Higgs boson mass limits in perturbative unification theories

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Motivated in part by recent demonstrations that electroweak unification into a simple group may occur at a low scale, we detail the requirements on the Higgs boson mass if the unification is to be perturbative. We do this for the standard model effective theory, minimal supersymmetry, and next-to-minimal supersymmetry with an additional singlet field. Within the standard model framework, we find that perturbative unification with $\sin^2\theta_W = 1/4$ occurs at $\Lambda = 3.8$ TeV and requires $m_h \lesssim 460$ GeV, whereas perturbative unification with $\sin^2\theta_W = 3/8$ requires $m_h \lesssim 200$ GeV. In supersymmetry, the presentation of the Higgs boson mass predictions can be significantly simplified, yet remain meaningful, by using a single supersymmetry breaking parameter Δ_S. We present Higgs boson mass limits in terms of Δ_S for the minimal supersymmetric model and the next-to-minimal supersymmetric model. We show that in next-to-minimal supersymmetry, the Higgs boson mass upper limit can be as large as 500 GeV even for moderate supersymmetry masses if the perturbative unification scale is low ($\Lambda \simeq 10$ TeV).

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I. INTRODUCTION

A large gap in our understanding of fundamental physics is the mechanism of electroweak symmetry breaking and fermion mass generation. Among the many ideas developed to explain this phenomena, the most economical explanation postulates the existence of a single scalar Higgs boson. This simple explanation has been remarkably successful, in that all precision electroweak data are compatible with it, yet not compatible with many other more complicated explanations of dynamical electroweak symmetry breaking.

Despite the success of the single Higgs boson theory, there are two challenges. First, there are theoretical problems explaining the large hierarchy of fundamental scales (e.g., $M_{\rm Pl} \gg m_W$). And second, we have yet to find the Higgs boson in experiment.

The standard model (SM) Higgs properties are completely fixed in terms of only one parameter, its mass. Unfortunately, the mass cannot be predicted. Precision measurements have constrained the Higgs boson mass to be below 222 GeV at the 95% C.L. (see p. 101 of [1]). Direct searches in $e^+e^- \to Z + $Higgs boson have constrained the Higgs boson to have mass above 114.1 GeV at the 95% C.L. [2]. The remaining 108 GeV window of possible Higgs boson mass is relatively narrow and proposals for future experiments have focused heavily on this region.

Nevertheless, a light Higgs boson below 222 GeV is not guaranteed for several reasons. First, precision electroweak data is sensitive mostly to the logarithm of the Higgs boson mass, and small changes in the χ^2 fit can yield large changes in the allowed Higgs boson mass. Using the "all data" fit from Table 13.2 of the LEP Electroweak Working Group summary report [1], which concludes that

$$\log_{10}(m_h/\text{GeV}) = 1.94^{+0.21}_{-0.22} \quad (\text{all data LEPEWWG fit}), \tag{1}$$

we can deduce that the 3σ (4σ) upper bound on the Higgs boson mass is about 372 GeV (603 GeV). A true value 4σ away from the experimentally determined central value is by no means out of the question. Furthermore, it is possible that a much heavier Higgs boson can conspire with new states (Z' bosons, new scalars, etc.) to be compatible with the precision electroweak data [3]. In this article we will not focus on the statistics and solidity of the present Higgs boson mass limits. Instead, our main purpose is to determine how much information can be learned about a theory from the Higgs boson mass whatever it might turn out to be.

We frame our discussion by first assuming that a Higgs boson exists that couples to SM states in the well-defined SM way. We then wish to explore what different values of the Higgs boson would imply for supersymmetric and non-supersymmetric gauge unification theories. Our last ingredient is to take into consideration an infinite set of possible unification scenarios parametrized by the value of $\sin^2\theta_W$ at the unification scale Λ. Most of the previous discussions on Higgs boson mass limits have relied strongly on high-energy unification ($\Lambda \sim 10^{16}$ GeV). In our analysis, the scale Λ ranges from ~ 1 TeV to $\sim 10^{18}$ GeV.

II. NON-SUPERSYMMETRIC UNIFICATION

Many years ago it was discovered that the SM fermions fit very nicely into two representations of $SU(5)$ and one representation of $SO(10)$. Unifying the SM gauge groups also gives explanation to the unusual values of the hypercharges. Grand unification theories (GUTs) based on these two groups gained considerable attention and are still of value today.

Non-supersymmetric $SU(5)/SO(10)$ GUTs have at least three major challenges. They have no explanation for the huge hierarchy between $M_{\rm GUT}$ and m_W. The gauge couplings do not precisely meet at any point at higher energy. The precise meeting point of the GUT-normalized hypercharge gauge coupling $g_1 \equiv g'\sqrt{5/3}$ and the $SU(2)_L$ gauge coupling g_2 is at a scale that would be too low ($\sim 10^{13}$ GeV) to satisfy proton decay constraints if the GUT gauge boson masses were nearby.

