Exactly solvable model of superstring in plane wave Ramond-Ramond background

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(Received 26 February 2002; published 28 May 2002)

We describe in detail the solution of type IIB superstring theory in the maximally supersymmetric plane-wave background with constant null Ramond-Ramond 5-form field strength. The corresponding light-cone Green-Schwarz action found by Metsaev is quadratic in both bosonic and fermionic coordinates. We obtain the light-cone Hamiltonian and the string representation of the corresponding supersymmetry algebra. The superstring Hamiltonian has a "harmonic-oscillator" form in both the string oscillator and the zero-mode parts and thus has a discrete spectrum. We analyze the structure of the zero-mode sector of the theory, establishing the precise correspondence between the lowest-lying "massless" string states and the type IIB supergravity fluctuation modes in the plane-wave background. The zero-mode spectrum has a certain similarity to the supergravity spectrum in $AdS_5\times S^5$ background of which the plane-wave background is a special limit. We also compare the plane-wave string spectrum with the expected form of the light-cone gauge spectrum of the $AdS_5\times S^5$ superstring.

DOI: 10.1103/PhysRevD.65.126004 PACS number(s): 11.25.Mj, 04.65.+e

I. INTRODUCTION

The simplest gravitational plane wave backgrounds

$$ds^2=2dx^+dx^-+K(x^+,x^I)dx^+dx^++dx^Idx^I,$$

$$K=k_{IJ}x^Ix^J,$$

supported by a constant Neveu-Schwarz–Neveu-Schwarz (NS-NS) 3-form background, provide examples of exactly solvable (super)string models: the string action becomes quadratic in the light-cone gauge $x^+=p^+\tau$ (see, e.g., [1–4]). It was recently pointed out [5] that this solvability property is shared also by a conformal model describing type IIB superstring propagating in a particular plane-wave metric supported by a *Ramond-Ramond* 5-form background [6]:

$$ds^2=2dx^+dx^--\mathrm{f}^2x_I^2dx^+dx^++dx^Idx^I,\quad I=1,\ldots,8, \tag{1.1}$$

$$F_{+1234}=F_{+5678}=2\mathrm{f}. \tag{1.2}$$

This background has several special properties. It preserves the maximal number of 32 supersymmetries [6], and it is related by a special limit (boost along a circle of S^5 combined with a rescaling of the coordinates and of the radius or α') to the $AdS_5\times S^5$ background [7]. The exactly solvable string theory corresponding to Eq. (1.1) may thus have some common features with a much more complicated string theory on $AdS_5\times S^5$ whose light-cone action contains nontrivial interaction terms [8,9].

In the present paper which is an extension of [5] we will present in detail the solution of this Ramond-Ramond (RR) plane-wave string model. In particular, we will explicitly identify the massless modes in its spectrum with small fluctuations of the type IIB supergravity fields in the background (1.1). The results will have an obvious similarity to those of [10] in the case of $AdS_5\times S^5$. In particular, a remarkable common feature of the RR plane wave supermultiplets and the AdS supermultiplets is that the massless fields with different spins belonging to the same supermultiplet have, in general, different lowest energy values. The same is true also for massive supermultiplets.[1]

Let us first recall the form of the light-cone gauge Green-Schwarz (GS) action for the type IIB superstring in the background (1.1). This action was found in [5] by using the supercoset method of [13], but there is a simple shortcut argument relating the presence of the fermionic "mass" term to the form of the generalized spinor covariant derivative in type IIB supergravity. In view of the special null Killing vector properties of the background (1.1), (1.2) it is possible to argue that the only non-vanishing fermionic contribution to the type IIB superstring action in the standard light-cone gauge

$$x^+=p^+\tau,\qquad \Gamma^+\theta^{\mathcal{I}}=0 \tag{1.3}$$

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[1]This is different from what one finds in the case of the non-supersymmetric bosonic plane wave backgrounds, where massless fields of different spins have, as in the case of the flat space, the same lowest energy values. This difference is related to supersymmetry and not to the definition of masslessness: in both cases we use the same definition of massless fields based on so-called *sim* invariance (invariance under transformations of the original plane-wave algebra supplemented by the dilatation) of the corresponding field equations [11,12].

comes from the direct covariantization

$$\mathcal{L}_{2F}=i(\eta^{ab}\delta_{\mathcal{I}\mathcal{J}}-\epsilon^{ab}\rho_{3\mathcal{I}\mathcal{J}})\partial_a x^{\underline{m}}\bar{\theta}^{\mathcal{I}}\Gamma_{\underline{m}}\mathcal{D}_b\theta^{\mathcal{J}} \qquad (1.4)$$

of the quadratic fermionic term in the flat-space GS [14] action. Here $\theta^{\mathcal{I}}$ ($\mathcal{I}=1,2$) are the two real positive chirality 10D MW spinors and $\rho_3=\mathrm{diag}(1,-1)$ (see the Appendix for notation). $\mathcal{D}$ is the generalized covariant derivative that appears in the Killing spinor equation (or gravitino transformation law) in type IIB supergravity [15]: acting on the real spinors $\theta^{\mathcal{I}}$ it has the form (we ignore the dilaton and RR scalar dependence)

$$\mathcal{D}_a=\partial_a+\frac{1}{4}\partial_a x^{\underline{m}}\left[\left(\omega_{\mu\nu\underline{m}}-\frac{1}{2}H_{\mu\nu\underline{m}}\rho_3\right)\Gamma^{\mu\nu} - \left(\frac{1}{3!}F_{\mu\nu\lambda}\Gamma^{\mu\nu\lambda}\rho_1+\frac{1}{2\cdot 5!}F_{\mu\nu\lambda\rho\kappa}\Gamma^{\mu\nu\lambda\rho\kappa}\rho_0\right)\Gamma_{\underline{m}}\right] \qquad (1.5)$$

where the ρ_s matrices in the $\mathcal{I},\mathcal{J}$ space are the Pauli matrices $\rho_1=\sigma_1$, $\rho_0=i\sigma_2$. In the light-cone gauge (1.3) the non-zero contribution to Eq. (1.4) comes only from the term where both the "external" and "internal" $\partial_a x^{\underline{m}}$ factors in Eq. (1.4) become $p^+\delta^{\underline{m}}_{\mp}\delta^0_a$. As is well known, in the flat-space light-cone GS action θ^1 and θ^2 become the right and the left moving 2D fermions. In the presence of the F_5 background (1.2) the surviving quadratic fermionic term is proportional to $\theta^1\Gamma^-\Gamma^{\mu_1\cdots\mu_4}\theta^2F_{+\mu_1\ldots\mu_4}$. While in the case of an NS-NS 3-form background the fermionic interaction term has a chiral 2D form (ρ_3 is diagonal), in the case of a RR background one gets a non-chiral 2D "mass-term" structure (ρ_1 and ρ_0 are off diagonal) out of the interaction term in $\mathcal{D}_a$ in Eqs. (1.4),(1.5).

The resulting quadratic light-cone action [5] can be written, like the flat-space GS action, in a 2D spinor form and describes 8 free massive 2D scalars and 8 free massive Majorana 2D fermionic fields $\psi=(\theta^1,\theta^2)$ propagating in a flat 2D world-sheet:

$$\mathcal{L}=\mathcal{L}_B+\mathcal{L}_F, \qquad \mathcal{L}_B=\frac{1}{2}(\partial_+x^I\partial_-x^I-\mathrm{m}^2x_I^2),$$

$$\mathrm{m}\equiv p^+\mathrm{f}, \qquad (1.6)$$

$$\mathcal{L}_F=\mathrm{i}(\theta^1\bar{\gamma}^-\partial_+\theta^1+\theta^2\bar{\gamma}^-\partial_-\theta^2-2\mathrm{m}\theta^1\bar{\gamma}^-\Pi\theta^2),$$

$$\bar{\gamma}^+\theta^{\mathcal{I}}=0. \qquad (1.7)$$

Here $\partial_\pm=\partial_0\pm\partial_1$ and we absorbed one factor of p^+ into $\theta^{\mathcal{I}}$. We use the spinor notation of [5] i.e., $\gamma^m,\bar{\gamma}^m$ are the 16×16 Dirac matrices which are the off-diagonal parts of 32×32 matrices Γ^m. The matrix Π in the mass term ($\Pi^2=1$) is the product of four γ matrices (see the Appendix) which originates from $\Gamma^{\mu_1\cdots\mu_4}F_{+\mu_1\ldots\mu_4}$ in Eqs. (1.4),(1.5).

In Sec. II A we shall review the solution of the classical equations corresponding to the light-cone gauge action (1.6),(1.7) and then (in Sec. II B) perform the straightforward canonical quantization of this quadratic system already sketched in [5]. In Sec. II C we shall present the light-cone string realization of the basic symmetry superalgebra of the plane-wave background. We shall then use this superalgebra to fix the vacuum-energy ("normal-ordering") constant in the zero-mode sector (Sec. II D). As we shall explain, the choice of the fermionic zero-mode vacuum is not unique with different (physically equivalent) choices depending on how one decides to describe the representation of the corresponding Clifford algebra. In particular, we note that a choice that leads to zero vacuum energy constant breaks the $SO(8)$ global symmetry down to $SO(4)\times SO'(4)$ [which is in fact the true symmetry of the plane-wave background (1.1),(1.2)] but is not the one that has a smooth flat-space limit.

In Sec. III we shall determine the spectrum of fluctuations of type IIB supergravity expanded near the plane-wave background (1.1),(1.2). Section III A will contain some general remarks on solutions of massless Klein-Gordon-type equations in the plane-wave metric (1.2). The bosonic (scalar, 2-form, graviton and 4-form field) spectra will be found in Sec. III B. The spin 1/2 and spin 3/2 cases will be analyzed in Sec. III C. Our analysis will be similar to the one carried out in [10] in the case of the $AdS_5\times S^5$ background. As a result, we will be able to give a space-time interpretation to the "massless" (zero-mode) sector of the string theory. The discreteness of the supergravity part of the light-cone energy spectrum will follow from the condition of square integrability of the solutions of the corresponding wave equations at fixed p^+. In Sec. III D we will summarize the results for the bosonic and fermionic spectra in the two tables and then explain how the corresponding physical modes can be interpreted as components of a single scalar type IIB superfield satisfying a massless (dilatation-invariant) equation in light-cone superspace.

In the concluding Sec. IV we shall make some comments on the parameters and possible limits of the plane-wave string theory, and also compare it with the expected form of the light-cone string theory spectrum in $AdS_5\times S^5$ background.

Our index and spinor notation and definitions as well as some useful relations will be given in the Appendix.

II. CANONICAL QUANTIZATION

A. Solution of classical equations

The equations of motion following from Eqs. (1.6),(1.7) take the form

$$\partial_+\partial_-x^I+\mathrm{m}^2x^I=0, \qquad (2.1)$$

$$\partial_+\theta^1-\mathrm{m}\Pi\theta^2=0, \quad \partial_-\theta^2+\mathrm{m}\Pi\theta^1=0. \qquad (2.2)$$

The parameter f in Eq. (1.1) which has dimension of mass, can be absorbed into rescaling of x^+,x^-, i.e. set to a given value.[2] We shall choose the length of the σ interval to be 1. The flat space limit corresponds to $\mathrm{m}\to 0$.

As follows from the structure of the covariant string action corresponding to the background (1.1),(1.2) one can absorb the dependence on the string tension into the following rescaling of the coordinates[3]: $x^-\to 2\pi\alpha' x^-, x^I \to (2\pi\alpha')^{1/2}x^I, \theta^{\mathcal{I}}\to(2\pi\alpha')^{1/2}\theta^{\mathcal{I}}$ with x^+ unchanged. Then all one needs to do to restore the dependence on the string tension is the following rescaling of p^+:

$$p^+\to 2\pi\alpha' p^+. \tag{2.3}$$

In particular, $\mathrm{m}\to\mathrm{m}=2\pi\alpha' p^+ \mathrm{f}$.

The general solutions to Eqs. (2.1),(2.2) satisfying the closed string boundary conditions

$$x^I(\sigma+1,\tau)=x^I(\sigma,\tau),\quad \theta(\sigma+1,\tau)=\theta(\sigma,\tau),\qquad 0\leqslant\sigma\leqslant 1, \tag{2.4}$$

are found to be

$$x^I(\sigma,\tau)=\cos\mathrm{m}\tau\; x_0^I+\mathrm{m}^{-1}\sin\mathrm{m}\tau\; p_0^I+\mathrm{i}\sum_{n\neq 0}\frac{1}{\omega_n}[\varphi_n^1(\sigma,\tau)\times\alpha_n^{1I}+\varphi_n^2(\sigma,\tau)\alpha_n^{2I}], \tag{2.5}$$

$$\theta^1(\sigma,\tau)=\cos\mathrm{m}\tau\;\theta_0^1+\sin\mathrm{m}\tau\Pi\,\theta_0^2+\sum_{n\neq 0}c_n\left(\varphi_n^1(\sigma,\tau)\,\theta_n^1+\mathrm{i}\frac{\omega_n-k_n}{\mathrm{m}}\varphi_n^2(\sigma,\tau)\Pi\,\theta_n^2\right), \tag{2.6}$$

$$\theta^2(\sigma,\tau)=\cos\mathrm{m}\tau\theta_0^2-\sin\mathrm{m}\tau\Pi\,\theta_0^1+\sum_{n\neq 0}c_n\left(\varphi_n^2(\sigma,\tau)\times\theta_n^2-\mathrm{i}\frac{\omega_n-k_n}{\mathrm{m}}\varphi_n^1(\sigma,\tau)\Pi\,\theta_n^1\right), \tag{2.7}$$

where the basis functions $\varphi_n^{1,2}(\sigma,\tau)$ are

$$\varphi_n^1(\sigma,\tau)=\exp[-\mathrm{i}(\omega_n\tau-k_n\sigma)],$$

$$\varphi_n^2(\sigma,\tau)=\exp[-\mathrm{i}(\omega_n\tau+k_n\sigma)] \tag{2.8}$$

and

$$\omega_n=\sqrt{k_n^2+\mathrm{m}^2},\quad n>0;\quad \omega_n=-\sqrt{k_n^2+\mathrm{m}^2},\quad n<0; \tag{2.9}$$

$$k_n\equiv 2\pi n,\quad c_n=\frac{1}{\sqrt{1+\left(\frac{\omega_n-k_n}{\mathrm{m}}\right)^2}},\qquad n=\pm1,\pm2,\ldots\,. \tag{2.10}$$

The canonical momentum $\mathcal{P}^I=\dot{x}^I$ takes the form

$$\mathcal{P}^I(\sigma,\tau)=\cos\mathrm{m}\tau\; p_0^I-\mathrm{m}\sin\mathrm{m}\tau\; x_0^I+\sum_{n\neq 0}(\varphi_n^1(\sigma,\tau)\alpha_n^{1I}+\varphi_n^2(\sigma,\tau)\alpha_n^{2I}). \tag{2.11}$$

The fermionic momenta given by $-\mathrm{i}\overline{\gamma^-}\theta^{\mathcal{I}}$ imply that there are the second class constraints which should be treated following the standard Dirac procedure (see, e.g., [5]).

