Reaction operator approach to multiple elastic scatterings

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We apply the Gyulassy-Levai-Vitev reaction operator formalism to compute the effects of multiple elastic scatterings of on-shell partons propagating through dense matter. We derive the elastic reaction operator and demonstrate that the recursion relations have a closed form solution that reduces to the familiar Glauber form. We also investigate the accuracy of the Gaussian dipole approximation for the parton transverse momentum broadening.

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I. INTRODUCTION

New experiments at the BNL Relativistic Heavy Ion Collider (RHIC) will provide high-statistics high-p_T measurements of photon, lepton, and hadron production in $A+A$ reactions [1,2]. These observables provide novel tests of perturbative QCD (PQCD) multiple scattering theory. Quantitative comparisons [3–5] to data require an understanding of nuclear effects such as parton shadowing [6], the Cronin effect [7–10], and non-Abelian jet energy loss [11–13]. In this paper we apply the reaction operator method [13], developed to solve the radiative energy loss problem, to derive the well known multiple elastic Glauber distribution [14] in order to illustrate the power of this recursion technique. The summation to all orders in the opacity, $\chi=\int\sigma(z)\rho(z)dz=L/\lambda$, of the matter is given in closed form. In addition we evaluate the accuracy of the dipole approximation, leading to the Gaussian approximation to Moliere scattering, for the case of moderate opacities $\chi<10$ relevant for applications in nuclear physics.

This work is focused on the case of fast $p^2\simeq 0$ on-shell partons with zero virtuality asymptotically prepared at time $t_0=-\infty$. It is in this context in which we use the term "jet" throughout the paper. We here only discuss elastic interactions inside a nuclear medium that conserve the initial integrated jet flux. The dynamical evolution of the jet cone (that includes inelasticity) for a highly virtual parton is beyond the scope of the present study. A physical situation that corresponds to our calculation is realized in heavy ion reactions where the partons in the nuclear wave function may interact semisoftly several times before the "hard" collision vertex. The scattering of gluons on a classical Yang-Mills system was discussed in [15].

We describe multiple jet scattering as a series expansion in χ, the mean number of interactions that a fast parton undergoes along its trajectory [13,16]. Jet production is modeled here by an initial wave packet $j(p)$ for a spinless parton in the color representation R prepared at time t_0 and centered at $\vec{\mathbf{x}}_0=(z_0,\mathbf{x}_0)$. The momentum space amplitude in the absence of interactions is then given by

$$M_0\equiv i e^{ipx_0} j(p)\mathbf{1}_{d_R\times d_R}, \tag{1}$$

where the d_R dimensional unit matrix accounts for the jet colors. Multiplying $|M_0|^2$ by the invariant one particle phase space element $d^3\vec{\mathbf{p}}/[(2\pi)^3 2|\vec{\mathbf{p}}|]$ and taking the color trace one arrives at the unperturbed inclusive distribution of jets in the wave packet

$$d^3N_0=\mathrm{Tr}|M_0|^2\frac{d^3\vec{\mathbf{p}}}{2|\vec{\mathbf{p}}|(2\pi)^3}=|j(p)|^2\frac{d_R d^3\vec{\mathbf{p}}}{2|\vec{\mathbf{p}}|(2\pi)^3}. \tag{2}$$

For asymptotically prepared ($t_0\to-\infty$) partonic states, the wave function is dominated by its on-shell momentum space Fourier components.

In the presence of nuclear matter the multiple interactions of a fast projectile propagating through the medium are modeled by a scattering potential assumed to be of the form

$$\begin{aligned} V_n&=v(q_n)e^{iq_nx_n}T_{a_n}(R)\otimes T_{a_n}(n)\\ &=2\pi\delta(q^0)v(\vec{\mathbf{q}}_n)e^{-i\vec{\mathbf{q}}_n\cdot\vec{\mathbf{x}}_n}T_{a_n}(R)\otimes T_{a_n}(n), \end{aligned} \tag{3}$$

where $T_{a_n}(R)$ is the generator of $SU(N_c)$ in the d_R dimensional representation of the jet, and $T_{a_n}(n)$ is the corresponding generator in the d_n dimensional representation of the target. The scattering center is localized at position $\vec{\mathbf{x}}_n$, and $v(\vec{\mathbf{q}}_n)$ is the Fourier transform of the spatial part of the potential. We here use $\vec{\mathbf{p}}$ to denote the 3D spatial part of a vector p and $\mathbf{p}$ represents its 2D transverse component. While there are no *a priori* limitations on the functional form of the scattering potential and the corresponding differential elastic scattering cross section

$$\frac{d\sigma_{el}(R,T)}{d^2\mathbf{q}} = \frac{C_R C_2(T)}{d_A}\frac{|v(\mathbf{q})|^2}{(2\pi)^2}, \tag{4}$$