Despite all the problems with these high-scale non-supersymmetric unification scenarios, there have been clever attempts to salvage them [4], and probably other clever ways that have yet to be discussed. The various attempts to save them may require intermediate thresholds, but here we do not admit all those uncertainties and instead analyze the SM evolution up to the unification scale $M_{\rm GUT}=\Lambda$ where $\sin^2\theta_W=3/8$ $(g_2/g'=\sqrt{5/3})$. Furthermore, the additional threshold corrections and symmetries required to make non-supersymmetric GUTs work may have very little impact on the Higgs sector. Or, more likely, there would exist an intermediate scale M_I, perhaps associated with the neutrino see-saw scale, such that below it one expects perturbative SM evolution. Therefore, we will keep this idea within the space of possibilities when discussing Higgs boson predictions, and we will make plots of Higgs boson mass assuming a perturbative SM evolution below an arbitrary scale Λ. The reader can then associate Λ with $M_{\rm GUT}$, M_I, or some other scale as he or she pleases.

A second idea that motivates unification at the low-energy scale is $SU(3)$ electroweak unification [5–8] (for some earlier attempts, see [9–11]). The generic prediction of this framework is that the hypercharge and $SU(2)_L$ couplings unify at $\sin^2\theta_W=1/4$ $(g_2/g'=\sqrt{3})$, which translates to a scale of about 3.8 TeV. It is relatively easy within these unification models to adjust the value of $\sin^2\theta_W$ at the unification scale to values above 1/4.

Our goal now is to demarcate the range of Higgs boson masses that allow for a perturbative unification theory, with a perturbative Higgs self-coupling, for each value of $\sin^2\theta_W$. We will consider all values of $\sin^2\theta_W$ from 1/4 to 3/8.

The reader should not get the impression that we are advocating theories with perturbative couplings as somehow more likely than theories with non-perturbative couplings. Nature's reality is probably independent of the pain it gives humans to understand it. Our emphasis here is only that by analyzing the Higgs sector coupling as given by its mass, we can determine if a perturbative theory is compatible with a particular unification scenario. One exception to this modest interpretative value to our work here is supersymmetric GUTs, where it appears desirable to keep all couplings perturbative so as not to feed into the gauge coupling renormalization group equations and spoil the extraordinary unification of all three gauge couplings of the SM at a high scale.

The Higgs potential for the doublet Higgs field Φ is

$$V(\Phi)=-m_\Phi^2\Phi^\dagger\Phi+\frac{\lambda}{2}(\Phi^\dagger\Phi)^2. \tag{2}$$

After spontaneous symmetry breaking, three of the four degrees of freedom are eaten by the weak gauge bosons, leaving one physical degree of freedom remaining with mass,

$$m_h^2=\lambda v^2 \tag{3}$$

where $v^2=1/\sqrt{2}G_F\simeq(246\ \ \mathrm{GeV})^2$.

Requiring the theory remain perturbative up to some scale Λ implies that the Higgs self-coupling $\lambda<\lambda_0$, where λ_0 is a non-perturbative value for the coupling constant. Choosing a numerical value for λ_0 is not easily justified in this analysis. One finds many conditions advocated for λ to remain perturbative: $\lambda\lesssim 2$, $\lambda\lesssim\pi$, $\lambda\lesssim 4\pi$, $\beta_\lambda\lesssim 1$, etc. These choices, unfortunately, depend on the numerical prefactor to the $(\Phi^\dagger\Phi)^2$ coupling. For example, if we were to rescale $V\supset 24\lambda(\Phi^\dagger\Phi)^2$ the conditions on λ set forth above are clearly much too weak to identify the onset of the non-perturbative regime.

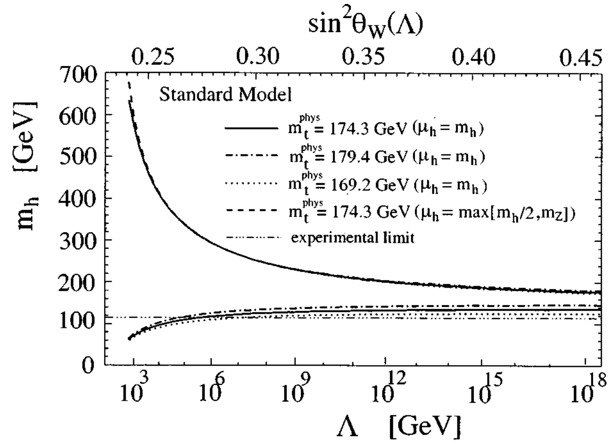

FIG. 1. The upper curve is the limit on the Higgs boson mass within the standard model such that the Higgs self-coupling remains perturbative, i.e., $\lambda<\lambda_0=8.2$, up the scale Λ. The lower curve is the Higgs limit such that $\lambda>0$ for all scales below Λ. The x axis can be equivalently expressed as Λ or the directly correlated value of $\sin^2\theta_W(\Lambda)$ which is labeled above.

