D3-D7 inflationary model and M theory

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A proposal is made for a cosmological D3-D7 model with a constant magnetic flux along the D7 world volume. It describes an $\mathcal{N}=2$ gauge model with Fayet-Iliopoulos terms and the potential of the hybrid *P*-term inflation. The motion of the D3-brane towards D7 in a phase with spontaneously broken supersymmetry provides a period of slow-roll inflation in the de Sitter valley, the role of the inflaton being played by the distance between D3- and D7-branes. After tachyon condensation a supersymmetric ground state is formed: a D3-D7 bound state corresponding to an Abelian non-linear (non-commutative) instanton. In this model the existence of a non-vanishing cosmological constant is associated with the resolution of the instanton singularity. We discuss a possible embedding of this model into a compactified M-theory setup.

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I. INTRODUCTION

Hybrid inflation [1] can be naturally implemented in the context of supersymmetric theories [2–5]. The basic feature of these inflationary models is the existence of two phases in the evolution of the universe: a slow-roll inflation in the de Sitter valley of the potential (the Coulomb phase of the gauge theory) and a tachyon condensation phase, or ''waterfall stage,'' towards the ground state Minkowski vacuum (a Higgs phase in gauge theory).

In $\mathcal{N}=1$ supersymmetric theories, hybrid inflation may arise as *F*-term inflation [2,3] or *D*-term inflation [4]. In $\mathcal{N}=2$ supersymmetric theories there is a triplet of Fayet-Iliopoulos (FI) terms, ξ^r, where $r=1,2,3$. Choosing the orientation of the triplet of FI terms, ξ^r, in directions 1,2, *F*-term inflation is promoted to $\mathcal{N}=2$ supersymmetry [6]. The more general case with $\mathcal{N}=2$, when all 3 components of the FI terms are present, is called *P-term inflation* [7]. When ξ^3 is non-vanishing, a special case of *D*-term inflation with Yukawa coupling related to gauge coupling is recovered. In this fashion, the two supersymmetric formulations of hybrid inflation are unified in the framework of $\mathcal{N}=2$ *P*-term inflation. This gauge theory has the potential [7]

$$V=\frac{g^2}{2}\Phi^\dagger\Phi(A^2+B^2) - \left[\frac{1}{2}(P^r)^2+P^r\left(\frac{g}{2}\Phi^\dagger\sigma^r\Phi+\xi^r\right)\right], \tag{1.1}$$

where P^r is a triplet of auxiliary fields of the $\mathcal{N}=2$ vector multiplet, $\Phi^\dagger,\Phi$ are 2 complex scalars forming a charged hypermultiplet, A,B are scalars from the $\mathcal{N}=2$ vector multiplet and g is the gauge coupling. The auxiliary field satisfies the equation $P^r=-(g\Phi^\dagger\sigma^r\Phi/2+\xi^r)$ and the potential simplifies to

$$V=\frac{g^2}{2}\left[\Phi^\dagger\Phi(A^2+B^2)+\left(\frac{1}{2}\Phi^\dagger\sigma^r\Phi+\frac{\xi^r}{g}\right)^2\right]. \tag{1.2}$$

An additional advantage of using $\mathcal{N}=2$ supersymmetric models for inflation is the possibility to link it to M or string theory where cases with $\mathcal{N}=2$ supersymmetry are simpler and less arbitrary than the cases with $\mathcal{N}=1$ supersymmetry.

In our first attempt[1] to link string theory to a gauge model with a hybrid potential [8], we used a system with a D4-brane attached to Neveu-Schwarz 5-branes (NS5-branes) that have a small angle relative to a D6-brane, so that the Coulomb phase of the theory is slightly non-supersymmetric and forces the D4 to move towards the D6-brane (see also [9]). This setup reproduces accurately the properties of the non-supersymmetric de Sitter vacuum of *P*-term inflation, for which $(P^r)_{\rm deSit}=-\xi^r$ and $(V)_{\rm deSit}=\vec{\xi}^2/2$. In particular, the mass splitting of the scalars in the hypermultiplet (e.g. for the case of $\xi^3=\xi\neq 0$),

$$M^2_{\rm hyper}=g^2(A^2+B^2)\pm g\xi, \tag{1.3}$$

is reproduced by the low-lying string states; the attractive force between the D4- and D6-brane is a one-loop effect from the open string channel, and correctly reproduces the one-loop gauge theory potential

$$\Delta V=\frac{\xi^2g^2}{16\pi^2}\ln\frac{|A^2+B^2|}{|A^2+B^2|_c} \tag{1.4}$$

for large values of the inflaton field. Notice the inflaton is the distance between D4 and D6 in the brane model.

When the distance between the branes becomes smaller than the critical distance

$$|A^2+B^2|_c=\frac{\xi}{g} \tag{1.5}$$

the spectrum of 4-6 strings develops a tachyon. The tachyon condensation is associated with a phase transition. A final

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[1]Earlier studies of brane inflation were performed in [10,11].

Higgs phase with unbroken $\mathcal{N}=2$ supersymmetry (susy) described in this model by a reconfiguration of branes: D6 cuts D4 into two disconnected parts, so that $\mathcal{N}=2$ supersymmetry is restored. From the viewpoint of the gauge theory it is described by a vanishing auxiliary field $(P^r)_{SUSY}=0$, a vanishing vacuum expectation value (VEV) of the inflaton field $A=B=0$, and a vanishing potential $(V)_{SUSY}=0$. The hypermultiplet VEV is not vanishing and has to satisfy

$$\frac{g}{2}\Phi^\dagger\sigma^r\Phi=-\xi^r. \tag{1.6}$$

One attractive feature of the D4-D6-NS5 model is that the deviation from supersymmetry in the Coulomb stage can be very small and supersymmetry is spontaneously broken. Nevertheless, a large number of e-foldings can be produced within a D-brane. Given the lessons from the D-brane approach to black hole physics, such small deviation from supersymmetry might be an advantage over brane or antibrane models [11], for which the deviation from supersymmetry is large.

The main difference between our model [8] and other models of brane inflation [9–13] is that our model provides the brane description of the full potential of hybrid P-term inflation. This includes both the logarithmic quantum corrections to the Coulomb branch potential and the exit from inflation with the corresponding supersymmetric Minkowski vacuum.

The first purpose of this paper is to suggest a new model (partially dual to [8]) which gives a better description of the Higgs branch (exit from inflation) while keeping the nice properties of the Coulomb branch (inflation). We will then argue that the stringy understanding of the exit from inflation provides a possible explanation for the existence of a nonvanishing cosmological constant. The second purpose is to suggest how this new model may be consistently compactified to four dimensions, without spoiling its cosmological properties.

The model consists of a D3-brane parallel to a D7-brane at some distance, which again is the inflaton field. The supersymmetry breaking parameter is related to the presence of the antisymmetric $\mathcal{F}_{mn}$ field on the world volume of the D7-brane, but transverse to the D3-brane. When this field is not self-dual in this four-dimensional space, the supersymmetry of the combined system is broken. This is (to some extent) a type IIB dual version of the cosmological model proposed in [8], which guarantees that the good properties in the Coulomb stage are kept; in particular the spectrum and the attractive potential should match the ones of P-term inflation. One immediate simplification is that the NS5-branes are not needed any longer.

More interestingly is the understanding of the Higgs stage. Equation (1.6) can be associated with the Atiyah-Drinfeld-Hitchin-Manin (ADHM) construction of instantons with gauge group $U(N)$. The moduli space of one instanton is the moduli space of vacua of a $U(1)$ gauge theory with N hypermultiplets and Eq. (1.6) is the corresponding ADHM equation. This suggests that in the brane construction of the cosmological model we may look for some instantons on the world volume of the brane in the Higgs phase of the theory. We will find an Abelian non-linear instanton solution with an associated ADHM equation (1.6). Moreover, the presence of a cosmological constant will translate as the resolution of the small size instanton singularity.

One other nice feature of the D3-D7 cosmological model is that it is well explained in terms of the κ symmetry of the D7-brane action, both in the Coulomb phase as well as in the Higgs phase.

Unbroken supersymmetry of bosonic configurations in supergravity has already proved to be an important tool in our understanding of M or string theory. To obtain Bogomol'nyi-Prasad-Sommerfield (BPS) solutions of supergravity one has to find Killing spinors, satisfying a condition $\delta\psi=0$ for all fermions, and to find out how many non-vanishing spinors solve this equation for a given solution of bosonic equations. This allows us to find configurations with some fraction of unbroken supersymmetry in supergravity.

Unbroken supersymmetry of the bosonic configurations on the world volume of the κ-symmetric branes embedded into some curved space (on shell superspace) was studied less so far. The condition of supersymmetric vacua in the context of the supermembrane was suggested in [14]. This condition was introduced more recently in the context of supersymmetric cycles in [15]. Later on, when more general κ-symmetric brane actions were discovered in [16], it was established in [17] that for all cases of D-branes, M2 and M5 κ-symmetric branes, a universal equation for the BPS configurations on the world volume is given by $(1-\Gamma)\epsilon=0$. Here $\Gamma(X^\mu(\sigma),\theta(\sigma),A_i(\sigma))$ is a generator of κ symmetry and it should be introduced into the equation for unbroken supersymmetry with a vanishing value of the fermionic world-volume field $\theta(\sigma)$. The existence or absence of solutions to these equations will fit naturally in the two stages of our cosmological model.

In addressing compactification of such a cosmological model to four dimensions, as to recover four-dimensional gravity, one is inevitably faced with the issue of anomalies. In many cases this is associated with the requirement that the overall charge in a compact space must vanish. We will point out that we can face our D3-D7 model within a more general setup, related to F theory [18–20], where the D7-brane charge is cancelled by orientifold 7-planes or (p,q) 7-branes. This seems to provide a setup where, in string theory, the compactification could be consistently performed.[2] This is a particularly relevant issue for the cosmological models with FI terms. It was shown by Freedman [21] that FI terms in $\mathcal{N}=1$, $D=4$ supergravity must be accompanied by an axial gauging of gravitino and gaugino which makes the theory anomalous. We then discuss the uplifting of the compactified, anomaly-free IIB model to M theory, where it becomes simpler. We close with a discussion.

