Gauge-gravity correspondence in an accelerating universe

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We discuss time-dependent backgrounds of type-IIB supergravity realizing gravitation duals of gauge theories formulated in de Sitter space-time as a tool of embedding de Sitter space in a supergravity. We show that only the gravitational duals to nonconformal gauge theories are sensitive to a specific value of a Hubble parameter. We consider two nontrivial solutions of this type: a gravity dual to six-dimensional (1, 1) little string theory, and to a four-dimensional cascading $SU(N+M)\times SU(N)$ supersymmetric gauge theory (related to fractional D3-branes on a singular conifold according to Klebanov and co-workers), in an accelerating universe. In both cases we argue that the IR singularity of the geometry is regulated by the expansion of the gauge theory background space-time.

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I. INTRODUCTION

Gauge-theory–gravity duality[1] relates a gauge theory on the world volume of a large number of D-branes to purely supergravity backgrounds where the branes are replaced by the corresponding fluxes. In the simplest case, the duality is realized [2] by a system of N D3-branes in a flat type-IIB string theory background. At small 't Hooft coupling $g_s N \ll 1$, the system is best described by open strings and realizes $SU(N)$ $\mathcal{N}=4$ supersymmetric gauge theory. In the limit of strong 't Hooft coupling, this gauge theory has a perturbative description as type-IIB supergravity compactified on $AdS_5\times S^5$, with N units of the Ramond-Ramond (RR) 5-form flux through the S^5. If this is a genuine equivalence, then phenomena observed on the gauge theory side should have a dual description in string theory on $AdS_5\times S^5$. In particular, *any* deformation on the gauge theory visible in the large-N limit should have a counterpart in the dual gravitational description, and vice versa.

As in [3], we use "deformations" in a generalized sense. For example, Klebanov-Witten duality [4] describing regular D3-branes placed at a conical singularity in type-IIB string theory can be thought of as a Z_2 orbifold of the original duality of Maldacena [2] along with a certain mass deformation that leaves only a quarter of the original supersymmetries unbroken. One could go a step further and consider deformations of a background space-time in which one formulates gauge dynamics. In [3], a gauge-gravity correspondence was considered in which Minkowski background space-time of the Klebanov-Strassler (KS) [5] cascading gauge theory was replaced with $R\times S^3$ or (in a Euclidean case) S^4. It was argued there that the curvature of the background geometry provided an infrared cutoff on the gauge theory dynamics and resolved the Klebanov-Tseytlin (KT) [6] naked singularity.

A natural extension to the proposal of [3] is to ask the following question: what would the gauge-gravity duality of Maldacena look like when the gauge theory space-time background is de Sitter? This is a perfectly valid "deformation" of the gauge theory background where one "turns on" a Hubble parameter. And thus, provided the original gauge-gravity correspondence was exact, one should be able to map this deformation onto the dual supergravity. In this paper, we describe such a map. We would like to emphasize that, much like in the original Maldacena correspondence, the gauge theory space-time is not dynamical on the gauge theory side of the correspondence. In other words, on the gauge theory side of the correspondence, we completely neglect the backreaction of the gauge theory dynamics on the background, and we ignore background fluctuations as well. The story on the gravity side of the correspondence is drastically different: here, as in the original Maldacena correspondence, what was the gauge theory background becomes a part of a dynamical type-IIB supergravity background. Thus, finding a gravity dual to a gauge theory on a (decoupled) dS background would provide an embedding of this space-time into dynamical supergravity. Put differently, we want to view "cosmological" deformation of the gauge-gravity correspondence as a tool of embedding a de Sitter space-time into a supergravity.[2]

The paper is organized as follows. In the next section, we describe a motivation for a time-dependent metric ansatz of type-IIB supergravity background dual to a gauge theory in an accelerating universe. We observe that de Sitter deformation applied to the $\mathcal{N}=4$ $SU(N)$ supersymmetric Yang-Mills (SYM) gauge-gravity correspondence does not give rise to a different geometry on the dual supergravity side: all we get is a de Sitter slicing of the AdS factor in the original Maldacena duality. Nonetheless, we expect the deformed gauge theory to be physically rather different from the undeformed one. In particular, because of the conformal coupling of the gauge theory scalars to the scalar curvature, in the $H\neq 0$ (H is the expansion rate of the universe) case, the SYM theory would

[1]For reviews and references see, e.g., [1].

[2]Related ideas of realizing de Sitter gravity in warped compactifications of type-IIB string theory were discussed in [7].

not have a moduli space.[3,4] We further show that conformal gauge theories are the only examples for which supergravity duals for the nonzero Hubble parameter H are related by some coordinate reparametrization to their $H=0$ supergravity duals.

