Constraining the variation of the coupling constants with big bang nucleosynthesis

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We consider the possibility of the coupling constants of the $SU(3)\times SU(2)\times U(1)$ gauge interactions at the time of big bang nucleosynthesis having taken different values from those we measure at present, and investigate the allowed difference requiring the shift in the coupling constants not to violate the successful calculation of the primordial abundances of the light elements. We vary the gauge couplings and Yukawa couplings (fermion masses) within the context of a model in which their relative variations are governed by a single scalar field, the dilaton, as found in string theory. The results include a limit on the fine structure constant $-6.0\times10^{-4}<\Delta\alpha_{\mathrm{e.m.}}/\alpha_{\mathrm{e.m.}}<1.5\times10^{-4}$, which is two orders of magnitude stricter than the limit obtained by considering the variation of $\alpha_{\mathrm{e.m.}}$ alone.

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I. INTRODUCTION

Big bang nucleosynthesis (BBN) is one of the most important tools for probing the early universe. Standard big bang nucleosynthesis (SBBN) predicts the primordial abundances of light elements (D, ^{3}He, ^{4}He, and ^{7}Li) with the only free parameter the baryon-to-photon ratio η. For $\eta\sim O(10^{-10})$, they are remarkably consistent with observations of the abundances which extend over nine digits. Requiring that they do not vitiate this consistency, stringent constraints on various theories that affect cosmology have been obtained.

Among such theories constrained by BBN, there are some in which the values of the coupling constants may vary. The SBBN prediction assumes, along with three light neutrino species and no lepton asymmetry, that the physical parameters involved in the calculation (the fine structure constant $\alpha_{\mathrm{e.m.}}$, the Fermi constant G_F, the electron mass m_e, etc.) are the same for now and the BBN time. However, this is not an obvious choice because there are a number of ways in which they could have varied. In addition to the theoretical possibility, the recent analysis of quasar absorption lines found possible evidence for a variation in $\alpha_{\mathrm{e.m.}}$ [1].

In this paper, we consider a model taken from string theory where these coupling constants are all related to the expectation values of a dilaton field Φ and could in principle vary with time. For example, very general arguments indicate that the dilaton cannot be stabilized at a value we would characterize as corresponding to weak coupling [2]. Also, in certain popular models for stabilizing the dilaton using gaugino condensates, the cosmological evolution would almost inevitably tend to overshoot the desired minimum of the dilaton potential and run off to an anti–de Sitter vacuum [3].

The notion that the coupling constants are determined by a single scalar field not only motivates the investigation of their time variation as a probe for physics beyond the standard model, but also makes the analysis of simultaneous changes in the couplings simple and concrete. In addition to the free parameter of the SBBN η, we have only one other parameter $\Delta\Phi/\Phi\equiv(\Phi_{BBN}-\Phi_{now})/\Phi_{now}$, the fractional variation of the dilaton field. After calculating the primordial abundances with various values of these two quantities, we search for a parameter region not excluded by the observations. The limit on $\Delta\Phi/\Phi$ thus obtained is readily translated into limits on the coupling constants.

Our analysis is positioned as an extension to Ref. [4]. There, the dilaton dependence of the coupling constants is determined by the action of the heterotic string in the Einstein frame. The constraint is obtained from ^{4}He whose abundance and coupling dependence can be estimated without recourse to a numerical calculation. The method of [4] is easy to calculate and appropriate for an order of magnitude estimation. But we can extract more information by performing a numerical calculation, whose advantages are (1) the abundances of the light elements other than ^{4}He are calculated so we can use more observational data, especially for D, to impose a constraint, (2) we can take into account the η dependence of the abundances, (3) we use realistic values for the reaction rates so we avoid the rough estimation of the weak reaction rate (both absolute value and temperature dependence) based on dimensional analysis, (4) by Monte Carlo simulation, we can estimate the theoretical uncertainty, and (5) by calculating a statistical measure (such as χ^2), we can objectively quantify the constraint. Of course, as the SBBN becomes more and more precise owing to progress in studies of nuclear reaction rates and of primordial abundances, numerical computation is necessary in general to reflect the recent developments and to produce a result close to the truth.