The coordinate x^- satisfies the equation

$$p^+\acute{x}^-+\mathcal{P}^I\acute{x}^I+\mathrm{i}(\theta^1\overline{\gamma^-}\acute{\theta}^1+\theta^2\overline{\gamma^-}\acute{\theta}^2)=0, \tag{2.12}$$

which leads to the constraint

$$\int d\sigma[\mathcal{P}^I\acute{x}^I+\mathrm{i}(\theta^1\overline{\gamma^-}\acute{\theta}^1+\theta^2\overline{\gamma^-}\acute{\theta}^2)]=0. \tag{2.13}$$

We get the following classical Poisson-Dirac brackets:

$$[p_0^I,x_0^J]_{P.B.}=\delta^{IJ},\qquad \{\theta_0^{\mathcal{I}\alpha},\theta_0^{\mathcal{J}\beta}\}_{P.B.}=\frac{\mathrm{i}}{4}(\gamma^+)^{\alpha\beta}\delta^{\mathcal{I}\mathcal{J}},$$

$$[\alpha_m^{\mathcal{I}I},\alpha_n^{\mathcal{J}J}]_{P.B.}=\frac{\mathrm{i}}{2}\omega_m\delta_{m+n,0}\delta^{IJ}\delta^{\mathcal{I}\mathcal{J}}, \tag{2.14}$$

$$\{\theta_m^{\mathcal{I}\alpha},\theta_n^{\mathcal{J}\beta}\}_{P.B.}=\frac{\mathrm{i}}{4}(\gamma^+)^{\alpha\beta}\delta^{\mathcal{I}\mathcal{J}}\delta_{m+n,0}. \tag{2.15}$$

The matrix γ^+ in Eq. (2.15) is reflecting the fact that we are using the light-cone gauge constrained fermionic coordinates, $\overline{\gamma^+}\theta^{\mathcal{I}}=0$. The coefficients c_n, Eq. (2.10), are chosen so that the Fourier modes of the fermionic coordinates satisfy the standard Poisson-Dirac brackets (2.15).

B. Quantization and space of states

We can now quantize 2D fields x^I and $\theta^{\mathcal{I}}$ by promoting as usual the coordinates and momenta or the Fourier components appearing in Eqs. (2.5),(2.6),(2.7) to operators and replacing the classical Poisson (anti)brackets (2.14),(2.15) by the equal-time (anti)commutators of quantum coordinates and momenta according to the rules $\{.,.\}_{P.B.}\to\mathrm{i}\{.,.\}_{quant}$, $[.,.]_{P.B.}\to\mathrm{i}[.,.]_{quant}$. This gives $(m,n=\pm1,\pm2,\ldots)$

$$[p_0^I,x_0^J]=-\mathrm{i}\delta^{IJ},\quad [\alpha_m^{\mathcal{I}I},\alpha_n^{\mathcal{J}J}]=\frac{1}{2}\omega_m\delta_{m+n,0}\delta^{IJ}\delta^{\mathcal{I}\mathcal{J}}, \tag{2.16}$$

$$\{\theta_0^{\mathcal{I}\alpha},\theta_0^{\mathcal{J}\beta}\}=\frac{1}{4}(\gamma^+)^{\alpha\beta}\delta^{\mathcal{I}\mathcal{J}},$$

$$\{\theta_m^{\mathcal{I}\alpha},\theta_n^{\mathcal{J}\beta}\}=\frac{1}{4}(\gamma^+)^{\alpha\beta}\delta^{\mathcal{I}\mathcal{J}}\delta_{m+n,0}. \tag{2.17}$$

[2] Note also that since the generator P^+ commutes with all other generators of the plane wave superalgebra we could fix p^+ to take some specific non-vanishing value. In what follows we shall p^+ arbitrary.

[3] After the rescaling x^-,x^I will be dimensionless (like τ and σ) but x^+ (and $\alpha' p^+$) will have dimension of length.

The light-cone superstring Hamiltonian is

$$H \equiv -P^{-}, \tag{2.18}$$

$$H=\frac{1}{p^{+}}\int_0^1 d\sigma\left[\frac{1}{2}(\mathcal{P}_I^2+\acute{x}_I^2+\mathrm{m}^2 x_I^2)+2\mathrm{i m}\theta^1\bar{\gamma}^{-}\Pi\theta^2 - \mathrm{i}(\theta^1\bar{\gamma}^{-}\acute{\theta}^1-\theta^2\bar{\gamma}^{-}\acute{\theta}^2)\right]. \tag{2.19}$$

Using the fermionic equations of motion it can be rewritten in the form

$$H=\frac{1}{p^{+}}\int d\sigma\left[\frac{1}{2}(\mathcal{P}_I^2+\acute{x}_I^2+\mathrm{m}^2 x_I^2)+\mathrm{i}(\theta^1\bar{\gamma}^{-}\dot{\theta}^1+\theta^2\bar{\gamma}^{-}\dot{\theta}^2)\right]. \tag{2.20}$$

Plugging in the above expressions for the coordinates and momenta we can represent the resulting light-cone energy operator as

$$H=E_0+E^1+E^2, \tag{2.21}$$

where E_0 is the contribution of the zero modes and E^1,E^2 are the contributions of the string oscillation modes:

$$E_0=\frac{1}{2p^{+}}(p_0^2+\mathrm{m}^2 x_0^2)+2\mathrm{if}\theta_0^1\bar{\gamma}^{-}\Pi\theta_0^2, \tag{2.22}$$

$$E^{\mathcal{I}}=\frac{1}{p^{+}}\sum_{n\neq 0}(\alpha_{-n}^{\mathcal{I}I}\alpha_n^{\mathcal{I}I}+\omega_n\theta_{-n}^{\mathcal{I}}\bar{\gamma}^{-}\theta_n^{\mathcal{I}}),\quad \mathcal{I}=1,2. \tag{2.23}$$

The constraint (2.13) takes the form

$$N^1=N^2,\quad N^{\mathcal{I}}\equiv\sum_{n\neq 0}\left(\frac{k_n}{\omega_n}\alpha_{-n}^{\mathcal{I}I}\alpha_n^{\mathcal{I}I}+k_n\theta_{-n}^{\mathcal{I}}\bar{\gamma}^{-}\theta_n^{\mathcal{I}}\right). \tag{2.24}$$

Let us introduce the following basis of creation and annihilation operators:

$$a_0^I=\frac{1}{\sqrt{2\mathrm{m}}}(p_0^I+\mathrm{im}x_0^I),\quad \bar{a}_0^I=\frac{1}{\sqrt{2\mathrm{m}}}(p_0^I-\mathrm{im}x_0^I), \tag{2.25}$$

$$\alpha_{-n}^{\mathcal{I}I}=\sqrt{\frac{\omega_n}{2}}a_n^{\mathcal{I}I},\quad \alpha_n^{\mathcal{I}I}=\sqrt{\frac{\omega_n}{2}}\bar{a}_n^{\mathcal{I}I},\quad n=1,2,\dots, \tag{2.26}$$

$$\theta_0=\frac{1}{\sqrt{2}}(\theta_0^1+i\theta_0^2),\quad \bar{\theta}_0=\frac{1}{\sqrt{2}}(\theta_0^1-i\theta_0^2), \tag{2.27}$$

$$\theta_{-n}^{\mathcal{I}}\equiv\frac{1}{\sqrt{2}}\eta_n^{\mathcal{I}},\quad \theta_n^{\mathcal{I}}\equiv\frac{1}{\sqrt{2}}\bar{\eta}_n^{\mathcal{I}},\quad n=1,2,\dots, \tag{2.28}$$

in terms of which the commutation relations (2.16),(2.17) take the form

$$[\bar{a}_0^I,a_0^J]=\delta^{IJ},\quad [\bar{a}_m^{\mathcal{I}I},a_n^{\mathcal{J}J}]=\delta_{m,n}\delta^{IJ}\delta^{\mathcal{I}\mathcal{J}}, \tag{2.29}$$

$$\{\bar{\theta}_0^{\alpha},\theta_0^{\beta}\}=\frac{1}{4}(\gamma^{+})^{\alpha\beta},$$

$$\{\bar{\eta}_m^{\mathcal{I}\alpha},\eta_n^{\mathcal{J}\beta}\}=\frac{1}{2}(\gamma^{+})^{\alpha\beta}\delta_{m,n}\delta^{\mathcal{I}\mathcal{J}}. \tag{2.30}$$

Here $\alpha=1,\dots,16$, and the spinors are subject to the $\bar{\gamma}^{+}\theta_0^{\mathcal{I}}=0,\bar{\gamma}^{+}\eta_n^{\mathcal{I}}=0$ constraint.

In this basis the light-cone energy operator (2.21) becomes the sum of E_0, E^1 and E^2 where

$$E_0=\mathrm{f}\mathcal{E}_0,\quad \mathcal{E}_0=a_0^I\bar{a}_0^I+2\bar{\theta}_0\bar{\gamma}^{-}\Pi\theta_0+4, \tag{2.31}$$

$$E^{\mathcal{I}}=\frac{1}{p^{+}}\sum_{n=1}^{\infty}\omega_n(a_n^{\mathcal{I}I}\bar{a}_n^{\mathcal{I}I}+\eta_n^{\mathcal{I}}\bar{\gamma}^{-}\bar{\eta}_n^{\mathcal{I}}). \tag{2.32}$$

We have normal-ordered the bosonic zero modes in $\mathcal{E}_0$ (getting extra term $\frac{1}{2}\times 8=4$) and both the bosonic and fermionic operators in $E^{\mathcal{I}}$ (here the normal-ordering constants cancel out as there are equal numbers of bosonic and fermionic oscillators). Note that because of the relation $\mathrm{Tr}(\gamma^{+}\bar{\gamma}^{-}\Pi)=0$, the contribution of the fermionic zero modes in Eq. (2.31) does not depend on ordering of θ_0 and $\bar{\theta}_0$.

To restore the dependence on α' we need to rescale p^{+} as in Eq. (2.3). The explicit form of the light-cone Hamiltonian is then

$$H=\mathrm{f}(a_0^I\bar{a}_0^I+2\bar{\theta}_0\bar{\gamma}^{-}\Pi\theta_0+4)+\frac{1}{\alpha' p^{+}}\sum_{\mathcal{I}=1,2}\sum_{n=1}^{\infty}\sqrt{n^2+(\alpha' p^{+}\mathrm{f})^2}(a_n^{\mathcal{I}I}\bar{a}_n^{\mathcal{I}I}+\eta_n^{\mathcal{I}}\bar{\gamma}^{-}\bar{\eta}_n^{\mathcal{I}}). \tag{2.33}$$

Note that the energy thus depends on the two parameters of mass dimension 1: the curvature (or RR field) scale f and the string scale $(p^{+}\alpha')^{-1}$. The flat-space limit corresponds to $\mathrm{f}=0$ (the zero-mode part recovers its flat-space form $p_I^2/2p^{+}$ as in the case of the standard harmonic oscillator; cf. Sec. III A).

The vacuum state is the direct product of a zero-mode vacuum and the Fock vacuum for string oscillation modes; i.e., it is defined by

$$\bar{a}_0^I|0\rangle=0,\quad \bar{\theta}_0^{\alpha}|0\rangle=0,\quad \bar{a}_n^{\mathcal{I}I}|0\rangle=0,\quad \bar{\eta}_n^{\mathcal{I}\alpha}|0\rangle=0,\quad n=1,2,\dots. \tag{2.34}$$

Generic Fock space vectors are then built up in terms of products of creation operators a_0^I, $a_n^{\mathcal{I}I}$, θ_0^{α}, $\eta_n^{\mathcal{I},\alpha}$ acting on the vacuum

$$|\Phi\rangle=\Phi(a_0,a_n,\theta_0,\eta_n)|0\rangle. \tag{2.35}$$

The subspace of physical states is obtained by imposing the constraint

$$N^1|\Phi_{phys}\rangle = N^2|\Phi_{phys}\rangle, \quad N^{\mathcal{I}} = \sum_{n=1}^{\infty} k_n(a_n^{\mathcal{I}I}\overline{a_n^{\mathcal{I}I}} + \eta_n^{\mathcal{I}}\overline{\gamma^-}\,\overline{\eta_n^{\mathcal{I}}}). \tag{2.36}$$

Note that in contrast to the flat space case here $E^{\mathcal{I}} \neq N^{\mathcal{I}}$.

Let us now make few remarks about the global symmetry of the above expressions. While the metric (1.1) and the bosonic part of the string action (1.6) have $SO(8)$ symmetry, the 5-form background (1.2) and thus the fermionic part of the classical action (1.7) are invariant only under $SO(4)\times SO'(4)$. The contribution of the string oscillators to the Hamiltonian (2.32) is $SO(8)$ invariant, but this invariance is broken down to $SO(4)\times SO'(4)$ by the contribution of the fermionic zero modes in Eq. (2.31). In general, the amount of global symmetry of the zero-mode Hamiltonian depends on the definition of the fermionic creation and annihilation operators, i.e. on the definition of the zero-mode vacuum. With the definition used in Eq. (2.27) the vacuum (2.34) preserves $SO(8)$ symmetry, but the fermionic part of the zero-mode Hamiltonian (2.31) is not $SO(8)$ invariant. One can instead introduce another set of fermionic creation-annihilation operators, i.e. use another definition of the fermionic zero-mode vacuum, which preserves only the $SO(4)\times SO'(4)$ invariance, but which formally restores the $SO(8)$ invariance of the zero-mode Hamiltonian (see Sec. II D below). In any case, the $SO(8)$ invariance is broken down to $SO(4)\times SO'(4)$ not only in the fermionic zero mode sector, but also explicitly by the string-mode contributions to the dynamical supercharges discussed in Sec. II C.

C. Light cone string realization of the supersymmetry algebra

In general, the choice of the light-cone gauge spoils part of manifest global symmetries, and in order to demonstrate that these global invariances are still present, one needs to find the (bosonic and fermionic) Noether charges that generate them. These charges play a crucial role in formulating superstring field theory in the light-cone gauge in flat space [16,17] and are of equal importance in the present plane-wave context (see also [5]).

In the light-cone formalism, the generators (charges) of the basic superalgebra can be split into the kinematical generators $P^+, P^I, J^{+I}, J^{ij}, J^{i'j'}, Q^+, \overline{Q}^+$, and the dynamical generators $P^-, Q^-, \overline{Q^-}$ [here $I=(i,i')$, $i=1,2,3,4$; $i'=5,6,7,8$].[4] It is important to find a free (quadratic) field representation for the generators of the basic superalgebra. The kinematical generators which effectively depend only on the zero modes are[5]

$$P^+ = p^+, \quad P^I = \int d\sigma(\cos \mathrm{f}x^+\mathcal{P}^I + \mathrm{f}\sin \mathrm{f}x^+ x^I p^+), \tag{2.37}$$

$$J^{+I} = \int d\sigma(\mathrm{f}^{-1}\sin \mathrm{f}x^+\mathcal{P}^I - \cos \mathrm{f}x^+ x^I p^+), \tag{2.38}$$

$$Q^+ = 2\sqrt{p^+}\int d\sigma \overline{\gamma^-} e^{\mathrm{i}\mathrm{f}x^+\Pi}\theta,$$

$$\overline{Q}^+ = 2\sqrt{p^+}\int d\sigma \overline{\gamma^-} e^{-\mathrm{i}\mathrm{f}x^+\Pi}\overline{\theta}. \tag{2.39}$$

The remaining kinematical charges $J^{IJ}=(J^{ij},J^{i'j'})$ have non-zero components which depend on all string modes and are

$$J^{ij} = \int d\sigma(x^i\mathcal{P}^j - x^j\mathcal{P}^i - \mathrm{i}\overline{\theta\gamma^-}\gamma^{ij}\theta),$$

$$J^{i'j'} = \int d\sigma(x^{i'}\mathcal{P}^{j'} - x^{j'}\mathcal{P}^{i'} - \mathrm{i}\overline{\theta\gamma^-}\gamma^{i'j'}\theta). \tag{2.40}$$

The dynamical charge P^- is given by Eq. (2.19), while the supercharges Q^- and $\overline{Q}^-$ are given by $[Q,\overline{Q}=(1/\sqrt{2})(Q^1 \pm iQ^2)]$

$$Q^{-1} = \frac{2}{\sqrt{p^+}}\int d\sigma[(\mathcal{P}^I - \acute{x}^I)\overline{\gamma^I}\theta^1 - \mathrm{m}x^I\overline{\gamma^I}\Pi\theta^2], \tag{2.41}$$

$$Q^{-2} = \frac{2}{\sqrt{p^+}}\int d\sigma[(\mathcal{P}^I + \acute{x}^I)\overline{\gamma^I}\theta^2 + \mathrm{m}x^I\overline{\gamma^I}\Pi\theta^1]. \tag{2.42}$$

The derivation of these supercharges is given in [5].