we here consider potentials with a range $1/\mu \ll \lambda$, the mean free path of the jet in the medium. This condition together with the hard jet approximation ensures the path ordering of the successive scatterings along the jet trajectory. Diagrams with crossed momentum transfers (at points typically separated by $z_n - z_{n-1} \simeq \lambda$) are suppressed by factors $\sim \exp(-\lambda\mu)$. One particular example used in Refs. [11,13] is the Yukawa color-screened potential

$$v(\vec{\mathbf{q}}_n) \equiv \frac{4\pi\alpha_s}{\vec{\mathbf{q}}_n^2 + \mu^2} = \frac{4\pi\alpha_s}{(q_{nz} + i\mu_n)(q_{nz} - i\mu_n)}, \tag{5}$$

where $\mu_n^2 = \mu_{n\perp}^2 = \mu^2 + \mathbf{q}_n^2$ and $\lambda\mu \gg 1$.

This paper is organized as follows. In Sec. II we present a systematic recursive way of keeping track of the multiple collision Feynman diagrams in terms of "direct" and "virtual" interactions. We discuss the color and kinematic structure of the single Born (direct) operator $\hat{D}$ and the double Born (virtual) operator $\hat{V}$. In Sec. III the corresponding elastic reaction operator $\hat{R} = \hat{D}^\dagger\hat{D} + \hat{V}^\dagger + \hat{V}$ is computed. We find a recursion relation for the observed inclusive jet distribution at any fixed order in opacity and use impact parameter space resummation to recover a classical Glauber-like formula. In Sec. IV we discuss the implications of our results in relation to the frequently employed Gaussian approximation of transverse momentum redistribution of jets. We show, however, that for $\chi < 10$ the Gaussian approximation fails to reproduce the power law tails of the jet p_T broadening as well as the $\sim \log p_{T\,\max}^2$ enhancement of $\langle \mathbf{p}_T^2 \rangle$.

II. TENSORIAL BOOKKEEPING AND DIAGRAMMATIC CALCULUS

The interaction Hamiltonian in a medium with N scattering centers in the high jet energy spinless approximation is given by

$$H_I(t) = \int d^3\vec{\mathbf{x}} \sum_{i=1}^{N} v(\vec{\mathbf{x}} - \vec{\mathbf{x}}_1) T_a(i)\phi^\dagger(\vec{\mathbf{x}},t) \times T_a(R)\hat{D}(t)\phi(\vec{\mathbf{x}},t), \tag{6}$$

where $\hat{D}(t) = i\overleftrightarrow{\partial}_t$. Each interaction of the colored parton propagating through the nuclear matter results in one power of the scattering potential Eq. (3) in the diagrammatic expansion. There is no limit on the number of scatterings the jet can have at a particular scattering center at position z_n. Using the terminology of Ref. [13] we call such interactions single Born, double Born, etc. It can be seen from Eq. (4) that two momentum transfers resulting from two interactions with the scattering potential are needed to produce one power of the elastic scattering cross section σ_{el}. In performing the average over the transverse (relative to the jet trajectory) position of the scattering center $\langle \cdots \rangle_{A_\perp} = \int d^2\mathbf{b}/A_\perp(\cdots)$ one order in the optical thickness (opacity) $N\sigma_{el}/A_\perp = L/\lambda$ is thus generated in three ways: (a) single Born or "direct" interaction in the amplitude and the complex conjugate amplitude; (b) double Born or "virtual" interaction in the amplitude and no interaction in the complex conjugate amplitude; (c) no interaction in the amplitude and double Born interaction in the complementary complex conjugate amplitude. More momentum transfers at a fixed longitudinal position z_n, e.g. a triple Born interaction, will produce $\mathcal{O}(\alpha_s^2)$ corrections to the opacity expansion and are here neglected.

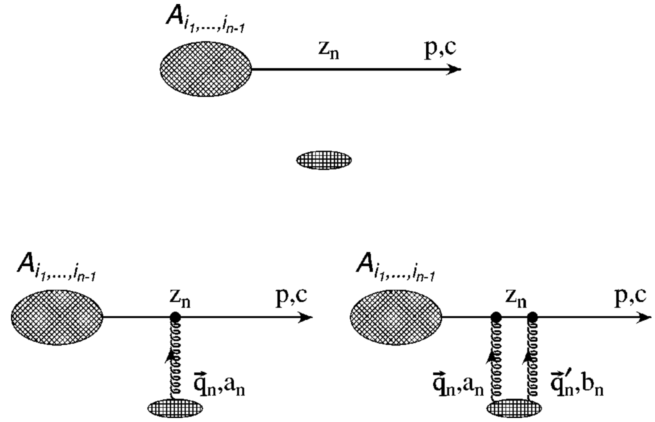

FIG. 1. The diagrammatic representation of the interactions with a fixed scattering center located at z_n up to the double Born term ($\hat{I}_n\mathcal{A}_{i_1\cdots i_{n-1}}$, $\hat{D}_n\mathcal{A}_{i_1\cdots i_{n-1}}$, and $\hat{V}_n\mathcal{A}_{i_1\cdots i_{n-1}}$) are shown.

