(2S1s)3/2^+$	4.027	3.944
$(1S1p)1/2'^-$	4.136	4.052	$(2P1s)1/2^-$	4.085	3.972
$(1S1p)5/2^-$	4.155	4.047	$(2P1s)3/2^-$	4.197	4.104
$(1S1p)3/2'^-$	4.196	4.034	$(3S1s)1/2^+$	4.154	4.072

TABLE VII. Mass spectrum of Ξ_{bb} baryons (in GeV).

State	Mass		State	Mass	
$(n_dLn_ql)J^P$	Our	[11]	$(n_dLn_ql)J^P$	Our	[11]
$(1S1s)1/2^+$	10.202	10.093	$(2S1s)1/2^+$	10.441	10.373
$(1S1s)3/2^+$	10.237	10.133	$(2S1s)3/2^+$	10.482	10.413
$(1S1p)1/2^-$	10.632	10.541	$(2S1p)1/2^-$	10.873	
$(1S1p)3/2^-$	10.647	10.567	$(2S1p)3/2^-$	10.888	
$(1S1p)5/2^-$	10.661	10.580	$(2S1p)1/2'^-$	10.902	
$(1S1p)1/2'^-$	10.675	10.578	$(2S1p)5/2^-$	10.905	
$(1S1p)3/2'^-$	10.694	10.581	$(2S1p)3/2'^-$	10.920	
$(1S2s)1/2^+$	10.832		$(2P1s)1/2^-$	10.563	10.493
$(1S2s)3/2^+$	10.860		$(2P1s)3/2^-$	10.607	10.533
$(1P1s)1/2^-$	10.368	10.310	$(3S1s)1/2^+$	10.630	10.563
$(1P1s)3/2^-$	10.408	10.343	$(3S1s)3/2^+$	10.673	
$(1P1p)1/2^+$	10.763		$(3P1s)1/2^-$	10.744	
$(1P1p)3/2^+$	10.779		$(3P1s)3/2^-$	10.788	
$(1P1p)5/2^+$	10.786		$(4S1s)1/2^+$	10.812	
$(1P1p)1/2'^+$	10.838		$(4S1s)3/2^+$	10.856	
$(1P1p)3/2'^+$	10.856		$(4P1s)1/2^-$	10.900	

TABLE VIII. Mass spectrum of Ω_{cc} baryons (in GeV).

$(n_dLn_ql)J^P$	Mass	$(n_dLn_ql)J^P$	Mass
$(1S1s)1/2^+$	3.778	$(1P1s)1/2^-$	4.002
$(1S1s)3/2^+$	3.872	$(1P1s)3/2^-$	4.102
$(1S1p)1/2^-$	4.208	$(2S1s)1/2^+$	4.075
$(1S1p)3/2^-$	4.252	$(2S1s)3/2^+$	4.174
$(1S1p)1/2'^-$	4.271	$(2P1s)1/2^-$	4.251
$(1S1p)5/2^-$	4.303	$(2P1s)3/2^-$	4.345
$(1S1p)3/2'^-$	4.325	$(3S1s)1/2^+$	4.321