Using the mode expansions of Sec. II A in Eqs. (2.37), (2.39) we get[6]

$$P^+ = p^+, \quad P^I = p_0^I, \quad J^{+I} = -\mathrm{i}x_0^I p^+, \tag{2.43}$$

$$Q^+ = 2\sqrt{p^+}\overline{\gamma^-}\theta_0, \quad \overline{Q}^+ = 2\sqrt{p^+}\overline{\gamma^-}\overline{\theta_0}. \tag{2.44}$$

The charges $J^{IJ}=(J^{ij},\ J^{i'j'})$ are given by

$$J^{IJ} = J_0^{IJ} + \sum_{\mathcal{I}=1,2}\sum_{n=1}^{\infty}\left(a_n^{\mathcal{I}I}\overline{a_n^{\mathcal{I}J}} - a_n^{\mathcal{I}J}\overline{a_n^{\mathcal{I}I}} + \frac{1}{2}\eta_n^{\mathcal{I}}\overline{\gamma^-}\gamma^{IJ}\overline{\eta_n^{\mathcal{I}}}\right), \tag{2.45}$$

where J_0^{IJ} is the contribution of the zero modes:

$$J_0^{IJ} = a_0^I\overline{a_0^J} - a_0^J\overline{a_0^I} + \frac{1}{2}\sum_{\mathcal{I}=1,2}\theta_0^{\mathcal{I}}\overline{\gamma^-}\gamma^{IJ}\theta_0^{\mathcal{I}}. \tag{2.46}$$

[4]At point $x^+=p^+\tau=0$ the kinematical generators in the superfield realization are quadratic in the physical string fields, while the dynamical generators receive higher-order interaction-dependent corrections.

[5]We define $\theta \equiv (1/\sqrt{2})(\theta^1 + i\theta^2)$, $\overline{\theta} \equiv (1/\sqrt{2})(\theta^1 - i\theta^2)$.

[6]While transforming the generators $J^{\mu\nu}$, Eqs. (2.38), (2.40), to the form given in Eqs. (2.43), (2.45) we multiply them by the factor $+\mathrm{i}$.

Note that the kinematical generators do not involve the matrix Π and formally look as if the $SO(8)$ symmetry were present.

The dynamical supercharges (2.41) have the following explicit form:

$$\sqrt{p^+}Q^{-1}=2p_0^I\overline{\gamma^I}\theta_0^1-2\mathrm{m}x_0^I\overline{\gamma^I}\Pi\theta_0^2+\sum_{n=1}^{\infty}\left(2\sqrt{\omega_n}c_n a_n^{1I}\overline{\gamma^I}\overline{\eta_n^1}+\frac{\mathrm{im}}{\sqrt{\omega_n}c_n}a_n^{2I}\overline{\gamma^I}\Pi\overline{\eta_n^2}+\mathrm{H.c.}\right),\tag{2.47}$$

$$\sqrt{p^+}Q^{-2}=2p_0^I\overline{\gamma^I}\theta_0^2+2\mathrm{m}x_0^I\overline{\gamma^I}\Pi\theta_0^1+\sum_{n=1}^{\infty}\left(2\sqrt{\omega_n}c_n a_n^{2I}\overline{\gamma^I}\overline{\eta_n^2}-\frac{\mathrm{im}}{\sqrt{\omega_n}c_n}a_n^{1I}\overline{\gamma^I}\Pi\overline{\eta_n^1}+\mathrm{H.c.}\right).\tag{2.48}$$

These expressions explicitly break the $SO(8)$ invariance down to $SO(4)\times SO'(4)$.

The requirement that the light-cone gauge formulation respect basic global symmetries amounts to the condition that the above generators satisfy the relations of the symmetry superalgebra of the plane wave RR background. The commutators of the bosonic generators are[7]

$$[P^-,P^I]=\mathrm{f}^2J^{+I},\quad [P^I,J^{+J}]=-\delta^{IJ}P^+,$$

$$[P^-,J^{+I}]=P^I,\tag{2.49}$$

$$[P^i,J^{jk}]=\delta^{ij}P^k-\delta^{ik}P^j,$$

$$[P^{i'},J^{j'k'}]=\delta^{i'j'}P^{k'}-\delta^{i'k'}P^{j'},\tag{2.50}$$

$$[J^{+i},J^{jk}]=\delta^{ij}J^{+k}-\delta^{ik}J^{+j},$$

$$[J^{+i'},J^{j'k'}]=\delta^{i'j'}J^{+k'}-\delta^{i'k'}J^{+j'},\tag{2.51}$$

$$[J^{ij},J^{kl}]=\delta^{jk}J^{il}+3\ \text{ terms},$$

$$[J^{i'j'},J^{k'l'}]=\delta^{j'k'}J^{i'l'}+3\ \text{ terms}.\tag{2.52}$$

The commutation relations between the even and odd generators are

$$[J^{ij},Q_\alpha^\pm]=\frac{1}{2}Q_\beta^\pm(\gamma^{ij})^\beta{}_\alpha,\quad [J^{i'j'},Q_\alpha^\pm]=\frac{1}{2}Q_\beta^\pm(\gamma^{i'j'})^\beta{}_\alpha,\tag{2.53}$$

$$[J^{+I},Q_\alpha^-]=\frac{1}{2}Q_\beta^+(\gamma^{+I})^\beta{}_\alpha,\tag{2.54}$$

$$[P^I,Q_\alpha^-]=\frac{1}{2}\mathrm{f}Q_\beta^+(\Pi\gamma^{+I})^\beta{}_\alpha,\quad [P^-,Q_\alpha^+]=\mathrm{f}Q_\beta^+\Pi^\beta{}_\alpha,\tag{2.55}$$

together with the commutators that follow from these by complex conjugation. The anticommutation relations are

$$\{Q_\alpha^+,\overline{Q_\beta^+}\}=2\gamma_{\alpha\beta}^-P^+,\tag{2.56}$$

$$\{Q_\alpha^+,\overline{Q_\beta^-}\}=(\overline{\gamma^-}\gamma^+\overline{\gamma^I})_{\alpha\beta}P^I-\mathrm{f}(\overline{\gamma^-}\gamma^+\overline{\gamma^i}\Pi)_{\alpha\beta}J^{+i}-\mathrm{f}(\overline{\gamma^-}\gamma^+\overline{\gamma^{i}}'\Pi')_{\alpha\beta}J^{+i'},\tag{2.57}$$

$$\{Q_\alpha^-,\overline{Q_\beta^+}\}=(\overline{\gamma^+}\gamma^-\overline{\gamma^I})_{\alpha\beta}P^I-\mathrm{f}(\overline{\gamma^+}\gamma^-\overline{\gamma^i}\Pi)_{\alpha\beta}J^{+i}-\mathrm{f}(\overline{\gamma^+}\gamma^-\overline{\gamma^{i}}'\Pi')_{\alpha\beta}J^{+i'},\tag{2.58}$$

$$\{Q_\alpha^-,\overline{Q_\beta^-}\}=2\gamma_{\alpha\beta}^+P^-+\mathrm{f}(\overline{\gamma^+}\gamma^{ij}\Pi)_{\alpha\beta}J^{ij}+\mathrm{f}(\overline{\gamma^+}\gamma^{i'j'}\Pi')_{\alpha\beta}J^{i'j'}.\tag{2.59}$$

One can check directly that our quantum generators expressed in terms of the creation-annihilation operators do satisfy these (anti)commutations relation. Note that one recovers the flat-space light-cone superalgebra in the limit $\mathrm{f}\to 0$. As in the flat superstring case the anticommutator relation between the dynamical generators Q^- and $\overline{Q^-}$, Eq. (2.59), is valid only on the physical subspace (2.36).

D. Choice of fermionic zero-mode vacuum

The states obtained by applying the fermionic zero-mode creation operators to the vacuum form a supermultiplet. States of that supermultiplet can be described in different ways depending on how one picks up a (‘‘Clifford’’) vacuum to construct the tower of other states on top of it. While it is natural to define ‘‘the’’ vacuum to have zero energy, this is not the only possible or necessary choice as we shall discuss below.

In general, the quantum counterpart of the zero-mode energy (2.22) may be written as [cf. Eq. (2.33)]

$$E_0=\mathrm{f}\mathcal{E}_0,\quad \mathcal{E}_0=a_0^I\overline{a_0^I}-2\theta_0\overline{\gamma^-}\Pi\overline{\theta_0}+e_0,\tag{2.60}$$

where $\theta_0=(1/\sqrt{2})(\theta_0^1+i\theta_0^2)$ [see Eq. (2.27)] and e_0 is a constant that should be fixed from the condition of the realization of the superalgebra (2.56)–(2.59) at the quantum level. Note that $E_0=0$ in the flat-space limit $\mathrm{f}\to 0$.

We shall need the following expressions for the zero-mode parts of some symmetry generators [see Eqs. (2.46), (2.47), (2.48)]:

$$J_0^{IJ}=a_0^I\overline{a_0^J}-a_0^J\overline{a_0^I}+\overline{\theta_0}\overline{\gamma^-}\gamma^{IJ}\theta_0,\tag{2.61}$$

$$\sqrt{p^+}Q_0^-=2p_0^I\overline{\gamma^I}\theta_0+2\mathrm{im}x_0^I\overline{\gamma^I}\Pi\theta_0,$$

$$\sqrt{p^+}\overline{Q_0^-}=2p_0^I\overline{\gamma^I}\overline{\theta_0}-2\mathrm{im}x_0^I\overline{\gamma^I}\Pi\overline{\theta_0}.\tag{2.62}$$

[7]Note that we use the Hermitian P^μ and the anti-Hermitian $J^{\mu\nu}$ generators. The supercharges $Q^\pm$ and $\overline{Q^\pm}$ are related to each other by the conjugation $(Q^\pm)^\dagger=\overline{Q^\pm}$.

Let us introduce instead of θ_0 the following complex fermionic zero-mode coordinates:

$$\theta_R = \frac{1+\Pi}{\sqrt{2}}\theta_0, \quad \theta_L = \frac{1-\Pi}{\sqrt{2}}\theta_0, \tag{2.63}$$

satisfying in view of Eqs. (2.15), (2.30) the relations

$$\{\overline{\theta_R},\theta_R\} = \frac{1}{4}(1+\Pi)\gamma^+,$$

$$\{\overline{\theta_L},\theta_L\} = \frac{1}{4}(1-\Pi)\gamma^+, \quad \{\overline{\theta_R},\theta_L\} = 0. \tag{2.64}$$

In terms of them

$$\mathcal{E}_0 = a_0^I\overline{a_0^I} + \theta_L\overline{\gamma^-}\,\overline{\theta_L} - \theta_R\overline{\gamma^-}\,\overline{\theta_R} + e_0, \tag{2.65}$$

and

$$Q_0^- = 2\sqrt{\mathrm{f}}(a_0^I\overline{\gamma^I}\theta_R + \overline{a_0^I}\,\overline{\gamma^I}\theta_L),$$

$$\overline{Q_0^-} = 2\sqrt{\mathrm{f}}(\overline{a_0^I}\,\overline{\gamma^I}\,\overline{\theta_R} + a_0^I\overline{\gamma^I}\,\overline{\theta_L}), \tag{2.66}$$

$$J_0^{IJ} = a_0^I\overline{a_0^J} - a_0^J\overline{a_0^I} + \frac{1}{2}\overline{\theta_R}\,\overline{\gamma^-}\gamma^{IJ}\theta_R + \frac{1}{2}\overline{\theta_L}\,\overline{\gamma^-}\gamma^{IJ}\theta_L. \tag{2.67}$$

Let us now discuss several possible definitions of the zero-mode vacuum (we shall always assume that $\overline{a_0^I}|0\rangle = 0$). In all the cases below the expression for J^{IJ} will imply that the vacuum is a scalar with respect to $SO(4)\times SO'(4)$.

First, we may define the fermionic zero-mode vacuum in the same way is in the case of the flat space background by imposing

$$\overline{\theta_0}|0\rangle = 0, \quad \text{i.e.} \quad \overline{\theta_R}|0\rangle = 0, \quad \overline{\theta_L}|0\rangle = 0. \tag{2.68}$$

This is the definition we used in Eq. (2.34). Then

$$\begin{aligned}\{Q_0^-,\overline{Q_0^-}\}|0\rangle &= 4\mathrm{f}\{a_0^I\overline{\gamma^I}\theta_R + \overline{a_0^I}\,\overline{\gamma^I}\theta_L, \overline{a_0^J}\,\overline{\gamma^J}\,\overline{\theta_R} + a_0^J\overline{\gamma^J}\,\overline{\theta_L}\}|0\rangle \\ &= 4\mathrm{f}(\overline{a_0^J}\,\overline{\gamma^J}\,\overline{\theta_R})(a_0^I\overline{\gamma^I}\theta_R)|0\rangle \\ &= 4\mathrm{f}\overline{\gamma^I}\,\overline{\theta_R}\,\overline{\gamma^I}\theta_R|0\rangle = \mathrm{f}\overline{\gamma^I}(1+\Pi)\gamma^+\overline{\gamma^I}|0\rangle \\ &= -8\mathrm{f}\overline{\gamma^+}|0\rangle,\end{aligned} \tag{2.69}$$

where we use the relation $\overline{\gamma^I}\Pi\gamma^I = 0$. On the other hand, from the supersymmetry algebra relation (2.59) we have

$$\{Q^-,\overline{Q^-}\}|0\rangle = 2\mathrm{f}\overline{\gamma^+}P^-|0\rangle = -2\mathrm{f}\overline{\gamma^+}e_0|0\rangle, \tag{2.70}$$

where we used that $J^{IJ}|0\rangle = 0$. Since for the zero modes $E_0 = -P^-$ we learn that here $e_0 = 4$.

Thus the normal ordering of bosons done in Eq. (2.31) is indeed consistent with the supersymmetry algebra. Then from Eq. (2.65) we see that acting with θ_L (θ_R) on $|0\rangle$ we increase (decrease) the energy by one unit. The generic fermionic zero-mode state is

$$(\theta_R)^{n_R}(\theta_L)^{n_L}|0\rangle, \quad n_L, n_R = 0,1,2,3,4. \tag{2.71}$$

The restriction on the values of n_R and n_L comes from $(\theta_R)^5 = 0$, $(\theta_L)^5 = 0$ (the projected fermions have only 4 independent components). The corresponding energy spectrum is thus

$$\mathcal{E}_0(n_R,n_L) = 4 - n_R + n_L. \tag{2.72}$$

The values of the energy of the lightest massless (type IIB supergravity) string modes with no bosonic excitations thus run from 0 to 8 (in units of f).