We then move on to consider nonconformal examples in Secs. III and IV. In Sec. III, we present supergravity dual to (1, 1) little string theory (LST) in an inflationary patch of the dS_6. The $H=0$ solution reproduces the Bogomol'nyi-Prasad-Summerfield (BPS) system of $N\gg 1$ Neveu-Schwarz 5-branes (NS5-branes), and thus has curvature singularity (in the Einstein frame) at the branes core. From the dual gauge theory perspective, this singularity is generated by the zero modes of the $d=6$ SYM theory, which is the infrared limit of (1, 1) LST. We explicitly demonstrate that de Sitter deformation of LST regulates this curvature singularity.

In Sec. IV, we briefly discuss gravitational dual to Klebanov-Tseytlin–Klebanov-Strassler (KT/KS) cascading gauge theory [6,5] in an accelerating universe. We show that the KT deformation is related by a Wick rotation plus some scaling of the KT gauge theory on S^4 previously considered in [3]. Thus the infrared singularity of the extremal KT geometry is resolved for $H\neq 0$ as explained in [3]. We conclude in Sec. V.

II. SUPERGRAVITY DUALS OF GAUGE THEORIES IN AN ACCELERATING UNIVERSE

We mentioned in the Introduction that given the original gauge-gravity duality of Maldacena, there is a simple way to embed dS space-time into supergravity. The reason for this is that since we can deform a background space-time of the gauge theory from Minkowski to a flat Robertson-Walker universe by simply "turning on" a Hubble parameter, we should be able to do this in the supergravity dual to this gauge theory.

Typically, in a gauge-gravity correspondence the dual supergravity metric[5] can be written as

$$ds_{10E}=c_1^2(dM_d)^2+c_2^2dr^2+(d\mu_{9-d})^2, \tag{2.1}$$

where M_d is a d-dimensional Minkowski space-time, which is related to the space-time background of the dual gauge theory, and μ_{9-d} (for a fixed τ) is a compact $(9-d)$-dimensional Riemannian manifold that encodes the gauge theory dynamics at energy scale $E\sim\rho$ with $c_2dr\sim d\rho/\rho$ as $\rho\to\infty$. From now on we consider only the cases where c_i depend only on r.[6] The metric on M_d does not depend on the angles of μ_{9-d} while $(d\mu_{9-d})^2$ does not depend on the M_d coordinates, though both M_d and μ_d can have explicit r dependence.[7] It seems natural to assume that such "separation of variables" would hold even when we start deforming the gauge theory space-time M_d. Specifically, taking the d-dimensional gauge theory in an accelerating universe,

$$(ds_d^H)^2=-dt^2+e^{2Ht}d\vec{x}^2, \tag{2.2}$$

which for $H=0$ has a dual supergravity background with the metric (2.1), we assume the metric ansatz of the dual supergravity for general H to be

$$(dM_d)^2\to(dM_d^H)^2\equiv(ds_d^H)^2. \tag{2.3}$$

In the orthonormal frame

$$e^1=c_1dt,$$

$$e^{i+1}=e^{Ht}c_1dx_i,\quad i=1,...d-1, \tag{2.4}$$

$$e^{d+1}=c_2dr,$$

$$e^j,\quad j=d+2,...,10\quad \text{such that}\quad e^je^j\equiv(d\mu_{9-d})^2, \tag{2.5}$$

the Ricci components of the metric are time-independent, so in this frame the supergravity fluxes and the dilaton would be time-independent as well.

We begin explicit examples by considering the case of the $H\neq 0$ deformation of the gauge-gravity correspondence discussed in [4] where the gauge theory is conformal, namely D3 branes at a conical singularity.[8] We observe that the dual supergravity background for $H\neq 0$ still remains $AdS_5\times S^5$: the only difference is that now we are doing a de Sitter slicing of the AdS factor in the metric.

Type-IIB equations of motion can be solved analytically in this case. We find

$$ds_{10}^2=\rho^2(-dt^2+e^{2Ht}d\vec{x}^2)+\frac{L^2d\rho^2}{L^2H^2+\rho^2}+L^2ds^2_{T^{1,1}}, \tag{2.6}$$

where $(ds_{T^{1,1}})^2$ is the standard metric on $T^{1,1}=[SU(2)\times SU(2)]/U(1)$ and

$$L^4=4\pi g_sN(\alpha')^{\frac{27}{16}}, \tag{2.7}$$

with N being the number of D3-branes. The metric (2.6) is supported by the following five-form flux:

$$F_5=\mathcal{F}_5+\star\mathcal{F}_5,\quad \mathcal{F}_5=-L^4d\,\mathrm{vol}_{T^{1,1}}. \tag{2.8}$$

[3]A similar phenomenon for the $R\times S^3$ deformation of the $\mathcal{N}=4$ SYM theory was emphasized in [3].

[4]It would be very interesting to study de Sitter deformations of gauge theories from a purely field-theoretic perspective. In this paper, we focus on the supergravity part of the de Sitter deformed gauge-gravity correspondence.

[5]We always work in the Einstein frame.

[6]This is not always the case, as, for example, in Polchinski-Strassler gauge-gravity correspondence [8].