We therefore seek a definition of the onset of non-perturbative behavior which is independent of the numerical factors out in front of operators. Our methodology that satisfies this aim is to turn off all couplings except the Higgs coupling and expand its β to successively higher loop order,

$$\frac{d\lambda}{dt}=\sum_i\ \beta_\lambda^{(i\ \mathrm{loop})}=\frac{L_1}{16\pi^2}\lambda^2+\frac{L_2}{(16\pi^2)^2}\lambda^3+\frac{L_3}{(16\pi^2)^3}\lambda^4+\cdots, \tag{4}$$

where in the $\overline{\rm MS}$ scheme, $L_{1,2,3}=12$, -78, and $897+504\zeta(3)\simeq 1503$, respectively [12,13]. We then identify the onset of non-perturbativity as when any higher loop order contribution to the beta function exceeds the value of any lower loop order contribution. That is,

$$|\beta_\lambda^{(j>i)}|<|\beta_\lambda^{(i)}|\quad \text{(perturbativity condition)} \tag{5}$$

implies perturbative coupling, and violation of the condition implies non-perturbative coupling. Therefore, given our definition of λ from Eqs. (2) and (4), λ remains perturbative as long as it is below $\lambda_0=8.2$.

In Fig. 1 we plot the upper limit on the Higgs boson mass that can be expected in a theory that remains perturbative (i.e., $\lambda<\lambda_0=8.2$) up to the scale Λ (similar SM analyses can be found in [14]). At the bottom of the figure we also plot a lower limit of the Higgs boson mass by requiring that λ remains positive for all scales below Λ. This gives an estimate for the Higgs boson mass requirement from vacuum stability [15].

The computation of the perturbative limit was done using full two-loop renormalization group equations (RGEs) for the SM gauge couplings, Higgs self-coupling and top quark Yukawa coupling. Other couplings are irrelevant to the analysis of the Higgs boson mass limit. In order to determine the initial condition for RGEs of gauge couplings at m_Z, we adopted experimental values [1] of the QED fine structure constant $\alpha^{-1}=137.06$, the hadronic contribution to the QED coupling at m_Z $\Delta\alpha^{(5)}_{\rm had}(m_Z)=0.02761$, the leptonic effective electroweak mixing angle $\sin^2\theta^{\rm lept}_{\rm eff}=0.23136$, and the QCD coupling $\alpha_s(m_Z)=0.118$. We then convert them into the $\overline{\rm MS}$ gauge couplings by using the formula in Refs. [16].

The one-loop corrections to the $\overline{\rm MS}$ top Yukawa coupling as a function of the top quark physical mass, $m_t^{\rm phys}$, are given in [17,18],

$$y_t(\mu)=2^{3/4}G_F^{1/2}m_t^{\rm phys}\{1+\delta_t(\mu)\}. \tag{6}$$

We use the full QCD and electroweak contributions to $\delta_t(\mu)$ as given by Eq. (2.12) and the Appendix of [18], and we match it at top quark mass scale $\mu=m_t^{\rm phys}$. The QCD corrections are the dominant contribution and have the value,

$$\delta_t^{\rm QCD}(\mu)=\frac{\alpha_s}{3\pi}\left[3\ln\left(\frac{m_t^{\rm phys}}{\mu}\right)^2-4\right]. \tag{7}$$

We also included all relevant one-loop finite corrections to set the $\overline{\rm MS}$ Higgs coupling as a boundary condition at some scale $\mu=\mu_h=\mathcal{O}(m_h)$:

$$\lambda(\mu)=\sqrt{2}G_F m_h^2\{1+\delta_h(\mu)\}, \tag{8}$$

where $\delta_h(\mu)$ is given by Eqs. (15a)–(15d) of Ref. [19], and reproduced in Eqs. (B1)–(B3) of Ref. [17]. In Fig. 1, taking $\mu_h=m_h$ or $\max[m_h/2,m_Z]$, we show the μ_h dependence on the Higgs boson mass upper limit. As the scale Λ gets lower, the μ_h dependence becomes larger. However, when $\Lambda>3.8$ TeV, the difference is less than 5 GeV for μ_h in the range $\max[m_h/2,m_Z]<\mu_h<m_h$.

In Fig. 1, we also showed a dependence of the top quark mass on the limits. We used the experimental result $m_t^{\rm phys}=174.3\pm5.1$ GeV (see p. 389 of Ref. [20]). Since the error on the top quark mass is now less than 3%, the induced variabilities on the Higgs boson mass upper limits are almost negligible.