It is therefore sufficient to consider single and double Born terms in the opacity expansion approach, i.e. when the jet propagates by a scattering center at position z_n it can either miss the center, interact once, or interact twice as illustrated in Fig. 1.

The kinematic and color structure of the scatterings is contained in the unit ($\hat{I}$), direct ($\hat{D}$), and virtual ($\hat{V}$) operators. The path ordering of the projectile interactions in the medium in the high energy eikonal regime facilitates the introduction of a convenient tensorial bookkeeping notation that classifies the diagrams according to the type of scatterings that the parton has experienced along its trajectory. At order n in the opacity expansion there are 3^n possible amplitudes classified by a set of n indices $i_1 \cdots i_n$. Here $i_m = 0$ indicates that the jet misses the mth target, $i_m = 1$ means that it scatters once, and $i_m = 2$ indicates that it scatters twice. The diagrams in Fig. 1 can be algebraically represented as follows:

$$\begin{aligned}
\mathcal{A}_{i_1\cdots i_{n-1},0}(p,c) &\equiv \hat{I}\mathcal{A}_{i_1\cdots i_{n-1}}(p,c),\\
\mathcal{A}_{i_1\cdots i_{n-1},1}(p,c) &\equiv \hat{D}\mathcal{A}_{i_1\cdots i_{n-1}}(p,c),\\
\mathcal{A}_{i_1\cdots i_{n-1},2}(p,c) &\equiv \hat{V}\mathcal{A}_{i_1\cdots i_{n-1}}(p,c).
\end{aligned} \tag{7}$$

The set of amplitude indices in Eq. (7) thus encodes the complete history of the jet interactions. Repeating the basic operator steps in Eq. (7) any amplitude that includes parton scatterings inside the medium can be iteratively derived from the unperturbed jet production amplitude

$$\mathcal{A}_{i_1\cdots i_n}(p,c)=\prod_{m=1}^{n}\left[\delta_{0,i_m}+\delta_{1,i_m}\hat{D}_m+\delta_{2,i_m}\hat{V}_m\right]M_0(p,c). \tag{8}$$

Time (or longitudinal coordinate) ordering is implicit in the above formula for the high energy eikonal limit under consideration here.

To proceed we need to know the color and kinematic structure of the direct and virtual interactions. We use the notation $\Delta(p)\equiv(p^2+i\epsilon)^{-1}$ and $\Gamma(p)=p^0$ for the momentum space representation of the propagator and the vertex factor. The vertex factor is treated here in the high energy spinless limit. The direct iteration step (with $z_n>z_{n-1}>\cdots$) in Eq. (7) reads

$$\begin{aligned}\mathcal{A}_{i_1\cdots i_{n-1},1}(p,c)=\int\frac{d^4q_n}{(2\pi)^4}&\mathcal{A}_{i_1\cdots i_{n-1}}(p-q_n,c)\\ &\times\Delta(p-q_n)v(q_n)\Gamma(2p-q_n)\\ &\times e^{iq_n(x_n-x_0)}T_{a_n}(n)T_{a_n}(R),\end{aligned} \tag{9}$$

where we have taken into account that the amplitude of order $n-1$ has to be evaluated at momentum $p-q_n$ if the jets emerges on shell with momentum p. The double Born amplitude at the *same* external potential is similarly given by

$$\begin{aligned}\mathcal{A}_{i_1\cdots i_{n-1},2}(p,c)=\int\frac{d^4q_n}{(2\pi)^4}\frac{d^4q_n'}{(2\pi)^4}&\mathcal{A}_{i_1\cdots i_{n-1}}(p-q_n-q_n',c)\\ &\times\Delta(p-q_n-q_n')\Gamma(2p-2q_n'-q_n)\\ &\times v(q_n)\Delta(p-q_n')\Gamma(2p-q_n')\\ &\times v(q_n')e^{i(q_n+q_n')(x_n-x_0)}\\ &\times T_{a_n}(R)T_{b_n}(R)T_{a_n}(n)T_{b_n}(n).\end{aligned} \tag{10}$$

III. ELASTIC RECURSION OPERATOR

The observed jet inclusive distribution $dN(p,c)=\Sigma_{n=0}^{\infty}dN^{(n)}(p,c)$ is expanded in the opacity series, where the contribution to order χ^n is given by

$$\begin{aligned}dN^{(n)}(p,c)&=C_n^N\overline{\mathcal{A}}^{i_1\cdots i_n}(p,c)\mathcal{A}_{i_1\cdots i_n}(p,c)\\ &\equiv C_n^N\,\mathrm{Tr}\sum_{i_1=0}^{2}\cdots\sum_{i_n=0}^{2}\overline{\mathcal{A}}^{i_1\cdots i_n}(p,c)\\ &\quad\times A_{i_1\cdots i_n}(p,c).\end{aligned} \tag{11}$$

