

$\mathcal{N}=3$ warped compactifications

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Orientifolds with three-form flux provide some of the simplest string examples of warped compactification. In this paper we show that some models of this type have the unusual feature of $D=4$, $\mathcal{N}=3$ spacetime supersymmetry. We discuss their construction and low energy physics. Although the local form of the moduli space is fully determined by supersymmetry, to find its global form requires a careful study of the Bogomol'nyi-Prasad-Sommerfield spectrum.

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I. INTRODUCTION

Warped compactifications are of great interest, due to the observation of Randall and Sundrum that warping in a higher dimensional space can produce a hierarchy of four-dimensional scales [1,2]. Becker and Becker [3] described a large class of warped three-dimensional M theory compactifications, in which four-form flux is the source for the warp factor. By duality these give rise to warped four-dimensional IIB compactifications, with three-form fluxes as the source [4,5].

In this paper we study some particularly simple examples of this type, which as we will show have $D=4$, $\mathcal{N}=3$ supersymmetry. These are of interest in part because of the rarity of $\mathcal{N}=3$ supersymmetry, but also because the supersymmetry strongly constrains their moduli spaces. The small-radius behavior of warped compactifications is likely to be quite complicated for $\mathcal{N}\leq 2$, as the warping becomes large in this limit and the application of T duality (or mirror symmetry) is complicated by the warping and fluxes. Also, such compactifications are intrinsically nonperturbative, in that the dilaton is fixed at a nonzero value. However, with $\mathcal{N}=3$ supersymmetry the local form of the moduli space is completely determined, and we can hope to deduce the global structure.

In Sec. II we describe these solutions, all of which are based on the T^6/\mathbf{Z}_2 orientifold, and discuss their supersymmetry. An interesting subtlety arises with the flux quantization. In Sec. III we study various aspects of the low energy physics—the massless spectrum, and the metric on moduli space—and show that it is consistent with the constraints of $\mathcal{N}=3$ supersymmetry. We argue that the breaking of $\mathcal{N}=4$ to $\mathcal{N}=3$ should appear to be spontaneous in the large radius limit. In Sec. IV we consider the duality groups. Because of the $H_{(3)}$ flux and the finite g_s , we have no tools to determine these directly, and so must try to deduce their form based on the spectrum of Bogomol'nyi-Prasad-Sommerfield (BPS) states. We find that, even though it may be possible to view

the duality group as a spontaneous breaking of the $\mathcal{N}=4$ dualities, the symmetry breaking is not straightforward.

While this work was in progress we learned of related work on T^6/\mathbf{Z}_2 orientifolds with flux [6]. We are grateful to those authors for communications.

II. $\mathcal{N}=3$ ORIENTIFOLDS

In this section we describe the specific orientifold solutions with three-form flux, and determine their supersymmetry. This overlaps the discussion in Ref. [4]; the T^6/\mathbf{Z}_2 orientifold was discussed briefly there, in its M theory avatar T^8/\mathbf{Z}_2 .

In Sec. II A we determine the action of the T^6/\mathbf{Z}_2 orientifold projection on the fields. In Sec. II B we discuss the quantization of three-form flux, which has an interesting subtlety. In Sec. II C we describe the solution to the Bianchi identities and equations of motion. In Sec. II D we identify a particularly simple class of models, in which only one complex component of the flux is nonvanishing. In Sec. II E we study the supersymmetry of these models and show that there are $\mathcal{N}=3$ unbroken supersymmetries.

A. Orientifold projection

All examples that we consider are based on the T^6/\mathbf{Z}_2 orientifold. Greek indices denote the noncompact directions $0, \dots, 3$, lower case roman indices denote the compact directions $4, \dots, 9$, and capital roman indices denote all directions $0, \dots, 9$. The coordinates x^m are each taken to be periodic with period 2π , and the \mathbf{Z}_2 is a simultaneous reflection of all compact coordinates x^m ,

$$R: (x^4, x^5, x^6, x^7, x^8, x^9) \rightarrow (-x^4, -x^5, -x^6, -x^7, -x^8, -x^9). \quad (2.1)$$

For now we take the toroidal metric to be rectangular,

$$\widetilde{ds}^2 = \sum_{m=4}^9 r_m^2 dx^m dx^m; \quad (2.2)$$

we will relax this in Sec. III.

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The action of the orientifold \mathbf{Z}_2 can be derived by using T duality to the type I theory, where g_{MN} , $C_{(2)}$, and Φ are even under world-sheet parity Ω and $B_{(2)}$, $C \equiv C_{(0)}$, and $C_{(4)}$ are odd. Alternately, one may derive it by noting that the orientifold \mathbf{Z}_2 must include a factor of $(-1)^{F_L}$, where F_L is the spacetime fermion number carried by the left-movers: $\mathcal{R} \equiv \Omega R (-1)^{F_L}$ [7,8]. This is necessary in order that it square to unity:

$$\mathcal{R}^2 = \Omega^2 R^2 (-1)^{F_L + F_R} = 1. \quad (2.3)$$

Note that $\Omega^2 = 1$, as Ω acts as ± 1 on all fields. R is equivalent to a rotation by π in each of three planes, so R^2 is a rotation by 2π in an odd number of planes and therefore equal to $(-1)^F$.

By either means one finds that \mathbf{Z}_2 acts on the various fields as follows:

$$\begin{aligned} \text{even: } & g_{\mu\nu}, g_{mn}, B_{\mu m}, C_{\mu m}, C_{mnpq}, C_{\mu\nu mn}, C_{\mu\nu\lambda\rho}, \Phi, C; \\ \text{odd: } & g_{\mu m}, B_{\mu\nu}, B_{mn}, C_{\mu\nu}, C_{mn}, C_{\mu mnp}, C_{\mu\nu\lambda m}. \end{aligned} \quad (2.4)$$

It follows that the fluxes H_{mnp} and F_{mnp} are even, and so constant three-form fluxes are allowed.

B. Flux quantization

The three-form fluxes must be appropriately quantized. The usual quantization conditions are¹

$$\frac{1}{2\pi\alpha'} \int_C H_{(3)} \in 2\pi\mathbf{Z}, \quad \frac{1}{2\pi\alpha'} \int_C F_{(3)} \in 2\pi\mathbf{Z} \quad (2.5)$$

for every three-cycle C . However, the orientifold presents some subtleties.

Consider first T^6 compactification. Letting C run over all T^3 's, one finds that constant fluxes

$$H_{mnp} = \frac{\alpha'}{2\pi} h_{mnp}, \quad F_{mnp} = \frac{\alpha'}{2\pi} f_{mnp}; \quad h_{mnp}, \quad f_{mnp} \in \mathbf{Z} \quad (2.6)$$

are allowed. Any cycle on the covering space T^6 descends to a cycle on T^6/\mathbf{Z}_2 , so the conditions (2.6) are still necessary. In addition, there are new 3-cycles on the coset space, such as

$$\begin{aligned} 0 \leq x^4 \leq 2\pi, \quad 0 \leq x^5 \leq 2\pi, \\ 0 \leq x^6 \leq \pi, \quad x^7 = x^8 = x^9 = 0. \end{aligned} \quad (2.7)$$

The conditions (2.5) on this cycle² would appear to require that h_{456} and f_{456} be even, and similarly for all other components. However, we claim that h_{mnp} and f_{mnp} can still be arbitrary odd or even integers.

To understand this, consider first the reduced problem of a charge moving in a constant magnetic field $F_{56} = F$ on a torus $0 \leq x^5, x^6 \leq 2\pi$. Let us work in the gauge

$$A_5 = 0, \quad A_6 = Fx^5. \quad (2.8)$$

The gauge field is periodic up to a gauge transformation,

$$\begin{aligned} A_m(x^5 + 2\pi, x^6) &= A_m(x^5, x^6) + \partial_m \lambda_5, \\ A_m(x^5, x^6 + 2\pi) &= A_m(x^5, x^6) + \partial_m \lambda_6, \end{aligned} \quad (2.9)$$

with $\lambda_5 = 2\pi Fx^6$ and $\lambda_6 = 0$. Similarly a field of unit charge satisfies

$$\begin{aligned} \psi(x^5 + 2\pi, x^6) &= e^{i\lambda_5} \psi(x^5, x^6), \\ \psi(x^5, x^6 + 2\pi) &= e^{i\lambda_6} \psi(x^5, x^6). \end{aligned} \quad (2.10)$$

The consistency of defining $\psi(x^5 + 2\pi, x^6 + 2\pi)$ implies the Dirac quantization

$$F = f/2\pi, \quad f \in \mathbf{Z}. \quad (2.11)$$

In other words,

$$\int_{T^2} F_{56} = 2\pi f \in 2\pi\mathbf{Z}. \quad (2.12)$$

Now let us form the orbifold $T^2/\mathbf{Z}_2 = S^2$ by identifying (x^5, x^6) with $(-x^5, -x^6)$. For any value of f we can define the quantum mechanics for the charged particle on the coset space simply by restricting to wave functions such that³

$$\psi(-x^5, -x^6) = +\psi(x^5, x^6). \quad (2.13)$$

However, the integral of F_{56} over S^2 is half of the integral over T^2 , so for f odd the flux is not quantized.