II. dE SITTER VALLEY: SEPARATED D3-D7 SYSTEM WITH FLUXES

We will start by reviewing the spectrum of the D3-D7 system with a gauge field [22,24]. The boundary conditions

[2]Some recent papers which also consider orientifold planes to describe the inflationary scenario are in [13].

TABLE I. Setup of branes and flux.

	0	1	2	3	4	5	6	7	8	9
D3	×	×	×	×						
D7	×	×	×	×			×	×	×	×
$\mathcal{F}$							×	×	×	×

on the D7 side will depend on a gauge field on its surface. To understand the dynamics of the branes we will consider a D7-brane in a background of the D3-brane. We will allow a world-volume gauge field on D7 and find that supersymmetry is broken when this gauge field is not self-dual. The potential between branes will be shown to have a logarithmic dependence on the distance, as expected for broken supersymmetry.

In an alternative picture, a D3-brane will probe the background of a D7-brane with a bulk B field. The combined system will be supersymmetric under the condition that the B field is self-dual. For the non-self-dual field we calculate the potential and find again a logarithmic dependence on the distance. Therefore we recover from the probe approximation, the gauge theory and open string theory results [8].

A. Perturbative string theory analysis

Consider a type IIB system with D3- and D7-branes plus a constant world-volume gauge field $\mathcal{F}$ field along the directions given in Table I. This is illustrated in Fig. 1. It describes a de Sitter stage of a hybrid inflation and it is T dual to the type IIA D4-D6 model of branes at an angle (without the NS5-branes), considered in [8].

We place D7 at $(x^4)^2+(x^5)^2=0$ and D3 is initially at some $d^2=(x^4)^2+(x^5)^2 \gg d_c^2$, where d_c is defined in Eq. (2.13). There is a constant world-volume gauge field $\mathcal{F}=dA-B$ present on D7:

$$\mathcal{F}_{67}=\tan\theta_1, \quad \mathcal{F}_{89}=\tan\theta_2, \tag{2.1}$$

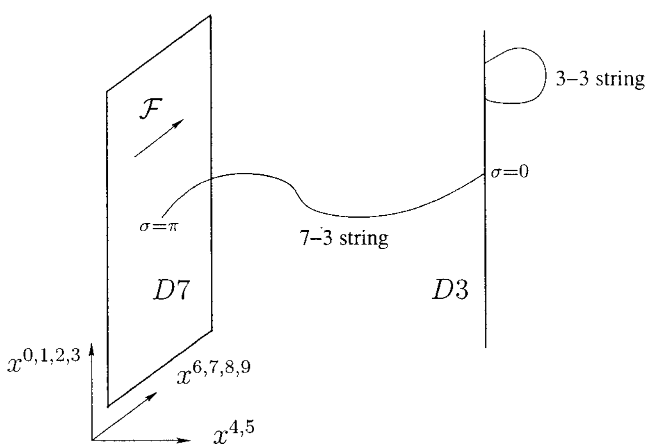

FIG. 1. The D3-D7 "cosmological" system. The 3-3 strings give rise to the $\mathcal{N}=2$ vector multiplet, the 7-3 strings to the hypermultiplet and the world-volume gauge field $\mathcal{F}$ to the FI terms of the $D=4$ gauge theory.

responsible for the spontaneous breaking of supersymmetry. For example, we may have $B=0$ in the bulk and the following vector fields on the brane:

$$A_6=-\frac{1}{2}\tan\theta_1 x^7, \quad A_7=\frac{1}{2}\tan\theta_1 x^6,$$
$$A_8=-\frac{1}{2}\tan\theta_2 x^9, \quad A_9=\frac{1}{2}\tan\theta_2 x^8. \tag{2.2}$$

Note that if $\mathcal{F}$ is self-dual, supersymmetry would be unbroken [22]. This will be explained via κ symmetry in Sec. II B. For future reference we define

$$\mathcal{F}^{\pm}=\mathcal{F}\pm\star\mathcal{F}, \tag{2.3}$$

and similarly for B.

Let us define complex coordinates:

$$z^1=x^6+ix^7, \quad z^2=x^8+ix^9. \tag{2.4}$$

The boundary conditions on the D3-D7 strings along the 6,7,8,9, directions are, therefore,

$$(\partial_\tau z^k)_{\sigma=0}=0, \quad (\partial_\sigma z^k+\tan\theta_k\partial_\tau z^k)_{\sigma=\pi}=0 \tag{2.5}$$

(no sum on k), where $k=1,2$, and similar conditions for $\overline{z^k}$ (with $\tan\theta_k \to -\tan\theta_k$). Let us now write the mode expansion as

$$z^k=\sum_n A^k_{n+\nu_k}e^{(n+\nu_k)(\tau+i\sigma)}+\sum_n B^k_{n+\nu_k}e^{(n+\nu_k)(\tau-i\sigma)}. \tag{2.6}$$

The first boundary condition yields

$$A^k_{n+\nu_k}=-B^k_{n+\nu_k}, \tag{2.7}$$

while the second one gives

$$e^{2\pi i\nu_k}=-\frac{1+i\tan\theta_k}{1-i\tan\theta_k}\Leftarrow\nu_k=\frac{1}{\pi}\left(\theta_k+\frac{\pi}{2}\right). \tag{2.8}$$

We take $-\pi/2<\theta_k<\pi/2$, which implies that $0<\nu_k<1$. It only remains to find out the zero-point energy as a function of ν_k, in the NS sector. We use

$$\sum_{n=1}^{\infty}(n-\nu)=-\frac{1}{12}(6\nu^2-6\nu+1). \tag{2.9}$$

The bosons along directions $x^{6,7}$ and $x^{8,9}$ are quantized with mode numbers $n\pm\nu_k$ and the fermions have mode numbers $n\pm(\nu_k-1/2)$. Thus the zero point energy of the system will be

$$E=-\frac{1}{2}\left(\left|\nu_1-\frac{1}{2}\right|+\left|\nu_2-\frac{1}{2}\right|\right). \tag{2.10}$$

TABLE II. Zero-point energies.

	ν_1	ν_2	E
1.	$\nu_1>1/2$	$\nu_2<1/2$	$-(\nu_1-\nu_2)/2$
2.	$\nu_1<1/2$	$\nu_2>1/2$	$(\nu_1-\nu_2)/2$
3.	$\nu_1>1/2$	$\nu_2>1/2$	$-(\nu_1+\nu_2-1)/2$
4.	$\nu_1<1/2$	$\nu_2<1/2$	$(\nu_1+\nu_2-1)/2$

From the above equation we have the four possible cases in Table II. We can choose the Gliozzi-Scheik-Olive (GSO) even states to be either 1,2 or 3,4. We will choose the former. Then the zero point energy becomes

$$E=\pm\frac{1}{2}(\nu_1-\nu_2)=\pm\frac{\theta_1-\theta_2}{2\pi}. \tag{2.11}$$

Therefore the lowest-lying multiplet of states of open strings consists of bosons whose masses are given by

$$M_\pm^2=\frac{d^2}{(\pi\alpha')^2}\pm\frac{\theta_1-\theta_2}{2\pi\alpha'}, \tag{2.12}$$

where we observe that the boson of mass M_-^2 becomes tachyonic at the critical distance[3]

$$d_c^2\equiv\pi\alpha'\left(\frac{\theta_1-\theta_2}{2}\right), \tag{2.13}$$

and the other boson remains at positive mass. The fermion in the multiplet have a mass

$$M_\psi^2=\frac{d^2}{(\pi\alpha')^2}, \tag{2.14}$$

as the contribution from the zero point energy to the Ramond sector is zero.

The higher states in the string spectrum satisfying the condition of spontaneously broken supersymmetry $StrM^2=0$, also have tachyonic masses whose values and splitting are given in [8]. One can check, however, that it is the very first tachyon which provides a critical distance between the branes; all other higher level tachyons always come at distances smaller than the first one. Therefore already at d_c^2 there is a phase transition and a waterfall stage sets on as the potential of the gauge theory suggests.

The $\mathcal{F}$ field plays the role of the Fayet-Illiopoulos term, from the viewpoint of the field theory living on the $D3$-brane. It creates an instability in the system driving the $D3$-brane into the $D7$-brane; this is the de Sitter stage. The evolution follows the description given in [8]. In particular, a tachyon will form and the system will end in a supersymmetric configuration which will be analyzed in Sec. III.

[3]We are assuming $\theta_1>\theta_2$.

B. D7-brane world-volume analysis and κ symmetry

In the presence of a Fayet-Illiopoulos parameter, the system of D3- and D7-branes at some distance from each other is unstable. It is instructive to see this instability from the viewpoint of the D7-brane world-volume action

$$\mathcal{S}=-T_7\int d^8\sigma e^{-\phi}\sqrt{-\det(g+\mathcal{F})}+T_7\int\sum \mathcal{C}_{p+1}\wedge e^{\mathcal{F}}, \tag{2.15}$$

where g is the metric induced on the brane and $\mathcal{F}=F-B$, where B is a pull-back of the NS-NS two-form and F is the two-form $(F=dA)$ Born-Infeld field strength. We place our D7-brane in the D3-brane background. The dilaton is constant while the metric and self-dual RR form read

$$\begin{aligned} ds^2&=H^{-1/2}ds^2(\mathbb{E}^{3,1})+H^{1/2}ds^2(\mathbb{E}^6),\\ \mathcal{F}^{RR}&=\partial_iH^{-1}dx^i\wedge\epsilon_{\mathbb{E}^{3,1}}+\star(\partial_iH^{-1}dx^i\wedge\epsilon_{\mathbb{E}^{3,1}}). \end{aligned} \tag{2.16}$$

H is the usual single center harmonic function on $\mathbb{E}^6$, $H=1+Q/r^4$, and $\epsilon_{\mathbb{E}^{3,1}}$ is the volume form on $\mathbb{E}^{3,1}$. The embedding functions for the D7-brane are $x^\mu=\sigma^\mu$, where $\mu=0,1,2,3,6,7,8,9$. We also turn on the world-volume gauge field, with components (2.1). Denoting by $\mathcal{V}_3$ the volume along $\sigma^1,\sigma^2,\sigma^3$ and allowing $i=6,7,8,9$, we find the effective potential,

$$V=T_7\mathcal{V}_3\int d\sigma^i\Big[\sqrt{(1+H^{-1}\tan\theta_1{}^2)(1+H^{-1}\tan\theta_2{}^2)}-(H^{-1}-1)\tan\theta_1\tan\theta_2\Big]. \tag{2.17}$$

Clearly, if $\theta_1=\theta_2$, the force between the D3 and D7 vanishes. This corresponds to a self-dual configuration in the Euclidean four-space $\mathbb{R}^4$ parametrized by directions 6 to 9. Let ρ be the radial coordinate in this four-space. Also denote $d^2\equiv(x^4)^2+(x^5)^2$. Change coordinates to $\lambda=\rho^2+d^2$. For large distances, $d^4\gg Q$, and up to a divergent term coming from the infinite volume of the brane, the potential can be rewritten as

$$\begin{aligned} V&\simeq-\frac{\pi^2}{2}QT_7\mathcal{V}_3(\sin\theta_1-\sin\theta_2)^2\int_{d^2}^{\Lambda}\frac{d\lambda}{\lambda}\\ &=\frac{\pi^2}{2}QT_7\mathcal{V}_3(\sin\theta_1-\sin\theta_2)^2\ln\frac{d^2}{\Lambda}, \end{aligned} \tag{2.18}$$

where we introduced the cutoff Λ. This reproduces the attractive potential of hybrid inflation in accordance with the results discussed in [8]. Note that the choice of gauge for the supergravity potential $\mathcal{C}_4$ is such that it vanishes at infinity and reproduces the right energy for the D3-D7 bound state.

