Nature of singularities in anisotropic string cosmology

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We study the nature of singularities in anisotropic string-inspired cosmological models in the presence of a Gauss-Bonnet term. We analyze two string gravity models—dilaton-driven and modulus-driven cases—in the Bianchi type-I background without an axion field. In both scenarios singularities can be classified in two ways—the determinant singularity where the main determinant of the system vanishes and the ordinary singularity where at least one of the anisotropic expansion rates of the Universe diverges. In the dilaton case, either of these singularities inevitably appears during the evolution of the system. In the modulus case, nonsingular cosmological solutions exist both in the asymptotic past and future with the determinants $D=+\infty$ and $D=2$, respectively. In both scenarios nonsingular trajectories in either the future or the past typically meet the determinant singularity in the past or future when the solutions are singular, apart from the exceptional case where the sign of the time derivative of the dilaton is negative. This implies that the determinant singularity may play a crucial role in leading to singular solutions in an anisotropic background.

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I. INTRODUCTION

Superstring theory continues to be of interest as a possible candidate to unify all fundamental interactions in nature [1]. It is known that there exist five supersymmetric perturbative string theories which are classified as type I, type IIA, type IIB, SO(32) heterotic and $E_8\times E_8$ heterotic string theory. Recently it was found that these theories are connected by dual symmetries, which leads to the conjecture that each theory appears as one of five branches of a unified theory, called M theory [2]. In particular Hořava and Witten [3] showed that the 10-dimensional $E_8\times E_8$ heterotic string theory is equivalent to an 11-dimensional M-theory compactified to $\mathbf{M}^{10}\times\mathbf{S}^1/Z_2$. Then the 10-dimensional spacetime is expected to be compactified into $\mathbf{M}^4\times\mathbf{CY}^6$, in which case the standard model particles are confined on the three-dimensional brane. This gives rise to the well-known brane world scenario [4] where the extra dimension is noncompact and gravity is effectively 3 dimensional.

For cosmologists it is very important to test the viability of string theories by extracting various cosmological implications from them [5]. One such attempt is the pre-big-bang (PBB) scenario [6] based on the low energy effective action of string theory. In this scenario there exist two branches of solutions by assuming a T duality, one of which ($t<0$) corresponds to the stage of polelike inflation driven by the kinetic term of the dilaton field and another ($t>0$) is the stage where the curvature continues to decrease. However, it is difficult to smoothly connect these two branches without a singularity in the tree-level string action [7,8].

One is required to take into account quantum loop or derivative corrections in order to overcome such singularity problems. In fact, Antoniadis, Rizos, and Tamvakis [9] included a Gauss-Bonnet term in the tree-level string effective action with dilaton and modulus fields, and showed the existence of nonsingular cosmological solutions. In this case the nonsingular behavior of the solutions is mainly determined by the evolution of the modulus field. Therefore, the allowed ranges of parameters were analyzed in the absence of the dilaton in a flat Friedmann-Robertson-Walker (FRW) background [10] (see also Ref. [11]). Since it is important to confirm the generality of singularity avoidance even starting from an anisotropic spacetime, several authors analyzed nonsingular cosmological solutions in the Bianchi type-I spacetime without a dilaton [12,13] and with a dilaton [14]. The presence of the modulus coupled to the Gauss-Bonnet term allows the existence of nonsingular solutions unless the dilaton controls the dynamics of the system.