To see how this can make sense, note that there are four fixed points $(x^5, x^6) = (0,0), (\pi,0), (0,\pi), (\pi,\pi)$. At the first three, the periodicities (2.10) and (2.13) are compatible, but at (π,π) they are incompatible and the wave function must vanish. If we circle this fixed point, from $(\pi - \epsilon, \pi)$ to the identified point $(\pi + \epsilon, \pi)$, the wave function is required to change sign: there is a half-unit of magnetic flux at the fixed point (π,π) . Thus the Dirac quantization condition is in fact satisfied.

Of course, the fixed point (π,π) is not special: the quantization condition is satisfied if there is a half-unit of flux at any one fixed point, or at any three. Similarly for f even there can be half-integer flux at zero, two, or four fixed points. In each case there are eight configurations, which can be obtained in the orbifold construction by including discrete Wilson lines on the torus, and a discrete gauge transformation in the orientifold projection.

This analysis extends directly to the quantum mechanics of an F-string or D-string wrapped in the 4-direction, moving

¹We follow the conventions of Ref. [9].

²The cycle (2.7) is unoriented, but the three-form fluxes can be integrated on it because they have odd intrinsic parity.

³We have chosen a gauge in which A_m is explicitly \mathbf{Z}_2 symmetric, so no gauge transformation is needed.

in the fluxes H_{456} and F_{456} . This is consistent for any integers h_{456} and f_{456} , but if either of these is odd then there must be Neveu-Schwarz–Neveu-Schwarz (NS-NS) or Ramond-Ramond (R-R) flux at some fixed points, for example all those with $x^4=x^5=x^6=\pi$. Indeed, there are four kinds of O3 plane, distinguished by the presence or absence of discrete NS-NS and R-R fluxes [10]; for recent reviews see Refs. [11,12]. The cycle (2.7), and each of the others obtained from it by a rotation of the torus, contains four fixed points. If the NS-NS flux through the cycle is even (odd) then an even (odd) number of the fixed points must have discrete NS-NS flux, and correspondingly for the R-R flux.

C. Bianchi identities and field equations

The Bianchi identities for the three-form flux, $dH_{(3)}=dF_{(3)}=0$, are trivially satisfied by constant fluxes. The Bianchi identity for the five-form flux is

$$d\tilde{F}_{(5)}=(2\pi)^4\alpha'^2\rho_3^{\text{loc}}dV_\perp+H_{(3)}\wedge F_{(3)}, \quad (2.14)$$

where ω_3^{loc} is the D3-brane density from localized sources and dV_\perp is the transverse volume form. The localized sources that we will consider are D3-branes and the various types of O3-plane. An O3-plane without discrete flux has D3 charge $-\frac{1}{4}$ [13], while an O3-plane with either discrete flux, or with both, has D3 charge $+\frac{1}{4}$ [10–12]. The integrated Bianchi identity then gives the tadpole cancellation condition

$$N+\frac{1}{2}\tilde{N}+\frac{1}{2\cdot 3!\cdot 3!}\hat{\epsilon}^{\text{mnpqrs}}h_{\text{mnp}}f_{\text{qrs}}=16. \quad (2.15)$$

Here N is the total number of D3-branes, \tilde{N} is the total number of O3 planes with any discrete flux, and $\hat{\epsilon}^{456789}=1$. The factor of $\frac{1}{2}$ in the flux term arises because the orientifold has half the volume of the original torus.

We are interested in compactifications to four-dimensional Minkowski space with supergravity fields plus D3-branes and O3-planes. In Ref. [14] it is shown that all such solutions must be of “smeared D3” form [15,16], which is dual to the M theory ansatz of Ref. [3]. That is, the flux

$$G_{(3)}=F_{(3)}-\tau H_{(3)}, \quad \tau=C+ie^{-\Phi}, \quad (2.16)$$

must be imaginary self-dual,

$$\frac{1}{3!}\epsilon_{\text{mnp}}^{\text{qrs}}G_{\text{qrs}}=iG_{\text{mnp}}. \quad (2.17)$$

This flux behaves as an effective D3-brane source for the remaining fields, which are therefore of black 3-brane form [17]

$$\tau=\text{const}\equiv C+\frac{i}{g_s},$$

$$ds_{\text{string}}^2=Z^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu+Z^{1/2}\tilde{g}_{mn}dx^m dx^n,$$

$$\tilde{F}_{(5)}=(1+*)d\chi_{(4)}, \quad \chi_{(4)}=\frac{1}{g_s Z}dx^0\wedge dx^1\wedge dx^2\wedge dx^3. \quad (2.18)$$

The warp factor Z is determined by

$$-\tilde{\nabla}^2 Z=(2\pi)^4\alpha'^2 g_s \tilde{\rho}_3+\frac{g_s}{12}G_{\text{mnp}}\tilde{G}^{\text{mnp}}, \quad (2.19)$$

where a tilde denotes the use of the unwarped metric (2.2). This is consistent provided that the net D3 charge (2.15) vanishes, and the Bianchi identity (2.14) and the field equations are then satisfied.

As discussed in [18], the warp factor can be obtained from Eq. (2.19) by the method of images. Under rescaling of the unwarped transverse metric, $\tilde{g}_{mn}\rightarrow\lambda^2\tilde{g}_{mn}$, the right-hand side of Eq. (2.19) scales as λ^{-6} (there is a factor of $\tilde{g}^{-1/2}$ in ρ_3), while $\tilde{\nabla}^2$ scales as λ^{-2} . It follows that in the large radius limit $Z=1+O(\lambda^{-4})$ and the warping becomes negligible. On the other hand, at small radius the warping is significant. Thus we might expect that in general the small radius region of moduli space is significantly modified—for example, the AdS radius of the warped region remains finite even as the radius of the unwarped manifold is taken to zero. Note also that due to the negative charge of the orientifold planes, the warp factor becomes negative and unphysical near the \mathbf{Z}_2 fixed points. Since the region of unphysical behavior is smaller than the string scale, the geometry cannot be taken literally, but it again suggests that the small-radius limit may be complicated.⁴ However, for the highly supersymmetric cases that we consider the small-radius limit is highly constrained.

D. Examples

There are many solutions based on the T^6/\mathbf{Z}_2 orientifold, distinguished by the three-form flux quanta and the discrete fluxes at orientifold points. Even with vanishing three-form fluxes there are many solutions to the tadpole cancellation condition (2.15) and the three-form flux quantization conditions. One extreme is to have 16 D3-branes and no discrete flux [19], which is the familiar T dual to the type I theory on T^6 . The other extreme is to have no D3-branes and 32 fixed points with discrete flux. For example, the configuration with discrete R-R flux at all fixed points in the plane $x^4=0$ satisfies the quantization conditions and is T -dual to a type I compactification without vector structure [20]. In these cases the supersymmetry is $D=4$, $\mathcal{N}=4$.

For simplicity we will restrict attention to a limited set of three-form flux configurations, where the nonzero fluxes are

$$\begin{aligned} h_{456} &= -h_{489} = -h_{759} = -h_{786} \equiv h_1, \\ f_{456} &= -f_{489} = -f_{759} = -f_{786} \equiv f_1, \\ h_{789} &= -h_{756} = -h_{486} = -h_{459} \equiv h_2, \\ f_{789} &= -f_{756} = -f_{486} = -f_{459} \equiv f_2, \end{aligned} \quad (2.20)$$

⁴This remark is due to S. Sethi.

and $f_{1,2}$ and $h_{1,2}$ are integers. The duality condition (2.17) implies that the T^6 is the product of three square T^2 's,

$$r_4=r_7, \quad r_5=r_8, \quad r_6=r_9, \quad (2.21)$$

and that the string coupling is fixed in terms of the integer fluxes,

$$\tau = \frac{f_2 - if_1}{h_2 - ih_1}. \quad (2.22)$$

This is therefore an intrinsically nonperturbative solution of type IIB string theory. It can be studied at large radius using supergravity, which becomes classical at low energy, but to understand the physics at small radius a high degree of supersymmetry will be essential. The tadpole cancellation condition is

$$N + \frac{1}{2}\tilde{N} = 16 - 2(h_1f_2 - h_2f_1) \leq 16. \quad (2.23)$$

The last inequality follows from the duality condition (2.22).