[7]The examples where $(dM_d)^2$ has an r dependence correspond to gauge theories formulated on compact manifolds as in [9,3].

[8]There is an obvious generalization to $AdS_5\times S^5$ and other conformal cases.

The above solution is related by a coordinate transformation to the extremal ($H=0$) D3-brane solution. Indeed, first introduce

$$\tau=\frac{1}{H}e^{-Ht}. \tag{2.9}$$

Then the change of variables that do the job is

$$r=\frac{\rho}{H\tau}=\rho e^{Ht}, \tag{2.10}$$

$$d\tilde{t}=-\frac{\sqrt{L^2H^2+\rho^2}}{\rho}d\tau-\frac{L^2H^2\tau}{\sqrt{L^2H^2+\rho^2}}\frac{d\rho}{\rho^2}, \tag{2.11}$$

where r and $\tilde{t}$ are the radial and the time coordinates of the $H=0$ solution. Note that in Eq. (2.11), $d^2\tilde{t}=0$, so this equation can indeed be integrated

$$\tilde{t}=\frac{\sqrt{L^2H^2+\rho^2}e^{-Ht}}{H\rho}. \tag{2.12}$$

From the coordinate transformations (2.10) and (2.12) we see that de Sitter slicing of AdS_5, as in Eq. (2.6), covers ‘‘half’’ ($\tilde{t}\geqslant 0$ region) of its Poincaré patch. It is easy to see that this slicing can be obtained from the analytical continuation (along with some scaling limits) of the Euclidean AdS in the ‘‘hyperboloid’’ parametrization.[9] Really, for the AdS_{d+1} the metric in this parametrization is given by

$$\begin{aligned} ds^2_{AdS_{d+1}}&=\sinh^2\rho(dS^d)^2+d\rho^2\\ &=\sinh^2\rho[d\tau^2+\sin^2\tau(dS^{d-1})]^2+d\rho^2. \end{aligned} \tag{2.13}$$

Now the Wick rotation of Eq. (2.13), $\tau\rightarrow i\tau$, and the ‘‘decompactification’’ limit of S^{d-1}, $(dS^{d-1})^2\rightarrow(dR^{d-1})^2$, along with $\tau\gg 1$ give

$$\begin{aligned} ds^2_{AdS_{d+1}}&=\sinh^2\rho[-d\tau^2+e^{2\tau}(dR^{d-1})]^2+d\rho^2\\ &=r^2[-d\tau^2+e^{2\tau}(dR^{d-1})]^2+\frac{dr^2}{1+r^2}, \end{aligned} \tag{2.14}$$

where $r\equiv\sinh\rho$. Thus, the coordinate transformations (2.10) and (2.12) must be represented by the (corresponding scaling limit of the) Wick rotation of local coordinate transformations relating Poincaré and ‘‘hyperboloid’’ parametrizations of the Euclidean AdS space.

In the rest of this section, we address the question of when the $H\neq 0$ deformation of a given gauge-gravity duality is related to the original ($H=0$) correspondence by some change of variables, as in the case above. We will argue that this is so only when the gauge theory in the duality correspondence is conformal. Let

$$\begin{aligned} (ds^0_{10E})^2=&(c^0_1)^2(-dt^2+d\vec{x}^2)\\ &+(c^0_2)^2dr^2+(d\mu^0_{9-d})^2 \end{aligned} \tag{2.15}$$

be a supergravity metric in the original[10] gauge-gravity correspondence, and

$$\begin{aligned} (ds^H_{10E})^2=&(c_1)^2(-d\tau^2+e^{2H\tau}d\vec{x}^2)\\ &+(c_2)^2d\rho^2+(d\mu_{9-d})^2 \end{aligned} \tag{2.16}$$

is the metric corresponding to its $H\neq 0$ deformation. We want to know when Eq. (2.16) is related by some coordinate reparametrization to Eq. (2.15). Replacing $\tau\rightarrow(1/H)e^{-H\tau}$ in Eq. (2.16), we get

$$\begin{aligned} (ds^H_{10E})^2=&\frac{c_1^2}{H^2\tau^2}(-d\tau^2+d\vec{x}^2)\\ &+c_2^2d\rho^2+(d\mu_{9-d})^2. \end{aligned} \tag{2.17}$$

Let us ignore for now the internal piece of the metric. Comparing $d\vec{x}^2$ pieces of the metric in Eqs. (2.17) and (2.15), we see that

$$c^0_1(r)=\frac{c_1(\rho)}{H\tau}, \tag{2.18}$$

so that

$$dr=\frac{[c_1(\rho)]'\tau d\rho-c_1(\rho)d\tau}{H\tau^2[c^0_1(r)]'}. \tag{2.19}$$