The amplitudes $\overline{\mathcal{A}}^{i_1\cdots i_n}(p,c)$ in Eq. (11) are the complementary amplitudes given by

$$\overline{\mathcal{A}}^{i_1\cdots i_n}(p,c)\equiv M_0^{\dagger}(p,c)\prod_{m=1}^{n}\left[\delta_{0,i_m}\hat{V}_m^{\dagger}+\delta_{1,i_m}\hat{D}_m^{\dagger}+\delta_{2,i_m}\right]. \tag{12}$$

The trace is taken over the color matrices and the binomial coefficient $C_n^N=N!/n!(N-n)!\approx N^n/n!$ is introduced to take into account the number of combinations of n scattering sites out of the total N.

Performing the sum over the first $n-1$ interaction points and using Eqs. (8),(12) we obtain a simple recursion identity which relates $dN^{(n)}$ to $dN^{(n-1)}$ through the reaction operator $\hat{R}$:

$$\begin{aligned}dN^{(n)}&=C_n^N\overline{\mathcal{A}}^{i_1\cdots i_{n-1}}\hat{R}_n\mathcal{A}_{i_1\cdots i_{n-1}},\\ \hat{R}_n&=\hat{D}_n^{\dagger}\hat{D}_n+\hat{V}_n+\hat{V}_n^{\dagger}.\end{aligned} \tag{13}$$

Consider first the direct part $dN^{(n)}(\mathrm{Dir.})\propto\overline{\mathcal{A}}^{i_1\cdots i_{n-1}}\hat{D}^{\dagger}\hat{D}\mathcal{A}_{i_1\cdots i_{n-1}}$ of the reaction operator. Performing the color traces we take into account the identity $\mathrm{Tr}\,T_a(i)T_b(j)=\delta_{ij}\delta_{ab}C_2(T)d_T/d_A$, which enforces that the positions of the ordered scattering centers in the amplitude and its complementary are identical. We use the form of the scattering potential specified by Eq. (3) as well as the high energy eikonal approximation where the deviations due to the in-medium interactions from the initial jet trajectory are small and $E^+\simeq 2E$:

$$\begin{aligned}dN^{(n)}(\mathrm{Dir.})=C_n^N\int\frac{dq_{zn}d^2\mathbf{q_n}}{(2\pi)^3}\frac{dq_{zn}'d^2\mathbf{q_n'}}{(2\pi)^3}&\overline{\mathcal{A}}^{i_1\cdots i_{n-1}}(p-q_n')\mathcal{A}_{i_1\cdots i_{n-1}}(p-q_n)\frac{C_RC_2(T)}{d_A}v(\vec{\mathbf{q}}_n)v^*(\vec{\mathbf{q}}_n')\\ &\times\frac{E^+}{E^+q_{zn}-q_{zn}^2-\mathbf{q}_n^2+i\epsilon}\frac{E^+}{E^+q_{zn}'-q_{zn}'^2-\mathbf{q}_n'^2-i\epsilon}\langle e^{-i(\vec{\mathbf{q}}_n-\vec{\mathbf{q}}_n')\cdot(\vec{\mathbf{x}}_n-\vec{\mathbf{x}}_0)}\rangle_{A_\perp}.\end{aligned} \tag{14}$$

The q_{zn},q_{zn}' integral can be performed by closing the contour in the lower or upper half plane. We note that due to the short range of the scattering potential relative to the mean free path the residues from the $q_{zn}=-i\sqrt{\mathbf{q}_n^2+\mu^2}$ and $q_{zn}'=+i\sqrt{\mathbf{q}_n'^2+\mu^2}$ poles are exponentially suppressed and only the residues at $q_{zn}\approx-i\epsilon+\mathbf{q}_n^2/E^+$ and $q_{zn}'\approx+i\epsilon+\mathbf{q}_n'^2/E^+$ contribute. In the high energy limit $E^+\gg\mu\gg\mathbf{q}_n^2/E^+$ the quadratic terms from the residues in the exponent can be neglected. This removes the correlations between the scattering centers and there are no coherence effects at the jet level. We note that this is an important difference from the case of induced gluon radiation discussed in Ref. [13]. Emitted gluons are much softer than the parent parton and non-negligible $\phi \sim (\mathbf{x}_i - \mathbf{x}_j)\cdot(\mathbf{k}-\mathbf{Q})^2/k^+$ phases control the non-Abelian analogue of the Landau-Pomeranchuk-Migdal effect [17]. Taking into account that the Glauber thickness function (at a fixed impact parameter $\mathbf{b_0}$) $T(\mathbf{b_0}) = \int dz \rho(\mathbf{b_0},z) = N/A_\perp$ is expected to vary slowly with the impact parameter:

$$\langle e^{-i(\mathbf{q}_n-\mathbf{q}_n')\cdot(\mathbf{b}-\mathbf{b}_0)}\rangle_{A_\perp} \simeq \frac{T(\mathbf{b}_0)}{N}(2\pi)^2\delta^2(\mathbf{q}_n-\mathbf{q}_n'). \tag{15}$$

Equation (15) is a key simplifying relation valid in the case $\sqrt{A_\perp} \gg 1/\mu$ because it diagonalizes Eq. (14) in the transverse momentum variables.