This configuration of fluxes has the simple feature that in terms of the complex coordinates

$$w^1 = \frac{x^4 + ix^7}{\sqrt{2}}, \quad w^2 = \frac{x^5 + ix^8}{\sqrt{2}}, \quad w^3 = \frac{x^6 + ix^9}{\sqrt{2}}, \quad (2.24)$$

there is a single component

$$G_{\bar{1}\bar{2}\bar{3}} = \frac{\sqrt{2}\alpha'}{\pi}(f_1 - \tau h_1). \quad (2.25)$$

That is, G_{mnp} is a (0,3)-form. Such solutions will be the focus of the remainder of this paper. The unwarped metric is

$$\tilde{d}s^2 = 2 \sum_{i=1}^3 r_{i+3}^2 dw^i d\bar{w}^i, \quad \tilde{g}_{i\bar{j}} = r_{i+3}^2 \delta_{i\bar{j}}. \quad (2.26)$$

If we restrict to even $f_{1,2}$ and $h_{1,2}$, and to O3-planes without flux, then it is easy to list all solutions, up to rotations and dualities:

$$\begin{aligned} \text{(A)} \quad & h_1=f_2=2, \quad h_2=f_1=0: \quad N=8, \quad g_s=1, \quad C=0; \\ \text{(B)} \quad & h_1=2, \quad f_2=4, \quad h_2=f_1=0: \quad N=0, \quad g_s=\frac{1}{2}, \quad C=0; \\ \text{(C)} \quad & h_1=-h_2=f_1=f_2=2: \quad N=0, \quad g_s=1, \quad C=0. \end{aligned} \quad (2.27)$$

For example, the solution $h_1=f_2=2$, $h_2=0$, $f_1=2m$, with $N=8$ and $\tau=i+m$, is S -dual to case (A).

With odd fluxes and discrete flux on the O3-planes the number of solutions is large. One example is $h_1=1$, $f_2=4$, $h_2=f_1=0$, $N=0$, $g_s=\frac{1}{4}$, $C=0$, with discrete NS-NS flux at the 16 fixed points at which exactly one of the following four conditions holds: $[x^4=x^5=x^6=0]$, $[x^4=x^8=x^9=0]$, $[x^7=x^5=x^9=0]$, $[x^7=x^8=x^6=0]$.

In the notation of Ref. [4] [Eqs. (3.18) and (3.19) of version 3], the ansatz (2.20), (2.25) corresponds to solutions with only A nonvanishing; in particular, case (C) is the solution $A=1+i$. The condition (3.18) in Ref. [4] is equivalent to f_{mnp} and h_{mnp} being even in our notation.

E. Supersymmetry counting

The supersymmetry of this class of type IIB solutions was discussed in Refs. [15,16]. Aside from the three-form fluxes, the background is a distribution of black 3-branes. Therefore the supersymmetries of the black or D3-brane,

$$SO(3,1) \times SO(6):$$

$$\varepsilon = \zeta \otimes \chi, \quad \Gamma_{(4)}\zeta = +\zeta, \quad \Gamma_{(6)}\chi = -\chi, \quad (2.28)$$

are broken only by terms that are linear in the three-form fluxes. Using the supersymmetry transformations from Refs. [21–23], the unbroken supersymmetries are those that satisfy

$$G\chi = G\chi^* = G\gamma^m\chi^* = 0,$$

$$G \equiv \frac{1}{6} G_{mnp} \gamma^{mnp} = G_{\bar{1}\bar{2}\bar{3}} \gamma^{\bar{1}\bar{2}\bar{3}}. \quad (2.29)$$

A spinor χ of chirality (2.28) is either χ_0 , where

$$\gamma^i \chi_0 = 0 \quad (\text{all } i), \quad (2.30)$$

or one of the three spinors $\gamma^{ij}\chi_0$. One readily verifies that for the latter three spinors the conditions (2.29) are satisfied and so the unbroken supersymmetry is $D=4$, $\mathcal{N}=3$. The number \mathcal{N} of solutions to the conditions (2.29) can be any of 0, 1, 2, 3, and 4 (the last is for vanishing fluxes); all but the case $\mathcal{N}=3$ have been discussed in the previous work.

The $\mathcal{N}=3$ supersymmetry can be understood simply as follows. The condition for an unbroken supersymmetry is that the flux $G_{(3)}$ be of type (2,1) and primitive [15,16]. The orientifold has several complex structures. If we choose the coordinates

$$(z^1, z^2, z^3) = (w^1, \bar{w}^2, \bar{w}^3) \quad (2.31)$$

then the nonzero flux $G_{\bar{z}^1\bar{z}^2\bar{z}^3}$ is indeed (2,1) and primitive. There are obviously two other such choices,

$$\begin{aligned} (z^1, z^2, z^3)' &= (\bar{w}^1, w^2, \bar{w}^3), \\ (z^1, z^2, z^3)'' &= (\bar{w}^1, \bar{w}^2, w^3). \end{aligned} \quad (2.32)$$

Each of these three complex structures leads to an unbroken supersymmetry.

$\mathcal{N}=3$ supersymmetry is unfamiliar but not unknown. Previous examples have been constructed as asymmetric orbifolds in type II theory [24], breaking half of the supersymmetry on one side and three-fourths on the other. The $\mathcal{N}=3$ matter multiplet (helicities $1, \frac{1}{2}, 0, -\frac{1}{2}$) plus its CPT conjugate form an $\mathcal{N}=4$ matter multiplet, but the supergravity multiplet (helicities $2, \frac{3}{2}, 1, \frac{1}{2}$ plus CPT conjugates) is

distinct. In the global case the renormalizable interactions are the same as for $\mathcal{N}=4$, but there are presumably higher-dimension operators allowed by $\mathcal{N}=3$ but not $\mathcal{N}=4$. The $\mathcal{N}=3$ supergravity was constructed in Ref. [25]. Like $\mathcal{N}=4$, the moduli space is a coset and its local form is completely determined,

$$\frac{U(3,n)}{U(3) \times U(n)}, \quad (2.33)$$

where n is the number of matter multiplets. Including the vectors in the supergravity multiplet, the gauge symmetry is $U(1)^{n+3}$.

III. LOW ENERGY EFFECTIVE THEORY

In this section we analyze the massless spectra of the models described in the previous section, to verify the structure required by $\mathcal{N}=3$ supergravity: with the supergravity multiplet plus n matter multiplets, there must be $6n$ moduli and $n+3$ vectors. We also verify, in the large-radius limit, that the metric on moduli space has the expected form (2.33). Note that, because g_s is fixed to be of order 1, we cannot use string perturbation theory to study these models. The one tool we have is low energy supergravity, which is valid in the large-radius limit. In this $\mathcal{N}=3$ case there is enough supersymmetry to extrapolate to the full moduli space, but for $\mathcal{N} \leq 2$ it will be very difficult to analyze the full moduli space.

A. Moduli

The massless scalars arise from the zero modes of the \mathbf{Z}_2 -even scalars in Eq. (2.4), namely g_{mn} , C_{mnpq} , Φ and C . However, not all of these are moduli, as the fluxes lift some of the directions of moduli space [4,5,18]. For example, we have already seen that the dilaton and R-R scalar are fixed. Their potential arises from the three-form flux and the resulting mass-squared is of order

$$G_{mnp} G^{mnp} \sim \frac{\alpha'^2}{R^6}. \quad (3.1)$$

We have assumed that all radii of the torus are of order R , so that $g_{mn} \sim R^2$, and have used the quantization conditions (2.5).

Now consider the scalars g_{mn} . These are partly fixed by the self-duality condition (2.17), through the dependence of the ϵ -tensor on g_{mn} . The zero mode of the three-form flux is fixed by the quantization conditions, so G_{mnp} remains a (0,3)-form in the w coordinates. The metric g_{mn} must therefore be Hermitean in these coordinates, otherwise there will be nonzero components $\epsilon_{\bar{i}\bar{j}\bar{k}} \bar{i}'\bar{j}'\bar{k}'$. The self-duality condition is satisfied for any Hermitean metric $g_{i\bar{j}}$. Thus, in terms of the w coordinates, the complex structure moduli are frozen while the Kähler moduli remain free. In terms of any of the supersymmetric complex structures (2.31), (2.32) these are a mix of Kähler and complex structure moduli.

The remaining bulk scalars are those from the four-form potential C_{mnpq} . The periodicity conditions on this potential

are slightly involved, and so the analysis is set aside below. The conclusion is that there is a field \tilde{c}_{mnpq} which is periodic and which appears in the field strength only through its exterior derivative. A constant shift of this field is then a new solution to the equations of motion. However, some of these are gauge-equivalent to the unshifted solution. It is shown below that the gauge variation around a given background is

$$\delta \tilde{c}_{(4)} = d\tilde{\chi} + i(\bar{\lambda}_A \wedge \hat{G}_{(3)} - \lambda_A \wedge \hat{\tilde{G}}_{(3)})/2 \text{Im}(\tau), \quad (3.2)$$

with $\tilde{\chi}$ periodic and λ_A a complex one-form. Since the background $\hat{G}_{(3)}$ is a (0,3) form, the (1,3) and (3,1) parts of $\tilde{c}_{(4)}$ can be gauged away. The (2,2) parts $\tilde{c}_{ij\bar{k}\bar{l}}$ are the moduli.

Finally, there is no restriction on the positions of any D3-branes that might be present, so their world-volume scalars are also moduli. It will be convenient to write these in complex form, as $W_I^i, \bar{W}_I^{\bar{j}}$ where I labels the D3-brane (perturbatively speaking, it would be a Chan-Paton factor diagonal on the two end points).