using perturbation theory, we are now able to calculate the mass spectra of $\Xi_{cc},\Xi_{bb},\Xi_{cb},\Omega_{cc},\Omega_{bb},\Omega_{cb}$ baryons with the account of $1/m_Q$ corrections. In Tables VI–IX we present mass spectra of ground and excited states of doubly heavy baryons containing both heavy quarks of the same flavor (c and b). The corresponding level orderings are schematically shown in Figs. 3–6. In these figures we first show our predictions for doubly heavy baryon spectra in the limit when all $1/M_d$ corrections are neglected [denoting baryon states by $n_dLn_ql(j)$]. We see that in this limit the p-wave excitations of the light quark are inverted. This means that the mass of the state with higher angular momentum $j=3/2$ is smaller than the mass of the state with lower angular momentum $j=1/2$ [13,14,27]. The similar p-level inversion was found previously in the mass spectra of heavy-light mesons in the infinitely heavy quark limit [14]. Note that the pattern of levels of the light quark and level separation in doubly heavy baryons and heavy-light mesons almost coincide in these limits. Next we switch on $1/M_d$ corrections. This results in splitting of the degenerate states and mixing of states with different j, which have the same total angular momentum J and parity, as it was discussed above. Since the diquark has spin one, the states with $j=1/2$ split into two different states with $J=1/2$ or 3/2, while the states with $j=3/2$ split into three different states with $J=1/2$,3/2 or 5/2. The fine splitting between p levels turns out to be of the same order of magnitude as the gap between $j=1/2$ and $j=3/2$ degenerate multiplets in the infinitely heavy diquark limit. The inclusion of $1/M_d$ corrections leads also to the relative shifts of the baryon levels further decreasing this gap. As a result, some of the p levels from different (initially degenerate) multiplets overlap; however, the heavy diquark spin averaged centers remain inverted. The resulting picture for the diquark in the ground state is very similar to the one for heavy-light mesons [14]. The purely inverted pattern of p levels is observed only for the B meson and Ξ_{bb},Ω_{bb} baryons, while in other heavy-light mesons (D,D_s,B_s) and doubly heavy baryons (Ξ_{cc},Ω_{cc}) p levels from different j multiplets overlap. The absence of the p-level overlap for the Ω_{bb} baryon in contrast to the B_s meson (where we predict a very small overlap of these levels [14]) is explained by the fact that the ratio m_s/M^d_{bb} is approximately two times smaller than m_s/m_b and thus it is of order m_q/m_b. As it was argued in [14], these ratios determine the applicability of the heavy quark limit.

TABLE IX. Mass spectrum of Ω_{bb} baryons (in GeV).

$(n_dLn_ql)J^P$	Mass	$(n_dLn_ql)J^P$	Mass
$(1S1s)1/2^+$	10.359	$(2S1s)1/2^+$	10.610
$(1S1s)3/2^+$	10.389	$(2S1s)3/2^+$	10.645
$(1S1p)1/2^-$	10.771	$(2S1p)1/2^-$	11.011
$(1S1p)3/2^-$	10.785	$(2S1p)3/2^-$	11.025
$(1S1p)5/2^-$	10.798	$(2S1p)1/2'^-$	11.035
$(1S1p)1/2'^-$	10.804	$(2S1p)5/2^-$	11.040
$(1S1p)3/2'^-$	10.821	$(2S1p)3/2'^-$	11.051
$(1S2s)1/2^+$	10.970	$(2P1s)1/2^-$	10.738
$(1S2s)3/2^+$	10.992	$(2P1s)3/2^-$	10.775
$(1P1s)1/2^-$	10.532	$(3S1s)1/2^+$	10.806
$(1P1s)3/2^-$	10.566	$(3S1s)3/2^+$	10.843
$(1P1p)1/2^+$	10.914	$(3P1s)1/2^-$	10.924
$(1P1p)3/2^+$	10.928	$(3P1s)3/2^-$	10.961
$(1P1p)5/2^+$	10.937	$(4S1s)1/2^+$	10.994
$(1P1p)1/2'^+$	10.971	$(4S1s)3/2^+$	11.031
$(1P1p)3/2'^+$	10.986	$(4P1s)1/2^-$	11.083

In Tables VI and VII we compare our predictions for the ground and excited state masses of Ξ_{cc} and Ξ_{bb} baryons

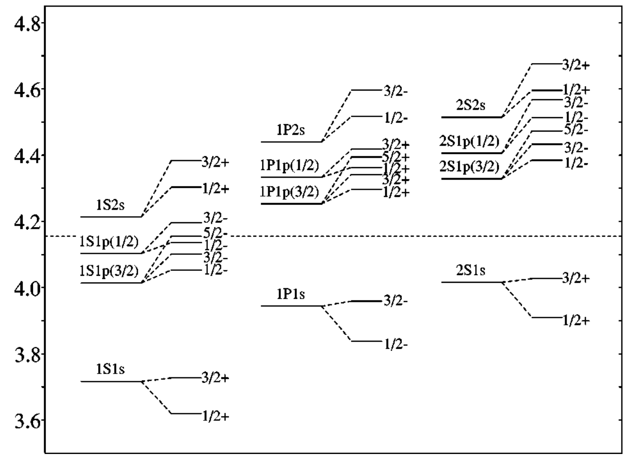

FIG. 3. Masses of Ξ_{cc} baryons (in GeV). The horizontal dashed line shows the Λ_cD threshold.