In order to understand how nonsingular or singular solutions appear, it is necessary to classify the nature of singularities in an anisotropic background. In particular, the main determinant D of the system is an important quantity to describe the singularities. When only the dilaton field ϕ is coupled to the Gauss-Bonnet term in the Bianchi type-I background, it was conjectured in Ref. [15] that nonsingular cosmological solutions in the future cross the determinant singularity ($D=0$) in the past when $\dot{\phi}$ is positive. While this singularity was found more than ten years ago [16], only now we begin to understand its importance. The similar kind of singularity also appears in the context of black hole inner solutions in the presence of dilaton coupled to gravity via the Gauss-Bonnet term [17,18]. In this paper we shall make detailed analysis about nature of singularities both in dilaton- and modulus-driven cosmologies in the Bianchi type-I background. We do not include an axion field in our analysis, but it is important to emphasize that its effect is generally vital as studied in Ref. [19]. We will classify other kinds of singularities where at least one expansion rate diverges. These investigations are important to understand how nonsingular solutions emerge in the modulus-driven case. In addition our analysis will be useful to construct more complicated non-

singular string-inspired models in the presence of other fields such as a axion.

This paper is organized as follows. In Sec. II we show background equations in anisotropic string-inspired models with dilaton or modulus fields. In Sec. III we study nature of singularities in dilaton-driven cosmology both for positive and negative $\dot{\phi}$ cases. Section IV is devoted to the modulus-driven cosmology where both of nonsingular and singular solutions exist. We present summary and discussions in the final section.

II. THE MODEL AND BACKGROUND EQUATIONS

We begin with the action [9–14]

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}R-\frac{1}{2}(\nabla\phi)^2+f(\phi)R^2_{\rm GB}\right], \quad (2.1)$$

written in the Einstein frame. Here R is the scalar curvature and ϕ denotes a scalar field which is either dilaton or modulus. $f(\phi)$ depends on string theories, whose explicit forms are given later. We do not consider the multifield system of dilaton and modulus fields [14] induced from the one-loop effective action of heterotic string theory. In addition we neglect the antisymmetric tensor $H_{\mu\nu\lambda}$ and the curvature terms higher than the second order. The Gauss-Bonnet term $R_{\rm GB}$ is defined as

$$R^2_{\rm GB}=R^2-4R^{\mu\nu}R_{\mu\nu}+R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}. \quad (2.2)$$

We normalize time and spatial coordinates by the string length scale $\sqrt{\lambda_s}$ as $\overline{x^\mu}=x^\mu/\sqrt{\lambda_s}$, and the scalar fields as $\overline{\phi}=\phi\sqrt{\lambda_s}$. Hereafter we drop bars for simplicity.

Let us consider the Bianchi type-I spacetime whose metric is given by

$$ds^2=-dt^2+a^2(t)dx^2+b^2(t)dy^2+c^2(t)dz^2, \quad (2.3)$$

where $a(t),b(t),c(t)$ are the scale factors in an anisotropic background. We define the anisotropic expansion rates $p(t),q(t),r(t)$ as

$$p(t)=\frac{\dot{a}}{a}, \quad q(t)=\frac{\dot{b}}{b}, \quad r(t)=\frac{\dot{c}}{c}, \quad (2.4)$$

where an overdot denotes the derivative with respect to t. It is also convenient to introduce new variables h,α,β:

$$p(t)=h+\alpha+\sqrt{3}\beta, \quad q(t)=h+\alpha-\sqrt{3}\beta, \quad r(t)=h-2\alpha. \quad (2.5)$$

Here $h=(p+q+r)/3$ is an average expansion rate, which is the generalization of the Hubble parameter in an isotropic case, and α and β correspond to parameters of anisotropy. The triangle in the (α,β) plane

$$\alpha+\sqrt{3}\beta>-h, \quad \alpha-\sqrt{3}\beta>-h, \quad \alpha<h/2, \quad (2.6)$$

which extends around the isotropic point $\alpha=\beta=0$, represents the regions where the universe expands in all directions. In the outside region the Universe is contracting at least in one direction.