This leads to a simple recursion relation for the direct contribution to the jet spectrum to the momentum-shifted distribution of order $n-1$,

$$dN^{(n)}(\text{Dir.}) = C_n^N \int d^2\mathbf{q_n} \overline{\mathcal{A}}^{i_1\cdots i_{n-1}}(p,c)\left[\frac{d\sigma_{el}(R,T)}{d^2\mathbf{q}_n}\frac{T(\mathbf{b}_0)}{N}e^{-i\mathbf{q}_n\cdot\hat{\mathbf{b}}^\dagger}\otimes e^{i\mathbf{q}_n\cdot\hat{\mathbf{b}}}\right]\mathcal{A}_{i_1\cdots i_{n-1}}(p,c), \tag{16}$$

where $\hat{\mathbf{b}} = i\overrightarrow{\nabla}_{\mathbf{p}}$ is the impact parameter operator conjugate to the transverse momentum $\mathbf{p}$ acting to the right, and $\hat{\mathbf{b}}^\dagger = -i\overleftarrow{\nabla}_{\mathbf{p}}$ its Hermitian conjugate acting to the left.

Next consider the virtual contribution $dN^{(n)}(\text{Vir.}) = C_n^N \overline{\mathcal{A}}^{i_1\cdots i_{n-1}}(\hat{V}+\hat{V}^\dagger)\mathcal{A}_{i_1\cdots i_{n-1}}$ of the reaction operator, Eq. (13),

$$\begin{aligned} dN^{(n)}(\text{Vir.}) = C_n^N 2\,\text{Re}\int \frac{dq_{zn}d^2\mathbf{q_n}}{(2\pi)^3}\frac{dq_{zn}'d^2\mathbf{q_n'}}{(2\pi)^3}\overline{\mathcal{A}}^{i_1\cdots i_{n-1}}(p)\mathcal{A}_{i_1\cdots i_{n-1}}(p-q_n-q_n')v(\vec{\mathbf{q}}_n)v(\vec{\mathbf{q}}_n')\frac{C_RC_2(T)}{d_A} \\ \times \frac{E^+}{E^+(q_{zn}+q_{zn}')-(q_{zn}+q_{zn}')^2-(\mathbf{q}_n+\mathbf{q}_n')^2+i\epsilon}\frac{E^+}{E^+q_{zn}'-q_{zn}'^2-\mathbf{q}_n'^2+i\epsilon}\langle e^{-i(\vec{\mathbf{q}}_n+\vec{\mathbf{q}}_n')\cdot(\vec{\mathbf{x}}_n-\vec{\mathbf{x}}_0)}\rangle_{A_\perp}. \end{aligned} \tag{17}$$

One notes that the momentum space integrations in this case could have been performed already at the amplitude level, Eq. (10). Using the same set of approximations as for the Direct part we first perform the q_{zn} integral as well as the impact parameter average and note that the latter constrains $\mathbf{q}_n + \mathbf{q}_n' = 0$. Picking the residue at $q_{zn} \approx -q_{zn}' - i\epsilon$:

$$dN^{(n)}(\text{Vir.}) = C_n^N 2\,\text{Re}\int \frac{dq_{zn}'d^2\mathbf{q_n'}}{(2\pi)^3}\overline{\mathcal{A}}^{i_1\cdots i_{n-1}}(p)\mathcal{A}_{i_1\cdots i_{n-1}}(p)(-i)\frac{T(\mathbf{b}_0)}{N}\frac{C_RC_2(T)}{d_A}v(\vec{\mathbf{q}}_n')v(-\vec{\mathbf{q}}_n')\frac{E^+}{E^+q_{zn}'-q_{zn}'^2-\mathbf{q}_n'^2+i\epsilon}. \tag{18}$$

It was shown in Ref. [13] that for a broad class of real spherically symmetric potentials of short range $\ll\lambda$ the remaining q_{zn}' integral can be performed. The screened Yukawa potential, Eq. (3), belongs in this category when $\mu\lambda \gg 1$. We note that the contour integration produces a real result and a factor of $\frac{1}{2}$ that cancels the factor of 2 arising from the virtual terms in both the amplitude and its complementary. The recursion relation for the virtual part of the reaction operator can then be written as

$$dN^{(n)}(\text{Vir.}) = C_n^N \int d^2\mathbf{q_n}\overline{\mathcal{A}}^{i_1\cdots i_{n-1}}(p,c)\left[\frac{d\sigma_{el}(R,T)}{d^2\mathbf{q}_n}\frac{T(\mathbf{b}_0)}{N}(-1)\right]\mathcal{A}_{i_1\cdots i_{n-1}}(p,c). \tag{19}$$