The breaking of supersymmetry manifest in the nonvanishing force between D3 and D7 can be deduced from general considerations of κ-symmetric D7-brane in the background of D3-brane. The bosonic part of the action is given in Eq. (2.15). Solutions of the equations following from this action may have some unbroken supersymmetry[4] if there are non-trivial spinor solutions to

$$(1-\Gamma)\epsilon=0, \tag{2.19}$$

where the κ-symmetry projector Γ for a D7-brane in a D3-brane background is given by [17]

$$\Gamma=e^{-a/2}i\sigma_2\otimes\Gamma_{01236789}e^{a/2}=e^{-a}i\sigma_2\otimes\Gamma_{01236789}. \tag{2.20}$$

Here $\epsilon^{\alpha I}$ is a spinor of type IIB theory which has 32 independent components. It is represented by two Majorana spinors $I=1,2$ and $\alpha=1,\ldots,32$, satisfying a chirality constraint; $(\sigma_2)_I{}^J$ is a Pauli matrix and in the absence of the "rotation" factor a this Killing equation would just reproduce the D7-brane projector $(1-i\sigma_2\otimes\Gamma_{01236789})\epsilon=0$, which would correspond to 1/2 of unbroken supersymmetry. We are looking for a configuration with an $F=dA$ on the world volume which is skew diagonal, as in Eq. (2.1). The general relation between $\mathcal{F}$ and a is rather complicated but it turns out to be very simple in our case since the choice of the skew-diagonal basis is possible for the matrix $\mathcal{F}$: it is antisymmetric and independent on the world-volume coordinates,

$$a=\sigma_3\otimes(\theta_1\gamma_{67}+\theta_2\gamma_{89})$$
$$=\sigma_3\otimes H^{1/2}(\theta_1\Gamma_{67}+\theta_2\Gamma_{89}), \tag{2.21}$$

where $\gamma_i=E_i{}^a\Gamma_a$, and the vielbeins are given by the metric of the D3-brane. Γ_a are space-time gamma matrices. The presence of the $\mathcal{F}$ field affects the unbroken supersymmetry projector of the D7-brane in D3 background and we find for the Killing spinors the following condition:

$$\exp\{-\sigma_3\otimes H^{1/2}(\theta_1\Gamma_{67}+\theta_2\Gamma_{89})\}i\sigma_2\otimes\Gamma_{01236789}\epsilon$$
$$=\epsilon. \tag{2.22}$$

Due to the presence of the D3-brane background, the Killing spinor has also to satisfy the following conditions:

$$\epsilon=H^{-1/4}\epsilon_0,\quad i\sigma_2\otimes\Gamma_{0123}\epsilon_0=\epsilon_0, \tag{2.23}$$

where ϵ_0 is a constant spinor which breaks half of the supersymmetry. Taking this into account in Eq. (2.22) we reduce the problem to the following one:

$$\exp\left\{-\frac{1}{2}\sigma_3\otimes H^{1/2}\Gamma_{67}[(\theta_1+\theta_2)(1-\Gamma_{6789})\right.$$
$$\left.+(\theta_1-\theta_2)(1+\Gamma_{6789})]\right\}\otimes\Gamma_{6789}\epsilon_0=\epsilon_0. \tag{2.24}$$

The harmonic function of the D3 background at the position of the D7-brane $d^2=(x^4)^2+(x^5)^2=0$ has the form $H=1+Q/\rho^4$, where $\rho^2=(\sigma^6)^2+(\sigma^7)^2+(\sigma^8)^2+(\sigma^9)^2$. Equation (2.24) shows that unless $\theta_1=\theta_2$ there is no solution and all supersymmetries are broken. Indeed if $\theta_1=\theta_2=\theta$ we find

$$\exp\{-\sigma_3\otimes H^{1/2}\theta\Gamma_{67}(1-\Gamma_{6789})\}\otimes\Gamma_{6789}\epsilon_0=\epsilon_0. \tag{2.25}$$

The solution is $\epsilon_0=\Gamma_{6789}\epsilon_0$. Thus for self-dual $\mathcal{F}$, $\mathcal{F}^-=0$ our exact non-linear Killing condition can be satisfied for the Killing spinors, subject to two projector equations

$$\epsilon=i\sigma_2\otimes\Gamma_{01236789}\epsilon$$
$$\epsilon=\Gamma_{6789}\epsilon. \tag{2.26}$$

However, if $\theta_1\neq\theta_2$, Eq. (2.24) cannot be satisfied for non-vanishing ϵ_0: the constraint on constant spinors cannot depend on a function of the world-volume coordinates $H(\sigma)$.

It is important to stress here that $H(\sigma)$ is a function of the world-volume coordinates which is fixed by the properties of the background. It will be quite different in the Higgs phase when an analogous situation will occur and it will be possible to use the independence of the corresponding function on the world-volume coordinates as a condition for the non-linear instanton equation.

C. D3-brane world-volume analysis

We may see the instability in the D3-D7 system still in a different fashion. We take a D7-brane geometry with a B field obeying $B^-\neq 0$, probed by a D3-brane. A corresponding supergravity solution can be taken in the following form (obtained by two T dualities at an angle from the D5-brane solution):

$$ds^2_{D7}=Z_7^{-1/2}ds^2(\mathbb{E}^{3,1})+Z_7^{1/2}ds^2(\mathbb{E}^2_{45})$$
$$+Z_7^{-1/2}H_1ds^2(\mathbb{E}^2_{67})+Z_7^{-1/2}H_2ds^2(\mathbb{E}^2_{89}),$$
$$e^{2\phi}=g_s^2Z_7^{-2}H_1H_2,$$
$$B_{67}=-\tan\theta_1Z_7^{-1}H_1,\quad B_{89}=-\tan\theta_2Z_7^{-1}H_2,$$
$$C_4=(Z_7^{-1}-1)\sin\theta_1\sin\theta_2\epsilon_{\mathbb{E}^{3,1}}, \tag{2.27}$$
$$C_6=(Z_7^{-1}-1)[H_1\cos\theta_1\sin\theta_2dx_6\wedge dx_7$$
$$+H_2\cos\theta_2\sin\theta_1dx_8\wedge dx_9]\wedge\epsilon_{\mathbb{E}^{3,1}},$$
$$C_8=(Z_7^{-1}-1)H_1H_2\cos\theta_1\cos\theta_2\epsilon_{\mathbb{E}^{3,1}}$$
$$\wedge dx_6\wedge dx_7\wedge dx_8\wedge dx_9,$$

where $ds^2(\mathbb{E}^2_{ij})$ denotes the Euclidean two-space parametrized by Cartesian coordinates x^i,x^j and we have defined

$$H_1=(\cos^2\theta_1+\sin^2\theta_1Z_7^{-1})^{-1},$$
$$H_2=(\cos^2\theta_2+\sin^2\theta_2Z_7^{-1})^{-1}, \tag{2.28}$$

[4]We are using notation of [17].

$$Z_7=\lim_{\epsilon\to 0^+}\left(1+c_7\frac{\Gamma(\epsilon/2)}{r^\epsilon}\right)$$
$$=1-2c_7\ln(r/e^{1/\epsilon}).$$

The $D3$-brane probe action is given by

$$S_{D3}=-T_{D3}\int d^4\sigma e^{-(\phi-\phi_0)}\sqrt{-\det(g+\mathcal{F})}$$
$$+T_{D3}\int C_4, \qquad (2.29)$$

where the zero mode of the dilaton ϕ_0 is defined by $e^{\phi_0}=g_s$.

For our application the gauge invariant two-form $\mathcal{F}$ is vanishing, and we do the approximation of $Z_7^{-1}\sim 1+2c_7\ln(r/\Lambda)$ with $\Lambda=e^{1/\epsilon}$ for large r, where we first employed the large r approximation and then took $\epsilon\to 0^+$ (the "dimensional regularization"), as can be understood from Eq. (2.28). Setting all the above results together, we have for the potential

$$V=T_{D3}V_{D3}[1+c_7(\sin\theta_1-\sin\theta_2)^2\ln(r/\Lambda)]. \qquad (2.30)$$

Thus we have again reproduced the logarithmic long range potential between the D3- and D7-brane. In the language of this section, the self-duality of the B field, i.e. $B^-=0$, implies supersymmetry of this system. This was to be expected from the gauge invariance of $\mathcal{F}=F-B$.

III. MINKOWSKI VACUUM: D3-D7 BOUND STATE

The attractive force leads the D3-brane towards the D7-brane. When the critical distance (2.13) is reached (where a hypermultiplet scalar becomes massless), tachyon condensation brings the system of these branes into a bound state: the D3-brane is dissolved on the world volume of the D7 brane. Supersymmetry is restored. In this section we describe this absolute ground state, show that it has an unbroken supersymmetry and that the D-flatness condition is satisfied. In our previous D4-D6 model [8] the picture of reconfigurated branes was suggested for the final state. In the present case, we are able to give a much more detailed and clear description of this state using the idea of a D3-D7 supersymmetric bound state which has an energy lower than that of D3 plus D7 when they are at a distance from each other.