The dynamical equations for the background are written as

$$(1+8r\dot{f})(\dot{q}+q^2)+(1+8q\dot{f})(\dot{r}+r^2)+(1+8\ddot{f})qr+\frac{1}{2}\dot{\phi}^2=0, \quad (2.7)$$

$$(1+8r\dot{f})(\dot{p}+p^2)+(1+8p\dot{f})(\dot{r}+r^2)+(1+8\ddot{f})rp+\frac{1}{2}\dot{\phi}^2=0, \quad (2.8)$$

$$(1+8q\dot{f})(\dot{p}+p^2)+(1+8p\dot{f})(\dot{q}+q^2)+(1+8\ddot{f})pq+\frac{1}{2}\dot{\phi}^2=0, \quad (2.9)$$

$$\ddot{\phi}+(p+q+r)\dot{\phi}-8f'\{\dot{p}qr+p\dot{q}r+pq\dot{r}+pqr(p+q+r)\}=0, \quad (2.10)$$

together with the constraint equation

$$pq+qr+rp+24pqr\dot{f}-\frac{1}{2}\dot{\phi}^2=0. \quad (2.11)$$

Defining a 4-dimensional vector, $\mathbf{x}=(\dot{p},\dot{q},\dot{r},\ddot{\phi})$, Eqs. (2.7)–(2.10) can be written in the matrix form

$$Z\mathbf{x}=\mathbf{y}, \quad (2.12)$$

where $\mathbf{y}=\mathbf{y}(p,q,r,\phi,\dot{\phi})$ and

$$Z=\begin{bmatrix} 0 & 1+8f'\dot{\phi}r & 1+8f'\dot{\phi}q & 8f'qr \\ 1+8f'\dot{\phi}r & 0 & 1+8f'\dot{\phi}p & 8f'rp \\ 1+8f'\dot{\phi}q & 1+8f'\dot{\phi}p & 0 & 8f'pq \\ -8f'qr & -8f'rp & -8f'pq & 1 \end{bmatrix}. \quad (2.13)$$

The determinant of Z yields

$$D=2+16f'\dot{\phi}(p+q+r)-64f'^2(p^2q^2+q^2r^2+r^2p^2)+128f'^2pqr(p+q+r)+128f'^2\dot{\phi}^2(pq+qr+rp)+1024f'^3\dot{\phi}pqr(pq+qr+rp+\dot{\phi}^2)+12288f'^4\dot{\phi}^2p^2q^2r^2. \quad (2.14)$$

In the case of $D\neq0$, the solutions of Eqs. (2.7)–(2.10) are given by $\mathbf{x}=Z^{-1}\mathbf{y}$. When D vanishes, however, we cannot proceed numerical calculations further. This ''determinant singularity'' plays an important role in the anisotropic background [15].

From Eq. (2.11) we find the constraint

$$\alpha^2+\beta^2 \leqslant h^2+96(pqrf')^2, \tag{2.15}$$

in which case $\dot{\phi}$ is solved as

$$\dot{\phi}=24pqrf' \pm \sqrt{(24pqrf')^2+2(pq+qr+rp)}. \tag{2.16}$$

When $f=0$ anisotropy parameters are restricted in the circle, $\alpha^2+\beta^2 \leqslant h^2$. If the Gauss-Bonnet term is taken into account, we have the wider allowed range of anisotropy parameters given by Eq. (2.15).

III. DILATON-DRIVEN CASE

Firstly we consider the dilaton-driven case with

$$f(\phi)=\frac{\lambda}{16}e^{-2\phi}, \tag{3.1}$$

where the string coupling λ takes a positive value. We set $\lambda=1$ in our numerical simulations. In the scenario (3.1) nature of singularities was analyzed in Ref. [15] in the Bianchi type-I background in the case of the plus sign in the right-hand side of Eq. (2.16). Hereafter we shall make a detailed analysis about the property of singularities in both signs of Eq. (2.16).