Periodicities of forms

The gauge transformations of the various potentials are

$$\begin{aligned} \delta C_{(2)} &= d\lambda_C \\ \delta B_{(2)} &= d\lambda_B \\ \delta C_{(4)} &= d\chi - \lambda_C \wedge H_{(3)}, \end{aligned} \quad (3.3)$$

in terms of one-forms λ_C and λ_B and three-form χ . The gauge transformation of $C_{(4)}$ corresponds to the field definition $\tilde{F}_{(5)} = dC_{(4)} + C_{(2)} \wedge H_{(3)}$. On T^6 these must be periodic up to a gauge transformation,

$$\begin{aligned} C_{(2)}(x + e^m) &= C_{(2)}(x) + d\lambda_C^m(x) \\ B_{(2)}(x + e^m) &= B_{(2)}(x) + d\lambda_B^m(x) \\ C_{(4)}(x + e^m) &= C_{(4)}(x) + d\chi^m(x) - \lambda_C^m(x) \wedge H_{(3)}(x). \end{aligned} \quad (3.4)$$

Here e^m is the lattice vector in the m -direction, $(e^m)^n = 2\pi \delta^{mn}$, and λ_C^m, λ_B^m , and χ^m are specified gauge transformations. To analyze these it is convenient to write each field as its background value plus a shift, for example $C_{(4)}(x) = \hat{C}_{(4)}(x) + c_{(4)}(x)$. The three-form flux backgrounds are constant, and so for the corresponding potentials we can choose a gauge

$$\hat{C}_{mn} = \frac{1}{3} \hat{F}_{mnp} x^p, \quad \hat{B}_{mn} = \frac{1}{3} \hat{H}_{mnp} x^p. \quad (3.5)$$

It follows that

$$\lambda_C^m = \frac{1}{6} \hat{F}_{mnp} x^n dx^p, \quad \lambda_B^m = \frac{1}{6} \hat{H}_{mnp} x^n dx^p. \quad (3.6)$$

The quantized fluxes cannot fluctuate, and so the $\lambda_{B,C}^m$ are fixed. It then follows that the two-form fluctuations are periodic,

$$c_{(2)}(x+e^m)=c_{(2)}(x), \quad b_{(2)}(x+e^m)=b_{(2)}(x). \quad (3.7)$$

The four-form must satisfy a more complicated boundary condition. This can be deduced from the condition that $C_{(4)}(x+e^m+e^n)$ be consistently defined, giving

$$\begin{aligned} d\chi^m(x+e^n)-d\chi^m(x)-d\chi^n(x+e^m)+d\chi^n(x) \\ = \frac{1}{3}\hat{F}_{nmq}dx^q \wedge H_{(3)}(x). \end{aligned} \quad (3.8)$$

Note that it is the full $H_{(3)}$ that appears on the right-hand side, so that χ^m has both a background piece and a field-dependent piece, $\chi^m = \hat{\chi}^m + \zeta^m$. Rather than solve directly for χ^m we first shift the four-form to one with a simpler periodicity. Define

$$\tilde{c}_{(4)} = c_{(4)} + \hat{C}_{(2)} \wedge b_{(2)} + \frac{1}{2} c_{(2)} \wedge b_{(2)}, \quad (3.9)$$

so that

$$\begin{aligned} \tilde{f}_{(5)} = d\tilde{c}_{(4)} - (\hat{F}_{(3)} + \frac{1}{2}f_{(3)}) \wedge b_{(2)} \\ + c_{(2)} \wedge (\hat{H}_{(3)} + \frac{1}{2}h_{(3)}). \end{aligned} \quad (3.10)$$

Then

$$\tilde{c}_{(4)}(x+e^m) = \tilde{c}_{(4)}(x) + d\tilde{\zeta}^m(x), \quad (3.11)$$

where $\tilde{\zeta}^m = \zeta^m + \lambda_C^m \wedge b_{(2)}$. It is consistent to take $\tilde{\zeta}^m = 0$, and we choose a gauge in which this is so. A $\tilde{\zeta}^m$ that could not be gauged away would correspond to a quantized five-form flux on T^6 , which is inconsistent with the \mathbf{Z}_2 projection (2.4).

The gauge variation of $\tilde{c}_{(4)}$ is

$$\begin{aligned} \delta\tilde{c}_{(4)} = d\chi - \lambda_C \wedge H_{(3)} + \hat{C}_{(2)} \wedge d\lambda_B \\ + \frac{1}{2} (c_{(2)} \wedge d\lambda_B + d\lambda_C \wedge b_{(2)}) \\ = d\tilde{\chi} - \lambda_C \wedge (\hat{H}_{(3)} + \frac{1}{2}h_{(3)}) + \lambda_B \wedge (\hat{F}_{(3)} + \frac{1}{2}f_{(3)}) \\ = d\tilde{\chi} - \frac{1}{2i \operatorname{Im}(\tau)} \{ \tilde{\lambda}_A \wedge (\hat{G}_{(3)} + \frac{1}{2}g_{(3)}) \\ - \lambda_A \wedge (\hat{G}_{(3)} + \frac{1}{2}\bar{g}_{(3)}) \}, \end{aligned} \quad (3.12)$$

where $\tilde{\chi} = \chi + C_{(2)} \wedge \lambda_B$ and $\lambda_A = \lambda_C - \tau \lambda_B$. (Note that the hatted background is defined to be fixed, so the gauge transformation goes entirely into the fluctuation.) The gauge transformation $\tilde{\chi}$ must be periodic. A nonperiodic gauge transformation would act on the periodic identification by conjugation, $\tilde{\zeta}^{m'}(x) = \tilde{\zeta}^m(x) + \tilde{\chi}(x+e^m) - \tilde{\chi}(x)$, so with fixed identification the gauge transformation must be periodic.

B. Gauge fields

The bulk vector fields that survive the orientifold projection (2.4) are $c_{\mu n}$ and $b_{\mu n}$. Form the complex linear combinations

$$A_{\mu m} = C_{\mu n} - \tau B_{\mu n}. \quad (3.13)$$

The gauge transformation is $\delta A_{\mu m} = \partial_\mu \lambda_{Am}$, where the one-form gauge parameter λ_A is as in Eqs. (3.2), (3.12). It follows from the transformation (3.2) that the (1,0) parts of λ_A leave the background invariant, so the unbroken gauge fields are $A_{\mu i}$. This is also evident from the linearized gauge field strength

$$\tilde{f}_{(5)} = d\tilde{c}_{(4)} - (a_{(2)} \wedge \hat{G}_{(3)} - \bar{a}_{(2)} \wedge \hat{G}_{(3)}) / 2i \operatorname{Im}(\tau). \quad (3.14)$$

The field $a_{\mu \bar{i}}$ appears in the $\mu \bar{i} jkl$ component. Comparing with the nonlinear Higgs covariant derivative $\partial_\mu \phi - A_\mu$, we see that $a_{\mu \bar{i}}$ is broken through the Higgs mechanism by $c_{\bar{i} jkl}$, so that $a_{\mu i}$ and $c_{\bar{i} jkl}$ remain as massless fields.

The real and imaginary parts of $a_{\mu i}$ give six gauge fields; for example when $\tau = i$, these are

$$\begin{aligned} \frac{C_{\mu 4} - B_{\mu 7}}{\sqrt{2}}, \quad \frac{B_{\mu 4} + C_{\mu 7}}{\sqrt{2}}, \quad \frac{C_{\mu 5} - B_{\mu 8}}{\sqrt{2}}, \\ \frac{B_{\mu 5} + C_{\mu 8}}{\sqrt{2}}, \quad \frac{C_{\mu 6} - B_{\mu 9}}{\sqrt{2}}, \quad \frac{B_{\mu 6} + C_{\mu 9}}{\sqrt{2}}. \end{aligned} \quad (3.15)$$

In addition each D3-brane adds a $U(1)$ gauge field, for total gauge group $U(1)^{6+N}$. The total number of moduli is nine from the metric, nine from $\tilde{c}_{(4)}$, and $6N$ from the D3-branes, for $6(3+N)$ in all. The counting matches $\mathcal{N}=3$ supergravity with $3+N$ matter multiplets; note that this agreement requires exactly six of the $U(1)$'s to be broken.