Combining Eqs. (16),(19) one recovers the full structure of the *elastic* reaction operator

$$\begin{aligned} dN^{(n)}(p,c) &= C_n^N \int d^2\mathbf{q_n}\overline{\mathcal{A}}^{i_1\cdots i_{n-1}}(p,c)\left[\frac{d\sigma_{el}(R,T)}{d^2\mathbf{q}_n}\frac{T(\mathbf{b}_0)}{N}(e^{-i\mathbf{q}_n\cdot\hat{\mathbf{b}}^\dagger}\otimes e^{i\mathbf{q}_n\cdot\hat{\mathbf{b}}}-1)\right]\mathcal{A}_{i_1\cdots i_{n-1}}(p,c) \\ &= \frac{1}{n!}\int\prod_{i=1}^n d^2\mathbf{q}_i\left[T(\mathbf{b}_0)\frac{d\sigma_{el}(R,T)}{d^2\mathbf{q}_i}(e^{-\mathbf{q}_i\cdot\vec{\nabla}_{\mathbf{p}}}-1)\right]dN^{(0)}(p,c), \end{aligned} \tag{20}$$

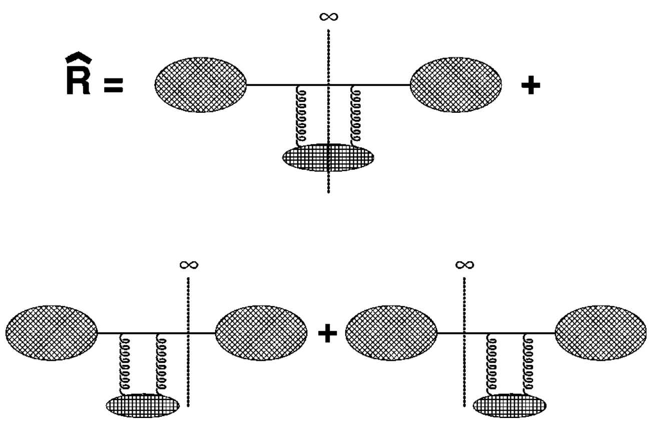

FIG. 2. The graphical structure of the reaction operator $\hat{R} = \hat{D}^\dagger \hat{D} + \hat{V} + \hat{V}^\dagger$ is illustrated. It represents all possible ($t = +\infty$) on-shell cuts through a new double Born insertion.

where we used the $C_n^N/N^n \approx 1/n!$ (Poisson) approximation.

Equation (20) is the main result of this paper and has a transparent physical interpretation illustrated in Fig. 2. The first term in the kinematic structure of the reaction operator corresponds to the direct elastic scattering part that leads to the deflection of the fact parton from its trajectory. The next two terms have delta function strength in the forward $\mathbf{p} = 0$ direction and correspond to the virtual corrections (in the amplitude and its complementary complex conjugate). These terms enforce unitarity in the Gyulassy-Levai-Vitev (GLV) reaction operator formalism. Given any initial parton distribution one can then recursively solve from Eq. (20) for the final inclusive distribution of jets that have penetrated the medium characterized here by its Glauber thickness function $T(\mathbf{b}_0)$. The method used here is quite general and has several important advantages. Within the eikonal approximation it provides exact answers to problems related to multiple elastic and inelastic processes in QCD media. In particular it was applied to the case of gluon radiation[1] from jets produced in heavy ion reactions [13]. For more details on its uses in jet tomography see Refs. [4,5]. Since the solution is known to all orders in opacity, cases ranging from thin media with a few scatterings to thick media with a formally very large number of scatterings can be studied.

In the case of multiple elastic scatterings, the reaction operator is so simple that it is possible to sum over all orders in closed form. As usual it is most convenient to perform the summation in impact parameter space. The distribution in the impact parameter space conjugate to the transverse momentum $\mathbf{p}$ is

$$dN(\mathbf{b}) = \sum_{n=0}^{\infty} \frac{1}{n!} \{T(\mathbf{b}_0)[\tilde{\sigma}_{el}(\mathbf{b}) - \sigma_{el}]\}^n dN^{(0)}(\mathbf{b})$$
$$= e^{T(\mathbf{b}_0)(\tilde{\sigma}_{el}(\mathbf{b}) - \sigma_{el}^{tot})} dN^{(0)}(\mathbf{b}), \qquad (21)$$

[1]In the case of medium-induced gluon bremsstrahlung the reaction operator $\hat{R}$ retains its general operator form Eq. (13) but its color and kinematic structures are different since the on-shell $t = +\infty$ cuts (see Fig. 2) go through both the jet and gluon lines that interact with the double Born term.