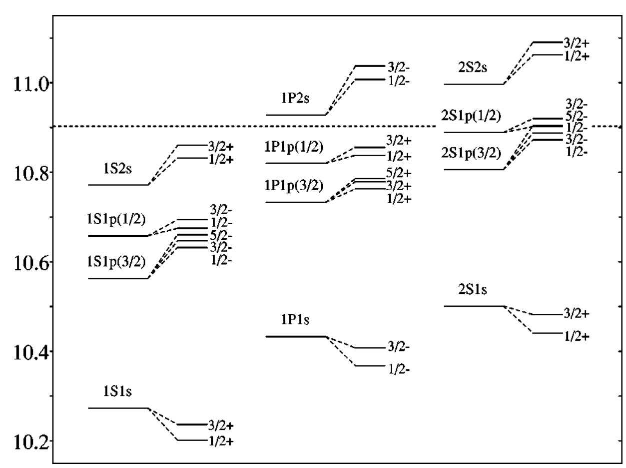

FIG. 4. Masses of Ξ_{bb} baryons (in GeV). The horizontal dashed line shows the Λ_bD threshold.

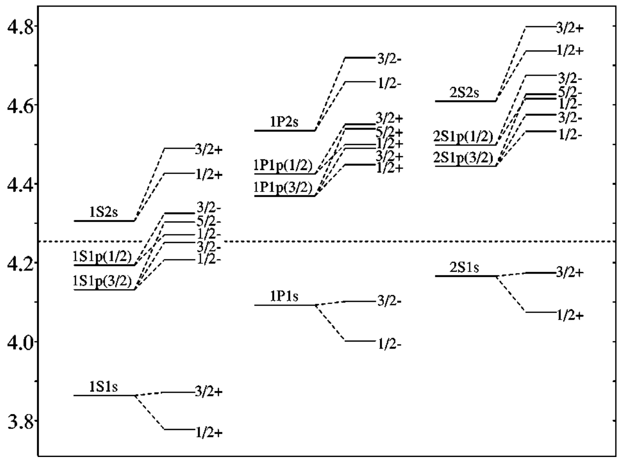

FIG. 5. Masses of Ω_{cc} baryons (in GeV). The horizontal dashed line shows the $\Lambda_c D_s$ threshold.

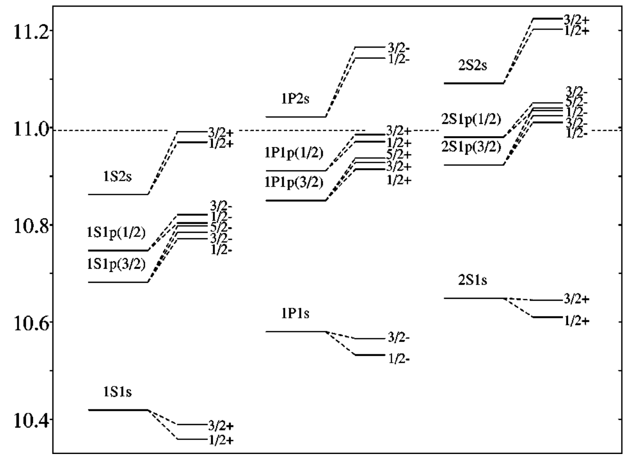

FIG. 6. Masses of Ω_{bb} baryons (in GeV). The horizontal dashed line shows the $\Lambda_b D_s$ threshold.

with the predictions of Ref. [11]. As we see from these tables our predictions are approximately 50–150 MeV higher than the estimates of Ref. [11]. One of the reasons for the difference between these two predictions for the masses of Ξ_{cc} baryons (which is the largest) is the difference in the c quark masses. The mass of the c quark in [11] is determined from fitting the charmonium spectrum in the quark model where all spin-independent relativistic corrections were ignored. However, our estimates show that due to the rather large average value of v^2/c^2 in charmonium [33] such corrections play an important role and give contributions to the charmonium masses of order of 100 MeV. As a result the c quark mass found in [11] is approximately 70 MeV less than in our model. For the calculation of the diquark masses we also take into account the spin-independent corrections (17) to the QQ potential. We find that their contribution is less than in the case of charmonium since $V_{QQ}=V_{Q\bar{Q}}/2$. Thus the cc diquark masses in [11] are approximately 50 MeV smaller than in our model. The other main source of the difference is the expansion in inverse powers of the light quark mass, which was used in [11] but is not applied in our approach, where the light quark is treated fully relativistically.