In the next section, we review briefly the SBBN and see what kinds of physical quantities are needed to make the prediction. In Sec. III, we estimate their coupling dependences. In Sec. IV, we introduce a model containing the dilaton and we investigate the dilaton dependence of the coupling constants. In Sec. V, we put a bound on $\Delta\Phi/\Phi$ by calculating χ^2 with the observational and theoretical uncertainty from the recent data (the former very much dominates). In terms of the limit on $\Delta\alpha_{\mathrm{e.m.}}/\alpha_{\mathrm{e.m.}}$, we find

$$-6.0\times10^{-4}<\frac{\Delta\alpha_{\mathrm{e.m.}}}{\alpha_{\mathrm{e.m.}}}<1.5\times10^{-4}, \tag{1}$$

two orders of magnitude more restrictive than the limits found in Refs. [5] and [6] where only the variation of $\alpha_{\text{e.m.}}$ was taken into account. We discuss whether this originates from the unified gauge couplings, which relate the electromagnetic sector to the strong sector. At the energy scale of BBN, the manifest quantities are $\alpha_{\text{e.m.}}$ and Λ_{QCD}, which appears instead of α_{strong} through dimensional transmutation. The former depend linearly on the dilaton but the latter exponentially. The ^{4}He abundance is determined by the magnitude of the neutron-proton mass difference Δm, which is in turn determined by Λ_{QCD}, and this dominates the constraints by BBN.

II. PHYSICAL QUANTITIES IN THE SBBN

In order to see how the effects of changing coupling constants arise, we summarize the main points of the SBBN calculation. Following Ref. [7], we divide it into three stages. The actual calculation is performed by computer which runs the code to solve the set of ordinary differential equations and so, of course, does not distinguish such stages, but we see that the important quantities which determine the primordial abundances of the light elements appear in this brief review.

First stage (statistical equilibrium) ($T \gg 1$ MeV;$t \ll 1$ s). The energy density and the number density are dominated by the relativistic electron e^-, positron e^+, neutrino ν, antineutrino $\overline{\nu}$, and photon γ. There are three types of neutrino and antineutrino. There is only a tiny fraction of protons and neutrons, just $\sim 10^{-9}$ number fraction. In this period, all these particles undergo elastic scattering so frequently that they are in thermal equilibrium and have equal temperatures. In particular, weak interaction cross sections are large and the neutrinos are in thermal equilibrium with the other particles. In addition, there are weak interaction processes interchanging protons and neutrons:

$$n + \nu_e \leftrightarrow p + e^-, \tag{2}$$

$$n + e^+ \leftrightarrow p + \overline{\nu}_e, \tag{3}$$

$$n \leftrightarrow p + e^- + \overline{\nu}_e, \tag{4}$$

so they are also in chemical equilibrium. Thus the number ratio of protons to neutrons at temperature T is $n_n/n_p = e^{-(m_n - m_p)/T} e^{(\mu_n - \mu_p)/T}$, where m_n (m_p) is the neutron (proton) mass and μ_n (μ_p) is its chemical potential, but the last factor $e^{(\mu_n-\mu_p)/T}$ is almost unity if the universe has no lepton asymmetry, which is the assumption of SBBN. So as long as equilibrium holds at temperature T, the neutron-to-proton ratio is determined by the neutron-proton mass difference $\Delta m = m_n - m_p$,

$$\frac{n_n}{n_p} = e^{-\Delta m/T}. \tag{5}$$

At this stage, the nuclear reaction rates are also fast enough for light nuclei to be in chemical equilibrium. Then the abundance of the nuclear species with mass number A and atomic number Z is [7]

$$Y_A = g_A[\zeta(3)^{A-1}\pi^{(1-A)/2}2^{(3A-5)/2}] \times A^{5/2}\left(\frac{T}{m_N}\right)^{3(A-1)/2} \eta^{A-1} Y_p^Z Y_n^{A-Z} \exp(B_A/T), \tag{6}$$

where $\zeta(3) \sim 1.202$, m_N is the nucleon mass, B_A is the binding energy, and g_A is the number of degrees of freedom. Since η is small ($\sim 10^{-10}$) and $B_A \ll T$, nuclear abundance is negligible at this temperature.

Second stage (neutron-proton freeze-out) ($T \sim 0.7$ MeV;$t \sim 2$ s). As the universe expands and cools, the rates of interactions involving neutrinos decrease. In particular, there is some point when the reactions (2)–(4) practically stop and the chemical equilibrium breaks down. After this point, the numbers of neutrons and protons do not change (in reality, small amount of neutrons turns into protons by beta decay) and their ratio is fixed (frozen out) at the value determined from Eq. (5) with the temperature of that epoch. We call this the ''freeze-out temperature'' and denote it T_f.