The equivalent definition of the vacuum is obtained by using the conjugate of Eq. (2.68)

$$\theta_0|0\rangle = 0, \quad \text{i.e.} \quad \theta_R|0\rangle = 0, \quad \theta_L|0\rangle = 0, \tag{2.73}$$

so that

$$e_0 = 4, \quad \mathcal{E}_0(n_R,n_L) = 4 + n_R - n_L. \tag{2.74}$$

One may instead define the vacuum by

$$\overline{\theta_R}|0\rangle = 0, \quad \theta_L|0\rangle = 0, \tag{2.75}$$

leading to

$$e_0 = 8, \quad \mathcal{E}_0(n_R,n_L) = 8 - n_R - n_L, \tag{2.76}$$

so that $\mathcal{E}_0$ again takes values in the range $0,1,\ldots,8$.

Finally, another possible choice is

$$\theta_R|0\rangle = 0, \quad \overline{\theta_L}|0\rangle = 0, \tag{2.77}$$

in which case one finds that

$$e_0 = 0, \quad \mathcal{E}_0(n_R,n_L) = n_R + n_L. \tag{2.78}$$

Here also $\mathcal{E}_0 = 0,1,\ldots,8$. Note that the two choices of the vacuum (2.68) and (2.73) preserve the $SO(8)$ symmetry but break the effective 2D supersymmetry of the light-cone string action (1.6) (the 2D vacuum energy does not vanish). At the same time, the choice (2.78) preserves the 2D supersymmetry, but breaks the $SO(8)$ symmetry down to $SO(4)\times SO'(4)$ [cf. Eq. (2.63)].

All these definitions of the vacuum are physically equivalent, being related by a relabeling of the states in the same ''massless'' supermultiplet. While in the last choice we discussed the vacuum energy constant e_0 is zero (i.e. the normal ordering constants of the bosonic and fermionic zero modes cancel as they do for the string oscillation modes), the advantage of the first definition we have used above in Eq. (2.34) is that it directly corresponds to the definition of the fermionic vacuum in flat space [16,18], i.e. with this definition one has a natural smooth flat space limit.

In the next section we shall determine the spectrum of the type IIB supergravity fluctuation modes in the background

(1.1), (1.2) and will thus be able to explicitly interpret the states (2.71) with energies $\mathcal{E}_0=0,1,\ldots,8$ in terms of particular supergravity fields.

III. TYPE IIB SUPERGRAVITY FLUCTUATION SPECTRUM IN THE RR PLANE-WAVE BACKGROUND

The string states obtained by acting by the fermionic and bosonic zero-mode operators on the vacuum should be in one-to-one correspondence with the fluctuation modes of type IIB supergravity fields expanded near the plane-wave background (1.1), (1.2). Assuming the choice of the zero-mode vacuum in Eq. (2.34) or (2.68) and acting by the products of the fermionic zero-mode operators one finds the lowest-lying states that can be symbolically represented as

$\|0\rangle$	complex scalar
$\theta_0\|0\rangle$	spin 1/2 field
$\theta_0\theta_0\|0\rangle$	complex 2- form field
$\theta_0\theta_0\theta_0\|0\rangle$	spin 3/2 field
$\theta_0\theta_0\theta_0\theta_0\|0\rangle$	graviton and self-dual 4-form field
..	complex conjugates to the above.

(3.1)

The complete type IIB supergravity spectrum is obtained by acting with the bosonic zero mode creation operators a_0^I on the above states.

The aim of this section is to explicitly derive the supergravity spectrum using the standard field-theoretic approach, analogous to the one used in [10] for the $AdS_5\times S^5$ background.

As a preparation, it is useful to present the decomposition of the 128+128 physical transverse supergravity degrees of freedom in the light-cone gauge using the $SO(8)\rightarrow SO(4)\times SO'(4)$ decomposition:[8]

$$\text{Graviton:}\quad h_{ij}^{\perp}(9),\quad h_{i'j'}^{\perp}(9),\quad h_{ij'}(16),\quad h(1);$$

$$N_{dof}=35 \tag{3.2}$$

($h_{ij}^{\perp}$, $h_{i'j'}^{\perp}$ are traceless and the $h_{ij'}$ is not symmetric in i,j').

$$\text{4-form field:}\quad a_{ij'}(16),\quad a_{iji'j'}(18),\quad a(1);$$

$$N_{dof}=35 \tag{3.3}$$

($a_{ij'}$ is not antisymmetric in i,j' and $a_{iji'j'}=-\frac{1}{4}\epsilon_{ijkl}\epsilon_{i'j'k'l'}a_{klk'l'}$).

$$\text{Complex 2-form field: } b_{ij}(12),\ b_{i'j'}(12),\ b_{ij'}(32);\ N_{dof}=56 \tag{3.4}$$

($b_{ij'}$ is not antisymmetric in i,j').

$$\text{Complex scalar field:}\quad \phi(2);\quad N_{dof}=2 \tag{3.5}$$

$$\text{spin 1/2 field:}\quad \lambda^{\oplus}(16);\quad N_{d.o.f}=16 \tag{3.6}$$

(λ is negative chirality complex spinor, and $\lambda^{\oplus}=\frac{1}{2}\overline{\gamma^-}\gamma^+\lambda$ is its light-cone projection).

$$\text{Spin 3/2 field:}\quad \psi_i^{\oplus\perp}(48),\quad \psi_{i'}^{\oplus\perp}(48),\quad \psi^{\oplus\|}(16);$$

$$N_{dof}=112 \tag{3.7}$$

(the gravitino is a positive chirality complex spinor, and $\psi^{\perp}$ and $\psi^{\|}$ are its γ-transverse and γ-parallel parts).

As we have already found in string theory (and will confirm directly from the supergravity equations below), here, as in the case of the AdS supermultiplets, the spectrum of the lowest eigenvalues of the light-cone energy operator is non-degenerate; i.e., different states have different values of E_0.

A. Massless field equations in plane-wave geometry

Our aim will be to find the explicit form of the type IIB equations of motion expanded to linear order in fluctuations near the plane-wave background (1.1), (1.2) and then to determine the corresponding light-cone energy spectrum. Let us first discuss the solutions of the simplest wave equations in the curved metric (1.1). The non-trivial components of the corresponding connection and curvature are ($g^{--}=\mathrm{f}^2x_I^2$)

$$\Gamma^{m}_{\mp I}=-\mathrm{f}^2x^I\delta^{m}_{-},\ \Gamma^{m}_{\mp +}=\mathrm{f}^2x^I\delta^{m}_{I},$$

$$R_{I++J}=-\mathrm{f}^2\delta_{IJ},\ R_{++}=8\mathrm{f}^2. \tag{3.8}$$

The massless scalar equation in the plane-wave geometry has the following explicit form:

$$\Box\varphi=0,$$

$$\Box\equiv\frac{1}{\sqrt{-g}}\partial_{\underline{m}}(\sqrt{-g}g^{\underline{mn}}\partial_{\underline{n}})$$
$$=2\partial^+\partial^-+\mathrm{f}^2x_I^2\partial^{+2}+\partial_I^2. \tag{3.9}$$

After the Fourier transform in x^-,x^I corresponding to the light-cone description where x^+ is the evolution parameter,

$$\varphi(x^+,x^-,x^I)=\int\frac{dp^+d^8p}{(2\pi)^{9/2}}e^{\mathrm{i}(p^+x^-+p^Ix^I)}\tilde{\varphi}(x^+,p^+,p^I), \tag{3.10}$$

it becomes

$$(2p^+P^--\mathrm{f}^2p^{+2}\partial_{p^I}^2+p_I^2)\tilde{\varphi}=0, \tag{3.11}$$

where $-P^-=\mathrm{i}\partial^-$ may thus be interpreted as the light-cone Hamiltonian appearing in the non-relativistic Schrödinger equation for the free harmonic oscillator in 8 dimensions with mass p^+ and frequency f:

[8] The number of independent components are indicated in brackets and N_{dof} is the total number of degrees of freedom.

$$H=-P^{-}=\frac{1}{2p^{+}}(p_I^2-\mathrm{m}^2\partial_{p^I}^2),\quad \mathrm{m}\equiv \mathrm{f}p^{+}. \tag{3.12}$$

Introducing the standard creation and annihilation operators

$$a^I\equiv\frac{1}{\sqrt{2\mathrm{m}}}(p^I-\mathrm{m}\partial_{p^I}),\quad \overline{a^I}\equiv\frac{1}{\sqrt{2\mathrm{m}}}(p^I+\mathrm{m}\partial_{p^I}),$$

$$[\overline{a^I},a^J]=\delta^{IJ}, \tag{3.13}$$

we get the following normal-ordered form of the Hamiltonian:

$$H=\frac{1}{2}\mathrm{f}(\overline{a^I}a^I+a^I\overline{a^I})=\mathrm{f}(a^I\overline{a^I}+4), \tag{3.14}$$

where $4=(D-2)/2,\ D=10$. As usual, the spectrum of states [and thus the solution of Eq. (3.9)] is then found by acting by a^I on the vacuum satisfying $\overline{a^I}|0\rangle=0$.

Below we will need the following simple generalization of this analysis: if a field φ satisfies the equation

$$(\Box+2\mathrm{ifc}\,\partial^{+})\varphi(x)=0, \tag{3.15}$$

where $\Box$ is defined in Eq. (3.9) and c is an arbitrary constant, then the corresponding light-cone Hamiltonian is

$$H=-P^{-}=\frac{p_I^2-\mathrm{f}^2p^{+2}\partial_{p^I}^2}{2p^{+}}+\mathrm{f}c=\mathrm{f}(a^I\overline{a^I}+4+c), \tag{3.16}$$

so that the lowest light-cone energy value is given by

$$E_0=\mathrm{f}\mathcal{E}_0,\quad \mathcal{E}_0=4+c. \tag{3.17}$$

In what follows we shall discuss in turn the equations of motion for various fields of type IIB supergravity, reducing them to the form (3.15) and thus determining the corresponding lowest energy values from Eq. (3.17).

B. Bosonic fields

1. Complex scalar field

The dilaton and RR scalar are decoupled from the 5-form background (1.2), i.e. satisfy

$$\Box\phi=0,\quad \text{i.e.}\quad \mathcal{E}_0(\phi)=4. \tag{3.18}$$

2. Complex 2-form field

The corresponding nonlinear equations are [15]

$$D^{\underline{m}}G_{\underline{m}\underline{m}_1\underline{m}_2}=P^{\underline{m}}G^{*}_{\underline{m}\underline{m}_1\underline{m}_2}-\frac{\mathrm{i}}{3}F_{\underline{m}_1\cdots\underline{m}_5}G^{\underline{m}_3\underline{m}_4\underline{m}_5} \tag{3.19}$$

where $G_{\underline{m}_1\underline{m}_2\underline{m}_3}=3\partial_{[\underline{m}_1}B_{\underline{m}_2\underline{m}_3]}$ is the field strength of the complex 2-form field $B_{\underline{mn}}$ and $P_{\underline{m}}$ is the complex scalar field strength. The aim is to derive the equation for small fluctuations $B_{\underline{mn}}=b_{\underline{mn}}$ in the plane-wave background (1.1), (1.2) (with $P_{\underline{m}}=0$) using the light-cone gauge

$$b_{-\underline{m}}=0. \tag{3.20}$$

It is sufficient to analyze Eqs. (3.19) for the following values of the indices $(\underline{m}_1,\underline{m}_2)$: $(-,I)$ and (I,J). We find

$$D^{\underline{m}}G_{\underline{m}IJ}=\partial_\mu G_{\mu IJ}+\mathrm{f}^2x_I^2\partial^{+}G_{-IJ},\quad D^{\underline{m}}G_{\underline{m}-I}=\partial_\mu G_{\mu-I}. \tag{3.21}$$

Taking into account that $F_{-\underline{m}_2\cdots\underline{m}_5}=0$ and the light-cone gauge condition (3.20) we find

$$\partial^{+}b_{+I}+\partial^{J}b_{JI}=0, \tag{3.22}$$

which allows us to express the non-dynamical modes b_{+I} in terms of the physical ones b_{IJ}. Then

$$D^{\underline{m}}G_{\underline{m}\,IJ}=\Box b_{IJ}. \tag{3.23}$$

Using that $F_{ij'\underline{m}_3\underline{m}_4\underline{m}_5}=0$ [cf. Eq. (1.2)] and $F_{ij\underline{m}_3\underline{m}_4\underline{m}_5}G^{\underline{m}_3\underline{m}_4\underline{m}_5}=6\mathrm{f}\epsilon_{ijkl}\partial^{+}b_{kl}$ we get from Eqs. (3.19), (3.23) the following equations for the physical modes b_{IJ}:

$$\Box b_{ij'}=0,\quad \Box b_{ij}+2\mathrm{if}\epsilon_{ijkl}\partial^{+}b_{kl}=0,$$

$$\Box b_{i'j'}+2\mathrm{if}\epsilon_{i'j'k'l'}\partial^{+}b_{k'l'}=0. \tag{3.24}$$

The equation for $b_{ij'}$ implies that $\mathcal{E}_0(b_{ij'})=4$ [see Eqs. (3.9), (3.17)]. To diagonalize the remaining equations we decompose the antisymmetric tensor field b_{ij} into the irreducible tensors of the $so(4)$ algebra:

$$b_{ij}=b_{ij}^{\oplus}+b_{ij}^{\ominus},\quad b_{ij}^{\oplus,\ominus}=\pm\frac{1}{2}\epsilon_{ijkl}b_{kl}^{\oplus,\ominus}. \tag{3.25}$$

Then

$$(\Box+4\mathrm{if}\partial^{+})b_{ij}^{\oplus}=0,\quad (\Box-4\mathrm{if}\partial^{+})b_{ij}^{\ominus}=0. \tag{3.26}$$

The same relations are found for $b_{i'j'}$. Then according to Eqs. (3.15), (3.17) we find the following lowest energy values:

$$\mathcal{E}_0(b_{ij}^{\ominus})=2,\ \mathcal{E}_0(b_{ij}^{\oplus})=6,\ \mathcal{E}_0(b_{i'j'}^{\ominus})=2,$$

$$\mathcal{E}_0(b_{i'j'}^{\oplus})=6,\ \mathcal{E}_0(b_{ij'})=4. \tag{3.27}$$

In the oscillator construction of Sec. II D [see Eqs. (2.68), (2.71), (2.72)] the monomials of the second order in $\theta_{L,R}$ with $\mathcal{E}_0=4$ are $\theta_R\theta_L$, which have 16 complex components; i.e., these monomials can be identified with the ground state of $b_{ij'}$. The second and sixth order monomials in θ_R, θ_L which can be identified with the ground states of $b_{ij}^{\ominus}$, $b_{i'j'}^{\ominus}$, $b_{ij}^{\oplus}$, $b_{i'j'}^{\oplus}$ may be found in Table I.