Massless vector solutions

It is an interesting exercise, though somewhat aside from our main point, to identify the massless vector solutions to the field equations, taking into account the warping of the internal space. We consider solutions without D3-branes. We take as an ansatz that the only nontrivial components of the fluctuations are the tensors, $g_{\mu\nu m}$ and $\tilde{f}_{\mu\nu mnp}$. The nontrivial field and Bianchi equations are

$$\begin{aligned} dg_{(3)} = 0, \quad d^*g_{(3)} = -ig_s(g_{(3)} \wedge \hat{F}_{(5)} + \hat{G}_{(3)} \wedge \tilde{f}_{(5)}), \\ \tilde{f}_{(5)} = *f_{(5)}, \quad d\tilde{f}_5 = \frac{ig_s}{2} (\hat{G}_{(3)} \wedge \bar{g}_{(3)} + g_{(3)} \wedge \hat{G}_{(3)}). \end{aligned} \quad (3.16)$$

We further take

$$\begin{aligned} g_{\mu\nu m}(x,y) = f_{\mu\nu}(x)u_m(y) + (*_4f)_{\mu\nu}(x)v_m(y), \\ \tilde{f}_{\mu\nu mnp} = f_{\mu\nu}(x)\gamma_{mnp}(y) + (*_4f)_{\mu\nu}(x) \\ \times (*_6\gamma)_{mnp}(y). \end{aligned} \quad (3.17)$$

Here u_m and v_m are complex, and γ_m and $f_{\mu\nu}$ are real. In this subsection and the next we use x for the noncompact coordinates and y for the compact coordinates. Subscripts of 4 (6)

on the Hodge star indicate that it is taken with respect to the spacetime (internal) indices only. Note that on two-forms $*_4$ is the same as the flat spacetime Hodge star.

Inserting this ansatz into the field equations gives the four-dimensional equations

$$d*_4 f_{(2)} = df_{(2)} = 0 \quad (3.18)$$

and the internal equations

$$\begin{aligned} du_{(1)} &= dv_{(1)} = 0, \\ d*_6 u_{(1)} &= -ig_s v_{(1)} \wedge \hat{F}_{(5)} - ig_s \hat{G}_{(3)} \wedge *_6 \gamma_{(3)}, \\ d*_6 v_{(1)} &= ig_s u_{(1)} \wedge \hat{F}_{(5)} + ig_s \hat{G}_{(3)} \wedge \gamma_{(3)}, \\ d\gamma_{(3)} &= \frac{ig_s}{2} (\hat{G}_{(3)} \wedge \bar{u}_{(1)} + u_{(1)} \wedge \hat{G}_{(3)}), \\ d*_6 \gamma_{(3)} &= \frac{ig_s}{2} (\hat{G}_{(3)} \wedge \bar{v}_{(1)} + v_{(1)} \wedge \hat{G}_{(3)}). \end{aligned} \quad (3.19)$$

The Bianchi identities for $u_{(1)}$ and $v_{(1)}$ are solved by

$$u_{(1)}(y) = \omega_{(1)} + da(y), \quad v_{(1)}(y) = \nu_{(1)} + db(y) \quad (3.20)$$

where $\omega_{(1)}$ and $\nu_{(1)}$ are constant one-forms on the internal space and $a(y)$ and $b(y)$ are periodic. The equations for $\gamma_{(3)}$ are then solved by

$$\gamma_{(3)} = \frac{ig_s}{2} (a \hat{G}_{(3)} - \bar{a} \hat{G}_{(3)}), \quad (3.21)$$

if

$$b(y) = -ia(y), \quad \omega_{\bar{i}} = \nu_{\bar{i}} = 0. \quad (3.22)$$

Finally, the field equations for $u_{(1)}$ and $v_{(1)}$ both become

$$Z \partial_m \partial_m a + 2 \partial_m Z \partial_m a + \partial_m Z (\omega_m + i \nu_m) = \frac{g_s^2}{12} a \hat{G}_{mnp} \hat{G}_{mnp}, \quad (3.23)$$

where all contractions are with the flat internal metric. There are then two solutions for each complex direction:

$$\begin{aligned} \omega_{(1)} &= -i \nu_{(1)} = dy^i, \quad a = \gamma_{(3)} = 0; \quad v_{(1)} = i u_{(1)}, \\ \omega_{(1)} &= i \nu_{(1)} = dy^i, \quad a \neq 0, \\ \gamma_{(3)} &\neq 0; \quad v_{(1)} = -i u_{(1)}. \end{aligned} \quad (3.24)$$

For the second solution we do not have a closed form, but can show by a variational argument that it exists. Thus we have the expected six internal solutions. Note that we do not get distinct solutions by choosing $\omega_{(1)} = i dy^i$, because the ansatz is invariant under $u \rightarrow v$, $v \rightarrow -u$, $f_{(2)} \rightarrow *_4 f_{(2)}$.

C. Metric on moduli space

In this section, we will find the low-energy action for the scalars and verify that it takes the form of a $U(3,n)/U(3) \times U(n)$ coset. We only consider the large-radius limit, where the warp factor Z becomes unity as discussed in Sec. II C. Thus we will drop the tildes on the internal metric. Four-dimensional geometric quantities will be denoted by a ‘‘4,’’ or by ‘‘E’’ in the four-dimensional Einstein frame; internal indices will always be raised with the string metric.

Let us first find the action for the metric moduli. The dimensional reduction of the ten-dimensional string frame Hilbert action gives

$$\begin{aligned} S_g &= \frac{1}{4\pi\alpha' g_s^2} \int d^4x \sqrt{-g_4} \Delta \left[R_4 + \Delta^{-2} \partial_\mu \Delta \partial^\mu \Delta \right. \\ &\quad \left. - \frac{1}{2} g^{\bar{j}i} g^{\bar{l}k} \partial_\mu g_{k\bar{j}} \partial^\mu g_{i\bar{l}} \right] \end{aligned} \quad (3.25)$$

where $\Delta = \alpha'^{-3} \det g_{i\bar{j}}$. The dimensional reduction includes a factor $\frac{1}{2} (2\pi)^6$ from the volume of T^6/\mathbf{Z}_2 . Switching to the four-dimensional Einstein frame, $2g_s g_{\mu\nu}^E = \Delta g_{\mu\nu}^4$, the action becomes

$$\begin{aligned} S_g &= \frac{1}{2\pi\alpha' g_s} \int d^4x \sqrt{-g_E} \left[R_E - \frac{1}{2} \Delta^{-2} \partial_\mu \Delta \partial^\mu \Delta \right. \\ &\quad \left. - \frac{1}{2} g^{\bar{j}i} g^{\bar{l}k} \partial_\mu g_{k\bar{j}} \partial^\mu g_{i\bar{l}} \right] \\ &= \frac{1}{2\pi\alpha' g_s} \int d^4x \sqrt{-g_E} \left[R_E - \frac{1}{2} \gamma^{\bar{j}i} \gamma^{\bar{l}k} \partial_\mu \gamma_{k\bar{j}} \partial^\mu \gamma_{i\bar{l}} \right], \end{aligned} \quad (3.26)$$

where all spacetime indices are raised with the Einstein metric. We have defined

$$\gamma_{i\bar{j}} = \frac{2g_s}{\alpha'} \frac{g_{i\bar{j}}}{\Delta} \quad (3.27)$$

in order to eliminate double trace terms from the derivatives of Δ ; the α' is included in order to make the moduli dimensionless.

The other bulk moduli are the R-R scalars, contained in the field strength fluctuation $\tilde{f}_{(5)}$. The moduli kinetic terms arise from $\tilde{f}_{\mu\nu\rho q r}$ and in Hodge dual form from $\tilde{f}_{\mu\nu\lambda q r}$; in order to avoid the problems of self-dual actions we include only the former, in terms of which

$$S_{RR} = - \frac{g_s}{8\pi\alpha'} \int d^4x \sqrt{-g_E} |\tilde{f}_{(5)}|^2. \quad (3.28)$$

In the absence of D3-branes, we have $f_{\mu i j k \bar{l}} = \partial_\mu c_{i j k \bar{l}}$, and the action is simply

$$S_{\text{RR}} = -\frac{g_s}{32\pi\alpha'} \int d^4x \sqrt{-g_E} g^{i\bar{i}'} g^{j\bar{j}'} g^{k\bar{k}'} g^{l\bar{l}'} \times \partial_\mu c_{ij\bar{k}\bar{l}} \partial^\mu c_{\bar{i}'\bar{j}'k'l'}. \quad (3.29)$$

To exhibit the coset structure we put these moduli in a two-index form,

$$\alpha' c_{ij\bar{k}\bar{l}} = 2\Delta^{-1} \epsilon_{ijklab} \beta^{ab}. \quad (3.30)$$

The action for all the bulk supergravity moduli is then

$$S_{\text{mod}} = -\frac{1}{4\pi\alpha'g_s} \int d^4x \sqrt{-g_E} \gamma_{k\bar{j}} \gamma_{i\bar{l}} (\partial_\mu \gamma^{i\bar{j}} \partial^\mu \gamma^{k\bar{l}} - \partial_\mu \beta^{i\bar{j}} \partial^\mu \beta^{k\bar{l}}). \quad (3.31)$$

This is just the $U(3,3)/U(3) \times U(3)$ moduli space metric, familiar from the untwisted moduli of the \mathbf{Z}_3 orbifold [26], with upper and lower indices exchanged.

