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Color confinement and dual superconductivity in full QCD

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We report on evidence that confinement is related to the dual superconductivity of the vacuum in full QCD, as in quenched QCD. The vacuum is a dual superconductor in the confining phase, while the $U(1)$ magnetic symmetry is realized *a la* Wigner in the deconfined phase.

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I. INTRODUCTION

In a series of papers [1–3], which we shall refer to as I, II, III respectively, we have demonstrated by numerical simulations that the quenched QCD vacuum is a dual superconductor in the confining phase, and goes to a normal state at the deconfinement transition (see also Ref. [4]). More precisely we have constructed an operator μ carrying a magnetic charge and we have measured its vacuum expectation value $\langle\mu\rangle$. For $T<T_c$ $\langle\mu\rangle\neq 0$, for $T>T_c$ $\langle\mu\rangle=0$ and, approaching T_c, $\langle\mu\rangle\simeq(1-T/T_c)^{\delta}$ [$\delta=0.50(3)$]. Magnetic charge is defined by a procedure called Abelian projection: it associates (N_c-1) $U(1)$ magnetic symmetries to any operator ϕ in the adjoint representation [5]. *A priori* magnetic symmetries corresponding to different Abelian projections (different choices of ϕ) are independent. In I, II, III we have shown that the behavior of $\langle\mu\rangle$, including the value of the index δ, is independent of the Abelian projection.

There is general agreement on the order-disorder nature of the deconfining transition in the quenched case. The popular order parameter is the Polyakov line $\langle L\rangle$; the symmetry involved is Z_N. Alternatively the dual ('t Hooft) line $\langle\tilde{L}\rangle$ [6] can be used as a disorder parameter (order parameter of the disordered phase) corresponding to the dual $\tilde{Z}_N$ symmetry. Our $\langle\mu\rangle$ is also a good disorder parameter, and in fact it coincides numerically with $\langle\tilde{L}\rangle$ [7,8].

In full QCD, i.e. in the presence of dynamical quarks, the situation is less clear. Z_N and $\tilde{Z}_N$ symmetries are explicitly broken by the very presence of the quarks. At zero quark mass there is a phase transition at some T_c involving chiral symmetry: for $T<T_c$ chiral symmetry is spontaneously broken, the pseudoscalar octet being the Goldstone particles, and for $T>T_c$ it is restored. Quark masses do break chiral symmetry explicitly. It is not clear theoretically what the chiral transition has to do with the deconfinement transition. However, the susceptibilities of different quantities (the Polyakov line $\langle L\rangle$, the chiral condensate) have been measured, and all of them have a maximum at the same value $T_c(m_q)$ for any value of m_q. Above a certain value of m_q ($m_q>3$ GeV) the transition is first order, as in the quenched case, and $\langle L\rangle$ still works as an order parameter. At $m_q\sim 0$ the transition is presumably second order. At intermediate values the susceptibilities which have been considered show a maximum at T_c, but it does not become large at increasing volume. The indication is then that there is no transition but only a crossover.

A natural question is then if dual superconductivity is a symmetry for the transition in full QCD as it is in the quenched case. In the spirit of the $N_c\to\infty$ limit, one would expect that the mechanism of confinement be the same as in quenched QCD, the idea being that the structure of the theory is the same as that in the limit $N_c\to\infty$ at $g^2N_c=\lambda$ fixed: at finite N_c small differences are expected with respect to the limiting case. Quark loops are nonleading in the expansion. The mechanism of confinement should be approximately N_c independent and the same with and without dynamical quarks.

The disorder parameter $\langle\mu\rangle$ can be constructed in full QCD exactly in the same way as in the quenched case (see Sec. II). At a given temperature T, $\langle\mu\rangle$ has to be computed in the infinite volume limit. We have investigated the region $T<T_c$, where we find

$$\lim_{V\to\infty}\langle\mu\rangle\neq 0 \tag{1}$$

and $T>T_c$, where we find

$$\lim_{V\to\infty}\langle\mu\rangle=0 \tag{2}$$

as will be shown in detail below. Notice that the limit Eq. (2) is not within errors but exact. Indeed we measure, instead of $\langle\mu\rangle$, the quantity $\rho=(d/d\beta)\ln\langle\mu\rangle$, and we find that it tends to $-\infty$ as $\rho=-kN_s+k'$ $(k>0)$ as the spatial size of the sample $N_s\to\infty$.

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The finite size scaling analysis in the critical region is under investigation, to study the nature and the order of the transition.