In Table X we compare our model predictions for the ground state masses of doubly heavy baryons with some other predictions [6,9,11,28] as well as our previous prediction [10], where the expansion in inverse powers of the heavy and light quark masses was used. In general we find a reasonable agreement within 100 MeV between different predictions [6,9–11,28] for the ground state masses of the doubly heavy baryons. The main advantage of our present approach is the completely relativistic treatment of the light quark and account for the nonlocal composite structure of the diquark.

For the Ξ_{cb} and Ω_{cb} baryons containing heavy quarks of different flavors (c and b) we calculate only the ground state masses. As it was argued in Ref. [11], the excited states of

TABLE X. Mass spectrum of ground states of doubly heavy baryons (in GeV). Comparison of different predictions. $\{QQ\}$ denotes the diquark in the axial vector state and $[QQ]$ denotes diquark in the scalar state.

Baryon	Quark content	J^P	Present work	[11]	[10]	[9]	[6]	[28]
Ξ_{cc}	$\{cc\}q$	$1/2^+$	3.620	3.478	3.66	3.66	3.61	3.69
Ξ_{cc}^*	$\{cc\}q$	$3/2^+$	3.727	3.61	3.81	3.74	3.68	
Ω_{cc}	$\{cc\}s$	$1/2^+$	3.778	3.59	3.76	3.74	3.71	3.86
Ω_{cc}^*	$\{cc\}s$	$3/2^+$	3.872	3.69	3.89	3.82	3.76	
Ξ_{bb}	$\{bb\}q$	$1/2^+$	10.202	10.093	10.23	10.34		10.16
Ξ_{bb}^*	$\{bb\}q$	$3/2^+$	10.237	10.133	10.28	10.37		
Ω_{bb}	$\{bb\}s$	$1/2^+$	10.359	10.18	10.32	10.37		10.34
Ω_{bb}^*	$\{bb\}s$	$3/2^+$	10.389	10.20	10.36	10.40		
Ξ_{cb}	$\{cb\}q$	$1/2^+$	6.933	6.82	6.95	7.04		6.96
Ξ_{cb}'	$[cb]q$	$1/2^+$	6.963	6.85	7.00	6.99		
Ξ_{cb}^*	$\{cb\}q$	$3/2^+$	6.980	6.90	7.02	7.06		
Ω_{cb}	$\{cb\}s$	$1/2^+$	7.088	6.91	7.05	7.09		7.13
Ω_{cb}'	$[cb]s$	$1/2^+$	7.116	6.93	7.09	7.06		
Ω_{cb}^*	$\{cb\}s$	$3/2^+$	7.130	6.99	7.11	7.12		

TABLE XI. Differences between spin-averaged masses of doubly heavy baryons defined in Eq. (44) (in GeV).

Baryon state	$\Delta\bar{M}_1(\Xi)$	$\Delta\bar{M}_2(\Xi)$	$\Delta\bar{M}_1(\Omega)$	$\Delta\bar{M}_2(\Omega)$	ΔM^d
$1S1s$	6.534		6.538		6.552
$1S1p$	6.512	6.532		6.519	6.552
$1P1s$	6.476		6.486		6.484
$2S1s$	6.480		6.492		6.480

heavy diquarks composed of the quarks with different flavors are unstable under the emission of soft gluons, and thus the calculation of the excited baryon (cbq and cbs) masses is not justified in the quark-diquark scheme. We get the following predictions for the masses of the ground state cbq baryons.