The freeze-out occurs when the rate of the reaction $\Gamma(T)$ and the expansion rate of the universe $H(T)$ become equal:

$$H(T_f) \approx \Gamma_{weak}(T_f). \tag{7}$$

The expansion rate is determined by the energy density ρ through the Friedmann equation

$$H^2 = \frac{8\pi G\rho}{3}. \tag{8}$$

At this stage, ρ is dominated by photons, neutrinos, and relativistic electrons ($T > m_e = 0.511$ MeV) and hence the cosmic density is given by $\rho = 10.75(\pi^2/30)T^4$. The rate of weak interaction is

$$\begin{aligned}\Gamma_{weak} &= \Gamma(n \to p)\\ &= \Gamma(n + \nu_e \to p + e^-)\\ &\quad + \Gamma(n + e^+ \to p + \overline{\nu}_e) + \Gamma(n \to p + e^- + \overline{\nu}_e).\end{aligned}$$

Using the Fermi theory of the weak interaction, this can be written as

$$\begin{aligned}\Gamma(n \to p) = A\left[\int_0^\infty + \int_{-\infty}^{-\Delta m - m_e} + \int_{-\Delta m + m_e}^{0}\right] dx\\ \times\left(1 - \frac{m_e^2}{(x+\Delta m)^2}\right)^{1/2} (x+\Delta m)^2 x^2\\ \times \frac{1}{1+e^{x/T_\nu}} \frac{1}{1+e^{-(x+\Delta m)/T_e}}.\end{aligned}$$

We calculate the normalization factor A by using $\Gamma(n \to p + e^- + \overline{\nu}_e)|_{T=0} = \tau_n^{-1}$ where τ_n is the neutron lifetime. Integration gives $A = 0.05606\tau_n^{-1}$ MeV^{-4}. From this expression and Eq. (7), we find $T_f \approx 0.7$ MeV and, from Eq. (5), the freeze-out ratio $(n_n/n_p)_f = e^{-\Delta m/T_f} \approx 0.158$.

For the nuclear reaction, the situation did not change from the previous stage. The abundances are still very small.

Third stage (light-element synthesis) (0.7 MeV$>T>$0.05 MeV;3 s$<t<$6 min). After the second stage, the electron-positron pair annihilation is completed so there remain the photons and the neutrinos as relativistic particles.

Since the early universe has lower density than that inside stars, there occur essentially only two-body reactions. Thus, unless the deuterons are synthesized by the reaction $p + n \to \mathrm{D} + \gamma$, larger nuclei are not synthesized. This reaction does not proceed until the photon density is low enough not to photodissociate D or, in other words, the factor $\eta \exp(B_{\mathrm{D}}/T)$ in Eq. (6) becomes $O(1)$. This occurs at about $T = 0.06$ MeV. After that, mainly charged-particle reactions such as $\mathrm{D}+\mathrm{D} \to t + p$ and $t + \mathrm{D} \to {}^4\mathrm{He} + n$ proceed, synthesizing almost all of the neutrons into ^{4}He because it has the largest binding energy among the light elements. Therefore, ^{4}He abundance (conventionally expressed by mass ratio) can be estimated by the frozen n-p ratio found at the second stage:

$$Y_{^4\mathrm{He}} \approx \frac{2}{1 + (n_p/n_n)_f} = \frac{2}{1 + e^{\Delta m/T_f}}. \tag{9}$$

In addition to ^{4}He, small amounts of D and ^{3}He and very small amounts of ^{7}Li are synthesized. Their abundances are determined by η and the reaction rates.

In summary, to perform the BBN calculation, we need to input the following values *at BBN time*: the neutron-proton mass difference Δm, the neutron lifetime τ_n, and the nuclear reaction rates. The abundance of ^{4}He is affected mainly by the first two as seen from Eqs. (7) and (9), and D, ^{3}He, and ^{7}Li by the last one.

III. THE COUPLING CONSTANT DEPENDENCE OF THE BBN INPUTS

We know the present value of the BBN input quantities pointed out in the previous section with some experimental uncertainty. To estimate their values during BBN, we try to express these quantities in terms of the coupling constants.

A. Neutron-proton mass difference Δm

The origin of the neutron-proton mass difference is traced to the electromagnetic self-energy difference and the d-u quark mass difference (QCD chiral symmetry breaking by the mass terms). The former makes the proton heavier than the neutron but the latter does the inverse and the total is measured accurately to be $\Delta m \equiv m_n - m_p = 1.2933318 \pm 0.0000005$ MeV [10].

As argued in Ref. [8] the largest contribution to the electromagnetic part comes from the Born term of the Cottingham formula [9] and hence it can be calculated with relatively less uncertainty. This formula expresses the nucleon self-energy in the first order of $\alpha_{\mathrm{e.m.}}$ via its electric and magnetic form factors. Using the formula and experimental data for the form factors, the difference between the neutron and the proton is -0.76 MeV. This value is proportional to (of course) $\alpha_{\mathrm{e.m.}}$ and to Λ_{QCD} to have a proper dimension.