Taking the most general ansatz for dt,

$$dt=g_1(\rho,\tau)d\tau+g_2(\rho,\tau)d\rho, \tag{2.20}$$

and matching Eqs. (2.17) and (2.15), we find

$$g_1(\rho,\tau)=\frac{c_1(\rho)\sqrt{[c^0_2(r)]^2+\tau^2[c^0_1(r)']^2}}{c^0_1(r)H\tau^2[c^0_1(r)]'}, \tag{2.21}$$

$$g_2(\rho,\tau)=-\frac{[c^0_2(r)]^2[c_1(\rho)]'}{H\tau c^0_1(r)[c^0_1(r)]'\sqrt{[c^0_2(r)]^2+\tau^2[c^0_1(r)']^2}}, \tag{2.22}$$

plus we have a constraint[11]

$$\begin{aligned} 0=&-[c^0_2(r)]^2[c_1(\rho)']^2+c_2(\rho)^2H^2[c^0_2(r)]^2\\ &+c_2(\rho)^2H^2\tau^2[c^0_1(r)']^2. \end{aligned} \tag{2.23}$$

Since we should be able to integrate Eq. (2.20),

[9]This is a Euclidean AdS_{d+1} parametrization where the constant radial slice is S^d.

[10]That is, a gauge theory is formulated in Minkowski space-time.

[11]We are assuming that the c^0_1 warp factor in Eq. (2.15) is nontrivial, that is, not a constant.

$$d^2 t \equiv 0. \tag{2.24}$$

It turns out, given the above expressions, that we can rewrite Eq. (2.24) as

$$0 = \frac{d}{dr}\left[\frac{[c_1^0(r)]'}{c_1^0(r)c_2^0(r)}\right]. \tag{2.25}$$

Without loss of generality, we can assume that in the original duality[12]

$$c_1^0(r) = r. \tag{2.26}$$

From Eq. (2.25) we find then

$$c_2^0 = \frac{L}{r}, \tag{2.27}$$

where L is some constant. Finally, the only way $(d\mu_{d-9})^2$ and $(d\mu_{d-9}^0)^2$ could ever match is when they are independent of ρ and r correspondingly. Thus we conclude that the metric (2.15) is actually

$$(ds_{10E}^0) = (ds_{\mathrm{AdS}_{d+1}})^2 + (d\mu_{9-d})^2, \tag{2.28}$$

where the metric on μ_{9-d} does not depend on the AdS_{d+1} radial coordinate. The AdS factor in Eq. (2.28) points to the conformal invariance of the dual gauge theory.

The above discussion suggests that for the embedding of a de Sitter space-time in supergravity, we should look for deformations of gauge-gravity duality where the gauge theory is not conformal. We will present explicit examples of such deformations in the next two sections.

III. (1, 1) LST IN AN ACCELERATING UNIVERSE AND THE IR SINGULARITY RESOLUTION BY INFLATION

In this section, we describe $H \neq 0$ deformations of the (1, 1) little string theory, realized on the world volume of NS5-branes in type-IIB string theory. The effective infrared description of the LST is in terms of $d=6$ $\mathcal{N}=2$ supersymmetric Yang-Mills theory. As this gauge theory is not conformal, we expect to get a nontrivial embedding of dS_6 from its $H \neq 0$ deformation.

In the extremal case, $H=0$, the supergravity approximation breaks down near the core of the branes. This curvature singularity can be thought of as being generated by the zero modes of the IR free $d=6$ SYM theory. Since a Hubble parameter provides an infrared cutoff on the dynamics of the theory, we expect that it should regulate the curvature singularity of the extremal background. We argue that this is indeed so.

We take the following ansatz for the metric of LST holographic dual in the inflationary patch of the dS_6:

$$(ds_{10E})^2 = c_1^2(-dt^2 + e^{2Ht}d\vec{x}^2) + c_2^2 d\rho^2 + \frac{c_3^2}{4}(g_1^2 + g_2^2 + g_3^2), \tag{3.1}$$

where $c_i = c_i(\rho)$, and g_i are the SU(2) left-invariant one-forms,

$$g_1 = \cos\phi\; d\theta + \sin\phi \sin\theta\; d\psi,$$
$$g_2 = \sin\phi\; d\theta - \cos\phi \sin\theta\; d\psi, \tag{3.2}$$
$$g_3 = d\phi + \cos\theta\; d\psi.$$

We assume the dilaton $\Phi \equiv \ln g_s$ to be a function of ρ only and the same NS-NS 3-form fluxes as in the extremal case,

$$H_3 = n g_1 \wedge g_2 \wedge g_3, \tag{3.3}$$

where n is related to the number of NS5-branes. Solving type-IIB supergravity equations, we get

$$0 = \left[\frac{g_s' c_1^6 c_3^3}{g_s c_2}\right]' + \frac{32 n^2 c_1^6 c_2}{c_3^3 g_3}, \tag{3.4}$$

$$0 = \left[\frac{c_1' c_1^5 c_3^3}{c_2}\right]' - \frac{c_1^4 c_2}{g_s c_3^3}(5H^2 g_s c_3^6 + 8n^2 c_1^2), \tag{3.5}$$

$$0 = \left[\frac{c_3' c_3^2 c_1^6}{c_2}\right]' - \frac{2c_1^6 c_2}{g_s c_3^3}(g_s c_3^4 - 12n^2), \tag{3.6}$$

along with the first-order constraint

$$0 = \frac{12 g_s^2 c_3^4}{c_1^4}[c_3 c_1]'[c_1^5 c_3]' - c_3^6 c_1^2 (g_s')^2 + 4 g_s c_2^2 (16 n^2 c_1^2 - 3 g_s c_3^4 [c_1^2 + 5H^2 c_3^2]). \tag{3.7}$$