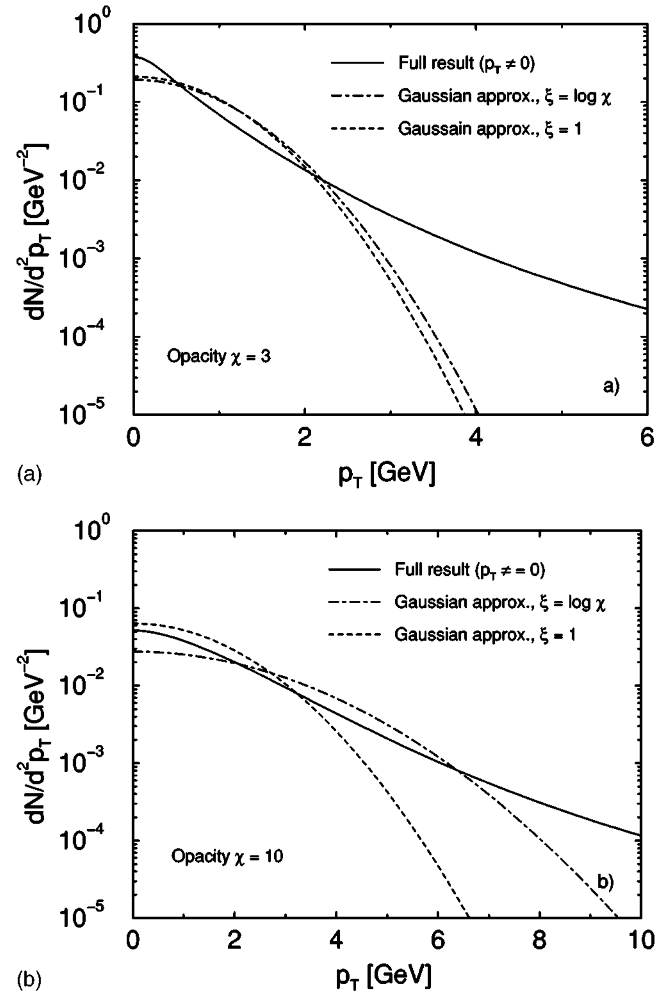

FIG. 3. The final parton p_T distribution is shown versus p_T for two different opacities $\chi = 3$ (a) and $\chi = 10$ (b). We compare the full result (without the delta function contribution at $p_T \sim 0$) to the Moliere Gaussian approximation with $\xi = 1$ and $\xi = \log\chi$. In this example we use $\mu^2 = 0.25$ GeV^2.

where the Fourier transform of the differential cross section was denoted by $(2\pi)^2 d\tilde{\sigma}_{el}/d^2\mathbf{q}(\mathbf{b}) \equiv \tilde{\sigma}_{el}(\mathbf{b})$ and $\tilde{\sigma}_{el}(\mathbf{0}) = \sigma_{el}$, the total elastic scattering cross section. The impact parameter space representation Eq. (21) depends only on the difference $\tilde{\sigma}_{el}(\mathbf{b}) - \tilde{\sigma}_{el}(\mathbf{0})$. Transforming back to momentum space, we recover the parton version of the Glauber multiple collision series [14]

$$dN(\mathbf{p}) = e^{-\sigma_{el}T(\mathbf{b}_0)} \int d^2\mathbf{b}\, e^{i\mathbf{p}\cdot\mathbf{b}} e^{\tilde{\sigma}_{el}(\mathbf{b})T(\mathbf{b}_0)} dN^{(0)}(\mathbf{b})$$
$$= \sum_{n=0}^{\infty} e^{-\chi} \frac{\chi^n}{n!} \int \prod_{i=1}^{n} d^2\mathbf{q}_i \frac{1}{\sigma_{el}} \frac{d\sigma_{el}}{d^2\mathbf{q}_i} dN^{(0)}$$
$$\times (\mathbf{p} - \mathbf{q}_1 - \cdots - \mathbf{q}_n), \qquad (22)$$

where $\chi = \sigma_{el} T(\mathbf{b}_0) = L/\lambda$. Equation (22) has the familiar physical interpretation of a Poisson random walk in transverse momentum space distributed according to $d\sigma_{el}/\sigma_{el}$.