As we run the gauge couplings up to higher scales, the value of $\sin^2\theta_W$ changes. We define $\sin^2\theta_W$ in the $\overline{\rm MS}$ scheme, and

$$\sin^2\theta_W(\Lambda)\equiv\frac{g'^2(\Lambda)}{g'^2(\Lambda)+g_2^2(\Lambda)}, \tag{9}$$

where $g'(m_Z)\simeq0.36$ and $g_2(m_Z)\simeq0.65$. The correspondence between scale Λ and the value of $\sin^2\theta_W(\Lambda)$ at that scale is plotted on this same graph. There are three cases of particular interest in this graph. These are the perturbative upper limit for m_h when $\Lambda=M_{\rm Pl}=2.4\times10^{18}$ GeV; the perturbative range for m_h at the scale where $\sin^2\theta_W=3/8$, which should be close to the unification scale of simple $SU(5)$ or $SO(10)$ GUT theories; and, the perturbative upper limit for m_h at the scale where $\sin^2\theta_W=1/4$, which is relevant for $SU(3)$ electroweak unification. We summarize the results of these three possibilities:

$$\sin^2\theta_W=1/4\Rightarrow\Lambda=3.8\ \text{TeV}, m_h<460\ \text{GeV}$$

$$\sin^2\theta_W=3/8\Rightarrow\Lambda\simeq10^{13}\ \text{GeV}, m_h<200\ \text{GeV}$$

$$\Lambda=M_{Pl}\Rightarrow m_h<180\ \text{GeV}.$$

When the scale Λ is low, we must be concerned that incalculable non-renormalizable operators might contribute significantly to the Higgs boson mass. However, even for a low scale such as $\Lambda=3.8$ TeV needed for $SU(3)$ electroweak unification, the non-renormalizable operators are not expected to have a large impact on the Higgs boson mass given our assumptions of perturbativity. For example, we can look at the simple dimension six operator,

$$\mathcal{L}_{NR}=f\frac{|\Phi|^6}{\Lambda^2} \tag{10}$$

and estimate the mass shift of the Higgs boson to be

$$\Delta m_h^2=f\left[31\ \text{GeV}\left(\frac{3.8\ \text{TeV}}{\Lambda}\right)\right]^2. \tag{11}$$

Since this adds in quadrature to the Higgs boson mass, a Higgs boson mass of 460 GeV is increased by only 1 GeV to 461 GeV if $f\simeq1$ and $\Lambda=3.8$ TeV, as would be appropriate when considering the $\sin^2\theta_W(\Lambda)=1/4$ case.

III. SUPERSYMMETRIC UNIFICATION

A. Minimal supersymmetric standard model

Within the small uncertainties of perturbative threshold corrections, all three gauge couplings meet at one point in a simple grand unified theory if we assume minimal supersymmetric standard model (MSSM). The unification scale is about 2×10^{16} GeV.

Unlike the SM, the Higgs sector in the MSSM faces very little direct constraint by enforcing perturbativity of couplings up to the high scale. Because there is no free parameter like λ in the Higgs potential of minimal supersymmetry, the only parameters that would be subject to the perturbativity constraint are the top and bottom Yukawa couplings,

$$y_t=\frac{\sqrt{2}\,\overline{m}_t(m_Z)}{v\sin\beta},\quad y_b=\frac{\sqrt{2}\,\overline{m}_b(m_Z)}{v\cos\beta},$$

$$y_\tau=\frac{\sqrt{2}\,\overline{m}_\tau(m_Z)}{v\cos\beta}, \tag{12}$$

where $\tan\beta$ is the ratio of the vacuum expectation values of the two Higgs doublets needed to give mass to the up and down quarks (see Sec. X of [21] for a discussion of the MSSM Higgs sector), and the dimensional reduction scheme

($\overline{\mathrm{DR}}$) masses $\overline{m}_t(m_Z)$, $\overline{m}_b(m_Z)$, and $\overline{m}_\tau(m_Z)$ are defined precisely the same as $\hat{m}_t(m_Z)$, $\hat{m}_b(m_Z)$, and $\hat{m}_\tau(m_Z)$ in Ref. [22]. The definitions of the Yukawa couplings in Eq. (12) are the same as Eq. (17) of Ref. [22], and the relationship between the physical masses and the $\overline{\mathrm{DR}}$ masses can be found in Sec. 3 of Ref. [22].

If $\tan\beta$ is too low (high), the top (bottom) Yukawa coupling will go non-perturbative before the unification scale. Perturbativity up to the high scale is motivated in this scenario because non-perturbative couplings would feed into the gauge coupling RGEs and disrupt the beautiful unification. Perturbativity up to this high scale then puts a constraint on $\tan\beta$ to be within the range $2 \lesssim \tan\beta \lesssim 65$, which in turn puts a constraint on the possible values of the lightest Higgs boson mass in supersymmetry.