The Higgs stage of a D3-D7 cosmological model is closely related to the known descriptions of a D0-D4 bound state in the framework of non-commutative gauge theory [23] as well as in string theory and Dirac-Born-Infeld theory [24–27]. The non-linear instantons of the κ-symmetric branes in [25] were constructed in the context of flat Euclidean branes. In particular, the most relevant case therein for our application is the Euclidean D3-brane instanton. There is no need to use the approximation of a double scaling limit, as in [24,25] to describe a D3-D7 bound state since we will employ the method to find solutions to the BPS equation for the non-linear instanton suggested in [27].

The setup is a D7-brane with normal Minkowski space signature in 0,1,2,3 directions and Euclidean signature in the four-dimensional space parametrized by 6,7,8,9, where the supersymmetric non-linear instanton solution for the gauge field will be found in the presence of a constant, non-self-dual magnetic flux F_{mn} (B_{mn}) field. If our F_{mn} (B_{mn}) field is self-dual to start with, $F^-=0$ ($B^-=0$) there is no Coulomb branch or inflation in our model. It would be a limit in which all three FI terms vanish. So we will keep a constant $F^-\neq 0$ ($B^-\neq 0$) which nevertheless allows for a Higgs branch with unbroken supersymmetry.

A. D3-D7 bound state and unbroken κ symmetry

In the context of the κ-symmetric D7-brane action (2.15), we will look for a D3-D7 supersymmetric bound state for fixed α'. We place our D7-brane in the background of the ten-dimensional target space, which is assumed to be flat Minkowski space with a non-vanishing constant B_{mn} field in $\mathbb{R}^4$ with coordinates X^m, $m=6,7,8,9$.

The equation for unbroken supersymmetry is $\Gamma\epsilon=\epsilon$, where [17]

$$\Gamma=e^{-a}i\sigma_2\otimes\Gamma_{01236789}, \qquad (3.1)$$

and

$$a=\frac{1}{2}Y_{ik}\Gamma^{ik}\otimes\sigma^3. \qquad (3.2)$$

Here the matrix Y is a non-linear function of the matrix $\mathcal{F}$. In the Coulomb phase, we could choose a skew diagonal basis for $\mathcal{F}$ everywhere, since $\mathcal{F}$ was constant. In Sec. II we chose $\mathcal{F}_{67}=\tan\theta_1$ and $\mathcal{F}_{89}=\tan\theta_2$, leading to $Y_{67}=\theta_1$ and $Y_{89}=\theta_2$. In the Higgs phase F is allowed to depend on the world-volume coordinates; therefore $\mathcal{F}=F-B$ cannot be brought to a skew diagonal form everywhere. The world-volume gauge field F is also assumed to vanish asymptotically. In such a case, the expression for the matrix $Y(\sigma)$ is a complicated non-linear expression in terms of $\mathcal{F}(\sigma)$:

$$\exp\left\{-\frac{1}{2}Y_{ik}\Gamma^{ik}\otimes\sigma^3\right\}=\frac{1}{\sqrt{|\eta+\mathcal{F}|}}\left[1-\frac{1}{2}\sigma_3\Gamma^{ik}\mathcal{F}_{ik}\right.$$
$$\left.+\frac{1}{8}\Gamma^{ikml}\mathcal{F}_{ik}\mathcal{F}_{ml}\right]. \qquad (3.3)$$

At large $\sigma^6,\sigma^7,\sigma^8,\sigma^9$ our matrix $Y(\sigma)|_{\sigma\to\infty}\equiv Y^0$ depends only on a constant matrix B

$$\exp\left\{-\frac{1}{2}Y^0_{ik}\gamma^{ik}\otimes\sigma^3\right\}=\frac{1}{\sqrt{|\eta-B|}}\left[1+\frac{1}{2}\sigma_3\gamma^{ik}B_{ik}\right.$$
$$\left.+\frac{1}{8}\gamma^{ikml}B_{ik}B_{ml}\right]. \qquad (3.4)$$

Now it will be useful to introduce a difference between σ-dependent Y and its asymptotic value:

$$\hat{Y} = Y[F(\sigma) - B] - Y[-B] \equiv Y - Y^0 = F(\sigma) + \cdots. \quad (3.5)$$

Here, the dots stand for terms which are non-linear in F and B. We can split Y into a self-dual and an anti-self-dual part $Y = Y^+ + Y^-$.

The Killing spinor equation $\Gamma\epsilon = \epsilon$ takes the form

$$\exp\left\{ -\frac{1}{4}\sigma_3 \otimes \Gamma^{ik}[Y^+_{ik}(1-\Gamma_{6789}) + Y^-_{ik}(1+\Gamma_{6789})] \right\} i\sigma_2 \otimes \Gamma_{01236789}\epsilon = \epsilon. \quad (3.6)$$

Our spinors ϵ are constant, therefore for Y, depending on σ in an arbitrary way, there is no solution with unbroken supersymmetry. However, in contrast to the Coulomb phase, the dependence on σ can now be constrained in a way such that Eq. (3.6) has a solution.

There are two possibilities to have unbroken supersymmetry. Choose the spinors in $\mathbb{R}^4$ to be antichiral (chiral) and require that Y^+ (Y^-) does not depend on σ, which in turn means that $Y^{\pm}(\sigma) - (Y^0)^{\pm} = 0$,

$$(1 \pm \Gamma_{6789})\epsilon = 0, \quad \frac{\partial Y^{\pm}}{\partial \sigma} = 0 \Rightarrow \hat{Y}^{\pm}(\sigma) = 0. \quad (3.7)$$

The remaining condition on spinors is

$$\exp\left\{ -\frac{1}{2}\sigma_3 \otimes \Gamma^{ik}(Y^0)^{\pm}_{ik} \right\} i\sigma_2 \otimes \Gamma_{01236789}\epsilon = \epsilon. \quad (3.8)$$

Now we have to decide which of these solutions is the correct description of the Higgs branch of our cosmological model. We can consider a limit in which there is no deformation due to B. With antichiral (chiral) spinors in $\mathbb{R}^4$ we find the following supersymmetric configuration:

$$i\sigma_2 \otimes \Gamma_{0123}\epsilon = \mp\epsilon, \quad i\sigma_2 \otimes \Gamma_{01236789}\epsilon = \epsilon, \quad F^{\pm} = 0. \quad (3.9)$$

This is a $\overline{D}3$- (D3-) brane projector and a D7-brane projector, in the notation of [17].

Thus, our solution has chiral spinors in $\mathbb{R}^4$. To have unbroken supersymmetry, in the presence of deformation B, requires that $\hat{Y}^-(\sigma) = 0$; this has an interpretation of a D3-brane (and not a $\overline{D}3$) dissolved into a D7-brane. It can be shown, using Eqs. (3.3) and (3.4), that $\hat{Y}^-(\sigma) = 0$ is equivalent to

$$\frac{\mathcal{F}^-_{ik}}{1 + \mathrm{Pf}\mathcal{F}} = -\frac{B^-_{ik}}{1 + \mathrm{Pf}B}. \quad (3.10)$$

Essentially the same BPS equation for the κ-symmetric Euclidean D3-brane was derived in [25].

B. D3-D7 bound state and deformed instantons

In this section we shall discuss a solution of the BPS equation (3.10), that is, a slight generalization of the nonlinear instanton solution of Seiberg and Witten [24], without taking their double scaling limit. This problem was worked out by Moriyama in [27]. Following the analysis therein,[5] we shall first show that the BPS equation (3.10) can be brought to the form

$$\underline{F}^-_{ab} = \underline{\theta}^-_{ab} F_{\mathrm{Pf}}\underline{F}. \quad (3.11)$$

We have defined $\mathcal{F}^-_{ij} = \mathcal{F}_{ij} - \frac{1}{2}\epsilon_{ijkl}\mathcal{F}^{kl}$ and $F_{\mathrm{Pf}}\mathcal{F} = \frac{1}{8}\epsilon^{ijkl}\mathcal{F}_{ij}\mathcal{F}_{kl}$, while $\underline{F}^-_{ab} = \underline{F}_{ab} - \frac{1}{2}\epsilon_{abcd}\underline{F}^{cd}$ and $F_{\mathrm{Pf}}\underline{F} = \frac{1}{8}\epsilon^{abcd}\underline{F}_{ab}\underline{F}_{cd}$. The indices $(i,j,\ldots)$ and $(a,b,\ldots)$ are associated, respectively, with the closed string metric $g_{ij} = \delta_{ij}$ and the frame metric δ_{ab}. The frame metric δ_{ab} is defined with respect to the open string metric

$$G_{ij} = \delta_{ij} - B_{ik}\delta^{km}B_{mj}. \quad (3.12)$$

The two types of indices, $(i,j,\ldots)$ and $(a,b,\ldots)$, are converted by the vierbein $e^a{}_i$ which is given by

$$e^a_i = \delta^a_i - B^a_i = (1-B)^a_i \quad \text{with} \quad G_{ij} = \delta_{ab}e^a e^b_j = (e^T e)_{ij}. \quad (3.13)$$

Likewise the noncommutative parameter θ^{ij} is defined by

$$\theta^{ij} = -\left(\frac{1}{1-B}\right)^{ik} B_{km}\left(\frac{1}{1+B}\right)^{mj}. \quad (3.14)$$

On the other hand, the frame quantities $\underline{F}_{ab}$ and $\underline{\theta}_{ab}$ in Eq. (3.11) are defined as

$$\underline{F}_{ab} = ((e^T)^{-1})_a{}^i F_{ij}(e^{-1})^j_b, \quad (3.15)$$

$$\underline{\theta}^{ab} = e^a_i\theta^{ij}(e^T)^b_j = -\delta^{ak}\delta^{mb}B_{km} = -B^{ab}. \quad (3.16)$$

Then the frame Pfaffian $F_{\mathrm{Pf}}\underline{F}$ is related to $F_{\mathrm{Pf}}F$ by

$$F_{\mathrm{Pf}}\underline{F} = (\det e)^{-1}F_{\mathrm{Pf}}F. \quad (3.17)$$

Now notice that an identity,

$$\underline{F}^- = (\det e)^{-1}\left\{ (1 + F_{\mathrm{Pf}}B)F^- - \frac{1}{4}\epsilon^{klmn}F_{kl}B_{mn}B^- - \frac{1}{2}(B^-F^- - F^-B^-) \right\}, \quad (3.18)$$

holds, while the BPS equation (3.10) can be rewritten as

$$(1 + F_{\mathrm{Pf}}B)F^-_{ij} - \frac{1}{4}\epsilon^{klmn}F_{kl}B_{mn}B^-_{ij} = -B^-_{ij}F_{\mathrm{Pf}}F. \quad (3.19)$$

[5]Our case is related to [27] by the replacements, $B \to -B$, $\theta \to -\theta$ and $\epsilon^{ijkl} \to -\epsilon^{ijkl}$.