On the other hand, the absolute values of the u and d quark masses are not well known, $m_u = 1$–5 MeV and $m_d = 3$–9 MeV [10]. But, knowing the electromagnetic contribution, the quark contribution should be 2.05 MeV. This is proportional to the difference between the Yukawa couplings y_u and y_d and the Higgs boson expectation value $\langle H \rangle$.

Now we can write

$$\Delta m = a\,\alpha_{\mathrm{e.m.}} \Lambda_{QCD} + b(y_d - y_u)\langle H \rangle, \tag{10}$$

where a and b are constants we assume not to depend on any coupling constants. Thus, we estimate the mass difference at the BBN epoch Δm_{bbn} as

$$\begin{aligned} \Delta m_{bbn} &= a\,\alpha_{\mathrm{e.m.},bbn} \Lambda_{QCD,bbn} + b y_{bbn} \langle H \rangle_{bbn} \\ &= -0.76 \frac{\alpha_{\mathrm{e.m.},bbn}}{\alpha_{\mathrm{e.m.},now}} \frac{\Lambda_{QCD,bbn}}{\Lambda_{QCD,now}} \\ &\quad + 2.05 \frac{y_{bbn}}{y_{now}} \frac{\langle H \rangle_{bbn}}{\langle H \rangle_{now}} \ (\mathrm{MeV}). \end{aligned} \tag{11}$$

B. Neutron lifetime

Neutron β decay $n \to p + e + \overline{\nu}$ is very well approximated by the one-point interaction of four particles: neutron, proton, electron, and neutrino. Its coupling constant is denoted G_F and called the Fermi coupling constant. Using this theory,

$$\begin{aligned} \tau_n^{-1} &\propto G_F^2 \int d^3p_e d^3p_\nu \delta(E_e + E_\nu + m_p - m_n) = G_F^2 16\pi^2 \int_{m_e}^{\Delta m} dE_e E_e \sqrt{E_e^2 - m_e^2}(\Delta m - E_e)^2 \\ &= G_F^2 16\pi^2 m_e^5 \frac{1}{60}\{\sqrt{q^2-1}(2q^4 - 9q^2 - 8) + 15q \log(q + \sqrt{q^2-1})\} \end{aligned} \tag{12}$$

where we defined $q \equiv \Delta m / m_e$. Denoting the factor $\frac{1}{60}\{\ldots\} \equiv f(q)$,

$$\tau_n \propto G_F^{-2} m_e^{-5} f(q)^{-1} = \langle H \rangle^4 (y_e \langle H \rangle)^{-5} f(q)^{-1} = \langle H \rangle^{-1} y_e^{-5} f(q)^{-1}. \tag{13}$$

The first equality follows from $G_F = g_2^2/M_W^2 = g_2^2/(g_2 \langle H \rangle)^2 = 1/\langle H \rangle^2$ where g_2 is the SU(2) coupling constant and M_W is the weak boson mass. Therefore, we obtain

$$\tau_{n,bbn} = \left[\frac{y_{e,bbn}}{y_{e,now}}\right]^{-5} \left[\frac{\langle H \rangle_{bbn}}{\langle H \rangle_{now}}\right]^{-1} \left[\frac{f(q_{bbn})}{f(q_{now})}\right]^{-1}, \tag{14}$$

where y_e is the electron Yukawa coupling and $f(q_{now}) = 1.63615$.

C. Charged-particle-induced reaction rates

Most of the reaction rates involved in the BBN calculation are charged-particle-induced reaction rates. Their $\alpha_{\mathrm{e.m.}}$ dependence is considered in Ref. [5] for the rates discussed in Ref. [11]. We implemented the $\alpha_{\mathrm{e.m.}}$ dependence in the same manner, updating the reaction rates recently compiled by Angulo *et al.* [12]. Since only a few reaction rates have resonance terms which are considered to be dependent on the strong coupling constant, or in other words since the charged-particle reaction rates influential for BBN are essentially determined by Coulomb barrier penetrability, we expect that neglecting their strong coupling dependence will not affect our results much.

D. Neutron-induced reaction rates

The cross section for $n + p \rightarrow \mathrm{D} + \gamma$ is calculated at energies relevant to BBN using the effective field theory that describes the two-nucleon sector [13]. The rate obtained by thermal averaging this theoretical cross section reproduces the abundance calculation using the rate shown in Ref. [11]. We exploit this theoretical formula to estimate the coupling dependence by assuming the parameters that have the dimension $[\text{length}]^n$ in the formula to be proportional to m_π^{-n}. The coupling dependence of the pion mass m_π is known from the Gell-Mann–Oakes–Renner relation to be $m_\pi^2 \propto m_{quark} \Lambda_{QCD}$. The other parameters in the formula are the nucleon mass m_N and the deuteron binding energy B_D. Their dependence is estimated by $m_N \propto \Lambda_{QCD}$ and $B_D \propto m_\pi^2/\Lambda_{QCD} \propto m_{quark}$. This B_D dependence is estimated by considering a simple square well potential for the deuteron. It is also estimated by the uncertainty principle $p \sim 1/\Delta x \sim m_\pi$, and $-B_D + p^2/2m_N \sim 0$.[1]