The asymptotic behavior of solutions in the past and future can be analyzed by assuming the following power-law forms for the expansion rates:

$$p=c_1|t|^s, \quad q=c_2|t|^s, \quad r=c_3|t|^s. \tag{3.2}$$

In order for the $\dot{f}$ term in Eqs. (2.7)–(2.10) to have a power-law dependence, the dilaton is required to take the form

$$\phi=\phi_0+c_4\ln|t|. \tag{3.3}$$

When the contribution from the Gauss-Bonnet term is negligible, one has $\dot{\phi}^2=2(c_1c_2+c_2c_3+c_3c_1)|t|^{2s}$ from Eq. (2.11). Comparing this with Eqs. (2.7)–(2.10) and (3.3), we find

$$s=-1, \quad c_1+c_2+c_3=\mathrm{sgn}(t), \quad c_1^2+c_2^2+c_3^2+c_4^2=1,$$
$$c_4^2=2(c_1c_2+c_2c_3+c_3c_1). \tag{3.4}$$

In the absence of the dilaton ($c_4=0$), the solution (3.4) for $t>0$ represents the vacuum Kasner solution where the universe is expanding in two directions and contracting in one direction. The interpretation of this solution is that large anisotropies are required at least in one dimension as $t\to 0$, in order to make the spacetime curved by anisotropies.

The situation is different when the dilaton is taken into account. For example, when $\sqrt{1/2} \leqslant c_4 \leqslant \sqrt{2/3}$ the universe is expanding in all directions for $t\to\infty$, while there exist both Friedmann- and Kasner-type solutions for $0 \leqslant c_4 < \sqrt{1/2}$. Note that in the limit where the Gauss-Bonnet term is negligible ($f'\to 0$) the determinant approaches a constant value $D=2$ from Eq. (2.14).

When the Gauss-Bonnet term is dominant in Eq. (2.11) ($|pq+qr+rp| \ll |24pqr\dot{f}|$), one has $\dot{\phi}=-6\lambda c_1c_2c_3|t|^{3s}e^{-2\phi}$ by using Eq. (3.2). Integrating this equation with respect to t, we easily find that $\dot{\phi}=\mathrm{sgn}(t)\times(3s+1)/(2|t|)$. Combining this with Eqs. (2.7)–(2.10) gives

$$s=-2, \quad c_4=-\frac{5}{2}, \quad c_1c_2c_3=\mathrm{sgn}(t)\frac{5e^{2\phi_0}}{12\lambda}. \tag{3.5}$$

Since $c_1c_2c_3<0$ for $t<0$, the universe is either contracting in all directions or expanding in two directions and contracting in one direction. When $t>0$, the universe is either expanding in all directions or expanding in one direction and contracting in two directions. For the asymptotic solution (3.5), we have that $\dot{\phi}\propto|t|^{-1}$ and $f'\propto|t|^5$, in which case the determinant is given by $D\propto|t|^6\to\infty$ for $|t|\to\infty$. In spite of this divergent behavior of the determinant, the solutions are nonsingular with $p,q,r\propto|t|^{-2}\to 0$ for $|t|\to\infty$.

We shall classify the cases where the solutions of Eqs. (2.7)–(2.10) exhibit singular behavior. When the system passes through the determinant singularity ($D=0$), Eq. (2.12) indicates that $\mathbf{x}=(\dot{p},\dot{q},\dot{r},\ddot{\phi})$ diverge. This singularity appears in an anisotropic background where three expansion rates are multiple-valued functions of time [13]. It is also a physical singularity where the curvature invariant $R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$ diverges due to the divergence of the time derivative of the expansion rates. Near the determinant singularity, the expansion rates and the scalar field can be expanded as [15]

$$h_i=h_{is}+h_{i1}\sqrt{|t-t_s|}+h_{i2}(\sqrt{|t-t_s|})^2+\cdots, \tag{3.6}$$

$$\phi=\phi_s+\phi_1(\sqrt{|t-t_s|})^2+\phi_2(\sqrt{|t-t_s|})^3+\cdots, \tag{3.7}$$

where $h_i=p,q,r(i=1,2,3)$, and t_s is the time at singularity. This means that $\dot{h}_i$ and $\ddot{\phi}$ diverge as $t\to t_s$, while $p,q,r,\dot{\phi}$ are finite. This property is different from the ordinary kind of singularity where p,q,r do not stay finite. The determinant singularity plays a crucial role in an anisotropic background.