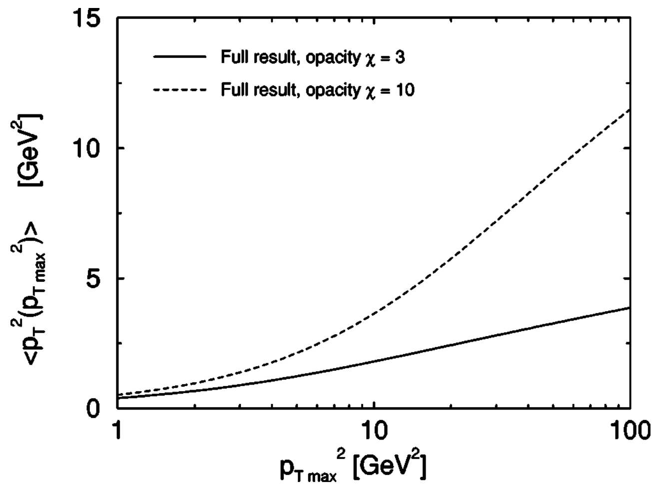

FIG. 4. The transverse momentum broadening of jets is shown versus $p^2_{\mathrm{T\,max}}$ for two different opacities $\chi = 3{,}10$ as in Figs. 3(a) and 3(b). We here use $\mu^2 = 0.25\ \mathrm{GeV}^2$ for illustration.

IV. DISCUSSION

Impact parameter space resummation is often used in conjunction with the small impact parameter approximation of the differential cross section, Eq. (4). We show below that although such an approach is analytically appealing, it gives jet distributions that may differ substantially from the exact formula.

The Fourier transform of the *normalized* cross section is given by

$$\frac{d\tilde{\sigma}_{el}}{d^2\mathbf{q}}(\mathbf{b}) = \int \frac{d^2\mathbf{q}}{(2\pi)^2} e^{-i\mathbf{q}\cdot\mathbf{b}} \frac{1}{\pi} \frac{\mu^2}{(\mathbf{q}^2+\mu^2)^2}$$

$$= \frac{\mu b}{4\pi^2} K_1(\mu b)$$

$$\approx \frac{1}{4\pi^2}\left(1 - \frac{\xi \mu^2 b^2}{2} + \mathcal{O}(b^3)\right), \tag{23}$$

where $b = |\mathbf{b}|$ and in the quadratic term in Eq. (23) the $\log 2/(1.08\mu b)$ multiplicative factor has been absorbed into a b-independent constant ξ due to its small logarithmic variation. We consider the transverse momentum broadening [18,19] of a fast parton propagating in the "$\hat{z}$" direction, i.e., $dN^{(0)}/d^2\mathbf{p} = \delta^2(\mathbf{p})$. In the approximation that dominantly small impact parameter scatterings are important, the momentum space distributions from Eqs. (21),(22) reduce to the classic Moliere form [16]

$$dN(\mathbf{p}) = \int d^2\mathbf{b} e^{i\mathbf{p}\cdot\mathbf{b}} \frac{1}{(2\pi)^2} \frac{e^{-\chi\mu^2\xi b^2/2}}{\chi\mu^2\xi} = \frac{1}{2\pi} \frac{e^{-p^2/2\chi\mu^2\xi}}{\chi\mu^2\xi}. \tag{24}$$

The resulting distribution is of Gaussian form and has a width of $\chi\mu^2\xi$, i.e., $\langle \mathbf{p}^2 \rangle = \chi\mu^2\xi$.

To assess the accuracy of the Gaussian ansatz for finite χ relevant in nuclear physics, we compare the Fourier transform of Eq. (21) to Eq. (24). Note that in going from Eq. (21) back to momentum space, an $e^{-\chi}\delta^2(\mathbf{p})$ component arises. This component is of course not displayed, and for finite p numerical integration converges much more rapidly if unity is subtracted from $\exp[\tilde{\sigma}(\mathbf{b})T(\mathbf{b}_0)]$ in the integrand (22).

In Figs. 3(a) and 3(b) numerical results are shown for opacities $\chi = 3$ and $\chi = 10$. For large opacities $\chi \geqslant 10$ [see Fig. 3(b)] the Gaussian approximation gives better results than for small ones $\chi \leqslant 5$. However, as is well known, at larger transverse momenta it fails to account for the power law p_T tail of the inclusive distribution. Unlike familiar Moliere scattering in atomic matter, there are no form factors to limit the growth of the high-p_T Rutherford tail out to the kinematic limit $p^2_{\mathrm{T\,max}} \sim E_0\mu$ in the case of QCD. Our results for the rms transverse momentum kick shown in Fig. 4 demonstrate that in semihard and hard PQCD processes there is a logarithmic in $p^2_{\mathrm{T\,max}}$ enhancement of the mean squared transverse momentum, $\langle \mathbf{p}^2 \rangle \simeq \chi\mu^2 \log(1 + p^2_{\mathrm{T\,max}}/\chi\mu^2)$ that is missed in the dipole approximation. Of course for large $p^2_{\mathrm{T\,max}}$ radiative energy loss also contributes substantially to the p_T broadening of jets and should be taken into account in applications.

In summary, we showed that the reaction operator formalism developed in [13] easily recovers the well known Glauber limit for elastic multiple collisions in PQCD matter and sheds light on the accuracy of the dipole approximation. In the future it would be interesting to combine both elastic and the Landau-Pomeranchuk-Migdal (LPM) suppressed inelastic processes in this method and apply the results to the jet acoplanarity observables suggested in Refs. [18,19].

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