3. Graviton and 4-form field

Since both the graviton and the 4-form field have nontrivial backgrounds, some of their fluctuation modes are mixed and need to be analyzed together. The full non-linear forms of the corresponding equations of motion are[9]

$$R_{\underline{mn}}=\frac{1}{24}F_{\underline{mm}_2\ldots\underline{m}_5}F_{\underline{n}}{}^{\underline{m}_2\ldots\underline{m}_5}, \tag{3.28}$$

$$F_{\underline{m}_1\ldots\underline{m}_5}=-\frac{1}{5!}\sqrt{-g}\,\epsilon_{\underline{m}_1\ldots\underline{m}_5\underline{n}_1\ldots\underline{n}_5}F^{\underline{n}_1\ldots\underline{n}_5}, \tag{3.29}$$

$$D^{\underline{m}}F_{\underline{mm}_2\ldots\underline{m}_5}=0,\quad F_{\underline{m}_1\ldots\underline{m}_5}=5\partial_{[\underline{m}_1}A_{\underline{m}_2\ldots\underline{m}_4]}\,. \tag{3.30}$$

Expanding near the plane wave RR background

$$g_{\underline{mn}}\rightarrow g_{\underline{mn}}+h_{\underline{mn}},\quad A_{\underline{m}_1\ldots\underline{m}_4}\rightarrow A_{\underline{m}_1\ldots\underline{m}_4}+a_{\underline{m}_1\ldots\underline{m}_4}, \tag{3.31}$$

$$R_{\underline{mn}}\rightarrow R_{\underline{mn}}+r_{\underline{mn}},\quad F_{\underline{m}_1\ldots\underline{m}_5}\rightarrow F_{\underline{m}_1\ldots\underline{m}_5}+f_{\underline{m}_1\ldots\underline{m}_5}, \tag{3.32}$$

we shall choose the light-cone gauges for the fluctuations $h_{\underline{mn}}$ and $a_{\underline{m}_1\ldots\underline{m}_4}$:

$$h_{-\underline{m}}=0,\quad a_{-\underline{m}_2\underline{m}_3\underline{m}_4}=0. \tag{3.33}$$

The linearized form of the Einstein equation is

$$r_{\underline{mn}}=\frac{1}{24}\Big(F_{\underline{m\,m}_1\ldots\underline{m}_4}f_{\underline{n}}{}^{\underline{m}_1\ldots\underline{m}_4}+f_{\underline{m\,m}_1\ldots\underline{m}_4}F_{\underline{n}}{}^{\underline{m}_1\ldots\underline{m}_4}-4F_{\underline{mn}_1\underline{m}_3\underline{m}_4\underline{m}_5}F_{\underline{nn}_2}{}^{\underline{m}_3\underline{m}_4\underline{m}_5}h^{\underline{n}_1\underline{n}_2}\Big) \tag{3.34}$$

where

$$r_{\underline{mn}}=\frac{1}{2}\Big(-D^2h_{\underline{mn}}+D_{\underline{m}}D^{\underline{k}}h_{\underline{kn}}+D_{\underline{n}}D^{\underline{k}}h_{\underline{km}}-D_{\underline{m}}D_{\underline{n}}h^{k}_{\underline{k}}+2R_{\underline{m\,m}_1\underline{m}_2\underline{n}}h^{\underline{m}_1\underline{m}_2}+R_{\underline{m\,k}}h^{k}_{\underline{n}}+R_{\underline{n\,k}}h^{k}_{\underline{m}}\Big). \tag{3.35}$$

The $(--)$ component gives $r_{--}=0$ and thus we find the zero-trace condition for the transverse modes of the graviton:

$$h_{II}=0. \tag{3.36}$$

The $(-I)$ components of Eq. (3.34) give $r_{-I}=0$ and this leads to the equation $D^{\underline{m}}h_{\underline{m}I}=0$ which allows us to express the non-dynamical modes in terms of the physical modes represented by the traceless tensor h_{IJ}:

$$h_{+I}=-\frac{1}{\partial^+}\partial_J h_{JI}\,. \tag{3.37}$$

Next, we need to consider the self-duality equation for the 5-form field whose $(I_1I_2I_3I_4-)$ component implies that $a_{+I_1I_2I_3}$ is expressed in terms of the physical modes a_{IJKL}:

$$a_{+I_1I_2I_3}=-\frac{1}{\partial^+}\partial_J a_{JI_1I_2I_3}. \tag{3.38}$$

In terms of a_{IJKL} the 5-form field strength self-duality condition becomes

$$a_{I_1\ldots I_4}=-\frac{1}{4!}\epsilon_{I_1\ldots I_4J_1\ldots J_4}a_{J_1\ldots J_4}. \tag{3.39}$$

The $(++)$ component of Eq. (3.34) leads to the expression for h_{++} (after taking into account the above results): $h_{++}=(1/(\partial^+)^2)\partial_I\partial_J h_{IJ}$. So far all is just as in the light-cone analysis near flat space.

Let us now do the $4+4$ split of the 8 transverse directions. The (i,j) components of Eq. (3.34) take the form

$$r_{ij}=\mathrm{f}\delta_{ij}\partial^+ a,\quad a\equiv\frac{1}{6}\epsilon_{i_1\ldots i_4}a_{i_1\ldots i_4}. \tag{3.40}$$

Using that $r_{ij}=-\frac{1}{2}\Box h_{ij}$ we get

$$\Box h_{ij}+2\mathrm{f}\delta_{ij}\partial^+ a=0. \tag{3.41}$$

Thus there is a mixing between the trace of the $SO(4)$ part of the graviton h_{ii} and the (pseudo) scalar part of the 4-form potential. From the $(i_1i_2i_3i_4)$ component of the $DF=0$ equation for the 4-form field in Eq. (3.30) we also find that

$$\Box a-8\mathrm{f}\partial^+ h_{ii}=0. \tag{3.42}$$

These equations are diagonalized by introducing the traceless graviton and the complex scalar,

$$h^{\perp}_{ij}\equiv h_{ij}-\frac{1}{4}\delta_{ij}h_{kk},\quad \mathrm{h}\equiv h_{ii}+\mathrm{i}a,\quad \bar{\mathrm{h}}\equiv h_{ii}-\mathrm{i}a, \tag{3.43}$$

so that we finish with

$$\Box h^{\perp}_{ij}=0,\quad (\Box-8\mathrm{i}\mathrm{f}\partial^+)\hat{\mathrm{h}}=0,\quad (\Box+8\mathrm{i}\mathrm{f}\partial^+)\bar{\mathrm{h}}=0. \tag{3.44}$$

According to Eq. (3.17) this implies

$$\mathcal{E}_0(h^{\perp}_{ij})=4,\quad \mathcal{E}_0(\mathrm{h})=0,\quad \mathcal{E}_0(\bar{\mathrm{h}})=8. \tag{3.45}$$

The same results are found of course in the other 4 directions, i.e. with $h_{ij}\rightarrow h_{i'j'}$ and $a\rightarrow a'=\frac{1}{6}\,\epsilon_{i'_1\ldots i'_4}a_{i'_1\ldots i'_4}$, $a'=-a$.

Let us now look at "mixed" components. Equations (3.34) in (ij') directions give

$$\Box h_{ij'}+4\mathrm{f}\partial^+ a_{ij'}=0,\quad a_{ij'}\equiv\frac{1}{3}\epsilon_{ii_1i_2i_3}a_{j'i_1i_2i_3}. \tag{3.46}$$

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[9]The equation $DF_5=0$ follows of course from the self-duality of F_5, but we will find it useful to use this second order form of the equation for A_4 below. Note that we ignore the quadratic 2-form correction term in F_5 [15] as it does not contribute to the linear fluctuation equations here.

We have used the self-duality (3.39) implying $\epsilon_{ii_2i_3i_4}a_{j'i_2i_3i_4}=\epsilon_{j'i'_2i'_3i'_4}a_{ii'_2i'_3i'_4}$. In addition, the $(ij'j'_1j'_2)$ components of the $DF=0$ equations (3.30) give

$$\Box a_{ij'}-4\mathrm{f}\partial^+h_{ij'}=0. \tag{3.47}$$

Again there is a mixing between the components of the graviton and the 4-form field. These equations are diagonalized by defining the complex tensor

$$\mathrm{h}_{ij'}\equiv h_{ij'}+\mathrm{i}a_{ij'}, \quad \overline{\mathrm{h}}_{ij'}\equiv h_{ij'}-\mathrm{i}a_{ij'}, \tag{3.48}$$

$$(\Box-4\mathrm{if}\partial^+)\mathrm{h}_{ij'}=0, \quad (\Box+4\mathrm{if}\partial^+)\overline{\mathrm{h}}_{ij'}=0, \tag{3.49}$$

so that the corresponding lowest eigenvalues of the energy are

$$\mathcal{E}_0(\mathrm{h}_{ij'})=2, \quad \mathcal{E}_0(\overline{\mathrm{h}}_{ij'})=6. \tag{3.50}$$

Finally, for $a_{iji'j'}$ satisfying, according to Eq. (3.39), the constraint

$$a_{iji'j'}=-\frac{1}{4}\epsilon_{ijkl}\epsilon_{i'j'k'l'}a_{klk'l'} \tag{3.51}$$

we find from Eq. (3.30) that

$$\Box a_{iji'j'}=0, \quad \text{i.e.} \quad \mathcal{E}_0(a_{iji'j'})=4. \tag{3.52}$$

Note that the self-dual tensor field $a_{iji'j'}$ is reducible with respect to the $SO(4)\times SO'(4)$ group. It can be decomposed into the irreducible parts $a^{\oplus\ominus}_{iji'j'}$, $a^{\ominus\oplus}_{iji'j'}$ satisfying

$$a^{\oplus\ominus}_{iji'j'}=\frac{1}{2}\epsilon_{ijkl}a^{\oplus\ominus}_{kli'j'}, \quad a^{\oplus\ominus}_{iji'j'}=-\frac{1}{2}\epsilon_{i'j'k'l'}a^{\oplus\ominus}_{ijk'l'}, \tag{3.53}$$

$$a^{\ominus\oplus}_{iji'j'}=-\frac{1}{2}\epsilon_{ijkl}a^{\ominus\oplus}_{kli'j'}, \quad a^{\ominus\oplus}_{iji'j'}=\frac{1}{2}\epsilon_{i'j'k'l'}a^{\ominus\oplus}_{ijk'l'}. \tag{3.54}$$

The $SO(4)\times SO'(4)$ labels of these irreducible parts may be found in Table I.

C. Fermionic fields

Let us now extend the above analysis to the fermionic fields of type IIB supergravity.

1. Spin 1/2 field

The equation of motion for the two Majorana-Weyl negative chirality spin 1/2 fields combined into one 32-component Weyl spinor field Λ [15],

$$\left(\Gamma^{\underline{m}}D_{\underline{m}}-\frac{\mathrm{i}}{480}\Gamma^{\underline{m}_1\cdots\underline{m}_5}F_{\underline{m}_1\ldots\underline{m}_5}\right)\Lambda=0, \tag{3.55}$$

can be rewritten in terms of the complex-valued 16-component spinor field λ (see the Appendix for notation):

$$\left(\gamma^{\underline{m}}D_{\underline{m}}-\frac{\mathrm{i}}{480}\gamma^{\underline{m}_1\cdots\underline{m}_5}F_{\underline{m}_1\ldots\underline{m}_5}\right)\lambda=0, \quad \Lambda=\begin{pmatrix}0\\ \lambda_\alpha\end{pmatrix}. \tag{3.56}$$

Here $\gamma^{\underline{m}}=e^{\underline{m}}_{\mu}\gamma^\mu$ where $e^{\underline{m}}_{\mu}$ is the (inverse) vielbein matrix. We use the following vielbein basis corresponding to the metric (1.1) ($e^\mu=e^\mu_{\underline{m}}dx^{\underline{m}}$):

$$e^+=dx^+, \quad e^-=dx^--\frac{\mathrm{f}^2}{2}x_I^2dx^+, \quad e^I=dx^I. \tag{3.57}$$

The spinor covariant derivative $D_{\underline{m}}=\partial_{\underline{m}}+\frac{1}{4}\omega^{\mu\nu}_{\underline{m}}\overline{\gamma}^{\mu\nu}$ then takes the following explicit form:

$$D_-=\partial_-, \quad D_I=\partial_I, \quad D_+=\partial_+-\frac{\mathrm{f}^2}{2}x^I\overline{\gamma}^{+I}. \tag{3.58}$$

Taking into account the background value of the 5-form field (1.2) we get

$$\left[\gamma^+\left(\partial^-+\frac{\mathrm{f}^2}{2}x_I^2\partial^+-\mathrm{if}\overline{\Pi}\right)+\gamma^-\partial^++\gamma^I\partial^I\right]\lambda=0, \tag{3.59}$$

where we used that

$$\gamma^{\underline{m}_1\cdots\underline{m}_5}F_{\underline{m}_1\ldots\underline{m}_5}=480\mathrm{f}\gamma^+\overline{\Pi}. \tag{3.60}$$

Decomposing λ as

$$\lambda=\lambda^\oplus+\lambda^\ominus, \quad \lambda^\oplus=\frac{1}{2}\overline{\gamma}^-\gamma^+\lambda, \quad \lambda^\ominus=\frac{1}{2}\overline{\gamma}^+\gamma^-\lambda, \tag{3.61}$$

we find that in the light-cone description $\lambda^\ominus$ is non-dynamical mode expressed in terms of the physical mode $\lambda^\oplus$:

$$\lambda^\ominus=\frac{1}{2\partial^+}\overline{\gamma}^I\partial^I\gamma^+\lambda^\oplus, \quad (\Box-2\mathrm{if}\overline{\Pi}\partial^+)\lambda^\oplus=0. \tag{3.62}$$

Decomposing $\lambda^\oplus$ further as [cf. Eq. (2.63)]

$$\lambda^\oplus=\lambda^\oplus_R+\lambda^\oplus_L, \quad \lambda_R\equiv\frac{1+\overline{\Pi}}{2}\lambda, \quad \lambda_L\equiv\frac{1-\overline{\Pi}}{2}\lambda, \tag{3.63}$$

we get the diagonal equations of the desired form (3.15):

$$(\Box-2\mathrm{if}\partial^+)\lambda^\oplus_R=0, \quad (\Box+2\mathrm{if}\partial^+)\lambda^\oplus_L=0. \tag{3.64}$$

Then from Eq. (3.17) we conclude that the lowest values of the light-cone energy for the fields $\lambda^\oplus_R$, $\lambda^\oplus_L$ are

$$\mathcal{E}_0(\lambda^\oplus_R)=3, \quad \mathcal{E}_0(\lambda^\oplus_L)=5. \tag{3.65}$$

2. Spin 3/2 field

The equation for the positive chirality gravitino in the 32-component notation is[10]

$$\Gamma^{\underline{m}\,\underline{m}_1\underline{m}_2}\left(D_{\underline{m}_1}+\frac{\mathrm{i}}{960}\Gamma^{\underline{n}_1\cdots\underline{n}_5}F_{\underline{n}_1\ldots\underline{n}_5}\Gamma_{\underline{m}_1}\right)\Psi_{\underline{m}_2}=0. \tag{3.66}$$

In the 16-component notation it becomes

$$\bar{\gamma}^{\underline{m}\,\underline{m}_1\underline{m}_2}\left(D_{\underline{m}_1}+\frac{\mathrm{i}}{960}\gamma^{\underline{n}_1\cdots\underline{n}_5}F_{\underline{n}_1\ldots\underline{n}_5}\bar{\gamma}_{\underline{m}_1}\right)\psi_{\underline{m}_2}=0,$$

$$\Psi_{\underline{m}}=\begin{pmatrix}\psi^{\alpha}_{\underline{m}}\\ 0\end{pmatrix}. \tag{3.67}$$