II. DISORDER PARAMETER

The operator μ is defined in full QCD exactly in the same way as in the quenched theory [1–3]

$$\langle \mu \rangle = \frac{\tilde{Z}}{Z},$$

$$Z = \int (\mathcal{D}U) e^{-\beta S}, \tag{3}$$

$$\tilde{Z} = \int (\mathcal{D}U) e^{-\beta \tilde{S}}.$$

$\tilde{Z}$ is obtained from Z by changing the action in the time slice x_0, $S \to \tilde{S} = S + \Delta S$. In the Abelian projected gauge the plaquettes

$$\Pi_{i0}(\vec{x}, x_0) = U_i(\vec{x}, x_0) U_0(x + \hat{i}, x_0) \times U_i^{\dagger}(\vec{x}, x_0 + \hat{0}) U_0^{\dagger}(\vec{x}, x_0) \tag{4}$$

are changed by substituting

$$U_i(\vec{x}, x_0) \to \tilde{U}_i(\vec{x}, x_0) \equiv U_i(\vec{x}, x_0) e^{iTb_i(\vec{x}-\vec{y})} \tag{5}$$

where $\vec{b}(\vec{x}-\vec{y})$ is the vector potential of a monopole configuration centered at $\vec{y}$ in the gauge $\vec{\nabla}\vec{b} = 0$, and T is the diagonal gauge group generator corresponding to the monopole species chosen. In SU(2) $T = \sigma_3/2$, in SU(3) $T = \lambda_3/2$ or $(\sqrt{3}\lambda_8 - \lambda_3)/2$. In the generic SU($N$) case the procedure is explained in Ref. [9]. Unlike the Z_N center symmetry, the $U(1)$ magnetic symmetry defined after Abelian projection is a good symmetry also in the presence of dynamical fermions. It can be shown that, as in the quenched case, μ adds to any configuration the monopole configuration $\vec{b}(\vec{x}-\vec{y})$. If the magnetic symmetry is realized *a la* Wigner, $\langle \mu \rangle = 0$ if μ carries nonzero net magnetic charge. Then $\langle \mu \rangle \neq 0$ means Higgs breaking of the U(1) symmetry. Therefore $\langle \mu \rangle$ can be a correct disorder parameter for the transition to dual superconductivity also in full QCD.

III. NUMERICAL RESULTS

We have measured $\langle \mu \rangle$ with two flavors of degenerate staggered fermions on $N_s^3 \times 4$ lattices, with different values of N_s ($N_s = 12,16,32$) and of the bare quark mass m_q. In particular we have chosen, in the transition region, to vary the temperature, $T = 1/(N_t a(\beta, m_q))$, moving in the (β, m_q) plane while keeping a fixed value of m_π/m_ρ. To do this and to extract the physical scale we have used fits to the m_ρ and m_π masses published in [10]. We present here results obtained at $m_\pi/m_\rho \simeq 0.505$: in this case, at $N_t = 4$, the β corresponding to the transition is approximately $\beta_c \sim 5.35$ [11]. Preliminary results have been already presented in [12].

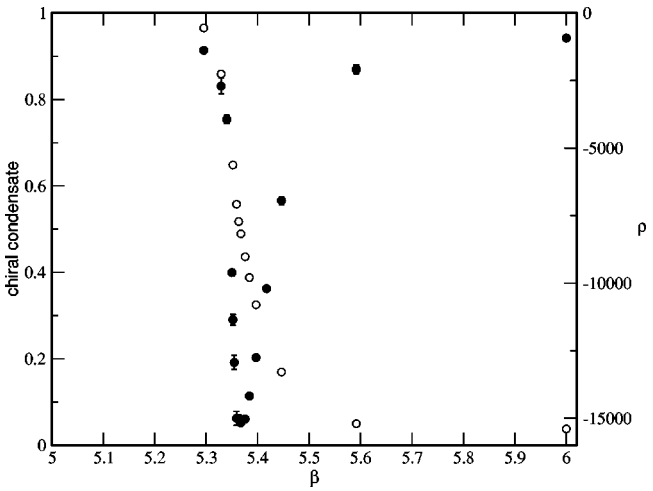

FIG. 1. Chiral condensate (open circles) and ρ (filled circles) on the $32^3 \times 4$ lattice.

Instead of $\langle \mu \rangle$ we measure the quantity

$$\rho = \frac{d}{d\beta} \ln \langle \mu \rangle. \tag{6}$$

It follows from Eq. (3) that

$$\rho = \langle S \rangle_S - \langle \tilde{S} \rangle_{\tilde{S}}, \tag{7}$$

the subscript meaning the action by which the average is performed. In terms of ρ,

$$\langle \mu \rangle = \exp\left(\int_0^\beta \rho(\beta') d\beta' \right). \tag{8}$$

A drop of $\langle \mu \rangle$ at the phase transition corresponds to a strong negative peak of ρ.