It is consistent with Eqs. (3.4)–(3.7) to choose an ansatz for the warp factors c_i similar to the extremal NS5-brane solution,

$$c_1 = f g_s^{-1/4}, \quad c_2 = c_3 = 2n^{1/2} g_s^{-1/4}. \tag{3.8}$$

We will end up with the following equations for f, g_s:

$$0 = \left[\frac{g_s' f^6}{g_s^3}\right]' + \frac{2f^6}{g_s^2}, \tag{3.9}$$

$$0 = \left[\left(\frac{g_s}{f^4}\right)' \frac{f^{10}}{g_s^3}\right]' + \frac{2f^4(40H^2 n + f^2)}{g_s^2}, \tag{3.10}$$

along with a first-order constraint

$$0 = g_s^2(60H^2 n + 2f^2) - (15 g_s^2 [f']^2 - 12 g_s' f' g_s f + [g_s']^2 f^2). \tag{3.11}$$

[12] This fixes an arbitrary choice of a radial coordinate in Eq. (2.15).

Though we cannot solve Eqs. (3.9)–(3.11) analytically, it is straightforward to exhibit a smooth solution. Really, a smooth solution as $\rho\to 0$ is

$$g_s = g_0[1-\tfrac{1}{7}\rho^2+\tfrac{5}{378}\rho^4+O(\rho^6)], \tag{3.12}$$

$$f = 2Hn^{1/2}[\rho-\tfrac{1}{63}\rho^3+\tfrac{1}{1470}\rho^5+O(\rho^7)], \tag{3.13}$$

where g_0 is an integration constant related to the string coupling. As $\rho\to\infty$, we rather find[13]

$$g_s\to g_0\rho^{3/4}e^{-\rho}, \quad f\to Hn^{1/2}\sqrt{20\rho}. \tag{3.14}$$

Note that the curvature of Eq. (3.1) can be maintained arbitrarily small by taking g_0 small. Thus the $H\neq 0$ deformation indeed regulates the strong curvature region of the extremal NS5-brane background. On the other hand, from Eq. (3.14) we see that turning on a Hubble parameter induces a logarithmic correction to the asymptotically linear dilaton background of the extremal NS5-branes. This should be contrasted with the finite-energy density regularization of this geometry, where one still recovers an asymptotically linear dilaton [10].

From the above analysis, it appears that given the Hubble parameter H, and for a fixed number of NS5-branes, there is a one-parameter family of the LST de Sitter deformations, characterized by g_0. Furthermore, it is g_0 and not H that controls the curvature of the geometry (3.1). This is surprising, as LST does not have any continuous coupling constant. Also, physically, we expect that the supergravity approximation describing deformed LST should break down for sufficiently small H (in string units), as this theory should still be weakly coupled at low energies. This suggests that g_0 cannot be a free parameter. In what follows, we argue that this is indeed so. We find that

$$g_0\sim 1/H^4, \tag{3.15}$$

so that small g_0 (necessary for the validity of the supergravity description) corresponds to a large Hubble parameter in string units, and thus the full picture is consistent with the general lore for the absence of a dual supergravity description to a weakly coupled gauge theory. Before we proceed with an argument for Eq. (3.15), we would like to mention that a somewhat similar phenomenon occurs in the near-extremal deformation of the NS5-branes [10]. Really, the near-extremal deformation of LST is characterized by a single parameter,[14] namely the energy density μ. On the other hand, its holographic dual naively has two parameters: r_0 (the location of the black five-branes horizon) and g_h (the value of the string coupling at the horizon). It turns out that by a simple change of a radial coordinate, the background geometry of the near-extremal NS5-branes can be shown to depend only on a combination r_0^2/g_h^2, which can be further identified with the energy density above the extremality in string units μ [10]. In our case, though we described a two-parameter family $\{g_0,H\}$ of the regular solutions of Eqs. (3.9)–(3.11), the H dependence of the geometry can also be eliminated by redefining the time coordinate $t\to\tau \equiv 1/He^{-Ht}$. This is not very illuminating, as in doing so we are changing the reference energy scale from the LST perspective. Rather, we continue measuring all energies in string units. To relate g_0 and H, we study the propagation of a minimally coupled scalar in the background (3.1) and on the NS5-brane probe. Specifically, consider a massless scalar χ minimally coupled to the Einstein metric (3.1) with zero angular momentum on S^3. The corresponding wave equation is