$(1S1s)1/2^+$ states with the axial vector and scalar cb diquarks respectively:

$$M(\Xi_{cb})=6.933 \text{ GeV}, \quad M(\Xi'_{cb})=6.963 \text{ GeV},$$

$(1S1s)3/2^+$ state

$$M(\Xi^*_{cb})=6.980 \text{ GeV},$$

and for cbs baryons:

$(1S1s)1/2^+$ states with the axial vector and scalar cb diquarks

$$M(\Omega_{cb})=7.088 \text{ GeV}, \quad M(\Omega'_{cb})=7.116 \text{ GeV},$$

$(1S1s)3/2^+$ state

$$M(\Omega^*_{cb})=7.130 \text{ GeV}.$$

Now we compare our results with the model-independent predictions of the heavy quark effective theory. The heavy quark symmetry predicts simple relations between the spin averaged masses of doubly heavy baryons with the accuracy of order $1/M_d$

$$\begin{aligned}\Delta\bar{M}_{1,2}&\equiv\bar{M}_1(\Xi_{bb})-\bar{M}_1(\Xi_{cc})=\bar{M}_2(\Xi_{bb})-\bar{M}_2(\Xi_{cc})\\&=\bar{M}_1(\Omega_{bb})-\bar{M}_1(\Omega_{cc})=\bar{M}_2(\Omega_{bb})-\bar{M}_2(\Omega_{cc})\\&=M^d_{bb}-M^d_{cc}\equiv\Delta M^d,\end{aligned} \tag{44}$$

where the spin-averaged masses are

$$\bar{M}_1=(M_{1/2}+2M_{3/2})/3,$$

$$\bar{M}_2=(M_{1/2}+2M_{3/2}+3M_{5/2})/6$$

and M_J are the masses of baryons with total angular momentum J. M^d_{QQ} are the masses of diquarks in definite states. The numerical results are presented in Table XI (only the states below threshold are considered). We see that the equalities in Eq. (44) are satisfied with good accuracy for the baryons with the heavy diquark and light quark both in ground and excited states.

It follows from the heavy quark symmetry that the hyperfine mass splittings of initially degenerate light quark states

$$\begin{aligned}\Delta M(\Xi_{QQ})&\equiv M_{3/2}(\Xi_{QQ})-M_{1/2}(\Xi_{QQ}),\\\Delta M(\Omega_{QQ})&\equiv M_{3/2}(\Omega_{QQ})-M_{1/2}(\Omega_{QQ}),\end{aligned} \tag{45}$$

should scale with the diquark masses:

$$\begin{aligned}\Delta M(\Xi_{cc})&=R\Delta M(\Xi_{bb}),\\\Delta M(\Omega_{cc})&=R\Delta M(\Omega_{bb}),\end{aligned} \tag{46}$$

where $R=M^d_{bb}/M^d_{cc}$ is the ratio of diquark masses. Our model predictions for these splittings are displayed in Table XII. Again we see that heavy quark symmetry relations are satisfied with high accuracy.

The close similarity of the interaction of the light quark with the heavy quark in the heavy-light mesons and with the heavy diquark in the doubly heavy baryons produces very simple relations between the meson and baryon mass splittings [7,29,30]. In fact for the ground state hyperfine splittings of mesons and baryons we obtain adopting the approximate relation $M^d_{QQ}\cong 2m_Q$

$$\begin{aligned}\Delta M(\Xi_{QQ})&=\frac{3}{2}\,\frac{m_Q}{M^d_{QQ}}\Delta M_{B,D}\cong\frac{3}{4}\Delta M_{B,D},\\\Delta M(\Omega_{QQ})&\cong\frac{3}{4}\Delta M_{B_s,D_s},\end{aligned} \tag{47}$$

where the factor 3/2 is just the ratio of the baryon and meson spin matrix elements. The numerical fulfillment of relations (47) is shown in Table XIII.

VI. CONCLUSIONS

In this paper we calculated the masses of the ground and excited states of the doubly heavy baryons on the basis of the

TABLE XII. Hyperfine splittings (in MeV) of the doubly heavy baryons for the states with the light quark angular momentum $j=1/2$.

Baryon state	R	$\Delta M(\Xi_{bb})$	$R\Delta M(\Xi_{bb})$	$\Delta M(\Xi_{cc})$	$\Delta M(\Omega_{bb})$	$R\ \Delta M(\Omega_{bb})$	$\Delta M(\Omega_{cc})$
$1S1s[3/2-1/2]$	3.03	35	106	107	30	91	94
$1S1p[3/2-1/2]$	3.03	19	58	60	17	52	54
$1P1s[3/2-1/2]$	2.87	40	115	121	34	98	100
$2S1s[3/2-1/2]$	2.83	41	116	117	35	99	99

TABLE XIII. Comparison of hyperfine splittings (in MeV) in doubly heavy baryons and heavy-light mesons. Experimental values for the hyperfine splittings in mesons are taken from Ref. [31].