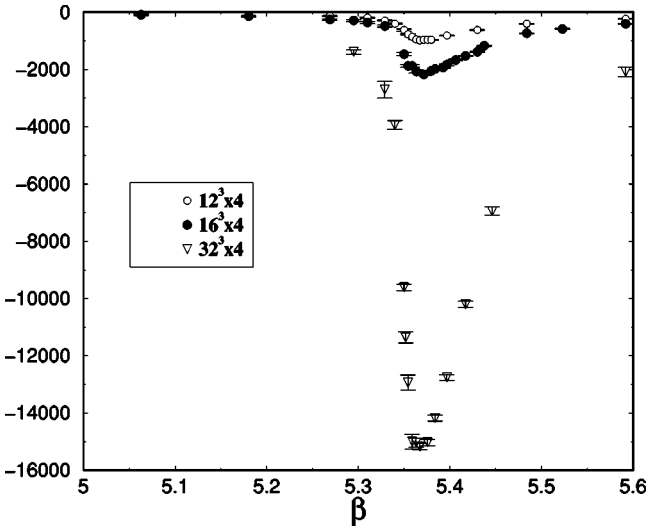

FIG. 2. Behavior of ρ around the phase transition at various lattice sizes.

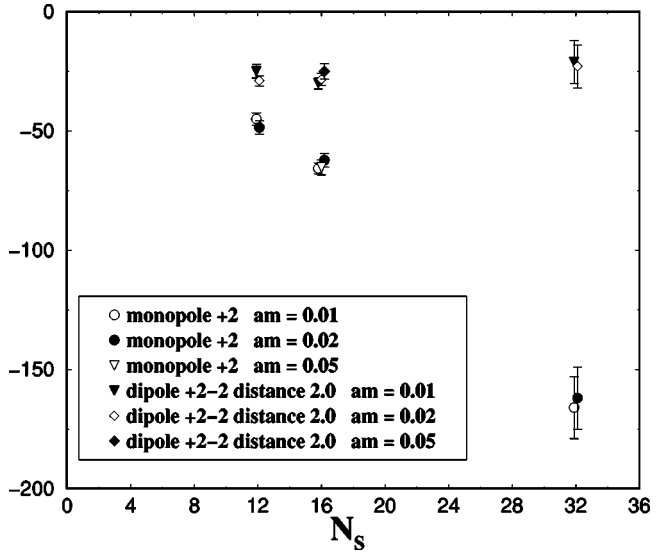

FIG. 3. Weak coupling behavior of ρ at various lattice sizes.

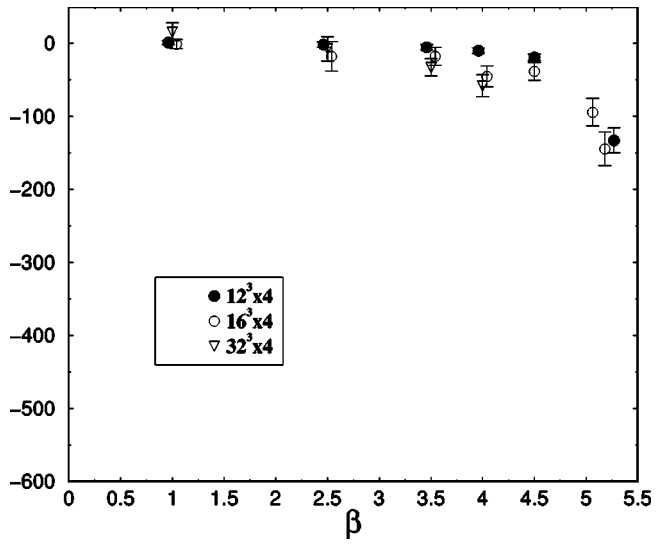

FIG. 4. Strong coupling behavior of ρ at various lattice sizes and $am=0.1335$.

We have used the R version of the hybrid Monte Carlo (HMC) algorithm for our simulations [13]. Some technical complications arise in the computation of the second term on the right-hand side of Eq. (7). In the evaluation of $\langle \tilde{S} \rangle_{\tilde{S}}$, $C^\star$-periodic boundary conditions in time direction have to be used for the gauge fields and this requires $C^\star$ boundary conditions in temporal direction also for fermionic variables (in addition to the usual antiperiodic ones), in order to ensure gauge invariance of the fermionic determinant. This implies relevant changes in the formulation and implementation of the HMC algorithm which are explained in detail in Ref. [14].

We have chosen the Polyakov line as the local adjoint operator which defines the Abelian projection. Actually, calling $L(\vec{x},x_0)$ the Polyakov line starting at point $(\vec{x},x_0)$, the Abelian projection is defined by the operator $L(\vec{x},x_0)L^\star(\vec{x},x_0)$, which transforms in the adjoint representation when using $C^\star$ boundary conditions.