Since the BPS equation (3.10) also implies that F^- is proportional to B^-, the third term $B^-F^- - F^-B^-$ in the identity (3.18) is vanishing. Thus we indeed find Eq. (3.11)

$$\underline{F}^-_{ab} = -(\det e)^{-1} B^-_{ij}\delta^i_a\delta^j_b F_{\mathrm{Pf}}F = \underline{\theta}^-_{ab}F_{\mathrm{Pf}}\underline{F}, \qquad (3.20)$$

as claimed. Now it is straightforward to solve this equation, following Seiberg and Witten [24]. First note that Eq. (3.15) for the field strength F can be restated, in terms of the frame coordinate x^a and the gauge potential $\underline{A}_a$, as

$$x^a = e^a_i x^i = (1-B)^a_i x^i, \qquad (3.21)$$

$$\underline{A}_a = ((e^T)^{-1})^i_a A_i = \left(\frac{1}{1+B}\right)^i_a A_i. \qquad (3.22)$$

Then the solution of Eq. (3.11) will be given by

$$\underline{A}_a = \underline{\theta}^-_{ab}x^b h(R), \qquad (3.23)$$

where we have defined

$$R^2 = \delta_{ab}x^a x^b. \qquad (3.24)$$

The function $h(R)$ takes the form, as in [24] and [26],

$$h(R) = -\frac{1}{2F_{\mathrm{Pf}}\underline{\theta}^-}\left(1 - \sqrt{1+\frac{4CF_{\mathrm{Pf}}\underline{\theta}^-}{R^4}}\right), \qquad (3.25)$$

where the constant C must be positive but otherwise arbitrary, as $F_{\mathrm{Pf}}\theta^-$ is negative. Below, we will take C to be $-8N$, with $\bar{N}$ being an integer.

In the presence of the noncommutative parameter θ^- or equivalently the B^- field, the singularity of the $U(1)$ instanton is improved, as discussed in [24]. In particular the solution (3.23) has a finite instanton number, though the gauge potential itself is singular as we go to the UV:

$$\frac{1}{16\pi^2}\int_{\mathbb{R}^4} F\wedge F = \frac{1}{8\pi^2}\int d^4x F_{\mathrm{Pf}}\underline{F}$$
$$= -\frac{1}{8}F_{\mathrm{Pf}}\underline{\theta}^- \lim_{R\to 0}(R^4h^2) = N. \qquad (3.26)$$

Note that the final result does not depend on the value of θ^-, but it was crucial to keep θ^- finite in the procedure of getting a finite instanton number. Had we taken the limit $\theta^- \to 0$ first, the instanton number would have been divergent, as $h^2(R)$ grows as R^{-8}, as R goes to zero, which corresponds to the fact that there is no Abelian instanton with finite energy.

In our cosmological scenario, the nonvanishing θ^- is the seed of the inflation, giving positive vacuum energy and producing an instability. In particular our model realizes the slow-roll inflation, as discussed in Sec. III A. After the slow-roll inflationary stage, the universe eventually goes through the tachyon condensation or "waterfall stage," and ends up with the Minkowski vacuum. As we mentioned earlier, this end-point vacuum is described by a non-marginal bound state of D3- and D7-branes that corresponds to the Higgs phase of the gauge theory of the D3-D7 system with the FI term provided by the nonvanishing θ^-. It is quite well known that the D3-branes on the D7-branes can be thought of as instantons on the D7-branes due to the Chern-Simons coupling,

$$S_{CS} = \frac{1}{16\pi^2}\int_{D7} C_4\wedge F\wedge F = N\int_{D3} C_4. \qquad (3.27)$$

In the above we have presented a D7-brane probe approximation of the Higgs phase where the D3-brane is described by a nonlinear or deformed instanton on the $U(1)$ DBI theory on the D7-brane.

However, given the fact that the Higgs phase of this D3-D7 system is actually identical to the noncommutative generalization of the ADHM construction of instantons, one may make a step further and argue that our Minkowski vacuum is described by the noncommutative instanton of Nekrasov and Schwarz [23], improving the nonlinear instanton solution that we have studied above. The noncommutative instanton solution is non-singular even at $R=0$, resolving the singularity of the zero-size instantons, and the action is of course finite.

It is to be stressed that we have thus found a possible connection of the cosmological "constant" in spacetime (0123-space) and the noncommutativity in internal space (6789-space):

$$\Lambda(\text{spacetime cosmological constant})$$
$$\Leftrightarrow \underline{\theta}^-(\text{noncommutativity in internal space}).$$

We have generated the cosmological constant by turning on B^- or equivalently θ^-, which can eventually be interpreted as the noncommutative parameter in the internal space at the end of our cosmological evolution, the Higgs phase, of our D3-D7 model. Also it may be somewhat suggestive that the non-vanishing noncommutativity parameter, i.e. the cosmological constant, serves, in a way, to resolve the singularity at the end of inflation.

IV. A COMPACTIFIED TYPE IIB SETUP

In Sec. II we studied a system of separated D3-D7 with world-volume two-form fluxes such that there is no supersymmetry. This was the Coulomb phase of our model. Because of spontaneously broken supersymmetry the D3-brane would be attracted towards the D7 and eventually it would fall into the D7 as an instanton. Due to the presence of fluxes (as the FI term) the instanton—which is the dissolved D3—will behave as a noncommutative instanton. The point-like singularity of the instanton is therefore resolved. This is the Higgs phase studied in Sec. III and now it is supersymmetric. Given such a setup, there are two interesting questions that we can ask at this stage:

(i) Is it possible to get a *compactified* four-dimensional model for this setup? We would then need to compactify our type IIB model on a six-dimensional base. But this raises the problem of anomalies or equivalently charge conservation. Therefore what we need is a configuration in four dimensions which is anomaly free.

(ii) Is it possible to study the metric for the system for both the Coulomb and the Higgs phase? In the Coulomb phase we expect the metric to have an inherent time dependence. In the Higgs phase—now because of supersymmetry—we expect a time-independent metric.

In this section we shall see that both these questions can be partially answered from a type IIB setup involving both branes and orientifold planes. In Sec. V we describe its M-theory lift.

A. The model

To get a model in four dimensions we have to compactify our D3-D7 system on a six-dimensional space. The branes are oriented as in Table I. Therefore we need to compactify directions $x^{4,5,6,7,8,9}$, which raises the question of charge conservation. Both the D3- and the D7-branes are *points* on the compact two space $x^{4,5}$. Therefore to cancel the charges of the branes we have to put *negatively* charged objects in our model. These have to be orientifold planes, since anti-branes would annihilate with the branes destroying the dynamics. However, in string theory an orientifold operation is an elaborate mechanism which requires various other branes and planes to get a completely consistent picture. We now describe one consistent setup which satisfies all the requirements of a compact model.

The model that we take to get a compactified type IIB setup has been studied earlier in a different context for supersymmetric cases. This involves compactifying type IIB on $K3\times T^2/\mathbb{Z}_2$, where the $\mathbb{Z}_2$ is an orientifold operation:

$$\mathbb{Z}_2=\Omega\cdot(-1)^{F_L}\cdot\mathcal{I}_{45}. \tag{4.1}$$

The torus T^2 is oriented along x^4,x^5; $\mathcal{I}_{45}$ is the orbifold projection with the action $x^{4,5}\to -x^{4,5}$; Ω is the world-sheet orientation reversal and $(-1)^{F_L}$ changes the sign of the left moving spacetime fermions. The $K3$ is oriented along $x^{6,7,8,9}$. Under the $\Omega\cdot(-1)^{F_L}$ operation the various fields of type IIB change as in Table III. Taking further into account $\mathcal{I}_{45}$, we see from the Table III that B fields along $x^{6,7,8,9}$ directions are projected out. The only B fields that survive must have one leg along $x^{4,5}$. These appear as gauge fields in four-dimensions or as scalars. Other fields which have one leg either along x^4 or x^5 are also removed from the spectrum. For example, if we denote

TABLE III. Parity of orientifold operations.

	Ω	$(-1)^{F_L}$	$\Omega\cdot(-1)^{F_L}$
g_{MN}	+	+	+
B^{NS}_{MN}	−	+	−
B^{RR}_{MN}	+	−	−
φ	+	+	+
$\tilde{\varphi}$	−	−	+
D^+	−	−	+

TABLE IV. Field decomposition upon compactification.