This can be rewritten as

$$\bar{\gamma}^{\underline{n}}D_{\underline{n}}\psi_{\underline{m}}-D_{\underline{m}}\psi-\frac{\mathrm{i}}{960}\bar{\gamma}^{\underline{n}}\gamma^{\underline{n}_1\cdots\underline{n}_5}F_{\underline{n}_1\ldots\underline{n}_5}\bar{\gamma}_{\underline{m}}\psi_{\underline{n}}=0,$$

$$\psi\equiv\bar{\gamma}^{\underline{m}}\psi_{\underline{m}}. \tag{3.68}$$

Making use of Eq. (3.60) we get

$$\bar{\gamma}^{\underline{n}}D_{\underline{n}}\psi_{\underline{m}}-D_{\underline{m}}\psi-\frac{\mathrm{if}}{2}\bar{\gamma}^{\underline{n}}\Pi\,\gamma^{+}\bar{\gamma}_{\underline{m}}\psi_{\underline{n}}=0, \tag{3.69}$$

and impose the light-cone gauge for the gravitino field

$$\psi_{-}=0. \tag{3.70}$$

Equation (3.69) for $\underline{m}=-$ then gives

$$\psi=\bar{\gamma}^{+}\psi_{+}+\bar{\gamma}^{I}\psi_{I}=0,\quad \text{i.e.}\quad \gamma^{+}\bar{\gamma}^{I}\psi_{I}=0. \tag{3.71}$$

As a consequence,

$$\bar{\gamma}^{J}\Pi\,\gamma^{+}\bar{\gamma}_{i}\psi_{J}=2\overline{\Pi\gamma}^{+}(\delta_{ij}-\gamma_{i}\bar{\gamma}_{j})\psi_{j},$$

$$\bar{\gamma}^{J}\Pi\,\gamma^{+}\bar{\gamma}_{i'}\psi_{J}=-2\overline{\Pi\gamma}^{+}(\delta_{i'j'}-\gamma_{i'}\bar{\gamma}_{j'})\psi_{j'}. \tag{3.72}$$

With the help of these relations the $\underline{m}=i$ component of Eq. (3.69) becomes

$$\left[\bar{\gamma}^{+}\left(\partial^{-}+\frac{\mathrm{f}^2}{2}x_I^2\partial^{+}\right)+\bar{\gamma}^{-}\partial^{+}+\bar{\gamma}^{J}\partial_{J}\right]\psi_{i}-\mathrm{if}\overline{\Pi\gamma}^{+}$$
$$\times(\delta_{ij}-\gamma_{i}\bar{\gamma}_{j})\psi_{j}=0. \tag{3.73}$$

Decomposing the gravitino field into the physical mode $\psi_i^{\oplus}$ and non-dynamical mode $\psi_i^{\ominus}$ as in Eq. (3.61) we get, from Eq. (3.73) (acting by γ^{+} or by γ^{-}),

$$\Box\,\psi_i^{\oplus}-2\mathrm{if}\Pi(\delta_{ij}-\gamma_i\bar{\gamma}_j)\partial^{+}\psi_j^{\oplus}=0,$$

$$\psi_I^{\ominus}=-\frac{1}{2\partial^{+}}\gamma^{+}(\bar{\gamma}^{J}\partial_J)\psi_I^{\oplus}. \tag{3.74}$$

The other non-dynamical mode ψ_{+} [split into $\psi_{+}^{\oplus}$ and $\psi_{+}^{\ominus}$ as in Eq. (3.61)] is found from Eq. (3.71) and the $\underline{m}=+$ component of the gravitino equation (3.69):

$$\psi_{+}^{\oplus}=-\frac{1}{\partial^{+}}\partial_I\psi_I^{\oplus},\qquad \psi_{+}^{\ominus}=-\frac{1}{2\partial^{+}}\gamma^{+}\bar{\gamma}^{I}\partial_I\psi_{+}^{\oplus}. \tag{3.75}$$

Decomposing the dynamical gravitino mode $\psi_I^{\oplus}$ into the γ-transverse and γ-parallel parts as

$$\psi_i^{\oplus\perp}\equiv\left(\delta_{ij}-\frac{1}{4}\gamma_i\bar{\gamma}_j\right)\psi_j^{\oplus},\qquad \psi^{\oplus\parallel}\equiv\bar{\gamma}_i\psi_i^{\oplus} \tag{3.76}$$

we find

$$(\Box-2\mathrm{if}\Pi\,\partial^{+})\psi_i^{\oplus\perp}=0,\qquad (\Box-6\mathrm{if}\bar{\Pi}\partial^{+})\psi^{\oplus\parallel}=0. \tag{3.77}$$

As in the spin 1/2 case, to diagonalize these equations we introduce [cf. Eq. (3.63)]

$$\psi_{iR}^{\oplus\perp}=\frac{1+\Pi}{2}\psi_i^{\oplus\perp},\qquad \psi_{iL}^{\oplus\perp}=\frac{1-\Pi}{2}\psi_i^{\oplus\perp},$$

$$\psi_R^{\oplus\parallel}=\frac{1+\bar{\Pi}}{2}\psi^{\oplus\parallel},\qquad \psi_L^{\oplus\parallel}=\frac{1-\bar{\Pi}}{2}\psi^{\oplus\parallel}. \tag{3.78}$$

This gives finally

$$(\Box-2\mathrm{if}\partial^{+})\psi_{iR}^{\oplus\perp}=0,\qquad (\Box+2\mathrm{if}\partial^{+})\psi_{iL}^{\oplus\perp}=0,$$

$$(\Box-6\mathrm{if}\partial^{+})\psi_R^{\oplus\parallel}=0,\qquad (\Box+6\mathrm{if}\partial^{+})\psi_L^{\oplus\parallel}=0. \tag{3.79}$$

These equations give, according to Eqs. (3.15), (3.17) the following values of the minimal energy $\mathcal{E}_0$ for the respective physical gravitino modes:

$$\mathcal{E}_0(\psi_{iR}^{\oplus\perp})=3,\qquad \mathcal{E}_0(\psi_{iL}^{\oplus\perp})=5,\qquad \mathcal{E}_0(\psi_R^{\oplus\parallel})=1,$$

$$\mathcal{E}_0(\psi_L^{\oplus\parallel})=7. \tag{3.80}$$

A similar analysis applies to the gravitino components $\psi_{i'}$. In this case we get [cf. Eq. (3.74)]

$$\Box\,\psi_{i'}^{\oplus}+2\mathrm{if}\Pi(\delta_{i'j'}-\gamma_{i'}\bar{\gamma}_{j'})\partial^{+}\psi_{j'}^{\oplus}=0, \tag{3.81}$$

and as a result

$$\mathcal{E}_0(\psi_{i'R}^{\oplus\perp})=5,\qquad \mathcal{E}_0(\psi_{i'L}^{\oplus\perp})=3. \tag{3.82}$$

As for the γ-parallel part $\psi'^{\,\oplus\parallel}=\bar{\gamma}_{i'}\psi_{i'}^{\oplus}$ of $\psi_{i'}$, it does not represent an independent dynamical mode being related to $\psi^{\oplus\parallel}$ through Eq. (3.71), i.e. $\bar{\gamma}^{I}\psi_I^{\oplus}=0$.

[10] The 5-form term in the gravitino equation was missing in [15] but its presence is implied by the supersymmetry transformations given there and in [19]. This term was explicitly included in [10].

TABLE I. Spectrum of bosonic physical on-shell fields.

$\mathcal{E}_0$	Field and N_{dof}	Energy spectrum k⩾0	$SO(4)\times SO'(4)$ labels	Term in superfield expansion
0	h(2)	k	(0,0)×(0,0)	θ_R^4
2	$\mathrm{h}_{ij'}(32)$	k+2	(1,0)×(1,0)	$\theta_R\overline{\gamma}^{-ik}\theta_R\theta_R\overline{\gamma}^{-j'k}\theta_L$
2	$\overline{b}_{ij}^{\oplus}(6)$	k+2	(1,1)×(0,0)	$\theta_R^4(\theta_L\pi^{\oplus}\overline{\gamma}^{-ij}\theta_L)$
2	$\overline{b}_{i'j'}^{\oplus}(6)$	k+2	(0,0)×(1,1)	$\theta_R^4(\theta_L\pi^{\oplus}\overline{\gamma}^{-i'j'}\theta_L)$
2	$b_{ij}^{\ominus}(6)$	k+2	(1,1)×(0,0)	$\theta_R\pi^{\ominus}\overline{\gamma}^{-ij}\theta_R$
2	$b_{i'j'}^{\ominus}(6)$	k+2	(0,0)×(1,−1)	$\theta_R\pi^{\ominus}\overline{\gamma}^{-i'j'}\theta_R$
4	$\phi(2)$	k+4	(0,0)×(0,0)	1
4	$\overline{\phi}(2)$	k+4	(0,0)×(0,0)	$\theta_R^4\theta_L^4$
4	$h_{ij}^{\perp}(9)$	k+4	(2,0)×(0,0)	$\theta_R\overline{\gamma}^{-k(i}\theta_R\theta_L\overline{\gamma}^{j)k-}\theta_L$
4	$h_{i'j'}^{\perp}(9)$	k+4	(0,0)×(2,0)	$\theta_R\overline{\gamma}^{-k'(i'}\theta_R\theta_L\overline{\gamma}^{j')k'-}\theta_L$
4	$a_{iji'j'}^{\oplus\ominus}(9)$	k+4	(1,1)×(1,−1)	$\theta_L\pi^{\oplus}\overline{\gamma}^{-ij}\theta_L\theta_R\pi^{\ominus}\overline{\gamma}^{-i'j'}\theta_R$
4	$a_{iji'j'}^{\ominus\oplus}(9)$	k+4	(1,−1)×(1,1)	$\theta_R\pi^{\ominus}\overline{\gamma}^{-ij}\theta_R\theta_L\pi^{\oplus}\overline{\gamma}^{-i'j'}\theta_L$
4	$b_{ij'}(32)$	k+4	(1,0)×(1,0)	$\theta_R\overline{\gamma}^{-ij'}\theta_L$
6	$b_{ij}^{\oplus}(6)$	k+6	(1,1)×(0,0)	$\theta_L\pi^{\oplus}\overline{\gamma}^{-ij}\theta_L$
6	$b_{i'j'}^{\oplus}(6)$	k+6	(0,0)×(1,1)	$\theta_L\pi^{\oplus}\overline{\gamma}^{-i'j'}\theta_L$
6	$\overline{b}_{ij}^{\ominus}(6)$	k+6	(1,−1)×(0,0)	$\theta_L^4(\theta_R\pi^{\ominus}\overline{\gamma}^{-ij}\theta_R)$
6	$\overline{b}_{i'j'}^{\ominus}(6)$	k+6	(0,0)×(1,−1)	$\theta_L^4(\theta_R\pi^{\ominus}\overline{\gamma}^{-i'j'}\theta_R)$
6	$\overline{\mathrm{h}}_{ij'}(32)$	k+6	(1,0)×(1,0)	$\theta_L\overline{\gamma}^{-ik}\theta_L\theta_R\overline{\gamma}^{-j'k}\theta_L$
8	$\overline{\mathrm{h}}(2)$	k+8	(0,0)×(0,0)	θ_L^4

D. Light-cone gauge superfield formulation of type IIB supergravity on the plane wave background

Before proceeding, let us first summarize the results of the above analysis in two tables: one for the bosonic modes and other for the fermionic modes. In Tables I and II in the $\mathcal{E}_0$ column we indicate lowest eigenvalues of the light-cone energy operator of the corresponding field. The energy spectrum of higher "Kaluza-Klein" modes (obtained by further action by the bosonic zero-mode creation operators a_0^I) is labeled by k, where k=0 corresponds to the ground state. Note, however, that these are not the usual Kaluza-Klein-type modes because the action of the symmetry algebra of the plane wave background mixes modes with different values of k. This algebra can be thus viewed as a spectrum generating algebra for the "Kaluza-Klein" modes.

In the fourth column we have given the Gelfand-Zetlin labels of the corresponding $SO(4)\times SO'(4)$ representations. In the last column we indicated the monomials in fermionic zero modes θ_L,θ_R which accompany the corresponding field components in the θ expansion of the light-cone superfield discussed below.

In the rest of this section we shall present the light-cone gauge superfield description of type IIB supergravity in the plane wave RR background. As in flat space, the equations for the physical modes we have found above can be summarized in a light-cone superfield form. The corresponding unconstrained scalar superfield $\Phi(x,\theta_0)$ will satisfy the "massless" equation, invariant under the dilatational invariance in superspace.

Finding even the quadratic part of the action for fluctuations of the supergravity fields in a curved background is a complicated problem.[11] We could in principle use the covariant superfield description of type IIB supergravity [21], starting with linearized expansion of superfields, imposing light-cone gauge on fluctuations and then solving the constraints to eliminate non-physical degrees of freedom in terms of physical ones. That would be quite tedious. The light-cone gauge approach is self-contained, i.e. does not rely upon existence of a covariant description, and provides a much shorter route to final results.

There are two methods of finding the light-cone gauge formulation of the type II supergravity. One [22] reduces the problem of constructing a new (light-cone gauge) dynamical system to finding a new solution of the commutation relations of the defining symmetry algebra. This method of Dirac was applied to the case of supergravity in $AdS_5\times S^5$ and $AdS_3\times S^3$ in [23] and [24].[12] The second method is based on finding the equations of motion by using the Casimir operators of the symmetry algebra. Here we shall fol-

[11] In the case of the $AdS_5\times S^5$ background in covariant gauge it was solved in [20].

[12] The application of this method to a superfield formulation of interaction vertices of $D=11$ supergravity may be found in [25] (see also [26] for various related discussions).

TABLE II. Spectrum of fermionic physical on-shell fields.

$\mathcal{E}_0$	Field and N_{dof}	Energy spectrum k$\geqslant$0	$SO(4)\times SO'(4)$ labels	Term in superfield expansion
1	$\psi_R^{\oplus\parallel}(8)$	k+1	$\left(\frac{1}{2},-\frac{1}{2}\right)\times\left(\frac{1}{2},\frac{1}{2}\right)$	θ_R^3
1	$\overline{\psi_L^{\oplus\parallel}}(8)$	k+1	$\left(\frac{1}{2},\frac{1}{2}\right)\times\left(\frac{1}{2},-\frac{1}{2}\right)$	$\theta_R^4\theta_L$
3	$\psi_{iR}^{\oplus\perp}(24)$	k+3	$\left(\frac{3}{2},-\frac{1}{2}\right)\times\left(\frac{1}{2},\frac{1}{2}\right)$	$(\theta_R\overline{\gamma}^{-ij}\theta_R)\overline{\gamma}^j\theta_L$
3	$\psi_{i'L}^{\oplus\perp}(24)$	k+3	$\left(\frac{1}{2},\frac{1}{2}\right)\times\left(\frac{3}{2},-\frac{1}{2}\right)$	$(\theta_R\overline{\gamma}^{-i'j'}\theta_R)\overline{\gamma}^{j'}\theta_L$
3	$\overline{\psi_{iL}^{\oplus\perp}}(24)$	k+3	$\left(\frac{3}{2},\frac{1}{2}\right)\times\left(\frac{1}{2},-\frac{1}{2}\right)$	$\theta_R^3\theta_L^2$
3	$\overline{\psi_{i'R}^{\oplus\perp}}(24)$	k+3	$\left(\frac{1}{2},-\frac{1}{2}\right)\times\left(\frac{3}{2},\frac{1}{2}\right)$	$\theta_R^3\theta_L^2$
3	$\lambda_R^{\oplus}(8)$	k+3	$\left(\frac{1}{2},-\frac{1}{2}\right)\times\left(\frac{1}{2},\frac{1}{2}\right)$	θ_R
3	$\overline{\lambda_L^{\oplus}}(8)$	k+3	$\left(\frac{1}{2},\frac{1}{2}\right)\times\left(\frac{1}{2},-\frac{1}{2}\right)$	$\theta_R^4\theta_L^3$
5	$\lambda_L^{\oplus}(8)$	k+5	$\left(\frac{1}{2},\frac{1}{2}\right)\times\left(\frac{1}{2},-\frac{1}{2}\right)$	θ_L
5	$\overline{\lambda_R^{\oplus}}(8)$	k+5	$\left(\frac{1}{2},-\frac{1}{2}\right)\times\left(\frac{1}{2},\frac{1}{2}\right)$	$\theta_R^3\theta_L^4$
5	$\psi_{iL}^{\oplus\perp}(24)$	k+5	$\left(\frac{3}{2},\frac{1}{2}\right)\times\left(\frac{1}{2},-\frac{1}{2}\right)$	$(\theta_L\overline{\gamma}^{-ij}\theta_L)\overline{\gamma}^j\theta_R$
5	$\psi_{i'R}^{\oplus\perp}(24)$	k+5	$\left(\frac{1}{2},-\frac{1}{2}\right)\times\left(\frac{3}{2},\frac{1}{2}\right)$	$(\theta_L\overline{\gamma}^{-i'j'}\theta_L)\overline{\gamma}^{j'}\theta_R$
5	$\overline{\psi_{iR}^{\oplus\perp}}(24)$	k+5	$\left(\frac{3}{2},-\frac{1}{2}\right)\times\left(\frac{1}{2},\frac{1}{2}\right)$	$\theta_R^2\theta_L^3$
5	$\overline{\psi_{i'L}^{\oplus\perp}}(24)$	k+5	$\left(\frac{3}{2},\frac{1}{2}\right)\times\left(\frac{1}{2},-\frac{1}{2}\right)$	$\theta_R^2\theta_L^3$
7	$\psi_L^{\oplus\parallel}(8)$	k+7	$\left(\frac{1}{2},\frac{1}{2}\right)\times\left(\frac{1}{2},-\frac{1}{2}\right)$	θ_L^3
7	$\overline{\psi_R^{\oplus\parallel}}(8)$	k+7	$\left(\frac{1}{2},-\frac{1}{2}\right)\times\left(\frac{1}{2},\frac{1}{2}\right)$	$\theta_L^4\theta_R$

low this second approach.