We can expand the lightest MSSM Higgs state in terms of a simple, but useful, equation:

$$m_h^2 = m_Z^2 \cos^2 2\beta + \eta \frac{3 G_F m_t^4}{\sqrt{2}\pi^2} \ln \frac{\Delta_S^2}{m_t^2}. \tag{13}$$

Here m_t denotes the running SM top-quark mass in the $\overline{\mathrm{MS}}$ scheme at the scale m_t^{phys}, as utilized in the Higgs boson mass computations of Ref. [23]. One can think of Eq. (13) as being valid in the limit of $m_A \gg m_Z$, or one can absorb the mixing effects between light and heavy Higgs bosons as being absorbed into the Δ_S definition. Since it has been known that $O(\alpha\alpha_s)$ two-loop contributions to the MSSM lightest Higgs boson mass reduce the one-loop upper limit on m_h [23], we introduce a suppression factor η in Eq. (13). To fix η, we match our expression Eq. (13) with the one in Ref. [23] at $\Delta_S^2 = m_{\tilde{t}}^2 \equiv (m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2)/2 = (1\ \ \mathrm{TeV})^2$ assuming no top squark mixing, and then we get

$$\eta = 1 - \frac{2\alpha_s}{\pi}\left(\ln \frac{m_{\tilde{t}}^2}{m_t^2} - \frac{2}{3}\right) = 0.78. \tag{14}$$

The numerical value of Δ_S is therefore a good indicator of the scale of superpartner masses.

Also, we remark that the quantity Δ_S has been introduced in analogy to T_{SUSY} in Refs. [24,25]. T_{SUSY} is useful because it remaps all superpartner threshold effects into one single mass scale for the purposes of matching gauge couplings between the SM effective theory below T_{SUSY} to the fully supersymmetric theory above T_{SUSY}. The purpose of Δ_S is similar. It can be defined as the matching scale that reproduces the correct Higgs boson mass by running supersymmetric RGEs above it and the one-loop SM RGE for λ below it, assuming (correctly) that the y_t^4 part of β_λ dominates.

Top squark mixing effects will begin to decorrelate the value of top squark masses from that of the correct value of Δ_S needed to recover an accurate Higgs boson mass using Eq. (13). For fixed top squark masses, the higher the left-right mixing effects the larger the Higgs boson mass becomes, and therefore the larger Δ_S. We demonstrate this effect in Fig. 2 by computing the needed value of Δ_S to reproduce the correct Higgs boson mass given various values of the mixing term in the top squark mixing matrix $X_t = A_t - \mu\cot\beta$ [see Eq. (4) of [23]]. We have computed the Higgs boson mass using Eqs. (46) and (47) of [23], and then recast the Higgs boson mass in the equivalent variable Δ_S using Eq. (13). For various values of the top squark mixing X_t we have plotted the correlation between required Δ_S and $m_{\tilde{t}}$. As we see, $\Delta_S > m_{\tilde{t}}$ for most of parameter space, which enables us to conclude that Δ_S generally overestimates the supersymmetry mass scales of the top squarks in the presence of mixing. This should be kept in mind when interpreting the parameter space for superpartners from Fig. 3; that is, superpartners can be significantly lighter than the Δ_S values needed for a Higgs boson mass above the experimental limit.

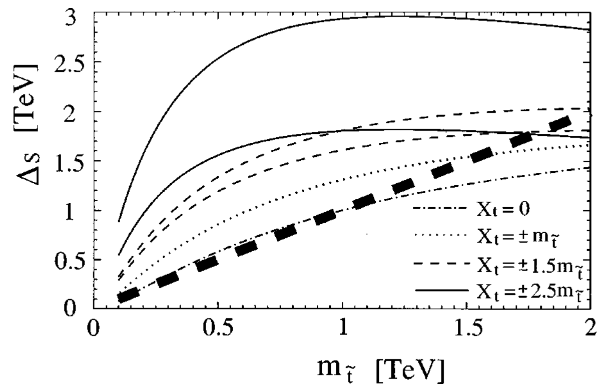

FIG. 2. The relationship between Δ_S defined by Eq. (13) and $m_{\tilde{t}} \equiv \sqrt{(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2)/2}$ for various top squark mixing $X_t = A_t - \mu\cot\beta$ in the limit $m_A \gg m_Z$ (no Higgs mixing effects). Since $\Delta_S > m_{\tilde{t}}$ for much of parameter space, superpartners are expected to be below the value of Δ_S that corresponds to the Higgs boson mass limit $m_h > 114.1$ GeV. For the reader's convenience a thick dashed line is plotted for the line $\Delta_S = m_{\tilde{t}}$.