For the other neutron-induced reactions ${}^3\mathrm{He}(n,p)t$ and ${}^7\mathrm{Be}(n,p){}^7\mathrm{Li}$, we use the rate fitted from the experimental data as found in Ref. [15] for the former and Ref. [11] for the latter, and they are assumed to be independent of the couplings because there are no theoretical derivations for these. Because ${}^3\mathrm{He}(n,p)t$ affects ${}^3\mathrm{He}$ abundance, which is not used for our analysis, our result does not change by neglecting its coupling dependence. On the other hand, ${}^7\mathrm{Be}(n,p){}^7\mathrm{Li}$ affects ${}^7\mathrm{Li}$ abundance a lot when η is large, so its effect may be large. However, since there is large observational uncertainty in ${}^7\mathrm{Li}$, it is expected that our result will not change but we have to know its coupling dependence to extract a reliable constraint from the ${}^7\mathrm{Li}$ data when the uncertainty of ${}^7\mathrm{Li}$ abundances decreases in the future.

IV. THE DILATON DEPENDENCE OF THE COUPLING CONSTANTS

Next, we introduce the action that governs the variation of the coupling constants as used in Ref. [4]. This is the tree level low energy action of the heterotic string in the Einstein frame:

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa^2} R - \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2} D_\mu \phi D^\mu \phi - \Omega^{-2} V(\phi) - \bar{\psi} \gamma_\mu D^\mu \psi - \Omega^{-1} m_\psi \bar{\psi} \psi - \frac{\alpha'}{16\kappa^2} \Omega^2 F_{\mu\nu} F^{\mu\nu} \right), \tag{15}$$

where Φ is the dilaton field, ϕ is an arbitrary scalar field, and ψ is an arbitrary fermion. D_μ is the gauge covariant derivative corresponding to gauge fields with field strength $F_{\mu\nu}$. $\kappa^2 = 8\pi G$ and $\Omega = e^{-\kappa\Phi/\sqrt{2}}$, which is the conformal factor used to move from the string frame. Powers of $\Omega(\Phi)$ multiplying terms in the action indicate the dilaton dependence of the coupling constants and masses.

More concretely, ϕ is the Higgs field and $V(\phi)$ is its potential, which we assume to be given by hand (as is done in the standard model). The overall Ω factor before the scalar potential means that the Higgs vacuum expectation value $\langle H \rangle$ is independent of the dilaton so it has the same value for now and the BBN time. $\langle H \rangle$ is taken to be constant in our calculation.

$F_{\mu\nu}$ is the gauge field with gauge group including $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$. We define its Lagrangian density as $-(1/4g^2) F_{\mu\nu} F^{\mu\nu}$ where g is the unified coupling constant. Comparing with Eq. (15),

$$\frac{1}{g^2(M_p^2)} = \frac{\alpha' e^{-\sqrt{2}\kappa\Phi}}{4\kappa^2} \equiv \frac{\alpha' S}{4\kappa}, \tag{16}$$

where we define

$$S \equiv \frac{e^{-\sqrt{2}\kappa\Phi}}{\kappa}. \tag{17}$$

For each value of S, we can calculate the gauge coupling constants at low energy using renormalization group equations. $\alpha_{\mathrm{e.m.}}$ almost does not run, so

[1] A more elaborate study of B_D's light quark mass dependence is found in Ref. [14] but a complete answer is not yet obtained.

$$\alpha_{\text{e.m.}}(M_{bbn}) \approx \alpha_{\text{e.m.}}(M_p) = \frac{g(M_p)^2}{4\pi} = \frac{\kappa}{\pi\alpha'} S^{-1}. \tag{18}$$

Therefore,

$$\frac{\alpha_{\text{e.m.},bbn}}{\alpha_{\text{e.m.},now}} = \left[\frac{S_{bbn}}{S_{now}}\right]^{-1} = \frac{1}{1+v_S}, \tag{19}$$

where we define the fractional S variation

$$v_S = \frac{S_{bbn} - S_{now}}{S_{now}}. \tag{20}$$

From the solution of the one-loop renormalization group equation (RGE) for the SU(3) coupling constant g_3, of which the integration constant is determined by $g_3(\Lambda_{QCD}) = \infty$,

$$\frac{\kappa}{\pi\alpha'} S^{-1} = \frac{g_3(M_p)^2}{4\pi} \approx \frac{12\pi}{27\log(M_p^2/\Lambda_{QCD}^2)} \tag{21}$$

or

$$\Lambda_{QCD} = M_p \exp\left(-\frac{2\pi^2\alpha' S}{9\kappa}\right). \tag{22}$$