The ordinary kind of singularities can be classified as

$$\text{(i)} \quad p\sim p_0/(t-t_s), \quad q\sim q_0, \quad r\sim r_0, \tag{3.8}$$

$$\text{(ii)} \quad p\sim p_0/(t-t_s), \quad q\sim p_0/(t-t_s), \quad r\sim r_0, \tag{3.9}$$

$$\text{(iii)} \quad p\sim p_0/(t-t_s), \quad q\sim p_0/(t-t_s), \quad r\sim p_0/(t-t_s), \tag{3.10}$$

where p_0, q_0, and r_0 are constants with $p_0>0$. If the time direction is forwards ($t-t_s\to -0$) one has $p\to-\infty$, while $p\to\infty$ for $t-t_s\to+0$. The asymptotic forms of the determinant (2.14) depend upon the cases presented above.

In the case (i) with a plus sign in RHS of Eq. (2.16), the signs of q_0 and r_0 are the same and the asymptotic form of $\dot{\phi}$ is given by $\dot{\phi}\sim(q_0+r_0)/(3\lambda e^{-2\phi_s}q_0r_0)$ for $t-t_s\to+0$, with ϕ_s being a constant. Then the determinant D yields, from Eq. (2.14),

$$D \sim -\frac{4\lambda^2 e^{-4\phi_s} p_0^2}{3(t-t_s)^2}\left[\left(q_0-\frac{r_0}{2}\right)^2+\frac{3}{4}r_0^2\right]\to-\infty \quad (\text{for } q_0 r_0>0). \tag{3.11}$$

In the minus sign of Eq. (2.16), one has $D\to-\infty$ for $q_0 r_0<0$ and $t-t_s\to+0$.

In the case (ii) with a plus sign of Eq. (2.16), one has $\dot{\phi}\sim 1/(3\lambda r_0 e^{-2\phi_s})$ for $r_0>0$ and $t-t_s\to+0$. Then the determinant should asymptotically take the form

$$D\sim-\frac{4\lambda^2 p_0^4 e^{-4\phi_s}}{3(t-t_s)^4}\to-\infty \quad (\text{for } r_0>0). \tag{3.12}$$

In numerical analysis we did not find this case for the parameter ranges and initial conditions we adopt. Typically ordinary singularities are dominated by the cases where one or three expansion rates tend to diverge. When one chooses the minus sign of Eq. (2.16), the asymptotic behavior is $D\to-\infty$ for $r_0<0$ and $t-t_s\to+0$.

In the case (iii) with a plus sign of Eq. (2.16), we find $\dot{\phi}\sim t/(\lambda p_0 e^{-2\phi_s})$ and

$$D\to 0, \tag{3.13}$$

for $t-t_s\to+0$. Strictly speaking this holds only for the isotropic case ($\alpha=\beta=0$) where all expansion rates are the same. In this case the solutions do not cross the determinant singularity, although they approach $D=0$ as $t-t_s\to+0$. When small anisotropies are included, the trajectories can pass through $D=0$. For the minus sign of Eq. (2.16) the asymptotic behavior of the determinant is not described by Eq. (3.13), as we will see later.

In another limit $t-t_s\to-0$, the signs of diverging expansion rates in Eqs. (3.8)–(3.10) are reversed, in which case the asymptotic forms of D are altered. Nevertheless the determinant of the cases (i) and (ii) generally approaches the asymptotic value $D=-\infty$. We shall confirm this by numerical investigations in subsequent sections.