We now consider D3-branes. Expanding the Dirac-Born-Infeld (DBI) action gives the kinetic term

$$S_{\text{DBI}} = -\frac{1}{(2\pi)^3 \alpha' g_s} \int d^4x \sqrt{-g_E} \gamma_{ij} \partial_\mu W_I^i \partial^\mu \bar{W}_I^{\bar{j}}, \quad (3.32)$$

with an implicit sum on I . In addition there is a dependence on the collective coordinates from the coupling of the D3-brane to $C_{(4)}$, which appears through a nontrivial five-form Bianchi identity. In the D3-brane rest frame,

$$d\tilde{F}_{(5)} = (2\pi)^4 \alpha'^2 \delta^6(y) d^6y \rightarrow \frac{\alpha'^2}{2\pi^2} d^6y, \quad (3.33)$$

where we have projected onto the zero mode; we omit the flux term in the Bianchi identity, which makes no contribution to the moduli kinetic terms. Boosting this gives

$$(d\tilde{f})_{\mu\nu ij\bar{k}\bar{l}} = \frac{1}{2\pi^2 \alpha' \Delta} \epsilon_{ijklab} (\partial_\mu W_I^{\bar{b}} \partial_\nu W_I^a - \partial_\mu W_I^a \partial_\nu W_I^{\bar{b}}),$$

$$f_{\nu ij\bar{k}\bar{l}} = \partial_\nu c_{ij\bar{k}\bar{l}} + \frac{1}{4\pi^2 \alpha' \Delta} \epsilon_{ijklab} (W_I^{\bar{b}} \partial_\nu W_I^a - W_I^a \partial_\nu W_I^{\bar{b}}). \quad (3.34)$$

The moduli space action is then

$$S_{\text{bulk}} = -\frac{1}{4\pi\alpha'g_s} \int d^4x \sqrt{-g_E} \left\{ \gamma_{k\bar{j}} \gamma_{i\bar{l}} (\partial_\mu \gamma^{i\bar{j}} \partial^\mu \gamma^{k\bar{l}} - \mathcal{D}_\mu \beta^{i\bar{j}} \mathcal{D}^\mu \beta^{k\bar{l}}) + \frac{1}{2\pi^2} \gamma_{ij} \partial_\mu W_I^i \partial^\mu \bar{W}_I^{\bar{j}} \right\}, \quad (3.35)$$

where

$$\mathcal{D}_\mu \beta^{i\bar{j}} = \partial_\mu \beta^{i\bar{j}} + \frac{1}{8\pi^2} (W_I^{\bar{j}} \partial_\mu W_I^i - W_I^i \partial_\mu W_I^{\bar{j}}). \quad (3.36)$$

With a bit of algebra, it is possible to show that the entire action on moduli space takes the form

$$S = \frac{1}{4} \frac{1}{2\pi\alpha'g_s} \int d^4x \sqrt{-g_E} \text{Tr}(\partial_\mu M \eta \partial^\mu M \eta) \quad (3.37)$$

where η is the $U(3,3+N)$ invariant metric ($\eta = \Omega^\dagger \eta \Omega$) and M is a Hermitean $U(3,3+N)$ matrix that behaves as $M \rightarrow \Omega M \Omega^\dagger$ under $U(3,3+N)$. We work in a basis with block diagonal form

$$\eta = \begin{bmatrix} I_3 & & \\ & I_3 & \\ & & I_N \end{bmatrix},$$

$$M = \begin{bmatrix} \gamma^{-1} & -\gamma^{-1} \mathcal{B} & -\gamma^{-1} \alpha^\dagger \\ -\mathcal{B}^\dagger \gamma^{-1} & \gamma + \mathcal{B}^\dagger \gamma^{-1} \mathcal{B} + \alpha^\dagger \alpha & \mathcal{B}^\dagger \gamma^{-1} \alpha^\dagger + \alpha^\dagger \\ -\alpha \gamma^{-1} & \alpha \gamma^{-1} \mathcal{B} + \alpha & I_N + \alpha \gamma^{-1} \alpha^\dagger \end{bmatrix} \quad (3.38)$$

with matrix notation $\gamma = \gamma^{\bar{i}i}$, $\alpha = W_I^i/2\pi$, and $\mathcal{B} = \mathcal{B} + (1/2)\alpha^\dagger \alpha$. To verify that this takes the appropriate coset form, note that we can write

$$M = V^\dagger V, \quad V = \begin{bmatrix} e & -e\mathcal{B} & -e\alpha^\dagger \\ 0 & e^{-1} & 0 \\ 0 & \alpha & I_N \end{bmatrix} \quad (3.39)$$

where e is the vielbein $e^\dagger e = \gamma^{-1}$. Following [27], we see that M indeed belongs to the coset $U(3,3+N)/U(3) \times U(3+N)$, precisely as we expected based on $\mathcal{N}=3$ supersymmetry.

D. Comparison to $\mathcal{N}=4$ heterotic string

The results of Sec. III C are notably similar to work done by Maharana and Schwarz on the $O(6,22)$ duality of the heterotic string on T^6 [27]. This is not an accident. Starting from the heterotic string, S duality maps to type I strings, and a further T duality on all six dimensions takes the theory to the type IIB model of [19]. Our $\mathcal{N}=3$ models are then obtained by nonperturbatively transforming D3-branes into self-dual $G_{(3)}$ flux, so we expect that our moduli space should simply be a subspace of the heterotic moduli space.

To make this more precise, we can follow the action of the S and T dualities on the moduli of the heterotic theory. For ease of comparison, we will use coordinates of radii equal to the string length $\sqrt{\alpha'}$. We will also choose duality conventions such that α' is the same in the heterotic, type I, and type IIB string theories. To get the normalization correct including numerical factors, we must be careful (see [28] for some factors in the type I theory, for example).

We start by considering the heterotic–type I S duality. Under this duality, the heterotic fundamental string maps to the type I D-string; in particular the actions must be equal. Since the D-string tension and charge are reduced by a factor of $\sqrt{2}$ by the orientifold projection in the type I theory, we therefore must have

$$\begin{aligned} & \frac{1}{2\pi\alpha'\sqrt{2}} \int d^2\xi e^{-\Phi(\text{I})} \sqrt{-\det g(\text{I})} \\ &= \frac{1}{2\pi\alpha'} \int d^2\xi \sqrt{-\det g(\text{het})} \\ \Rightarrow g_{MN}(\text{het}) &= \frac{e^{-\Phi(\text{I})}}{\sqrt{2}} g_{MN}(\text{I}) \end{aligned} \quad (3.40)$$

and likewise $B_2(\text{het}) = C_2(\text{I})/\sqrt{2}$. The 10D supergravity actions then map into each other if we take the gauge theory potentials to be equal.

In the T duality between type I on T^6 and IIB on T^6/\mathbf{Z}_2 , the dilaton picks up a well-known factor of $\sqrt{2}$ [28], so the T duality is

$$\begin{aligned} e^\Phi(\text{I}) &= \frac{\sqrt{2}}{\det^{1/2} g_{mn}} e^\Phi(\text{IIB}), \\ g_{mn}(\text{I}) &= g^{mn}(\text{IIB}), \quad g_{\mu\nu}(\text{I}) = g_{\mu\nu}(\text{IIB}). \end{aligned} \quad (3.41)$$

There is an additional factor in the RR sector, as follows. Taking the prefactor of the 10D action to be the same in the two theories, T duality tells us that we should have the same dimensionally reduced actions, or

$$\begin{aligned} & \frac{(2\pi)^6 \alpha'^3}{2 \cdot 2} \int d^4x \sqrt{-g_4} \Delta \partial_\mu C_{mn} \partial^\mu C^{mn}(\text{I}) \\ &= \frac{(2\pi)^6 \alpha'^3}{2 \cdot 2 \cdot 4!} \int d^4x \sqrt{-g_4} \Delta \partial_\mu C_{mnpq} \partial^\mu C^{mnpq}(\text{IIB}) \end{aligned} \quad (3.42)$$

for the moduli. Here, $\Delta = \det^{1/2} g_{mn}$ and g_4 is the string frame metric. The additional factor of 2 in the type IIB case again comes from the volume. This equality holds if we take

$$C_{mn}(\text{I}) = \frac{1}{\sqrt{2 \cdot 4!}} \Delta \epsilon^{mnpqrs} C_{pqrs}. \quad (3.43)$$

Then the heterotic moduli (using the notation of [27]) map to the type IIB $\mathcal{N}=4$ moduli as follows:

$$\begin{aligned} g_{\mu\nu} &\rightarrow g_{E\mu\nu}, \quad g_{mn} \rightarrow \gamma^{-1 mn}, \\ B_{mn} &\rightarrow \beta^{mn}, \quad a_m^I \rightarrow \alpha_I^m, \end{aligned} \quad (3.44)$$

following the notation of Sec. III C for the type IIB side, up to factors of α' from coordinate rescaling. The $\mathcal{N}=3$ moduli

are then clearly the (anti-)Hermitian subset of the gravitational and R-R moduli along with all the D-brane positions in complex form.

There is an additional complex modulus in the $\mathcal{N}=4$ case which corresponds on the heterotic side to the four-dimensional dilaton and $B_{\mu\nu}$ axion, and on the type IIB side to the ten-dimensional dilaton and R-R scalar. In the $\mathcal{N}=3$ theories this modulus is fixed.