Therefore, we obtain

$$\frac{\Lambda_{QCD,bbn}}{\Lambda_{QCD,now}} = \exp\left(-\frac{2\pi^2\alpha'[S_{bbn} - S_{now}]}{9\kappa}\right) = \exp\left(-\frac{8\pi^2}{9g(M_p)^2} v_S\right), \tag{23}$$

where we use $g(M_p)^2 = 0.1$.[2]

Finally, the ψ's are the ordinary standard model leptons and quarks. As we take $\langle H \rangle = \text{const}$, the Yukawa couplings y depend on the dilaton as $\propto e^{\kappa\Phi/\sqrt{2}}$. In terms of S,

$$\frac{y_{bbn}}{y_{now}} = \frac{1}{\sqrt{1+v_S}}. \tag{24}$$

V. CONSTRAINTS ON THE VARIATION OF THE COUPLING CONSTANTS

Using the model described in the previous section, we can express the coupling dependence of the BBN input parameters considered in Sec. III on the fractional variation of the dilaton v_S defined in Eqs. (17) and (20). In order to quantify how much variation is consistent with the observations, we calculate the abundances for different values of v_S in addition to η with the standard BBN code [19]. Then we calculate $\chi^2(\eta, v_S)$ as

$$\chi^2 = \sum_i \frac{(a_i^{th} - a_i^{obs})^2}{(\sigma_i^{th})^2 + (\sigma_i^{obs})^2} \tag{25}$$

where i is the type of element with which we try to impose a constraint. To estimate theoretical errors, we have performed 1000 Monte Carlo simulations using the values of Ref. [15] for the nuclear reaction rate uncertainty and of the Particle Data Group [10] for the neutron lifetime 885.7 ± 0.8 s. For the observational errors for D, ^{4}He, and ^{7}Li, we adopt

$$(\text{D/H})^{obs} = (3.0 \pm 0.4) \times 10^{-5}, \tag{26}$$

$$Y^{obs} = 0.238 \pm (0.002)_{stat} \pm (0.005)_{syst}, \tag{27}$$

$$\log_{10}[(^7\text{Li/H})^{obs}] = -9.76 \pm (0.012)_{stat} \pm (0.05)_{syst} \pm (0.3)_{add}. \tag{28}$$

Equation (26) is taken from Ref. [20] and Eq. (27) from Ref. [21] where the first error is the statistical uncertainty and the second error is the systematic one. Equation (28) is from Ref. [22] with the error we have added for the uncertainty in chemical evolution [23].

The results are shown in Fig. 1. The shapes of the contour lines are easily understood. The contours drawn from the χ^2 of the three elements [Fig. 1(a)] are just the product set of D [Fig. 1(b)] and ^{4}He [Fig. 1(c)] because ^{7}Li [Fig. 1(d)] does not give much constraint.

D and ^{7}Li do not vertically constrain much and ^{4}He alone constrains v_S. The reason is that D and ^{7}Li abundances are determined mainly by charged-particle reactions and ^{4}He by the freeze-out ratio. The former is determined by the Coulomb barrier penetrability or the electromagnetic coupling $\alpha_{\text{e.m.}}$ and the latter by the neutron-proton mass difference Δm. Δm depends linearly on Λ_{QCD} as described in Sec. III A. $\alpha_{\text{e.m.}}$ and Λ_{QCD} are related to each other through Eqs. (19) and (21) which come from general notions of the gauge coupling unification. This relation tells that $\Delta\Lambda_{QCD}/\Lambda_{QCD,now} \approx (80\pi^2/9)\Delta\alpha_{\text{e.m.}}/\alpha_{\text{e.m.},now}$ where $\Delta\alpha_{\text{e.m.}} = \alpha_{\text{e.m.},bbn} - \alpha_{\text{e.m.},now}$, etc., so a small increase in $\alpha_{\text{e.m.}}$ is accompanied by a great increase in Λ_{QCD} and Δm. This is the reason why ^{4}He constrains v_S much more than D and ^{7}Li do in our model.

To make sure, we explain the trend of the ^{4}He contour. As v_S increases, Λ_{QCD} decreases and hence Δm increases. This makes $(n_n/n_p)|_{freeze\text{-}out}$ decrease, so the ^{4}He abundance decreases. Since the ^{4}He abundance is an increasing function of η, the increase in v_S relaxes the constraint for higher η. This shows up in the trend that the contour goes up in the direction of increasing η.