In Fig. 3 we have plotted the lightest Higgs boson mass in the MSSM as a function of the supersymmetry mass scale Δ_S. Although presented in a slightly different way here, the results of this plot are well known [26]. Low $\tan\beta$ requires large supersymmetry breaking mass in order to evade the current experimental limits on the lightest Higgs boson mass. Large $\tan\beta$ enables the MSSM to be comfortably within all

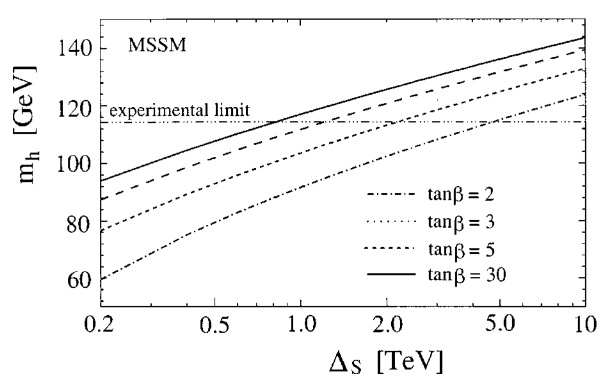

FIG. 3. The lines plot the lightest Higgs boson mass in the MSSM as a function of the supersymmetry scale Δ_S, whose leading log value is $\Delta_S^2 = m_{\tilde{t}}^2$. The four lines from bottom to top represent $\tan\beta = 2,3,5,30$.

experimental constraints for moderately small supersymmetry breaking mass.

We view it as a success that supersymmetry predicts that its lightest Higgs boson mass can naturally reside in the squeezed window of 114.1 GeV from direct experimental searches and 222 GeV from precision electroweak measurements. Superpartners could disrupt the precision electroweak predictions, but it is well known that supersymmetry decouples rapidly from Z-pole observables. Attempts to make the global fits to the data better by resolving some small discrepancies between leptonic and hadronic observables also demonstrate the rapid decoupling of supersymmetry, since the active superpartners in these studies must be very light (e.g., $m_{\tilde{\nu}} \lesssim m_Z$ as in Ref. [27]).

B. The next-to-minimal supersymmetric standard model

As soon as one goes beyond the most minimal supersymmetric theory, the constraints of perturbativity become very significant again, just as they were in our SM analysis [28–31]. The reason is because non-minimal supersymmetric theories add additional Yukawa couplings that contribute directly to the mass of the lightest Higgs boson, but are not usefully constrained by any known measurement.

The most important example of non-minimal supersymmetry is the NMSSM (next-to-minimal MSSM), which adds another singlet S to the theory. This approach has been used by many authors to make the μ term more natural within supersymmetry. That is, in the MSSM there exists a term in the superpotential $\mu H_u H_d$ which might be best explained by an NMSSM term, $\lambda_s S H_u H_d$, where $\mu = \lambda_s \langle S \rangle$.

We can write the mass of the lightest scalar of the NMSSM theory in a very similar way as we did for the MSSM:

$$m_h^2 = m_Z^2 \cos^2 2\beta + \frac{\lambda_s^2}{2\sqrt{2} G_F} \sin^2 2\beta + \eta \frac{3 G_F m_t^4}{\sqrt{2}\pi^2} \ln \frac{\Delta_S^2}{m_t^2}, \tag{15}$$

where we take $\eta = 0.78$. The scale Δ_S is close but not precisely the same as it is in the MSSM. This is because there are some more states and parameters in the NMSSM that feed into Δ_S, such as the additional masses and mixings in the Higgs sector. There could also be additional contributions to the lightest mass, and to Δ_S, if S is charged under another gauge group. In that case, H_u and/or H_d would be charged too, leading to additional contributions to the mass [32]. Furthermore, a large Yukawa coupling of a fourth generation to the Higgs boson can add substantial radiative corrections to the Higgs boson mass, just as the top Yukawa does in the MSSM [33]. For the purposes of being conservative and illustrative of how even the smallest deviation from the MSSM can affect the lightest Higgs boson mass, we will ignore additional gauge charges or additional states that may contribute to the radiative corrections of the Higgs boson mass.

The numerical value of λ_s is arbitrary. If it is large it contributes significantly to the Higgs boson mass via Eq. (15) and raises it to a much higher value than the MSSM