	Scalars	Vectors
g_{MN}	$g_{mn}, g_{\alpha\beta}$	$g_{\alpha\mu}$
$B^{NS,RR}_{MN}$	$B^{NS,RR}_{m\alpha}$	$B^{NS,RR}_{m\mu}$
$\varphi,\tilde{\varphi}$	$\varphi,\tilde{\varphi}$	−
D^+_{MNPQ}	$D^+_{mn\alpha\beta}, D^+_{0123}, D^+_{\mu\nu\alpha\beta}$	$D^+_{\alpha\beta\gamma\mu}$

$$m,n=x^{4,5},\quad \alpha,\beta=x^{6,7,8,9},\quad \mu,\nu=x^{0,1,2,3}, \tag{4.2}$$

then reducing over $T^2/\mathbb{Z}_2$ we have in Table IV, in addition to a metric $g_{\mu\nu}$, the spectrum of scalars and vectors from the supergravity fields (in terms of four-dimensional multiplets). However, $K3$ further cuts down the spectrum as the first Betti number is zero (so that no one form can exist). In particular it kills $g_{\alpha\mu}, B^{NS,RR}_{m\alpha}$ and $D^+_{\alpha\beta\gamma\mu}$. It also restricts the number of two-forms on it to be

$$b_2=b_{11}+2b_{20}=22, \tag{4.3}$$

corresponding to the number of four-dimensional scalars from the metric. This is the supersymmetric spectrum of the model. This spectrum is, as we know, incomplete. The complete spectrum involves additional 16 D7-branes as 4 O7 planes. The O7 planes are located at the four orbifold singularities of $T^2/\mathcal{I}_{45}$. To cancel charges locally we need 4 D7-branes on top of each orientifold planes. Therefore we have 7 branes wrapping the $K3$. This is a familiar model in the F-theory context. As discussed in [19] the axion dilaton is constant and could be arbitrary.[6]

1. The orbifold limit

Naively from the supergravity point of view it is difficult to argue the existence of the tetrahedron $T^2/\mathcal{I}_{45}$ since the total *source* is zero at the orbifold point. However, perturbative string theory "feels" it. Thus, a D3-brane feels the orientifold background, even if both charge and tension are cancelled by the presence of both orientifolds and D-branes.[7] Let us concentrate on one bunch of 4 D7 + O7. The gauge theory on the D3 is a $Sp(1)\equiv SU(2)$ Seiberg-Witten theory with 8 supercharges whose matter hypermultiplets are given by the D3-D7 strings. The strings which go around the O7 plane and come back give the massive $W^{\pm}$ bosons charged under the $U(1)$ gauge multiplets (the string starting from and ending on the D3).

[6]There are, however, some special points in the constant coupling moduli space where the seven branes are distributed in some specific way, which gives rise to exceptional global symmetry on the four-dimensional gauge theory [28]. These points are at strong coupling. We ignore these subtleties because they will not affect the result.

[7]At this point putting a probe looks arbitrary and it seems like we have not cancelled its charge on the compact space. In the next section we shall argue the consistency of this from the M-theory point of view.

The location of the D3-brane in the $x^{4,5}$ corresponds to the expectation value of the complex chiral field in the adjoint of $SU(2)$—this is the *inflaton* field in our model—which belongs to the vector multiplet. Denoting this as $\Phi = x^4 + ix^5$ we can diagonalize this to

$$\Phi = \begin{pmatrix} d & 0 \\ 0 & -d \end{pmatrix}. \tag{4.4}$$

The classical massless point is $d=0$ and the $W^{\pm}$ have masses $2d$.

There are also strings which start from and end on the D7-branes. They give rise to the $SO(8)$ gauge fields on the D7-branes. These strings survive the orientifold projection because the vertex operators are neutral under Ω projections. However, there could be strings which start from a D7, cross an O7 and come back to the D7. For these strings Ω projections remove the first massless vector multiplets charged under $SO(8)$. The surviving states are the next stringy oscillations which are massive and non-BPS [29].

The *Coulomb* branch of our model is when we switch on gauge flux $\mathcal{F}$ on the seven branes. As discussed earlier this flux is *non-primitive*, i.e. non-self-dual. This breaks supersymmetry spontaneously and therefore the D3 probe would be attracted towards the D7. This motion creates an instability in the system which triggers off non-perturbative corrections on the background.

2. Away from orbifold limit

Under non-perturbative corrections each O7 plane splits into two (p,q) seven branes [where (p,q) seven branes are defined as branes on which (p,q) strings can end—p,q denoting the two background values of B fields]. In a theory with 16 bulk supercharges this is explicitly demonstrated in [19,20]. We expect something similar for our case too. However, our main concern is to study an *isolated* and compactified D3-D7 system. This can be achieved by pulling a D7 from the bunch and studying its dynamics in the presence of a probe D3-brane. This way we can isolate the other O7-D7 dynamics from our inflationary model.

Our last concern is the anomaly cancellation. For the SUSY setup of our model (with self-dual $\mathcal{F}$) the argument of anomaly cancellation is simple. Let us make two T dualities along directions $x^{4,5}$ of the torus T^2. Under this

$$\Omega \cdot (-1)^{F_L} \cdot \mathcal{I}_{45} \rightarrow \Omega. \tag{4.5}$$

This is now type I theory on a torus which is further dual to heterotic string on a torus. As we know all anomalies—gauge and gravity—are cancelled for this theory and therefore we expect the same for a type IIB background.

In our case, however, the background is non-supersymmetric and hence T-dualities are subtle. So anomaly cancellation has to be checked by explicitly doing the one-loop graph for the given background. However, as we shall discuss in the ensuing section, anomaly cancellation can be equated in this case to charge cancellation on a compact manifold in M-theory. For this case therefore we can argue that anomalies do get cancelled.

Before we end this section let us summarize the situation. First, the full matter content of our theory:

(1) *Gauge fields on D3-branes*. These are the $SU(2)$ gauge fields broken to $U(1)$ at any generic point of our model. In fact there is never an enhancement to $SU(2)$ in this setup, so it remains $U(1)$ all through [30,31,19,20]. If we take a large number of D3-branes then the situation can be more subtle.

(2) *Gauge fields on the D7-branes*. These are the $SO(8)$ gauge fields on the seven branes. In our case we have switched on a background $U(1)$ field on one of the D7-branes which breaks supersymmetry in our model. This field, $\mathcal{F}$, acts as a FI term in the D3 world-volume theory.

(3) *Massive hypermultiplets*. These are generated by 3–7 strings which survive the Ω projections. In general they are massive.

(4) *Bulk modes*. These come from the bulk supergravity modes. Under Ω projection many of the bulk fields are removed from the spectrum as discussed above. The surviving ones undergo further projection due to the $K3$ manifold. In the end we get a metric, scalars and vectors in four dimensions. These vectors are in addition to the $SO(8)$ and the $U(1)$ vectors of the D7- and D3-branes, respectively.

(5) *Stable non-BPS states*. These are actually the modes that survive Ω projection on the D7-branes coming from the 7-7 strings that go around the O plane. They are all massive and receive sizable quantum correction.

(6) *Modes on O planes*. They are non-existent when we neglect non-perturbative corrections. However, in our model we expect the corrections to be sizable as we have isolated a D7 from the rest of the dynamics. Therefore the O planes no longer remain non-dynamical. Now (p,q) strings can end on them and therefore they support matter fields.

(7) *Junctions and networks*. Additional states can also arise in our setup. A D7-brane can be connected to two different (p,q) seven branes via a junction or a string network. They contribute additional states. But these are in general massive, so we can neglect them.

From the above analysis we see that most of the other matter fields coming from branes and planes are actually massive, and in the limit when we separate one D7-brane from the whole crowd, we can in principle study a compactified theory with few massless matter fields (including gravity). This is what we have tried to achieve here.

V. M THEORY ON A FOURFOLD WITH G FLUXES

The above setup can be further simplified by lifting it to M theory. All the complicacies of branes and planes now disappear in M theory. The system of 16 D7 + 4 O7 located at four points of $T^2/\mathcal{I}_{45}$ in type IIB theory simply becomes a $T^4/\mathbb{Z}_2$ orbifold of $K3$ in M theory. In other words, the whole setup in type IIB theory is actually just M theory compactified on $T^4/\mathbb{Z}_2 \times K3$. Denoting the T^4 directions as $x^{3,10,4,5}$ the $\mathbb{Z}_2$ operation sends (notice this is distinct from the $\mathbb{Z}_2$ of Sec. IV):

$$x^{3,10,4,5} \overset{\mathbb{Z}_2}{\longrightarrow} -x^{3,10,4,5}. \tag{5.1}$$

The original positions of the branes and planes become orbifold singularities in M theory. The non-supersymmetric flux on one of the seven branes in type IIB theory becomes localized G flux near one of the orbifold singularities. Supersymmetry is broken by choosing a non-primitive G flux. Before we go into the details of the M-theory setup let us summarize the situation in Table V. The model that we are going to use is M theory compactified on a fourfold with G fluxes switched on. To get a $\mathcal{N}=2$ theory we have to compactify M theory on $K3\times K3$. We shall take one of the $K3$ to be a torus fibration over a $\mathbb{C}P^1$ base (see Fig. 2). The Weierstrass equation governing the background is given by

$$y^2=x^3+xf(z)+g(z), \tag{5.2}$$

where $x,y,z\in\mathbb{C}P^1$, $f(z)$ is a polynomial of degree eight, and $g(z)$ is a polynomial of degree 12 in z. As a concrete example let us consider [19]

$$g(z)=\prod_{i=1}^{4}(z-z_i)^3,\quad f(z)=a\prod_{i=1}^{4}(z-z_i)^2, \tag{5.3}$$

where a is a constant. This basically describes a torus at every point of $\mathbb{C}P^1$ labeled by the coordinate z. The modular parameter $\tau(z)$ of the torus is determined in terms of f^3/g^2 through the relation

$$j(\tau)=\frac{[\theta_1^8(\tau)+\theta_2^8(\tau)+\theta_3^8(\tau)]^3}{\eta(\tau)^{24}} =\frac{4\cdot(24f)^3}{27g^2+4f^3}=\frac{55296a^3}{27+4a^3}. \tag{5.4}$$

The number of points at which the torus degenerates is given in terms of the zeroes of the polynomial

$$\Delta\equiv 4f^3+27g^2. \tag{5.5}$$

In terms of our choice of f and g the number of points at which the torus degenerates is given as

$$\Delta=(4a^3+27)\prod_{i=1}^{4}(z-z_i)^6=0. \tag{5.6}$$

To go to type IIB theory we have to shrink the fiber torus to zero size. Let us study this decomposition carefully. We first denote the base $K3$ by coordinates $x^{6,7,8,9}$ and the other $K3$ by $x^{4,5,3,10}$ in M theory. The spacetime is $x^{0,1,2}$. In type IIB therefore we get 16 D7 and 4 O7 planes located at points on a $\mathbb{C}P^1$ along directions $x^{4,5}$. These branes and planes wrap the other $K3$ along $x^{6,\ldots,9}$. The spacetime is now *four* dimensional: $x^{0,1,2,3}$. From our choice of f,g the $K3$ is elliptically fibered and therefore the $\mathbb{C}P^1$ base in IIB theory is given by $T^2/(-1)^{F_L}\cdot\Omega\cdot\mathcal{I}_{45}$ where Ω is the orientifold operation, $(-1)^{F_L}$ changes the sign of the left moving fermions and $\mathcal{I}_{45}:x^{4,5}\to -x^{4,5}$.