$$0=-\partial_t[e^{5Ht}\partial_t\chi]+e^{3Ht}\partial_i^2\chi + \frac{e^{5Ht}g_s^2(\rho)}{4nf^4(\rho)}\partial_\rho\left[\frac{f^6(\rho)}{g_s^2(\rho)}\partial_\rho\chi\right], \tag{3.16}$$

where i denotes the spatial directions on the NS5-brane. The last term in Eq. (3.16) can be interpreted as a ρ-dependent mass term operator on the LST space-time. Using Eqs. (3.9)–(3.11), we can explicitly factor out the $\{g_0,H\}$ dependence of this operator,

$$\frac{e^{5Ht}g_s^2(\rho)}{4nf^4(\rho)}\partial_\rho\left[\frac{f^6(\rho)}{g_s^2(\rho)}\partial_\rho\cdots\right]\equiv e^{5Ht}H^2O(\rho)[\cdots]. \tag{3.17}$$

Assuming the factorized dependence of χ on ρ,

$$\chi(t,\overline{x};\rho)=\kappa(\rho)\tilde{\chi}(t,\overline{x}), \tag{3.18}$$

we get from Eq. (3.16)

$$0=-\partial_t[e^{5Ht}\partial_t\tilde{\chi}]+e^{3Ht}\partial_i^2\tilde{\chi}+e^{5Ht}H^2\lambda(\rho)\tilde{\chi}, \tag{3.19}$$

where $\lambda(\rho)\equiv 1/\kappa(\rho)O(\rho)[\kappa(\rho)]$. As in the original gauge-gravity correspondence of Maldacena, we would like to interpret ρ as a (measured in string units) holographic renormalization group (RG) scale. Thus the dynamics of $\tilde{\chi}$ should be qualitatively similar to the dynamics of the generically *massive* scalar η propagating along a probe NS5-brane oriented along $\{t,\overline{x}\}$, and sitting at a fixed radial coordinate ρ. With a scalar η minimally coupled to the induced Einstein frame metric on the probe, we find its wave equation to be

$$0=-\partial_t[e^{5Ht}\partial_t\eta]+e^{3Ht}\partial_i^2\eta+e^{5Ht}m^2g_s^{-1/2}(\rho)\eta, \tag{3.20}$$

where m is a constant mass of η. Extracting the g_0 dependence of the last term in Eq. (3.20) and comparing with Eq. (3.19), we are led to the identification (3.15),

$$g_0^{-1/2}\sim H^2.$$

We believe that the above arguments relating g_0 and H are qualitatively correct, and apparently lead to the expected physical picture. It is important to find a more precise understanding of this relation, or in other words the map between the supergravity parameters $\{g_0,H\}$ and their LST dual. This will likely require an understanding of how to measure en-

[13]These asymptotics can also be verified by numerical integration.

[14]Classically, the temperature of the LST is independent of the energy density.

ergies in nonstatic, asymptotically nonflat supergravity backgrounds. Finally, it is well known that string propagation in the throat geometry of the near-extremal NS5-branes corresponds to an exact conformal field theory (CFT).[15] It would be interesting to see whether there is a CFT description of the (1,1) LST in the de Sitter background presented here.

IV. de SITTER DEFORMATIONS OF THE KTKS BACKGROUNDS

Our aim in this section will be to explore dS embedding in the supergravity in the context of the corresponding deformation of the KT model [6]. Here, the conformal invariance of the gauge theory on the D3-branes at a conical singularity [4] is broken by adding fractional D3-branes [12]. We also comment on the de Sitter deformation of the KS background [5].

We will start with the same ansatz as in [6] and simply replace $1+3$ "longitudinal" directions by the Robertson-Walker metric with flat spacelike hypersurfaces,

$$(dM_4^H)^2=-dt^2+e^{2Ht}d\vec{x}^2. \tag{4.1}$$

As we show,[16] there will be a direct relation to the KT model on S^4 considered in [3].

As in [6], we will impose the requirement that the background has Abelian symmetry associated with the U(1) fiber of $T^{1,1}$ as we will consider a phase where chiral symmetry is restored.[17] Our general ansatz for a ten-dimensional (10D) Einstein-frame metric will involve three functions *y, z,* and *w* of radial coordinate *u*,[18]

$$ds_{10E}^2=e^{2z}(dM_4^H)^2+e^{-2z}[e^{10y}du^2+e^{2y}(dM_5)^2]. \tag{4.2}$$

Here M_5 is a deformation of the $T^{1,1}$ metric,

$$(dM_5)^2=e^{-8\omega}e_\psi^2+e^{2\psi}(e_{\theta_1}^2+e_{\phi_1}^2+e_{\phi_1}^2+e_{\phi_2}^2), \tag{4.3}$$

$$e_\psi=\tfrac{1}{3}(d\psi+\cos\theta_1 d\phi_1+\cos\theta_2 d\phi_2),$$

$$e_{\theta_i}=\frac{1}{\sqrt{6}}d\theta_i,\quad e_{\phi_i}=\frac{1}{\sqrt{6}}\sin\theta_i dgf_i.$$