The basic light-cone gauge superfield will be denoted as $\Phi(x,\theta)$ and will have the following expansion in powers of the Grassmann coordinates θ:[13]

$$\Phi(x,\theta)=\partial^{+2}A+\theta^a\partial^+\psi_a+\theta^{a_1}\theta^{a_2}\partial^+A_{a_1a_2}+\theta^{a_1}\theta^{a_2}\theta^{a_3}\psi_{a_1a_2a_3}+\theta^{a_1}\dots\theta^{a_4}A_{a_1\dots a_4}-(\epsilon\theta^5)_{a_1a_2a_3}\frac{\mathrm{i}}{\partial^+}\psi^{a_1a_2a_3*}-(\epsilon\theta^6)_{a_1a_2}\frac{1}{\partial^+}A^{a_1a_2*}+(\epsilon\theta^7)_a\frac{\mathrm{i}}{\partial^{+2}}\psi^{a*}+(\epsilon\theta^8)\frac{1}{\partial^{+2}}A^*, \tag{3.83}$$

where $\epsilon_{a_1\dots a_8}$ is the spinorial Levi-Civitá tensor, i.e.

$$(\epsilon\theta^{8-n})_{a_1\dots a_n}\equiv\frac{1}{(8-n)!}\epsilon_{a_1\dots a_na_{n+1}\dots a_8}\theta^{a_{n+1}}\dots\theta^{a_8}. \tag{3.84}$$

Here we use the following Hermitian conjugation rule: $(\theta_1\theta_2)^\dagger=\theta_2^\dagger\theta_1^\dagger$. This superfield has a certain reality property: the component field for the monomial θ^n is complex conjugated to the one for θ^{8-n}. This reality constraint can be written in the superfield notation as

$$\Phi(x,\theta)=\int d^8\theta^\dagger e^{\mathrm{i}(\partial^+)^{-1}\theta\theta^\dagger}(\partial^+)^4(\Phi(x,\theta))^\dagger. \tag{3.85}$$

In what follows we will use again the 16-component spinor

$$\theta^\alpha=\begin{pmatrix}\theta^a\\ 0\end{pmatrix}.$$

Decomposing it into θ_R and θ_L as in Eq. (2.63) we can expand the superfield Φ in terms of these anticommuting coordinates.

The expansion in this basis can be used to identify the superfield components with physical on-shell modes of type IIB supergravity fields found earlier in this section. The corresponding monomials in $\theta_{L,R}$ are shown in Tables I and II. The dilaton field ϕ is the lowest superfield component, while its complex conjugate $\overline{\phi}$ appears in the last component multiplying $\theta_R^4\theta_L^4$. As another example, consider the antisymmetric second rank complex tensor field modes $b_{ij}^{\oplus}$ and $b_{ij}^{\ominus}$. According to Table I, they correspond to the monomials $\theta_L\pi^{\oplus}\gamma^{-ij}\theta_L$ and $\theta_R\pi^{\ominus}\gamma^{-ij}\theta_R$ where we used the following notation for the self-dual projectors ($\pi^{\oplus}\gamma^{-ij}\equiv\pi_{ij;kl}^{\oplus}\gamma^{-kl}$):

$$\pi_{ij;kl}^{\oplus}=\frac{1}{4}(\delta_{ik}\delta_{jl}-\delta_{il}\delta_{jk}+\epsilon_{ijkl}),$$

$$\pi_{ij;kl}^{\ominus}=\frac{1}{4}(\delta_{ik}\delta_{jl}-\delta_{il}\delta_{jk}-\epsilon_{ijkl}). \tag{3.86}$$

Let us now determine the equations of motion for the scalar superfield Φ. For this we will need the explicit form of the second-order Casimir operator for the plane wave superalgebra described in Sec. II C:

[13]Here we omit the index 0 on the light-cone fermionic zero-mode variable θ_0, denoting it simply as θ. To simplify the expressions for the superfield expansion and its reality constraint we solve the light-cone gauge constraint $\overline{\gamma}^+\theta=0$ in terms of eight fermions θ^a ($a=1,\dots,8$) by using the representation for γ^0 in Eq. (A8) and $\gamma^9=\mathrm{diag}(1_8,-1_8)$.

$$\mathcal{C}=2P^{+}P^{-}+P^{I}P^{I}+\mathrm{f}^{2}J^{+I}J^{+I}-\frac{1}{2}\mathrm{f}\overline{Q}^{+}\Pi\gamma^{+}Q^{+}. \tag{3.87}$$

The representations of the generators of the plane-wave superalgebra in terms of differential operators acting of $\Phi(x,\theta)$ may be found by using the standard supercoset method[14] [cf. Eqs. (2.37)–(2.39)]:

$$P^{+}=\partial^{+},\quad P^{-}=\partial^{-},\quad P^{I}=\cos \mathrm{f}x^{+}\partial^{I}+\mathrm{f}\sin \mathrm{f}x^{+}x^{I}\partial^{+}, \tag{3.88}$$

$$J^{+I}=\mathrm{f}^{-1}\sin \mathrm{f}x^{+}\partial^{I}-\cos \mathrm{f}x^{+}x^{I}\partial^{+},$$

$$J^{IJ}=x^{I}\partial^{J}-x^{J}\partial^{I}+\frac{1}{2}\partial_{\theta}\gamma^{IJ}\theta, \tag{3.89}$$

$$Q^{+}=-2\mathrm{i}\partial^{+}\overline{\gamma}^{-}e^{\mathrm{i}\mathrm{f}x^{+}\Pi}\theta,\quad \overline{Q}^{+}=\frac{1}{2}\overline{\gamma}^{-}e^{-\mathrm{i}\mathrm{f}x^{+}\Pi}\gamma^{+}\partial_{\theta},\quad \{\partial_{\theta\beta},\theta^{\alpha}\}=\frac{1}{2}(\gamma^{+}\overline{\gamma}^{-})^{\alpha}{}_{\beta}. \tag{3.90}$$

The projector on the right-hand side (RHS) of the definition of the fermionic derivatives ∂_{θ} in Eq. (3.90) reflects the fact that θ satisfies the light-cone gauge condition. Plugging these expressions into Eq. (3.87) we find

$$\mathcal{C}=\Box-2\mathrm{i}\mathrm{f}\partial^{+}\theta\overline{\Pi}\partial_{\theta}, \tag{3.91}$$

where $\Box$ was defined in Eq. (3.9).

In a general curved background the equations of motion for the superfield Φ take the form $(\mathcal{C}-\mathcal{C}_0)\Phi=0$, where the constant term $\mathcal{C}_0$ should be fixed by an additional requirement. For example, in the case of AdS space, $\mathcal{C}_0$ is expressed in terms of constant curvature of the background. In the present case of the plane wave background the $\mathcal{C}_0$ can be fixed by using the so-called ''*sim*'' invariance—the invariance under the original plane-wave superalgebra supplemented by the scale-invariance condition, i.e. by the condition of dilatational invariance in superspace [12].[15] The generator D of dilatations in the light-cone superspace ($\lambda=\mathrm{const}$)

$$\delta x^{+}=0,\quad \delta x^{-}=2\lambda x^{-},\quad \delta x^{I}=\lambda x^{I},\quad \delta\theta=\lambda\theta, \tag{3.92}$$

has the obvious form

$$\mathrm{D}=2x^{-}\partial^{+}+x^{I}\partial^{I}+\theta\partial_{\theta}. \tag{3.93}$$

The requirement of *sim* invariance of the superfield equations of motion amounts to the condition $[\mathrm{D},\mathcal{C}]\Phi=0$. Since, as it is easy to see from Eq. (3.91), $[\mathrm{D},\mathcal{C}]=-2\mathcal{C}$, it follows then that the only *sim*-invariant equation of motion is simply

$$\mathcal{C}\Phi=0,\quad \text{i.e.}\quad (\Box-2\mathrm{i}\mathrm{f}\partial^{+}\theta\overline{\Pi}\partial_{\theta})\Phi(x,\theta)=0. \tag{3.94}$$

The corresponding quadratic term in the superfield light-cone gauge action is then

$$S_{l.c.}=\frac{1}{2}\int d^{10}x d^{8}\theta\Phi(x,\theta)(\Box-2\mathrm{i}\mathrm{f}\partial^{+}\theta\overline{\Pi}\partial_{\theta})\Phi(x,\theta). \tag{3.95}$$

Splitting the fermionic coordinate θ into θ_R and θ_L parts as in Eq. (2.63) one can rewrite Eq. (3.94) as

$$[\Box+2\mathrm{i}\mathrm{f}\partial^{+}(\theta_L\partial_{\theta_L}-\theta_R\partial_{\theta_R})]\Phi(x,\theta_R,\theta_L)=0. \tag{3.96}$$

This remarkably simple equation summarizes all the field equations for the physical fluctuation modes of type IIB supergravity fields in the present RR plane-wave background [i.e. the components of Φ, Eq. (3.83)] which were derived earlier in this section. In particular, the universal expression for the lowest values of the light-cone energy operator can be found by applying Eqs. (3.15), (3.17) to the case of Eq. (3.96):

$$E_0=\mathrm{f}(4+\theta_L\partial_{\theta_L}-\theta_R\partial_{\theta_R}). \tag{3.97}$$

This reproduces the values of $\mathcal{E}_0$ in Tables I and II.

IV. CONCLUDING REMARKS

In this paper we presented the quantization of type IIB string theory in the maximally supersymmetric RR plane-wave background of [6] whose light-cone gauge action was found in [5]. We explicitly constructed the quantum light-cone Hamiltonian and the string representation of the corresponding supersymmetry algebra. The superstring Hamiltonian has the standard ''harmonic-oscillator'' form, i.e. is quadratic in creation-annihilation operators in all 8 transverse directions, so that its spectrum can be readily obtained.

We have discussed in detail the structure of the zero-mode sector of the theory, giving it the space-time field-theoretic interpretation by establishing the precise correspondence between the lowest-lying ''massless'' string states and the type IIB supergravity fluctuation modes in the plane-wave background.

The ''massless'' (supergravity) part of the spectrum has certain similarities with the supergravity spectrum found [10] in the case of another maximally supersymmetric type IIB background: $\mathrm{AdS}_5\times \mathrm{S}^5$ [15] (this may not be completely surprising given that the two backgrounds are related by a special limit [7]). In particular, the light-cone energy spectrum of a superstring in the RR plane-wave background is discrete. As in the AdS case [27], the discreteness of the spec-

[14]In this section we use the anti-Hermitean representation for the generators P^{μ}. The corresponding commutation relations in this representation can be found from Eqs. (2.49)–(2.59) by the substitutions $P^{\mu}\to -\mathrm{i}P^{\mu}$.

[15]In the usual 4 dimensions scale transformations (dilatations) combined with the Poincaré group form the maximal subgroup of the conformal group, or similitude group $SIM(3,1)$. Dilatation invariance ensures masslessness, so the direct generalization to the supergroup case should give a criterion of masslessness for the superfields.

trum depends on a particular natural choice of the boundary conditions. In the present case they are the same as in the standard harmonic oscillator problem: the square integrability of the wave functions in all 8 transverse spatial directions.

An interesting feature of the plane-wave string spectrum is its non-trivial dependence on p^+. This is possible due to the fact that the generator P^+ commutes with all other generators of the symmetry superalgebra. We defined the spectrum in terms of the light-cone energy $H=-P^-$, which does not depend on p^+ for the massless (zero-mode) states but does depend on it for the string oscillator modes. In general, one may define the string spectrum in curved space in terms of the second-order Casimir operator of the corresponding superalgebra. In the present case the eigen-values of this operator depend on discrete quantum numbers as well as on p^+ (through the dimensionless combination $\mathrm{m}=2\pi\alpha' p^+ \mathrm{f}$ with the curvature scale f and the string scale α').

Given the exact solvability of this plane-wave string theory, there are many standard flat-space string calculations that can be straightforwardly repeated in this case. One can determine the vertex operators for the "massless" superstring states and compute the 3-point and 4-point correlation functions, following the same strategy as in the light-cone Green-Schwarz approach to flat superstring theory.[16] It would be interesting to compare (the $\alpha'\to 0$ limits of) the plane-wave string results to the corresponding correlation functions in the type IIB supergravity on $AdS_5\times S^5$. One can also find possible D-brane configurations, by imposing open-string boundary conditions in some directions and repeating the analysis of Sec. II.[17]

Let us comment on some limits of this plane-wave string theory. It depends on the two mass parameters which enter the Hamiltonian (2.33): the curvature scale f and the string scale $(\alpha' p^+)^{-1}$. The limit $\mathrm{f}\to 0$ is the flat-space limit: the discrete spectrum then becomes the standard type IIB flat-space string spectrum (in the same sense in which the harmonic oscillator spectrum reduces to the spectrum of a free particle in the zero-frequency limit). The $\mathrm{f}\to\infty$ limit is not special: it corresponds simply to a rescaling of the light-cone energy and p^+ [recall that f in Eq. (1.1) can be set to 1 by a rescaling of x^+ and x^-].