To verify the above analysis one can make a further T-duality along, say, x^3. Using the fact that [32]

$$\Omega\ \text{in IIB}\ \overset{T_3}{\to}\ \mathcal{I}_3\Omega\ \text{in IIA} \tag{5.7}$$

TABLE V. Dictionary between type IIB and M-theory descriptions.

Type IIB	M theory
$T^2/[\Omega\cdot(-1)^{F_L}\cdot\mathbb{Z}_2]\times K3$	$T^4/\mathbb{Z}_2\times K3$
4(O7 + 4 D7)	4 orbifold fixed points
D3 brane	$M2$
$\mathcal{F}$ on D7 brane	Localized G flux at fixed points
Coulomb phase	Non-primitive G flux
Higgs phase	Primitive G flux
Away from orientifold limit	$T^4/\mathbb{Z}_2\to$ Smooth $K3$

we can show that IIA theory is now on

$$\frac{T^3}{(-1)^{F_L}\cdot\Omega\cdot\mathcal{I}_{345}}. \tag{5.8}$$

This, however, is dual to M theory on $K3$ [33], thus proving our result.

Equation (5.8) describes the orbifold limit of $K3$. We assume this is the initial stage of our dynamical process. In terms of type IIB language, we place a D3-brane at the center of mass of the $4\times(4\ \text{D7/O7})$ setup on one side of the "pillow" $T^2/\mathbb{Z}_2$ and another on its diametrically opposite side, as in Fig. 3. We turn on the same gauge fluxes on all of the four fixed points. The logarithmic potential of Sec. II creates a force between each pair of D3 and D7 branes, if we assume the size of the $T^2/\mathbb{Z}_2$ to be large enough. However, in this configuration the total force between D3 and D7 branes is balanced, and the logarithmic potential being approximately flat leads to a nearly de Sitter evolution. Quantum fluctuations will destabilize the system allowing both the D3-brane to move towards some D7-brane and the D7-branes to move away from the orbifold fixed points. This is the beginning of the Coulomb phase or equivalently the inflationary stage (Fig. 4). As the system starts evolving we do not expect the coupling to remain constant anymore. The whole system moves away from the orientifold limit, which in M-theory

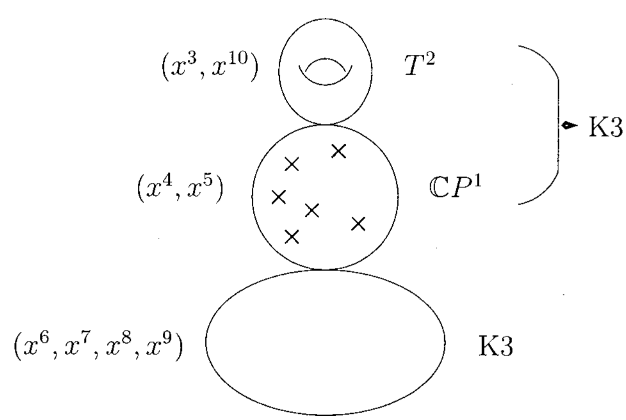

FIG. 2. The "snowman" fibration of the $K3\times K3$ fourfold. The crosses indicate points on the $\mathbb{C}P^1$ basis at which the fiber tori degenerate. In the orbifold limit of the "top" $K3$ there will be four such points.

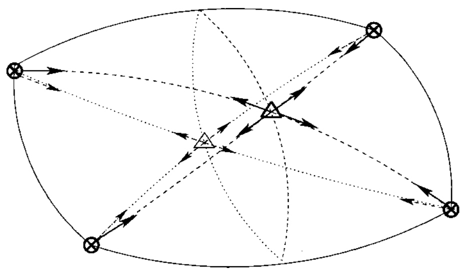

FIG. 3. The initial brane configuration on the "pillow" $T^2/\mathbb{Z}_2$.

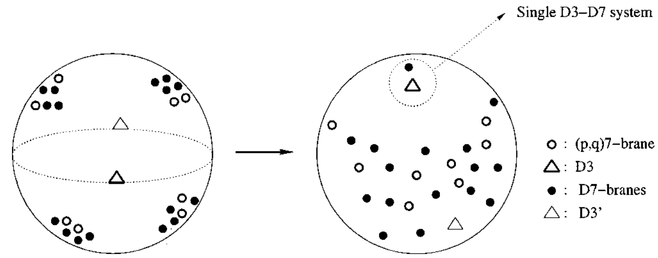

FIG. 4. The Coulomb branch. As the system is driven away from the unstable point of Fig. 2, $T^2/\mathbb{Z}_2 \to \mathbb{C}P^1$, the orientifold planes split into (p,q) 7-branes, and the D3-brane will eventually fall into one D7-brane as an instanton.

language means we have a generic $K3$. Finally, we expect the D3 to fall into one particular D7-brane as a noncommutative instanton as explained in Sec. III. This is the supersymmetric configuration that has been studied in M theory and which we now describe.

A. Supersymmetric setup: Higgs phase

Let us first summarize the known properties of the fourfold that allows for a supersymmetric configuration:

The fourfold vacua has a tadpole anomaly given by $\chi/24$ where χ is the Euler characteristics of the fourfold. If $\chi/24$ is integral, then the anomaly can be cancelled by placing a sufficient number of spacetime filling M2-branes n on points of the compactification manifold. There is also another way of cancelling the anomaly and this is through G flux. The G flux contributes a C tadpole through the Chern-Simons coupling $\int C\wedge G\wedge G$. When $\chi/24$ is not integral then we need both the branes and the G flux to cancel the anomaly. The anomaly cancellation formula becomes

$$\frac{\chi}{24}=\frac{1}{8\pi^2}\int G\wedge G+n, \tag{5.9}$$

which must be satisfied for type IIA or M theory.

If we denote the spacetime coordinates by x^μ where $\mu=0,1,2$ and the internal space by the complex coordinates $y^a, a=1,\dots,4$ then in the presence of G flux the metric becomes a warped one

$$ds^2=e^{-\phi(y)}\eta_{\mu\nu}dx^\mu dx^\nu+e^{\phi(y)/2}g_{a\bar{b}}dy^a dy^{\bar{b}}, \tag{5.10}$$

with the G flux satisfying the condition

$$G={}^*G, \quad J\wedge G=0, \tag{5.11}$$

where the Hodge star acts on the internal fourfold with metric g and J is the Kahler form of the fourfold. There is also another non-vanishing G given in terms of the warp factor as $G_{\mu\nu\rho a}=\epsilon_{\mu\nu\rho}\partial_a e^{-3\phi/2}$. The warp factor satisfies the equation

$$\Delta e^{3\phi/2}={}^*\left[4\pi^2 X_8-\frac{1}{2}G\wedge G -4\pi^2\sum_{i=1}^{n}\delta^8(y-y_i)\right], \tag{5.12}$$

where the Laplacian and the Hodge $*$ is defined with respect to g, and X_8 is an eight-form constructed out from curvature tensors given in Eq. (5.26).

The G flux can now decompose in two ways to give different *supersymmetric* backgrounds in type IIB. Let us choose the following basis of one-forms for the fiber torus (with modular parameter τ) [34].

$$dz=dx+\tau dy, \quad d\bar{z}=dx+\bar{\tau}dy. \tag{5.13}$$

In terms of spacetime coordinates the directions y,x are x^3,x^{10}. The $\mathbb{C}P^1$ base has coordinates x^4,x^5 and the other $K3$ is oriented along $x^{6,7,8,9}$. The "flat" directions are $x^{0,1,2,3}$.

Decomposing G flux as

$$\frac{G}{2\pi}=dz\wedge\omega-d\bar{z}\wedge{}^*\omega, \tag{5.14}$$

where ω has one leg along $\mathbb{C}P^1$ and two along $K3$, we get in type IIB the following three forms:

$$H_{NS}=\omega-{}^*\omega, \quad H_{RR}=\omega\tau-{}^*\omega\bar{\tau}. \tag{5.15}$$

The anomaly cancellation condition will now become, in type IIB theory

$$\frac{\chi}{24}=n+\int H_{RR}\wedge H_{NSNS}, \tag{5.16}$$

where n is the number of D3-branes.

We can also decompose G fluxes as

$$\frac{G}{2\pi}=\sum_{i=1}^{k}F_i\wedge\Omega_i, \tag{5.17}$$

where $i=1,2,\dots,k$ are the number of points at which the fiber torus degenerates, and Ω_i are the normalizable harmonic forms localized at the singularities. The physical significance of F_i are the *gauge* fields that would appear on the 7-branes when we go to type IIB by shrinking the fiber torus. The fact that locally a $K3$ looks like a Taub-NUT space implies that there exist a harmonic two form which is anti-self-dual and normalizable. It is not difficult to identify the harmonic two-forms with Ω_i. The anomaly cancellation condition or equivalently the warp factor equation will now become in type-IIB theory:

$$\Delta e^{3\phi/2} = *_{\mathcal{B}} \sum_{i=1}^{k} [F_i \wedge F_i + \text{tr}(R\wedge R)]\delta^2(z-z_i), \tag{5.18}$$

where z_i are the positions of the seven branes.

Hence the general G-flux background will be

$$\frac{G}{2\pi} = \sum_i F_i \wedge \Omega_i + dz \wedge \omega_1 + d\bar{z} \wedge \omega_2, \tag{5.19}$$

where i denotes the number of singularities or an equivalent number of seven branes (in IIB).

The self-duality of G flux does not imply F to be self-dual. This is the precise condition for the existence of a non-commutative instanton.

Since the background is supersymmetric we now expect the metric to be given by the analysis of [35,36]. The metric has the form of a D3-brane metric and the cosmological constant vanishes.

B. Coulomb phase

As discussed in [36] a (2,2) G-flux background is non-supersymmetric if the G flux does not satisfy the primitivity condition, i.e. we require

$$g^{a\bar{b}}G_{a\bar{b}c\bar{d}} \neq 0. \tag{5.20}$$

Therefore let us choose a generic background for the G-flux as

$$\frac{G}{2\pi} = dz \wedge \tilde{\omega}_1 + d\bar{z} \wedge \tilde{\omega}_2. \tag{5.21}$$

We do not impose any conditions on $\tilde{\omega}_{1,2}$ as $g^{a\bar{b}}G_{a\bar{b}c\bar{d}} \neq 0$. The number n of D3-branes can be tuned by choosing some appropriate values for $\tilde{\omega}_{1,2}$, satisfying

$$n = 24 - \int (\tilde{\omega}_1 + \tilde{\omega}_2) \wedge (\tau\tilde{\omega}_1 - \bar{\tau}\tilde{\omega}_2). \tag{5.22}$$

Since the system is *not* supersymmetric we expect the D3-brane to be moving towards the seven branes. And therefore the metric ansatz cannot be a static one as in [35,36].