Consider the $\mathcal{N}=4$ states which become massive due to the fluxes. These include one gravitino, so we must have a massive spin-3/2 multiplet. This must be a large representation because these supergravity states are all neutral under the $U(1)$ central charges, and so the helicities are

$$\frac{3}{2}, \quad 1^6, \quad \frac{1}{2}^{15}, \quad 0^{20}, \quad -\frac{1}{2}^{15}, \quad -1^6, \quad -\frac{3}{2}. \quad (3.45)$$

This agrees with the finding that six gauge symmetries are broken. The twenty spin-zero components are the dilaton-axion, the six zero-helicity components of the massive vectors (from $C_{(4)}$), and the twelve real components of $g_{w^i w^j}$. Note that at large radius these states, with masses α'^2/R^6 , lie parametrically below the Kaluza-Klein scale of R^{-2} . Thus we can truncate to an effective field theory in which only these and the massless states survive. Since the mass scale is parametrically below the Planck scale as well, the SUSY breaking from $\mathcal{N}=4$ to $\mathcal{N}=3$ must be spontaneous. There has been some discussion of such breaking in supergravity [29–31].

IV. DUALITIES

In this section, we discuss the stringy duality group of these compactifications. In particular, we are interested in the dual description that governs the physics when the radii become small.

A. Dualities of the $\mathcal{N}=4$ theory with 16 D3-branes

As a warmup, let us first consider the dualities of the $\mathcal{N}=4$ theory with 16 D3-branes, which is the T dual of type I on T^6 and the TS dual of the heterotic theory on T^6 . The duality of the latter theory is $SO(22,6,\mathbf{Z}) \times SU(1,1,\mathbf{Z})$ [32]. Consider first the perturbative $SO(22,6,\mathbf{Z})$ factor. This group is generated by discrete shifts of the Wilson lines, Weyl reflections in the gauge group, discrete shifts of B_{mn} , large coordinate transformations on the torus, and the inversion of one or more directions on the torus (this is not meant to be a minimal set of generators). We will call this last operation R duality to distinguish it from the full perturbative T duality. The first four operations are manifest in the type IIB description, as the periodicities of the D3-brane collective coordinates, permutations of the D3-branes, discrete shifts of the C_{mnpq} , and large coordinate transformations respectively. The R duality is not manifest in the type IIB description. Note that this is not the same as type IIB R duality, because it leaves fixed the ten-dimensional type IIB coupling and not the four-dimensional coupling. Rather, it is the image of the heterotic R duality; therefore we will henceforth designate it R_{het} .

To see R_{het} in the type IIB description it is useful to focus on its action on the BPS states. In the heterotic description R_{het} interchanges Kaluza-Klein (KK) states and winding F-strings. In the type I description these become KK states and winding D-strings, and then in type IIB they become winding F-strings and D5-branes. Similarly it interchanges winding D-strings and NS5-branes.

To analyze the duality carefully we need the masses of these objects, taking for simplicity a rectangular torus $ds^2 = r_m^2 dx^m dx^m$, and vanishing R-R backgrounds. We take the F- and D-strings to be wound in the 4-direction, and the D5- and NS5-branes to be wound in the 56789-directions. Then (in the string frame)

$$m_{\text{F1}} = \frac{r_4}{\alpha'}, \quad m_{\text{D1}} = \frac{r_4}{\alpha' g_s},$$

$$m_{\text{D5}} = \frac{v}{2r_4 \alpha'^3 g_s}, \quad m_{\text{NS5}} = \frac{v}{2r_4 \alpha'^3 g_s^2}, \quad (4.1)$$

where $v = \Pi_m r_m$. The factors of 2 come about because the strings must be wound on cycles of T^6 , while the 5-branes can be wound on the fixed cycle $x^4=0$ whose volume is halved. For the F-string this represents the fact that in an orientifold the closed strings are obtained by projection; for the NS5-brane it is simply the \mathbf{Z}_2 reduction of an NS 5-brane solution at $x^4=0$ on the original T^6 . For the D1- and D5-branes, these statements are T -dual to the fact that in the type I string the D5-brane has two Chan-Paton values while the D1-brane has one [33,28]: thus, the IIB D1-brane can move off the fixed plane, while the D5-brane is fixed. For future reference let us also give the masses in the type I description, where $r'_m = \alpha'/r_m$; the couplings are related by $v'/g'^2 = v/2g_s^2$, the factor of 2 being from the orientifold volume. Then

$$m_{\text{KK}'} = \frac{1}{r'_4}, \quad m_{\text{D5}'} = \frac{v' \sqrt{2}}{r'_4 \alpha'^3 g'_s}, \quad m_{\text{D1}'} = \frac{r'_4}{v' g'_s \sqrt{2}}. \quad (4.2)$$

The factors of $\sqrt{2}$ are as found in Ref. [28].

In units of the four-dimensional Planck mass $m_4 = (v/2)^{1/2} \alpha'^{-2} g_s^{-1}$ the BPS masses are

$$\frac{m_{\text{F1}}}{m_4} = \frac{r_4 \alpha' g_s \sqrt{2}}{v^{1/2}} = \frac{g_s^{1/2}}{\rho_4},$$

$$\frac{m_{\text{D1}}}{m_4} = \frac{r_4 \alpha' \sqrt{2}}{v^{1/2}} = \frac{1}{\rho_4 g_s^{1/2}},$$

$$\frac{m_{\text{D5}}}{m_4} = \frac{v^{1/2}}{r_4 \alpha' \sqrt{2}} = \rho_4 g_s^{1/2},$$

$$\frac{m_{\text{NS5}}}{m_4} = \frac{v^{1/2}}{r_4 \alpha' g_s \sqrt{2}} = \frac{\rho_4}{g_s^{1/2}}. \quad (4.3)$$

We have defined $\rho_4 = v^{1/2}/r_4 \alpha' g_s^{1/2} \sqrt{2}$, which is just the radius in the heterotic string picture, in heterotic string units. The masses of strings and 5-branes interchange under inversion of ρ_4 , as expected.

The $SU(1,1,\mathbf{Z})$ of the heterotic theory maps to the $SU(1,1,\mathbf{Z})$ of the ten-dimensional type IIB theory. In particular, $g_s \rightarrow g_s^{-1}$ interchanges strings with strings and 5-branes with 5-branes.

B. Dualities of the $\mathcal{N}=3$ theories

We expect that the duality group will be an integer version of the continuous low energy symmetry $U(3,3+N)$. The simplest guess would be that it is the intersection of this continuous group with the discrete symmetry $SO(6,22,\mathbf{Z}) \times SU(1,1,\mathbf{Z})$ of the $\mathcal{N}=4$ theory. In other words, the fluxes break the duality symmetry to a subgroup, just as they do with the supersymmetry. However, we will see that this guess is incorrect.

Let us consider the BPS states discussed in Sec. IV A. Note that these do not have a perturbative description, because g_s is of order 1, but we can study them using the effective low energy description when the radii are large. In the $\mathcal{N}=4$ theory, these states are invariant under eight supersymmetries; one finds that four of these supersymmetries lie in the $\mathcal{N}=3$ subalgebra of interest.⁵ Thus these are “1/3-BPS” states, in agreement with the result that BPS particles in $\mathcal{N}=3$ preserve four supersymmetries [34].

When the torus is rectangular, the R-R backgrounds zero, and all D3-brane coincident, the central charges are from the bulk $U(1)$'s $A_{\mu i}$. For simplicity let us focus on the case in which $g_s=1$. The unbroken gauge fields associated with the 4-7 torus are

$$\frac{B_{\mu 4} + C_{\mu 7}}{\sqrt{2}}, \quad \frac{C_{\mu 4} - B_{\mu 7}}{\sqrt{2}}, \quad (4.4)$$

while the broken symmetries are

$$\frac{B_{\mu 4} - C_{\mu 7}}{\sqrt{2}}, \quad \frac{C_{\mu 4} + B_{\mu 7}}{\sqrt{2}}. \quad (4.5)$$

Thus a D-string in the 4-direction, or an F-string in the 7-direction, have the same BPS charge, electric charge in the first $U(1)$. A D5-brane in the 56789-directions, and an NS5-brane in the 89456-directions, carry the analogous magnetic charge.⁶

There is, however, an important subtlety: not all of these states actually appear in the spectrum. Each of these objects couples both to a massless and a massive vector. The discussion of Eq. (3.14) shows that the vector mass arises from electric breaking through the Higgs mechanism. For the elec-

⁵More details, and further analysis, will be presented in future work.