The limit $\alpha' p^+\to 0$ corresponds to the supergravity in the plane-wave background: the string Hamiltonian (2.21), (2.31), (2.32) becomes infinite on all states that contain non-zero string oscillators; i.e., it effectively reduces to E_0, Eq. (2.31), restricted to the subspace of the zero-mode states. The opposite ("zero-tension") limit $\alpha' p^+\to\infty$ is also regular: it follows from Eq. (2.33) that here we are left with

$$H_{\alpha' p^+\to\infty}=\mathrm{f}\Bigg[(a_0^I\overline{a_0^I}+2\,\overline{\theta_0}\,\overline{\gamma^-}\,\Pi\,\theta_0+4)+\sum_{\mathcal{I}=1,2}\sum_{n=1}^{\infty}(a_n^{\mathcal{I}I}\overline{a_n^{\mathcal{I}I}}+\eta_n^{\mathcal{I}}\overline{\gamma^-}\,\overline{\eta_n^{\mathcal{I}}})\Bigg]. \tag{4.1}$$

The constraint (2.36) remains the same as it does not involve α'. This provides an interesting example of a non-trivial "*null-string*" spectrum which is worth further study. Note, in particular, that here the energies do not grow with the oscillator level number *n*; i.e., there are no Regge-type trajectories.[18]

Let us now compare the plane-wave string spectrum with the expected form of the light-cone spectrum of the superstring in an $AdS_5\times S^5$ background. In general, the spectrum of the light-cone Hamiltonian $\mathcal{H}=-\mathcal{P}^-$ in $AdS_5\times S^5$ [9] should depend on two characteristic mass parameters: the curvature scale R^{-1} (the inverse AdS radius),[19] which is the analogue of f in Eq. (1.1), and the string mass scale $\sqrt{\alpha'}$. In the context of the standard AdS conformal field theory (CFT) correspondence the coordinates should be rescaled so that R is always combined with α' into the effective dimensionless tension parameter $\mathrm{T}=R^2T=R^2/2\pi\alpha'=\sqrt{\lambda}/2\pi$. In contrast to the plane-wave case, here the dependence of $\mathcal{H}$ on p^+ can only be the trivial one, i.e. only through the $1/p^+$ factor [in Poincaré coordinates the $AdS_5\times S^5$ background has Lorentz invariance in the $(+,-)$ directions]. Let us recall the form of the light-cone string Hamiltonian using the "conformally flat" 10D coordinates (x^a,Z^M) in which the $AdS_5\times S^5$ metric is (here $a=0,1,2,3;M=1,\dots,6$)

$$ds^2=R^2Z^{-2}(dx^a dx^a+dZ^M dZ^M). \tag{4.2}$$

Splitting the 4D coordinates as $x^a=(x^+,x^-,x^\perp)$ and using the appropriate light-cone gauge one finds the following phase space Lagrangian [9]:

$$\mathcal{L}=\mathcal{P}_\perp\dot{x}_\perp+\mathcal{P}_M\dot{Z}^M+\frac{\mathrm{i}}{2}(\theta^i\dot{\theta}_i+\eta^i\dot{\eta}_i-\mathrm{H.c.})-\mathcal{H}, \tag{4.3}$$

$$\mathcal{H}=\frac{1}{2p^+}\{\mathcal{P}_\perp^2+\mathcal{P}_M\mathcal{P}_M+\mathrm{T}^2Z^{-4}(\acute{x}_\perp^2+\acute{Z}^M\acute{Z}^M)+Z^{-2}[(\eta^2)^2+2\mathrm{i}\eta^i\rho_{ij}^{MN}\eta^jZ_M\mathcal{P}_N]-2\mathrm{T}[|Z|^{-3}\eta^i\rho_{ij}^{M}Z^M(\acute{\theta}^j-\mathrm{i}\sqrt{2}|Z|^{-1}\eta^j\acute{x}_\perp)+\mathrm{H.c.}]\}. \tag{4.4}$$

Compared to [9] we have rescaled the fermions θ^i,η^i ($i=1,2,3,4$) by $\sqrt{p^+}$ (thus absorbing all spurious p^+ dependence). $\mathcal{P}_\perp,\mathcal{P}_M$ are the momenta and ρ^{MN} is a product of Dirac matrices. Here the coordinates and momenta (includ-

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[16]Note that in the present plane-wave case we do not have the standard *S*-matrix setup: the string spectrum is discrete in all 8 transverse directions; i.e., the string states with non-zero p^+ are localized near $x_I=0$ and cannot escape to infinity.

[17]One obvious candidate is a D-string along the x^9 direction. For a light-cone gauge description of D-branes in flat space see [28].

[18]Note that the parameter f may be viewed as a "regularization" introduced to define a non-trivial tensionless string limit of the flat superstring.

[19]In the context of the standard AdS/CFT the radius R is related to the 't Hooft coupling λ by [29] $R=\lambda^{1/4}\sqrt{\alpha'}$.

ing $\mathcal{H}$ and p^+) are all dimensionless (measured in units of R), reflecting the rescaling done in Eq. (4.2). Restoring the canonical mass dimensions ($\mathcal{H}\to RH$, $p^+\to Rp^+$) the corresponding analogue of the plane-wave result (2.33) should thus have the structure

$$H=\frac{1}{p^+R^2}[\mathcal{E}_0+\mathrm{T}\mathcal{E}_{str}(\mathrm{T})]$$
$$=\frac{1}{p^+}\left[\frac{1}{R^2}\mathcal{E}_0+\frac{1}{2\pi\alpha'}\mathcal{E}_{str}\left(\frac{R^2}{2\pi\alpha'}\right)\right], \tag{4.5}$$

where $\mathcal{E}_0,\mathcal{E}_{str}$ are dimensionless functions of the parameters and discrete quantum numbers.

Here the limit $\alpha'\to 0$ or $\mathrm{T}\to\infty$ for fixed p^+R^2 corresponds to the type IIB supergravity $AdS_5\times S^5$ background with only the $\mathcal{E}_0$ part (known explicitly [10,23]) surviving on the subspace of finite mass states. The limit $R\to\infty$ with fixed p^+ should reproduce the flat space string spectrum [this suggests that $\mathcal{E}_{str}(\mathrm{T}\to\infty)$ should be finite]. The limit $\mathrm{T}\to 0$ for fixed p^+R^2 is a ‘‘null-string’’ limit [30]. Like the corresponding limit in the plane-wave case (4.1) it is expected to be well defined.

A formal correspondence between Eqs. (4.5) and (2.33) is established by identifying f with $1/(p^+R^2)$, so that m $=2\pi\alpha' p^+$f in Eq. (2.33) goes over to $2\pi\alpha'/R^2=\mathrm{T}^{-1}$. This rescaling of R^2 by p^+ ‘‘explains’’ why Eq. (4.5) does not have a non-trivial dependence on p^+ while Eq. (2.33) does.

The dependence of the string-mode part $\mathcal{E}_{str}$ of Eq. (4.5) on T should of course be much more complicated than dependence on $\alpha' p^+$f in Eq. (2.33). To determine it remains an outstanding problem.

While this work was nearing completion there appeared an interesting paper [31] which provides a gauge-theory interpretation of this plane-wave string theory based on a special limit of the AdS/CFT correspondence.

ACKNOWLEDGMENTS

This work of R.R.M. was supported by INTAS project 00-00254, by RFBR Grant 02-02-16944 and RFBR Grant 01-02-30024 for Leading Scientific Schools. The work of A.A.T. was supported in part by the DOE DE-FG02-91ER-40690, PPARC PPA/G/S/1998/00613, INTAS 991590, CRDF RPI-2108 grants and the Royal Society. We are grateful to S. Frolov and J. Russo for useful discussions.

APPENDIX: NOTATION AND DEFINITIONS

We use the following conventions for the indices:

$\underline{m},\underline{n},\underline{k}=0,1,\ldots,9$

10D space-time coordinate indices

$\mu,\nu,\rho=0,1,\ldots,9$

$so(9,1)$ vector indices (tangent space indices)

$I,J,K,L=1,\ldots,8$

$so(8)$ vector indices (tangent space indices)

$i,j,k,l=1,\ldots,4$

$so(4)$ vector indices (tangent space indices)

$i',j',k',l'=5,\ldots,8$

$so'(4)$ vector indices (tangent space indices)

$\alpha,\beta,\gamma=1,\ldots,16$ $so(9,1)$

spinor indices in chiral representation

$a,b=0,1$ 2D world-sheet coordinate indices

$\mathcal{I},\mathcal{J}=1,2$ labels of the two real MW spinors.

We identify the transverse target indices with tangent space indices, i.e. $x^{\underline{I}}=x^I$, and avoid using the underlined indices in $+$ and $-$ light-cone directions, i.e. adopt simplified notation x^+, x^-. We suppress the flat space metric tensor $\eta_{\mu\nu}=(-,+,\ldots,+)$ in scalar products, i.e. $X^\mu Y^\mu\equiv\eta_{\mu\nu}X^\mu Y^\nu$. We decompose x^μ into the light-cone and transverse coordinates: $x^\mu=(x^+,x^-,x^I)$, $x^I=(x^i,x^{i'})$, where

$$x^\pm\equiv\frac{1}{\sqrt{2}}(x^9\pm x^0). \tag{A1}$$

The scalar products of tangent space vectors are decomposed as

$$X^\mu Y^\mu=X^+Y^-+X^-Y^++X^IY^I,\quad X^IY^I=X^iY^i+X^{i'}Y^{i'}. \tag{A2}$$

The notation $\partial_\pm$, ∂_I is mostly used for target space derivatives:[20]

$$\partial_+\equiv\frac{\partial}{\partial x^+}\quad \partial_-\equiv\frac{\partial}{\partial x^-},\quad \partial_I\equiv\frac{\partial}{\partial x^I}. \tag{A3}$$

We also use

$$\partial^+=\partial_-,\quad \partial^-=\partial_+,\quad \partial^I=\partial_I. \tag{A4}$$

The $SO(9,1)$ Levi-Cività tensor is defined by $\epsilon^{01\cdots 9}=1$, so that in the light-cone coordinates $\epsilon^{+-1\cdots 8}=1$. The derivatives with respect to the world-sheet coordinates (τ,σ) are denoted as

$$\dot{x}^I\equiv\partial_\tau x^I,\quad \acute{x}^I\equiv\partial_\sigma x^I. \tag{A5}$$

We use the chiral representation for the 32×32 Dirac matrices Γ^μ in terms of the 16×16 matrices γ^μ:

[20]In Secs. I and II A $\partial_\pm$ indicate world-sheet derivatives.

$$\Gamma^{\mu}=\begin{pmatrix} 0 & \gamma^{\mu} \\ \bar{\gamma}^{\mu} & 0 \end{pmatrix}, \tag{A6}$$

$$\gamma^{\mu}\bar{\gamma}^{\nu}+\gamma^{\nu}\bar{\gamma}^{\mu}=2\eta^{\mu\nu},\quad \gamma^{\mu}=(\gamma^{\mu})^{\alpha\beta},$$

$$\bar{\gamma}^{\mu}=\gamma^{\mu}_{\alpha\beta}, \tag{A7}$$

$$\gamma^{\mu}=(1,\gamma^{I},\gamma^{9}),$$

$$\bar{\gamma}^{\mu}=(-1,\gamma^{I},\gamma^{9}),\quad \alpha,\beta=1,\ldots,16. \tag{A8}$$

We adopt the Majorana representation for Γ matrices, $C=\Gamma^0$, which implies that all γ^{μ} matrices are real and symmetric, $\gamma^{\mu}_{\alpha\beta}=\gamma^{\mu}_{\beta\alpha}$, $(\gamma^{\mu}_{\alpha\beta})^{*}=\gamma^{\mu}_{\alpha\beta}$. As in [5] $\gamma^{\mu_1\cdots\mu_k}$ are the antisymmetrized products of k gamma matrices, e.g., $(\gamma^{\mu\nu})^{\alpha}{}_{\beta}\equiv\frac{1}{2}(\gamma^{\mu}\bar{\gamma}^{\nu})^{\alpha}{}_{\beta}-(\mu\leftrightarrow\nu)$, $(\gamma^{\mu\nu\rho})^{\alpha\beta}\equiv\frac{1}{6}(\gamma^{\mu}\bar{\gamma}^{\nu}\gamma^{\rho})^{\alpha\beta}\pm5$terms. Note that $(\gamma^{\mu\nu\rho})^{\alpha\beta}$ are antisymmetric in α, β. We assume the normalization

$$\Gamma_{11}\equiv\Gamma^{0}\ldots\Gamma^{9}=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\quad \gamma^{0}\bar{\gamma}^{1}\ldots\gamma^{8}\bar{\gamma}^{9}=I. \tag{A9}$$

We use the following definitions:

$$\Pi^{\alpha}{}_{\beta}\equiv(\gamma^{1}\bar{\gamma}^{2}\gamma^{3}\bar{\gamma}^{4})^{\alpha}{}_{\beta},\quad (\Pi')^{\alpha}{}_{\beta}\equiv(\gamma^{5}\bar{\gamma}^{6}\gamma^{7}\bar{\gamma}^{8})^{\alpha}{}_{\beta}, \tag{A10}$$

$$\bar{\Pi}_{\alpha}{}^{\beta}\equiv(\bar{\gamma}^{1}\gamma^{2}\bar{\gamma}^{3}\gamma^{4})_{\alpha}{}^{\beta},\quad (\bar{\Pi}')_{\alpha}{}^{\beta}\equiv(\bar{\gamma}^{5}\gamma^{6}\bar{\gamma}^{7}\gamma^{8})_{\alpha}{}^{\beta}. \tag{A11}$$

Note that $\Pi^{\alpha}{}_{\beta}=\bar{\Pi}_{\beta}{}^{\alpha}$. Because of the relation $\gamma^{0}\bar{\gamma}^{9}=\gamma^{+-}$, the normalization condition (A9) takes the form $\gamma^{+-}\Pi\Pi'=1$. Note also the following useful relations (see also [5]):

$$(\gamma^{+-})^{2}=\Pi^{2}=(\Pi')^{2}=1, \tag{A12}$$

$$\gamma^{+-}\gamma^{\pm}=\pm\gamma^{\pm},\quad \bar{\gamma}^{\pm}\gamma^{+-}=\mp\bar{\gamma}^{\pm},\quad \gamma^{+}\bar{\gamma}^{+}=\gamma^{-}\bar{\gamma}^{-}=0, \tag{A13}$$

$$\bar{\gamma}^{+}(\Pi+\Pi')=(\Pi+\Pi')\gamma^{-}=0,$$

$$\bar{\gamma}^{-}(\Pi-\Pi')=(\Pi-\Pi')\gamma^{+}=0, \tag{A14}$$

$$\gamma^{\pm}\bar{\Pi}=\Pi\gamma^{\pm},\quad \gamma^{i}\bar{\Pi}=-\Pi\gamma^{i},\quad \bar{\gamma}^{i}\Pi=-\overline{\Pi\gamma^{i}},$$

$$\gamma^{i}\bar{\Pi}'=\Pi'\gamma^{i},\quad \bar{\gamma}^{i}\Pi'=\bar{\Pi}'\bar{\gamma}^{i}. \tag{A15}$$

The 32-component positive chirality spinor θ and the negative chirality spinor Q are decomposed in terms of the 16-component spinors as

$$\theta=\begin{pmatrix} \theta^{\alpha} \\ 0 \end{pmatrix},\quad Q=\begin{pmatrix} 0 \\ Q_{\alpha} \end{pmatrix}. \tag{A16}$$

The complex Weyl spinor θ is related to the two real Majorana-Weyl spinors θ^1 and θ^2 by

$$\theta=\frac{1}{\sqrt{2}}(\theta^{1}+\mathrm{i}\theta^{2}),\quad \bar{\theta}=\frac{1}{\sqrt{2}}(\theta^{1}-\mathrm{i}\theta^{2}). \tag{A17}$$

The shorthand notation like $\overline{\theta\gamma^{\mu}}\theta$ and $\bar{\gamma}^{\mu}\theta$ stands for $\bar{\theta}^{\alpha}\gamma^{\mu}_{\alpha\beta}\theta^{\beta}$ and $\gamma^{\mu}_{\alpha\beta}\theta^{\beta}$ respectively.

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