$\Delta M(\Xi_{cc})$	$\frac{3}{4}\Delta M_D^{\rm expt}$	$\Delta M(\Xi_{bb})$	$\frac{3}{4}\Delta M_B^{\rm expt}$	$\Delta M(\Omega_{cc})$	$\frac{3}{4}\Delta M_{D_s}^{\rm expt}$	$\Delta M(\Omega_{bb})$	$\frac{3}{4}\Delta M_{B_s}^{\rm expt}$
107	106	35	34	94	108	30	35

quark-diquark approximation in the framework of the relativistic quark model. The orbital and radial excitations both of the heavy diquark and the light quark were considered. The main advantage of the proposed approach consists in the fully relativistic treatment of the light quark (u,d,s) dynamics and in the account for the internal structure of the diquark in the short-range quark-diquark interaction. We apply only the expansion in inverse powers of the heavy diquark mass (M^d_{bb},M^d_{cc}), which considerably simplifies calculations. The infinitely heavy diquark limit as well as the first order $1/M_d$ spin-independent and spin-dependent contributions were considered. A close similarity between excitations of the light quark in the doubly heavy baryons and heavy-light mesons was demonstrated. In the infinitely heavy (di)quark limit the only difference originates from the internal structure of the diquark which is important at small distances. The first order contributions to the heavy (di)quark expansion explicitly depend on the values of the heavy diquark (boson) and heavy quark (fermion) spins and masses $(M^d_{QQ}\approx 2m_Q)$. This results in the different number of levels to which the initially degenerate states split as well as their ordering. Our model respects the constraints imposed by heavy quark symmetry on the number of levels and their splittings.

We find that the p-wave levels of the light quark which correspond to heavy diquark spin multiplets with $j=1/2$ and $j=3/2$ are inverted in the infinitely heavy diquark limit. The origin of this inversion is the following. The confining potential contribution to the spin-orbit term in Eq. (30) exceeds the one-gluon exchange contribution. Thus the sign before the spin-orbit term is negative, and the level inversion emerges. However, the $1/M_d$ corrections, which produce the hyperfine splittings of these multiplets, are substantial. As a result the purely inverted pattern of p levels for the heavy diquark in the ground state occurs only for the doubly heavy baryons Ξ_{bb} and Ω_{bb}. For Ξ_{cc} and Ω_{cc} baryons the levels from these multiplets overlap. The similar pattern was previously found in our model for the heavy-light mesons [14].

We plan to use the found wave functions of doubly heavy baryons for the calculation of semileptonic and nonleptonic Ξ_{bb} decays to Ξ_{cb} and Ξ_{cb} to Ξ_{cc} baryons. The corresponding baryonic Isgur-Wise functions [6,32] will be determined.

ACKNOWLEDGMENTS

The authors express their gratitude to M. Müller-Preussker, V. Savrin and H. Toki for support and discussions. Two of us (R.N.F. and V.O.G.) were supported in part by the *Deutsche Forschungsgemeinschaft* under contract Eb 139/2-1 and *Russian Foundation for Basic Research* under Grant No. 00-02-17768. The work of R.N.F., V.O.G. and A.P.M. was supported in part by *Russian Ministry of Education* under Grant No. E00-3.3-45.