⁶More generally we can consider (p,q) -strings and 5-branes, at various angles—a full accounting of the BPS states is an interesting exercise.

trically charged 1-branes the massive charge is screened and there is no great effect. However, the 5-branes carry the corresponding magnetic charge and so must be confined: the Higgs mechanism breaks the symmetry between these two sets of states. We can understand this in two other ways as well. First, the breaking through Higgs mechanism reduces the long-ranged interaction between the electric and magnetic objects by a factor of 2. Since they had the minimum relative Dirac quantum in the $\mathcal{N}=4$ theory, they are no longer correctly quantized. Second, the gauge invariant flux on the D5-brane is $\mathcal{F}_{(2)}=F_{(2)}-B_{(2)}/2\pi\alpha'$, which satisfies

$$d\mathcal{F}_{(2)}=-H_{(3)}/2\pi\alpha'. \quad (4.6)$$

The integral of this over any 3-cycle should then vanish, but this is inconsistent because our background includes at least one of H_{678} or H_{567} , among others. In order for the Bianchi identity to be consistent, there must be another source. This would be a D3-brane, which is localized in the 3-cycle in question and extended in the other two compact directions and one noncompact direction: this is a confining flux tube.

It follows that the duality R_{het} that interchanges the basic 1- and 5-branes does not survive in the $\mathcal{N}=3$ theory.⁷ There are magnetic objects in this theory, but they are bound states. For example, a 56789 D5-brane and a 89456 NS5-brane have the same BPS $U(1)$ charge and the opposite broken charge, and so their bound state is unconfined and is a BPS state of twice the minimum $\mathcal{N}=4$ mass. In a perturbative description, the D5-brane ends twice on the NS5-brane, as in the (p,q) -5-brane webs of [35,36].

The simplest conjecture would then be that the duality group interchanges the objects of minimum electric and magnetic charge. With the D5-brane masses (4.3) doubled, this would now mean that $\rho'_m=1/2\rho_m$; it is not clear whether this symmetry could be inherited from the $\mathcal{N}=4$ theory.⁸ To be precise, this symmetry can act independently on any set of paired indices, 4-7, 5-8, or 6-9: it must preserve Eq. (2.21). This conjectured symmetry relates rather different objects, and so for example the total number of BPS states of a D-string in the 4-direction and an F-string in the 7-direction must equal that of the D5-NS5 bound states. It is an interesting exercise, to be studied in future work, to determine the BPS spectra of these objects as a function of the background fluxes. It is possible that this will reveal a more intricate pattern of dualities, in which the various $\mathcal{N}=3$ models mix. It is conceivable that the dualities might involve other types

⁷Note that this duality interchanges electric and magnetic objects, while the $SO(6,22,\mathbf{Z})$ of the heterotic theory acts separately on each. This is because the unbroken gauge fields (4.4) are a linear combination of electric and magnetic gauge fields in the heterotic picture: the nonlinear Higgs field has both electric and magnetic charges.

⁸Such a duality does exist in the heterotic string for a nonzero axion [20], but it has not been determined if it can be combined with the heterotic strong-weak coupling duality [37] to generate the proper action on the BPS states. This possibility also requires that the axion of the “heterotic” description of the $\mathcal{N}=3$ theory be shifted by half a unit, and it is not immediately clear that this is so.

of $\mathcal{N}=3$ construction, such as those of Ref. [24], though we have no particular reason to expect this. Note also that there is no reason to expect an effective heterotic description anywhere in the moduli space. For the $\mathcal{N}=4$ theories such a description holds when the type IIB radii are small and the ten-dimensional IIB coupling is large, but in the $\mathcal{N}=3$ models the latter coupling is always of order 1.

The remainder of the duality group would be generated by large coordinate transformations mixing the holomorphic coordinates, periodicities of the D3-brane coordinates, permutations of the D3-branes, and shifts of the R-R backgrounds. We conclude this section with a few remarks about these.

When discussing large coordinate transformations on the torus, we should distinguish between U dualities, which leave the background invariant, and “string-string”-like dualities, which take one background into a different but equivalent background. The transformations that give “string-string” dualities are discussed in [6]; here we are interested in finding those that give U dualities.

A large coordinate transformation will leave the background $G_{\overline{123}}$ invariant if its determinant is unity. Nevertheless, the duality also includes elements of nontrivial determinant. For example, at $\tau=i$, rotation of a single coordinate $w^1 \rightarrow iw^1$ changes the background 3-form $G_{\overline{123}} \rightarrow iG_{\overline{123}}$, but this can be undone by one of the broken $SL(2;\mathbf{Z})$ dualities of the type IIB string, $\tau \rightarrow -1/\tau$. Note that this combined operation leaves the background invariant and so does not act on the moduli space, but it does mix the BPS states and so is a nontrivial duality. Also, if the fluxes are chosen so that $\tau \neq i$, this duality is not a U duality, so we find that different $\mathcal{N}=3$ backgrounds have slightly different U -duality groups. Note that in models with fluxes on the orientifold planes, we must restrict ourselves to transformations that take O3-planes of a given type into the same type. If we insist that all the fixed points map to themselves under dualities, then the off-diagonal elements of the linear transformation must be even and the diagonal elements must be unity (or -1 with a translation). Again, different backgrounds will have different U -duality groups.

The D3-brane gauge charges do not appear in the type IIB superalgebra, and a zero-length F- or D-string stretched between coincident D3-branes is massless, giving an enhanced gauge symmetry. When the D3-branes are separated the stretched string begins to couple to the bulk gauge fields, and acquires a BPS mass and charge. When the D3-branes shift fully around the 1-cycles of the torus, the attached F- and D-strings acquire integer winding charges. Since the electric charges on the D3-branes are the end points of F-strings, this duality shifts the bulk electric charges by the D3-brane electric charges. Note that since the magnetic D3-brane charges are D-string end points, the shift also depends on the D3-brane magnetic charges: as noted in footnote 7, the duality group is nontrivially embedded in the low energy electric-magnetic duality group.

In order to understand the R-R shift dualities in detail one needs to consider two other classes of BPS objects. The first are Euclidean D3-branes wrapped entirely on the internal torus. These are instantons under the unbroken gauge symmetries, and their phases depend on the R-R moduli. The

magnetic analogs to these are spacetime strings, D3-branes wrapped on the appropriate 2-cycles of the torus and extended in one direction of the external space; we have already encountered these above, as confining flux tubes. As one circles such a string one traces a closed loop in moduli space. The discrete shift dualities must leave all instanton amplitudes invariant, and one expects that all such shifts will be generated by the dual strings. Note that the instantons wrap enough directions for the identity (4.6) to be relevant, so their spectrum will be subject to restrictions.

There are two physically distinct cases of these instantons and strings. The simpler case couples to the diagonal $\beta^{\bar{i}i}$ ($i = \bar{i}$) moduli, as these moduli correspond to a single real component of $\tilde{c}_{(4)}$. For example, $\beta^{\bar{1}1}$ couples to an instantonic D3-brane wrapped on the 5689 directions and a string D3-brane partially wrapped on the 47 directions. Notice that these instantons do not wrap any 3-cycle including $H_{(3)}$ or $F_{(3)}$ flux. Additionally, we have checked that these strings preserve supersymmetry; in fact, they preserve 6 supercharges in common with the background. The other case corresponds to the off-diagonal moduli, which have real and imaginary parts constructed from two components of $\tilde{c}_{(4)}$ each. The instantons do wrap 3-cycles with flux, so they must come in bound states, much as the magnetic BPS charges discussed above, and the corresponding strings would then fill half a supermultiplet each. These strings preserve four supersymmetries in common with the background.

We consider here just the diagonal case. In the $\mathcal{N}=4$ theory the wrapped D3-branes are dual to type I instantonic D-strings. These have a single Chan-Paton index, so there exist D3-brane instantons wrapping one of the special half-volume 4-cycles. Their action is given by

$$\frac{1}{(2\pi)^3 \alpha'^2} \int \tilde{c}_{5689} dx^5 dx^6 dx^8 dx^9 = \frac{\pi}{\alpha'^2} \tilde{c}_{5689}. \quad (4.7)$$

This implies that \tilde{c}_{5689} can shift by even integral multiples of α'^2 without changing the path integral. As this shifts $\beta^{\bar{1}1}$ by $i/2$ times that integer, we see that the shift duality has been broken by the instantons to \mathbf{Z} for each axion.

Let us check that this is consistent with the spacetime strings. A D3-brane wrapped on 47 is dual to a D5-brane in the $\mathcal{N}=4$ type I theory. Since the type I D5-brane must have two Chan-Paton indices, these D3-branes can only wrap 2-cycles of volume $(2\pi)^2$. Using the relative coefficients of terms in the action, the 10-dimensional Bianchi identity for the 5-form integrates to

$$\frac{1}{(2\pi)^7 \alpha'^4} \oint_M \tilde{F}^{(5)} = \frac{1}{(2\pi)^3 \alpha'^2}. \quad (4.8)$$

The surface surrounding the string is $M = S^1 \times T^4 / \mathbf{Z}_2$. Integrating over the latter factor gives

$$\oint_{S^1} d\tilde{c}_{5689} = 2\alpha'^2, \quad (4.9)$$

which is the minimum shift consistent with the instanton amplitude.

A complete analysis of the duality group is left for future work.

In conclusion, we see that although supersymmetry strongly constrains these $\mathcal{N}=3$ models, there remain interesting dynamical issues. Thus these models may be a useful preliminary to the study of less symmetric and more realistic warped compactifications.

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