As for the matter fields, we will assume that the dilaton Φ may depend on *u*, and our ansatz for the *p*-form fields will be exactly as in the extremal KT case [6] and in [13,3],

$$F_3=Pe_\psi\wedge(e_{\theta_1}\wedge e_{\phi_1}-e_{\theta_2}\wedge e_{\phi_2}),$$
$$B_2=f(u)(e_{\phi_1}\wedge e_{\phi_1}-e_{\theta_2}\wedge e_{\phi_2}), \tag{4.4}$$

$$F_5=\mathcal{F}+*\mathcal{F},$$

$$\mathcal{F}=K(u)e_\psi\wedge e_{\theta_1}\wedge e_{gf_1}\wedge e_{\theta_2}\wedge e_{\phi_2}, \tag{4.5}$$

$$K(u)=Q+2Pf(u),$$

where, as in [6], the expression for *K* follows from the Bianchi identity for the 5-form. The constants *Q* and *P* are proportional to the numbers of standard and fractional D3-branes.

We could now directly derive the corresponding system of type-IIB supergravity equations of motion describing the radial evolution of the five unknown functions of *u*: y,z,w,K,Φ. A better approach is to notice that the background we consider here could be obtained from the KT model on S^4 discussed in [3]. Really, the only difference of our case from the S^4 compactification of [3] is the replacement of the "longitudinal" directions (4.1) with $(dS^4)^2$,

$$(dM_4^H)^2\to(dM_4)^2\equiv(dS^4)^2$$
$$=d\alpha^2+\sin^2\alpha[d\beta^2+\sin^2\beta(d\gamma^2+\sin^2\gamma d\delta^2)]. \tag{4.6}$$

Now, Wick rotation of Eq. (4.6), $\alpha\to i\alpha$, and the scaling limit on S^3 parametrized by β,γ,δ, $(dS^3)^2\to d\vec{x}^2$, along with $\alpha\gg1$ gives

$$(dS^4)^2\to-da^2+e^{2\alpha}d\vec{x}^2, \tag{4.7}$$

which is precisely Eq. (4.1) with $H=1$. Thus, the resulting equations[19] are just the straightforward modification of Eqs. (4.7)–(4.12) of [3],

$$10y''-8e^{8y}(6e^{-2w}-e^{-12w})-30H^2e^{10y-4z}+\Phi''=0, \tag{4.8}$$

$$10w''-12e^{8y}(e^{-2w}-e^{-12w})-\Phi''=0, \tag{4.9}$$

[15]See, for example, [11].

[16]I would like to thank Arkady Tseytlin for pointing this out.

[17]Much like in the case of the LST on the de Sitter space-time, we expect the Hubble scale to realize an IR cutoff on the gauge theory dynamics. Thus for sufficiently high *H* (which we take to be the case in this section), we expect restoration of the chiral symmetry in the dual gauge theory.

[18]For $H=0$, this metric can be brought into a more familiar form $ds_{10E}^2=h^{-1/2}(r)(dM_4^{H=0})^2+h^{1/2}(r)[dr^2+r^2ds_5^2]$, where $h=e^{-4z}$, $r=e^{y+w}$, and $e^{10y}du^2=dr^2$. When $w=0$ and $e^{4y}=r^4=1/4u$, the transverse 6D space is the standard conifold with $M_5=T^{1,1}$. Small *u* thus corresponds to large distances in 5D and vice versa. In the AdS_5 region, large *u* is near the origin of AdS_5 space, while $u=0$ is its boundary.

[19]We also obtained these equations directly in the background (4.2).

$$\Phi'' + e^{-\Phi+4z-4y-4w}\left(\frac{K'^2}{4P^2} - e^{2\Phi+8y+8w}P^2\right) = 0, \tag{4.10}$$

$$4z'' - K^2e^{8z} - e^{-\Phi+4z-4y-4w} \times\left(\frac{K'^2}{4P^2} + e^{2\Phi+8y+8w}P^2\right) - 12H^2e^{10y-4z} = 0, \tag{4.11}$$

$$(e^{-\Phi+4z-4y-4w}K')' - 2P^2Ke^{8z} = 0, \tag{4.12}$$

with the first-order constraint

$$5y'^2 - 2z'^2 - 5w'^2 - \tfrac{1}{8}\Phi'^2 - \tfrac{1}{4}e^{-\Phi+4z-4y-4w}\frac{K'^2}{4P^2} - 3H^2e^{10y-4z} - e^{8y}(6e^{-2w} - e^{-12w}) + \tfrac{1}{4}e^{\Phi+4z+4y+4w}P^2 + \tfrac{1}{8}e^{8z}K^2 = 0. \tag{4.13}$$