The use of a modified gauge action also implies changes in the molecular dynamics equations. One has to maintain the modified Hamiltonian containing $\tilde{S}$ constant. A change in any temporal link indeed induces a change in $L(\vec{x},x_0)$ and hence in the Abelian projection defining the monopole field. Therefore the dependence of $\tilde{S}$ on temporal links is nontrivial and the equations of motion for the temporal momenta become more complicated.

Figure 1 shows ρ for a $32^3\times 4$ lattice, and the chiral condensate as a function of β. The negative peak of ρ is clearly at the same value of β where $\langle \overline{\psi}\psi \rangle$ drops to zero.

Figure 2 shows the plot of ρ for different spatial sizes N_s. For larger lattices the peak becomes deeper and the value of ρ at high β lower.

An analysis of ρ at large β's as a function of N_s is shown in Fig. 3, for different masses of the staggered fermions used in the simulations. For net magnetic charge $\neq 0$

$$\rho \simeq -kN_s + k' \quad (k>0) \tag{9}$$

and is practically independent of the quark mass within errors. For net charge zero (e.g. monopole-antimonopole pair) ρ stays constant at large N_s. Going back to Eq. (8) this means that $\langle \mu \rangle$ is strictly zero in the infinite volume limit for nonzero magnetic charge, and can be $\neq 0$ for excitations with zero net magnetic charge. This statement is based on the analysis of many different excitations with different magnetic charges, and Fig. 3 is only an example. The magnetic symmetry is therefore realized *a la* Wigner for $T>T_c$ and the Hilbert space is superselected. Notice that

(1) $\langle \mu \rangle$ can only be strictly zero in the infinite volume limit (Lee-Yang theorem [15]), and (2) if we were measuring $\langle \mu \rangle$ directly we would find zero within large errors. Looking instead at ρ we can unambigously check Eq. (9), which means that $\langle \mu \rangle$ is strictly zero as $N_s \to \infty$.

For $T<T_c$ $\langle \mu \rangle \neq 0$ if ρ stays constant and finite with increasing volume. This is what indeed happens as shown in Fig. 4. Nothing spectacular can happen at larger volumes, since no larger length scale exists in the system.

Around T_c a finite size scaling analysis is required to get information on the order of the transition as well as to measure the critical indices. The problem is more complicated than in the quenched case, since an extra scale, the quark mass, is present. The program is on the way on a set of APEmille machines. Some qualitative features are shown in Fig. 2.

IV. CONCLUSIONS

The preliminary data reported in this paper contain enough information to state that dual superconductivity is at work as a confinement mechanism in QCD with dynamical quarks, in the same way as in the quenched theory [I, II, III]. For $T>T_c$ the Hilbert space is superselected with respect to

magnetic charge, for $T<T_c$ the symmetry is broken by the Higgs mechanism.

Dependence of the disorder parameter on the choice of the Abelian projection and the nature of the transition are under investigation.

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[1] A. Di Giacomo, B. Lucini, L. Montesi, and G. Paffuti, Phys. Rev. D **61**, 034503 (2000).
[2] A. Di Giacomo, B. Lucini, L. Montesi, and G. Paffuti, Phys. Rev. D **61**, 034504 (2000).
[3] J.M. Carmona, M. D'Elia, A. Di Giacomo, B. Lucini, and G. Paffuti, Phys. Rev. D **64**, 114507 (2001).
[4] P. Cea and L. Cosmai, Phys. Rev. D **62**, 094510 (2000).
[5] G. 't Hooft, Nucl. Phys. **B190**, 455 (1981).
[6] G. 't Hooft, Nucl. Phys. **B138**, 1 (1978).
[7] L. Del Debbio, A. Di Giacomo, and B. Lucini, Nucl. Phys. **B594**, 287 (2001).
[8] L. Del Debbio, A. Di Giacomo, and B. Lucini, Phys. Lett. B **500**, 326 (2001).
[9] L. Del Debbio, A. Di Giacomo, B. Lucini, and G. Paffuti, IFUP-TH-2002-11, hep-lat/0203023.
[10] T. Blum *et al.*, Phys. Rev. D **51**, 5153 (1995).
[11] S. Aoki *et al.*, Phys. Rev. D **57**, 3910 (1998).
[12] J.M. Carmona, M. D'Elia, L. Del Debbio, A. Di Giacomo, B. Lucini, and G. Paffuti, Nucl. Phys. B (Proc. Suppl.) **106**, 607 (2002).
[13] S. Gottlieb, W. Liu, D. Toussaint, R.L. Renken, and R.L. Sugar, Phys. Rev. D **35**, 2531 (1987).
[14] J.M. Carmona, M. D'Elia, A. Di Giacomo, and B. Lucini, Int. J. Mod. Phys. C **11**, 637 (2000).
[15] T.D. Lee and C. N Yang, Phys. Rev. **87**, 404 (1952); **87**, 410 (1952).