[1] J.-M. Richard, Phys. Rep. **212**, 1 (1992).
[2] G.S. Bali, Phys. Rep. **343**, 1 (2001).
[3] D.S. Kuzmenko and Yu.A. Simonov, Phys. Lett. B **494**, 81 (2000); Yad. Fiz. **64**, 110 (2001) [Phys. At. Nucl. **64**, 107 (2001)].
[4] M. Anselmino, E. Predazzi, S. Ekelin, S. Fredriksson, and D.B. Lichtenberg, Rev. Mod. Phys. **65**, 1199 (1993).
[5] A.V. Manohar and M.B. Wise, *Heavy Quark Physics* (Cambridge University Press, Cambridge, England, 2000).
[6] J.G. Körner, M. Kramer, and D. Pirjol, Prog. Part. Nucl. Phys. **33**, 787 (1994).
[7] M.J. Savage and M.B. Wise, Phys. Lett. B **248**, 177 (1990).
[8] E. Bagan, H.G. Dosch, P. Godzinsky, and J.-M. Richard, Z. Phys. C **64**, 57 (1994).
[9] R. Roncaglia, A. Dzierba, D.B. Lichtenberg, and E. Predazzi, Phys. Rev. D **51**, 1248 (1995); R. Roncaglia, D.B. Lichtenberg, and E. Predazzi, *ibid.* **52**, 1722 (1995).
[10] D. Ebert, R.N. Faustov, V.O. Galkin, A.P. Martynenko, and V.A. Saleev, Z. Phys. C **76**, 111 (1997).
[11] S.S. Gershtein, V.V. Kiselev, A.K. Likhoded, and A.I. Onishchenko, Phys. Rev. D **62**, 054021 (2000).
[12] V.V. Kiselev and A.K. Likhoded, hep-ph/0103169.
[13] N. Isgur, Phys. Rev. D **62**, 014025 (2000).
[14] D. Ebert, V.O. Galkin, and R.N. Faustov, Phys. Rev. D **57**, 5663 (1998); **59**, 019902(E) (1999).
[15] A.A. Logunov and A.N. Tavkhelidze, Nuovo Cimento **29**, 380 (1963).
[16] A.P. Martynenko and R.N. Faustov, Teor. Mat. Fiz. **64**, 179 (1985) [Theor. Math. Phys. **64**, 765 (1985)].
[17] V.O. Galkin, A.Yu. Mishurov, and R.N. Faustov, Yad. Fiz. **55**, 2175 (1992) [Sov. J. Nucl. Phys. **55**, 1207 (1992)].
[18] S. Weinberg, in *Lectures on Elementary Particles and Quantum Field Theory*, edited by S. Deser, M. Grisaru, and H. Pendleton (MIT Press, Cambridge, MA, 1970); I.B. Khriplovich, Zh. Éksp. Teor. Fiz. **96**, 385 (1989) [Sov. Phys. JETP **69**, 217 (1989)]; S. Ferrara, M. Porrati, and V.L. Telegdi, Phys. Rev. D **46**, 3529 (1992).
[19] V.O. Galkin and R.N. Faustov, Yad. Fiz. **44**, 1575 (1986) [Sov. J. Nucl. Phys. **44**, 1023 (1986)]; V.O. Galkin, A.Yu. Mishurov, and R.N. Faustov, *ibid.* **51**, 1101 (1990) [*ibid.* **51**, 705 (1990)].
[20] R.N. Faustov and V.O. Galkin, Z. Phys. C **66**, 119 (1995).
[21] D. Ebert, R.N. Faustov, and V.O. Galkin, Phys. Rev. D **62**, 034014 (2000).

[22] R.N. Faustov, Ann. Phys. (N.Y.) **78**, 176 (1973); Nuovo Cimento A **69**, 37 (1970).
[23] A.P. Martynenko and V.A. Saleev, Phys. Lett. B **385**, 297 (1996).
[24] H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Springer-Verlag, Berlin, 1957).
[25] A.P. Martynenko and R.N. Faustov, Teor. Mat. Fiz. **66**, 399 (1986).
[26] I.B. Khriplovich and A.I. Milstein, Report No. 96-49, hep-ph/9607374; P. Labelle, G.P. Lepage, and U. Magnea, Phys. Rev. Lett. **72**, 2006 (1994).
[27] H.J. Schnitzer, Phys. Lett. **76B**, 461 (1978).
[28] I.M. Narodetskii and M.A. Trusov, Yad. Fiz. **65**, 944 (2002) [Phys. At. Nucl. **65**, 917 (2002)].
[29] R. Lewis, N. Mathur, and R.M. Woloshyn, Phys. Rev. D **64**, 094509 (2001).
[30] H.J. Lipkin, Phys. Lett. B **171**, 293 (1986).
[31] Particle Data Group, D.E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).
[32] D. Ebert, T. Feldmann, C. Kettner, and H. Reinhardt, Z. Phys. C **71**, 329 (1996); Int. J. Mod. Phys. A **13**, 1091 (1998).
[33] The spin-dependent relativistic corrections, which are of the same order in v^2/c^2, produce the level splittings of order of 120 MeV.