The constraint on the dilaton field variation is obtained from Fig. 1(a). For 95% confidence level,

$$-1.5 \times 10^{-4} < v_S < 6.0 \times 10^{-4}, \tag{29}$$

or, using $S = e^{-\sqrt{2}\kappa\Phi}/\kappa$ with $\kappa = (8\pi G)^{1/2} = (2.43 \times 10^{18}\ \text{GeV})^{-1}$, we obtain

$$-1.0 \times 10^{15}\ \text{GeV} < \Delta\Phi < 2.6 \times 10^{14}\ \text{GeV}. \tag{30}$$

[2]Our RGE argument above to derive the relations between the dilaton vacuum expectation value and the gauge coupling constants [Eqs. (18) and (22)] follows Ref. [4] and contains some approximations. More detailed arguments are found in Refs. [16,17] and [18].

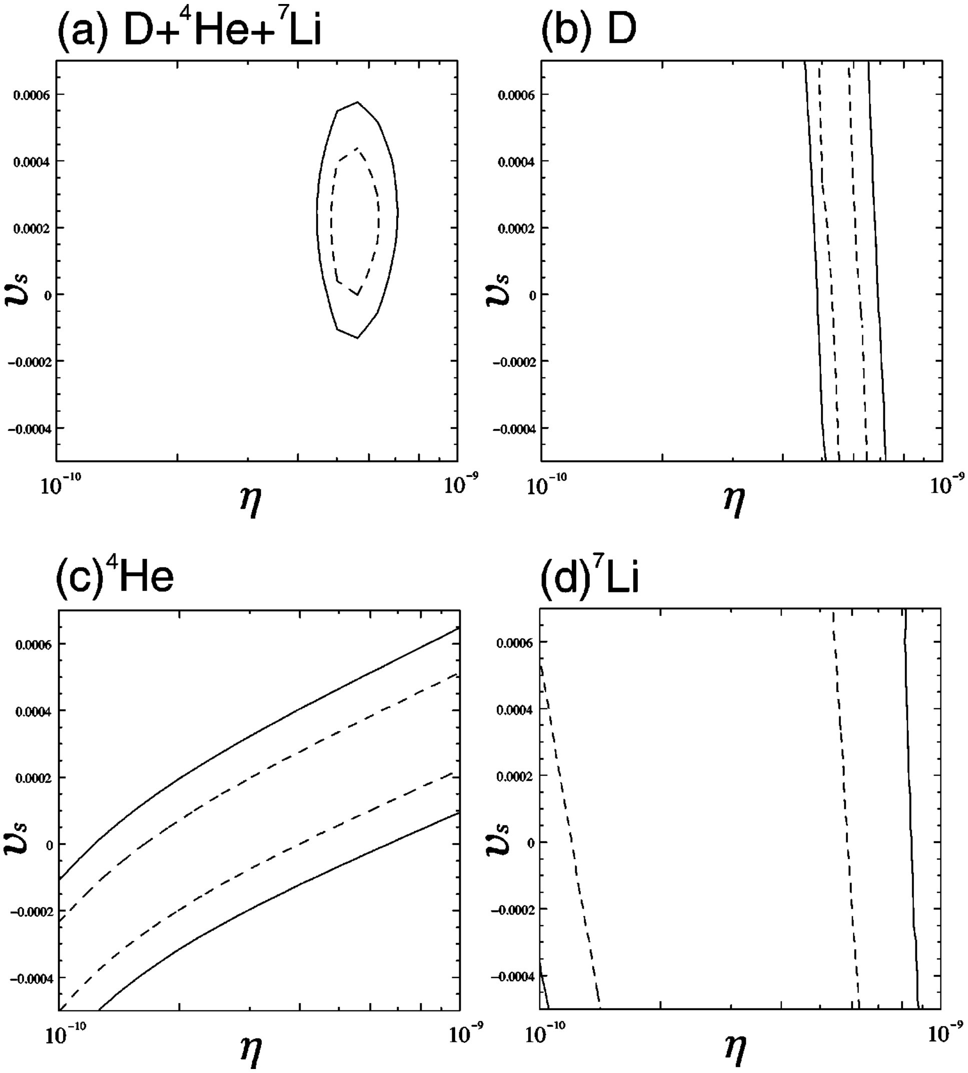

FIG. 1. The contour lines on the η-v_S plane drawn (a) using the data of the three light elements D, ^{4}He, and ^{7}Li, (b) using D, (c) using ^{4}He, and (d) using ^{7}Li. The vertical axis is $v_S = (S_{bbn} - S_{now})/S_{now}$. The solid line is the 95% confidence level and the dotted line 70%.