C. Higher order corrections

Before we end this section let us make some remarks on higher order corrections in 11-dimensional supergravity which were briefly alluded to in the earlier sections. In our model there are two different sources of higher order corrections [37].

(1) *Curvature corrections*. In the M-theory metric the higher curvature corrections come from the following terms in the 11-dimensional Lagrangian:

$$\int d^{11}x \sqrt{-g}\left[J_0 - \frac{1}{2}E_8\right] - \int C \wedge X_8. \tag{5.23}$$

The various terms appearing in the above equation are defined as

$$J_0 = 3\times 2^8 \left(R^{MNPQ}R_{RNPS}R_M{}^{TUR}R^S{}_{TUQ} + \frac{1}{2}R^{MNPQ}R_{RSPQ}R_M{}^{TUR}R^S{}_{TUN}\right) + \mathcal{O}(R_{MN}), \tag{5.24}$$

$$E_8 = \frac{1}{3!}\epsilon^{ABCN_1\ldots N_8}\epsilon_{ABCM_1\ldots M_8}R^{M_1M_2}{}_{N_1N_2}\cdots \times R^{M_7M_8}{}_{N_7N_8}, \tag{5.25}$$

$$X_8 = \frac{1}{3\times 2^{10}\cdot \pi^4}\left[\text{tr}\,\mathcal{R}^4 - \frac{1}{4}(\text{tr}\,\mathcal{R}^2)^2\right]. \tag{5.26}$$

Now assuming that our warp factor is everywhere smooth, then integrating out (5.12) we have the required anomaly cancellation equation for the fourfold. However, if we set $X_8 = 0$ we then observe that there could be *no* non-trivial background fluxes in the model. The only consistent solution then is to take $G = 0$ and $n = 0$. This is the key issue which we think is important to have the required background.

Another point which follows immediately is that since J_0 and E_8 are of the same form (and order) as X_8 it will be inconsistent to neglect them and consider only X_8. Therefore to have a consistent picture in the presence of branes and planes we *cannot* neglect higher order curvature terms. Also, this is a possible way to go around the no-go theorems of Gibbons [38] and Maldacena and Nuñez [39].

(2) *G-flux corrections*. In our model G flux could also contribute higher order corrections. These corrections have not been worked out in any detail. However, they are believed to be of the general form

$$\sum_{m,n,p} \alpha_{mnp}\sqrt{-g}(\partial G)^m G^n R^p, \tag{5.27}$$

where the coefficients α_{mnp} are not known in general.[8] As long as we take the background G fluxes to be smooth and small we can in principle neglect these corrections. For sharply varying G fluxes, however, there could be sizable corrections which may destabilize the background.

VI. DISCUSSION

In our paper the main emphasis was on the derivation of the gauge theory with the particular potential and FI terms from type IIB string theory. When this gauge theory is coupled to gravity in four dimension it leads to hybrid inflation. As briefly discussed in Secs. IV and V we expect the configuration of D7-branes and O7 planes in the presence of

[8]For some special cases when $n = p = 0$ and $m = 4$ they are calculated in [40].

D3-branes (at the equilibrium points on the ‘‘pillow’’) will lead to a dS_4 space as the beginning of inflation. Quantum fluctuation will trigger off inflation in this system.

The Coulomb phase of the cosmological D3-D7 setup describes a period of slow-roll inflation in de Sitter valley and in the Higgs phase describes a supersymmetric ground state with vanishing cosmological constant. As discussed in the text, there is a compactified picture in M theory on a $K3\times K3$ manifold with a choice of G flux on it. When the flux is non-primitive the background is non-supersymmetric and when the background is primitive, supersymmetry is restored. We believe the various moduli in this setup may be fixed by the choice of G flux and higher derivative corrections described in Sec. V [41,36]. We will leave this problem for future work.

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[1] A.D. Linde, Phys. Lett. B **259**, 38 (1991); Phys. Rev. D **49**, 748 (1994).
[2] E.J. Copeland, A.R. Liddle, D.H. Lyth, E.D. Stewart, and D. Wands, Phys. Rev. D **49**, 6410 (1994).
[3] G.R. Dvali, Q. Shafi, and R. Schaefer, Phys. Rev. Lett. **73**, 1886 (1994).
[4] P. Binetruy and G. Dvali, Phys. Lett. B **388**, 241 (1996); E.D. Stewart, Phys. Rev. D **51**, 6847 (1995); E. Halyo, Phys. Lett. B **387**, 43 (1996).
[5] A.D. Linde and A. Riotto, Phys. Rev. D **56**, 1841 (1997); Phys. Lett. B **412**, 28 (1997); Phys. Rep. **314**, 1 (1999).
[6] T. Watari and T. Yanagida, Phys. Lett. B **499**, 297 (2001).
[7] R. Kallosh, hep-th/0109168.
[8] C. Herdeiro, S. Hirano, and R. Kallosh, J. High Energy Phys. **12**, 027 (2001).
[9] B.S. Kyae and Q. Shafi, Phys. Lett. B **526**, 379 (2002).
[10] G.R. Dvali and S.H. Tye, Phys. Lett. B **450**, 72 (1999); E. Halyo, hep-ph/0105341.
[11] S.H. Alexander, Phys. Rev. D **65**, 023507 (2002); C.P. Burgess, M. Majumdar, D. Nolte, F. Quevedo, G. Rajesh, and R.J. Zhang, J. High Energy Phys. **07**, 047 (2001); G.R. Dvali, Q. Shafi, and S. Solganik, hep-th/0105203.
[12] J. Garcia-Bellido, R. Rabadan, and F. Zamora, J. High Energy Phys. **01**, 036 (2002).
[13] C.P. Burgess, P. Martineau, F. Quevedo, G. Rajesh, and R.J. Zhang, J. High Energy Phys. **03**, 052 (2002); R. Blumenhagen, B. Kors, D. Lust, and T. Ott, hep-th/0202124.
[14] E. Bergshoeff, M.J. Duff, C.N. Pope, and E. Sezgin, Phys. Lett. B **199**, 69 (1987).
[15] K. Becker, M. Becker, and A. Strominger, Nucl. Phys. **B456**, 130 (1995).
[16] M. Cederwall, A. von Gussich, B.E. Nilsson, and A. Westerberg, Nucl. Phys. **B490**, 163 (1997); M. Aganagic, C. Popescu, and J.H. Schwarz, Phys. Lett. B **393**, 311 (1997); M. Cederwall, A. von Gussich, B.E. Nilsson, P. Sundell, and A. Westerberg, Nucl. Phys. **B490**, 179 (1997); E. Bergshoeff and P.K. Townsend, *ibid.* **B490**, 145 (1997); M. Aganagic, C. Popescu, and J.H. Schwarz, *ibid.* **B495**, 99 (1997); I. Bandos, K. Lechner, A. Nurmagambetov, P. Pasti, D. Sorokin, and M. Tonin, Phys. Rev. Lett. **78**, 4332 (1997).
[17] E. Bergshoeff, R. Kallosh, T. Ortin, and G. Papadopoulos, Nucl. Phys. **B502**, 149 (1997)
[18] C. Vafa, Nucl. Phys. **B469**, 403 (1996).
[19] A. Sen, Nucl. Phys. **B475**, 562 (1996).
[20] T. Banks, M.R. Douglas, and N. Seiberg, Phys. Lett. B **387**, 278 (1996).
[21] D.Z. Freedman, Phys. Rev. D **15**, 1173 (1977).
[22] M. Berkooz, M.R. Douglas, and R.G. Leigh, Nucl. Phys. **B480**, 265 (1996).
[23] N. Nekrasov and A. Schwarz, Commun. Math. Phys. **198**, 689 (1998).
[24] N. Seiberg and E. Witten, J. High Energy Phys. **09**, 032 (1999).
[25] M. Marino, R. Minasian, G.W. Moore, and A. Strominger, J. High Energy Phys. **01**, 005 (2000).
[26] S. Terashima, Phys. Lett. B **477**, 292 (2000).
[27] S. Moriyama, J. High Energy Phys. **08**, 014 (2000).
[28] K. Dasgupta and S. Mukhi, Phys. Lett. B **385**, 125 (1996).
[29] A. Sen, J. High Energy Phys. **06**, 007 (1998); A. Sen and B. Zwiebach, *ibid.* **03**, 036 (2000).
[30] N. Seiberg and E. Witten, Nucl. Phys. **B426**, 19 (1994); **B430**, 485(E) (1994).
[31] N. Seiberg and E. Witten, Nucl. Phys. **B431**, 484 (1994).
[32] A. Dabholkar, hep-th/9804208.
[33] A. Sen, J. High Energy Phys. **10**, 002 (1997).
[34] K. Dasgupta, G. Rajesh, and S. Sethi, J. High Energy Phys. **08**, 023 (1999).
[35] K. Becker and M. Becker, Nucl. Phys. **B477**, 155 (1996).
[36] K. Becker and M. Becker, J. High Energy Phys. **07**, 038 (2001).
[37] A.A. Tseytlin, Nucl. Phys. **B584**, 223 (2000).
[38] G.W. Gibbons, in *Supersymmetry, Supergravity and Related Topics*, edited by F. del Aguila, J. A. de Azcárraga, and L.E. Ibañez (World Scientific, Singapore, 1985), pp. 346–351.
[39] J. Maldacena and C. Nuñez, Int. J. Mod. Phys. A **16**, 822 (2001).
[40] S. Deser and D. Seminara, Phys. Rev. D **62**, 084010 (2000).
[41] S. Gukov, C. Vafa, and E. Witten, Nucl. Phys. **B584**, 69 (2000); **B608**, 477 (2000); S.B. Giddings, S. Kachru, and J. Polchinski, ‘‘Hierarchies from fluxes in string compactifications,’’ hep-th/0105097.