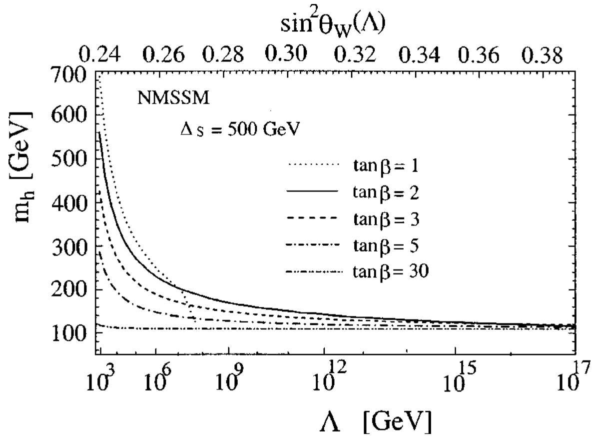

FIG. 4. The five lines are for the same values of $\tan\beta$ in the NMSSM. The λ_s coupling of the superpotential $\lambda_s S H_u H_d$ term is assumed to be at its maximum allowed value without blowing up before the scale Λ ($\lambda_s < 5.1$). Since $\sin^2\theta_W(\Lambda)$ correlates directly with Λ we provide the $\sin^2\theta_W(\Lambda)$ values on the upper axis.

prediction for the same values of $\tan\beta$ and Δ_S. However, if we do not wish to spoil perturbative gauge coupling unification we must require that λ_s and the other remaining couplings, such as y_t and y_b, remain perturbative so as not to disrupt too much the RGE evolution of the gauge couplings.

The one-loop β functions of λ_s, y_t and y_b all depend on each other. Therefore, we must insure that all remain perturbative. To determine what values of λ_s, y_t and y_b are perturbative, we employ the condition of Eq. (5) on each of these three couplings. To do this we compute the three-loop β functions using Refs. [34] for each of the couplings in the limit that all other couplings are turned off:

$$\beta_{y_t}^{(3\text{ loop})} = \frac{6}{16\pi^2} y_t^3 - \frac{22}{(16\pi^2)^2} y_t^5 + \frac{(102+36\zeta(3))}{(16\pi^2)^3} y_t^7 \tag{16}$$

$$\beta_{\lambda_s}^{(3\text{ loop})} = \frac{4}{16\pi^2} \lambda_s^3 - \frac{10}{(16\pi^2)^2} \lambda_s^5 + \frac{(32+24\zeta(3))}{(16\pi^2)^3} \lambda_s^7 . \tag{17}$$

β_{y_b} is the same as β_{y_t} after replacing $y_t \to y_b$. Applying the perturbativity conditions of Eq. (5) we find that perturbative couplings must satisfy $y_t < 4.9$, $y_b < 4.9$, and $\lambda_s < 5.1$.

In Fig. 4 we plot the mass of the lightest Higgs boson as a function of scale Λ, requiring that all couplings remain perturbative below Λ. In this analysis, we use two-loop RGEs for all gauge, top and bottom Yukawa and Higgs couplings including full one-loop supersymmetry corrections to the Yukawa couplings discussed below Eq. (12), and one-loop supersymmetry logarithmic corrections to all gauge couplings at m_Z. For this computation we set all supersymmetry masses to Δ_S. To be consistent with our definition of Δ_S given above, the scale at which λ_s is evaluated in Eq. (15) is Δ_S. The five different curves in the figure represent different values of $\tan\beta = 1$, 2, 3, 5, and 30. In the MSSM

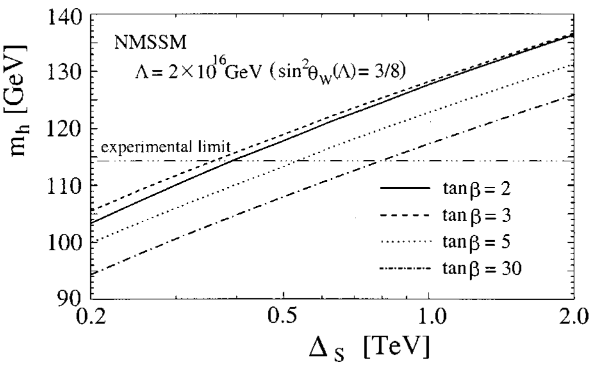

FIG. 5. The lines plot the lightest Higgs boson mass in the NMSSM as a function of the supersymmetry scale Δ_S. The leading log value for $\Delta_S = m_{\tilde{t}}$. The value of λ_s used in Eq. (15) is at its maximum consistent with $\lambda_s(\Lambda) < 5.1$ (perturbative). Here $\Lambda = 2\times10^{16}$ GeV, which corresponds to the simple grand unification scenario of $\sin^2\theta_W(\Lambda) = 3/8$.