VI. CONCLUSION

In summary, we have considered a BBN calculation with the coupling constants taking different values from the present ones and whose variation is determined by the dilaton field Φ as in the action (15). The setup is the same as in Ref. [4] but we have performed a full nuclear reaction network calculation with the BBN code so the abundances of D and ^{7}Li in addition to those of ^{4}He are available to compare with the observations and make a constraint. This is done by calculating χ^2 as a function of the baryon-to-photon ratio η and the fractional variation of the dilaton v_S defined in Eqs. (17) and (20), and we have obtained the constraint (29), which is of the same order as the value found in Ref. [4]. It might be interesting that the standard BBN ($v_S=0$) is outside the allowed region at the 70% confidence level as seen in Fig. 1(a). However, this result relies on the high-η (low-D and/or low-^{4}He) observations which are not yet unambiguous and its error may be underestimated. If the error turned out to be larger or η to be lower (D and/or ^{4}He to be higher), $v_S=0$ is allowed at the 70% confidence level.

The constraint on v_S is readily converted to that on $\alpha_{\text{e.m.}}$ using Eq. (19), which becomes $\Delta\alpha_{\text{e.m.}}/\alpha_{\text{e.m.}} = -v_S$ when v_S is small, and gives

$$-6.0\times 10^{-4} < \frac{\Delta\alpha_{\text{e.m.}}}{\alpha_{\text{e.m.}}} < 1.5\times 10^{-4}, \tag{31}$$

which is two orders of magnitude stricter than what was found in Ref. [5] where only $\alpha_{\text{e.m.}}$ is varied. This feature should be common in the models in which variations of the gauge couplings are related to each other through gauge unification, so the variation in Λ_{QCD} is much larger than that of $\alpha_{\text{e.m.}}$.

For comparison, we draw a contour diagram when only $\alpha_{\text{e.m.}}$ is varied. The result is Fig. 2. The shape is similar but, noticing the vertical scale, it is two orders of magnitude larger than in Fig. 1(a). The constraint becomes $-5.0\times 10^{-2} < \Delta\alpha_{\text{e.m.},only}/\alpha_{\text{e.m.},only} < 1.0\times 10^{-2}$ and this is similar to the result obtained in Refs. [5] and [6] (the factor difference is attributed to the difference in the adopted observational data). The contours in Fig. 2 completely cover the contours in Fig. 1(a). This is another demonstration that the constraint is not obtained by the variation of $\alpha_{\text{e.m.}}$ directly but by Λ_{QCD}.

Finally, we notice that the limit (31) is consistent and has a favorable trend with the result found in Ref. [1] from quasar absorption systems, that is, $\Delta\alpha_{\text{e.m.},only}/\alpha_{\text{e.m.},only}$

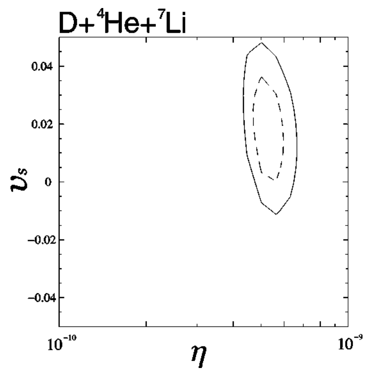

FIG. 2. The contour lines when only $\alpha_{\text{e.m.}}$ is varied drawn using the data of the three light elements D, ^{4}He, and ^{7}Li. The relation $v_S \approx -\Delta\alpha_{\text{e.m.}}/\alpha_{\text{e.m.}}$ holds.

$=(-0.72\pm0.18)\times10^{-5}$. This shows that $\alpha_{\text{e.m.}}$ was *smaller* at $z=O(1)$. So if the variation is monotonic, it should be even smaller at the BBN time. This is just the tendency of Eq. (31) (the allowed region is wider on the negative side). A more quantitative comparison needs a model for the time evolution of the coupling constants (for example, a model for some dilaton potential) and the analysis of the quasar data in the situation where couplings other than $\alpha_{\text{e.m.}}$ vary.

We have not reached a stage where such a constraint as Eq. (30) can be used to say something about particle physics. In this paper, we bear string theory in mind and treat the time variation of the coupling constants as caused by the dilaton, one of the scalar fields found in the theory. However, without referring to string theory, scalar fields flourish in cosmology to attack important problems (e.g., inflation, quintessence) so a similar analysis (with a different action) can be used to constrain such scenarios. In order to find useful applications, the analysis itself has to be sharpened because there still remain unsatisfactory estimates concerning the nuclear force. This sector will be improved with a field theoretic (QCD based) understanding of the nuclear force.

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