Lacking the exact solution of the above system, it was nonetheless demonstrated in [3] the existence of a smooth interpolation (in radial coordinate only) between (i) a nonsingular short-distance region where the 10D background is approximately $AdS_5 \times T^{1,1}$ written in the coordinates where the $u = \text{const}$ slice is S^4, and (ii) a long-distance region where the 10D background approaches the KT solution. This was shown by starting with the short-distance ($u=\infty$ or $\rho=0$) region, i.e., $AdS_5\times T^{1,1}$ space (with the radius determined by the effective charge K_*) and demonstrating that by doing perturbation theory in the small parameter $P^2/K_* \ll 1$ one can match it onto the KT asymptotics at large distances ($u \to 0$ or $\rho\to\infty$). The crucial point was that $O(P^2/K_*)$ perturbations were regular at small distances. One can literally repeat this analysis of [3] in our case to argue that for a large enough Hubble parameter H, the naked singularity of the KT geometry will be resolved. Here the short-distance region is a direct product of approximately a de Sitter slicing of AdS_5 (as in Sec. II) and a $T^{1,1}$ coset.

The de Sitter deformations of the KT model above and of the LST in the previous section are similar in that as one turns off a Hubble parameter (or, rather, sufficiently lowers it), one ends up with a singular geometry. One way to turn on small (vanishingly small) de Sitter deformation is to start with a gauge-gravity correspondence for a confining gauge theory such as, say, the KS model [5]. It is straightforward to repeat the above analysis for the deformed KS background and obtain a consistent system of equations. We do not present this system here due to its complexity and the fact that we could not find an analytical solution. The added difficulty (compare to the extremal KS background) comes from the fact that it is inconsistent (on the level of equations of motion) to demand $H\neq 0$ along with a constant dilaton. A similar phenomenon has been observed in studies of the near-extremal deformation of the KS background [14]: it was shown there that a black hole with a regular Schwarzschild horizon in the KS geometry necessarily has a nonconstant dilaton. This observation has a simple physical interpretation. In the extremal KS solution, the string coupling g_s was an exact modulus of the cascading gauge theory, dual to the sum of the individual gauge couplings

$$\frac{1}{g_s} = \frac{4\pi}{g_1^2} + \frac{4\pi}{g_2^2} = \text{const.} \tag{4.14}$$

As both the finite temperature and the Hubble parameter breaks supersymmetry, this modulus is expected to be lifted, thus developing a nontrivial radial dependence in the dual supergravity.

V. CONCLUDING REMARKS

In this paper, we presented a simple framework on how one can embed an accelerating universe in the supergravity. The idea is to start with a gauge-gravity duality of Maldacena, and consider deformations of this duality where Minkowski background space-time of the gauge theory is replaced with a de Sitter space-time.

We argued that to get nontrivial time-dependent solutions (i.e., unrelated by coordinate reparametrization to a static solution), the starting point for the deformation must be a gravitational dual to a nonconformal gauge theory. We discussed two examples of such deformations: the little string theory and the KT model. In both cases, conformal invariance is broken by considering (adding) NS5- (D5-) branes. We argued that the expansion of the background geometry on the gauge theory side serves as an infrared cutoff in the dual supergravity. In particular, for a sufficiently high expansion rate this resolves a naked singularity of the KT solution [6].

There are several interesting future directions. The vacuum state in an accelerating universe has a nonzero Gibbons-Hawking temperature $T_{GH} = H/2\pi$, analogous to the Hawking temperature of a black hole. The KT deformation discussed here is very similar to the finite temperature deformation of the KT solution considered in [13,15]. By comparing a critical expansion rate for the $H\neq 0$ deformation of the KT model with the critical temperature for its finite-temperature deformation, one should be able to relate the Gibbons-Hawking temperature of the expanding universe with the temperature of the gauge theory in the standard near-extremal deformation.

Another interesting question is the dynamical stability of the deformed backgrounds discussed here. Since de Sitter deformation breaks supersymmetry, one has to worry about potential tachyons. The similarity of this deformation with the near-extremal one suggests that the KT model is likely to be stable, while there could be a tachyon in the LST deformation, in analogy with [11]. It would be nice to explicitly verify these conjectures. In the case of the KS deformation, at least for small values of the Hubble parameter, we expect to get a stable nonsupersymmetric background. The argument is identical to the one given in [16]: the original supergravity background had a mass gap, and thus a small deformation should not produce a tachyon.

In this paper we *only* constructed de Sitter backgrounds in supergravity. It is important to understand the spectrum of density fluctuations and the physics of D-brane probes in these geometries.

Recently, Giddings, Kachru, and Polchinski [17] studied the embedding of the KS model in the type-IIB string compactifications in the context of moduli stabilization and generation of large hierarchies of physical scales. It would be very interesting to explore de Sitter deformation of these models.

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