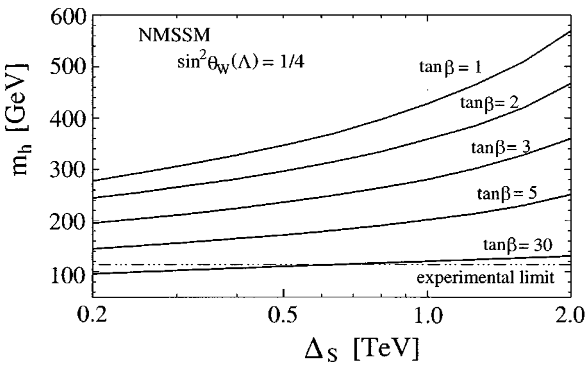

FIG. 6. The lines plot the lightest Higgs boson mass in the NMSSM as a function of the supersymmetry scale Δ_S. The leading log value for $\Delta_S = m_{\tilde{t}}$. The value of λ_s used in Eq. (15) is such that $\lambda_s(\Lambda) < 5.1$ (perturbative). Here 8 TeV $< \Lambda <$ 110 TeV (precise value depends on Δ_S), which corresponds to the $SU(3)$ electroweak unification scenario of $\sin^2\theta_W(\Lambda) = 1/4$.

(without GUT), $\tan\beta < 1$ is excluded by the Higgs search. However, in the NMSSM with low Λ, such low values of $\tan\beta$ are allowed as long as constraints such as $b \to s\gamma$, for example, are satisfied, which would perhaps require a very heavy charged Higgs boson mass (see, e.g., Fig. 12 of Ref. [35]).

We can then look again at the most interesting scales in this plot related to $\sin^2\theta_W = 1/4$ $SU(3)$ electroweak unification, and $\sin^2\theta_W = 3/8$ $SU(5)/SO(10)$ grand unification. In the NMSSM the Λ scales associated with this unification are different than they were in the SM case. Furthermore, the unknown Yukawa coupling λ_s contributes to the mass of the lightest Higgs boson in a very different way than the SM Higgs self-coupling λ, and the RGEs are very different. The result, with $\Delta_S = 500$ GeV is

$$\sin^2\theta_W = 1/4 \Rightarrow \Lambda = 37 \text{ TeV}, m_h < 350 \text{ GeV},$$

$$\sin^2\theta_W = 3/8 \Rightarrow \Lambda \simeq 2\times10^{16} \text{ GeV}, \quad m_h < 120 \text{ GeV}.$$

We plot in Figs. 5 and 6 the lightest Higgs boson mass in the NMSSM as a function of Δ_S confining ourselves to the two scenarios $\sin^2\theta_W = 1/4$ ($\Lambda \sim 8$–110 TeV) and $\sin^2\theta_W = 3/8$ ($\Lambda = 2\times10^{16}$ GeV). For low values of $\tan\beta$ the Higgs boson mass prediction is very well separated between the two theories because the λ_s contribution is not suppressed much by $\sin^2 2\beta$ and the difference between the $\lambda_s(\Delta_S)$ allowed such that λ_s is still perturbative at Λ is dramatically different for $\Lambda \sim 10$ TeV [$\lambda_s(\Delta_S = 500 \text{ GeV}) \sim 2$ allowed] and $\Lambda = 2\times10^{16}$ GeV [$\lambda_s(\Delta_S = 500 \text{ GeV}) \sim 0.7$ allowed]. However, as we go to higher values of $\tan\beta$ the Higgs boson mass has very little dependence on the $\lambda_s^2 \sin^2 2\beta$ term since it is suppressed by $1/\tan\beta$ at high $\tan\beta$. For that reason, the two $\tan\beta = 30$ lines are very nearly on top of each other in the plot.

IV. CONCLUSIONS

In this article we have examined Higgs boson mass upper limits in theories that are perturbative up to a scale Λ. After discovery of a Higgs boson, the results of these computations can tell us at what scale a perturbative description of our low-scale theory (SM, MSSM, NMSSM, etc.) breaks down. The results can also help determine if unification with $\sin^2\theta_W(\Lambda) = 1/4$ or $\sin^2\theta_W(\Lambda) = 3/8$ can occur perturbatively. In the several unification scenarios we studied, we found that it is not expected to have a Higgs boson above about 500 GeV and still remain perturbative. This is not a theorem, but a highly suggestive result based on the simple theories that are currently attractive.

Fortunately, the CERN Large Hadron Collider (LHC) will be able to see all SM-like Higgs bosons easily up to 500 GeV and probably to at least as high as 800 GeV [36]. The extra dynamics that go along with the explanation for electroweak symmetry breaking, such as supersymmetry or extra dimensions, should also be detectable at the LHC. Finding an NMSSM Higgs boson in some regions of parameter space could be a significant challenge at the LHC, but there are good indications that the LHC will cover those possibilities also [37].

A future linear collider will be able to see and carefully study a Higgs boson with mass $m_h \lesssim \sqrt{s} - m_Z$ in e^+e^- mode, and perhaps slightly higher in $\gamma\gamma$ mode [38–40]. Indications of $SU(3)$ electroweak unification could also come from direct collider probes and precision electroweak studies that match expectations [41] of minimal models and beyond. Directly confirming the unification scenarios may be difficult to do, but additional clues from measurements of superpartner masses and the complete Higgs sector, for example, would be critical information if we are to be successful.

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