A. Plus sign of Eq. (2.16)

We first analyze the case of the plus sign in Eq. (2.16). When ϕ is largely positive, the term $f'=-(\lambda/8)e^{-2\phi}$ is negligible in Eq. (2.16), implying that $\dot{\phi}$ is positive as long as $pq+qr+rp>0$. In this case ϕ increases toward the future, which results in nonsingular asymptotic solutions with determinant $D\approx 2$. When we go back to the past, there are two possibilities for the evolution of the determinant. One is the case where the solutions pass through the determinant singularity ($D=0$) and another is the one where D goes toward infinity ($D\to+\infty$). Our numerical investigations suggest that the latter case does *not* occur for the plus sign in Eq. (2.16), which means that solutions nonsingular in the future ($D\to 2$) meet the determinant singularity in the past, irrespective of the initial values of ϕ.

In the left panel of Fig. 1(a) we plot the evolution of the expansion rates for the anisotropy parameters $\alpha=0.05, \beta=0.05$ with initial conditions $h=0.16$ and $\phi=0$, corresponding to $D\approx 1.7$ at $t=0$. The determinant continues to grow until it approaches the finite value $D=2$ as $t\to\infty$, which indicates that this solution belongs to the nonsingular solution in the future given by Eq. (3.4). Note that for $t>0$ the universe is expanding in all directions. When we solve the equations of motion backwards ($t<0$), the solution meets the determinant singularity around $t=-0.82$, thereby leading to the divergence of $\dot{p}$, $\dot{q}$, $\dot{r}$, and $\ddot{\phi}$. If we introduce a new time parameter τ defined by

$$\tau\equiv\int\frac{dt}{D}, \tag{3.14}$$

it becomes possible to enter the region of the negative sign of D by overpassing the determinant singularity [13]. This does not mean that we can remove the singularity by coordinate transformations. The determinant singularity is a physical one where the divergence of the curvature invariant is unavoidable even in other coordinates. After entering another branch, the solution turns back forwards and the determinant begins to decrease rapidly toward $D\to-\infty$ [see the right panel of Fig. 1(a)]. From Fig. 1(a) we find that this belongs to the class of the case (i) with $p\sim p_0$, $q\sim q_0$, $r\sim r_0/(t-t_s)$, and $p_0 q_0>0$ [see Eq. (3.11)]. The universe is rapidly contracting in one direction ($r\to-\infty$) for $t-t_s\to-0$. As is claimed in Ref. [13], these trajectories can be understood as the pair creation of two branches ($D>0$ and $D<0$) at the determinant singularity. For the nonsingular solutions in the future, it is inevitable to cross the determinant singularity in the past in an anisotropic background. Notice that in the isotropic case D is always positive and decreases toward zero as $t\to-\infty$. In the presence of small anisotropies, however, the solution reaches the determinant singularity at finite past and fall into the $D=-\infty$ singularity as shown in Fig. 1(a).

When ϕ is large, the allowed anisotropy parameters in the (α,β) plane lie inside a circle, given by

$$\alpha^2+\beta^2\lesssim h^2, \tag{3.15}$$

which comes from the constraint (2.15). In Fig. 2(a) we show the nature of singularities in the past and future in the (α,β) plane for $h=0.16$ and $\phi=0$ at $t=0$. In this case the allowed region is approximately described by Eq. (3.15). We find that all solutions inside this region exhibit the determinant singularity in the past (region II in Fig. 2) while they are nonsingular ($D\to 2$) in the future (region I).

With the decrease of ϕ, the allowed region gets larger as found by Eq. (2.15). The nature of singularities becomes more complicated due to the appearance of ordinary singularities. If the system is close to the isotropic case ($|\alpha|,|\beta|\ll 1$), one has $D\to 2$ for $t\to\infty$ and $D\to 0$ at the finite past as in the case of Fig. 2(a). However, larger anisotropies alter this picture. For example, we show the evolution of the expansion rates in the left panel of Fig. 1(b) for $\alpha=0.15$ and $\beta=0.05$ with $h=0.16$ and $\phi=-1$ at $t=0$. In this case p is singular at finite past while q,r are positive constants, which means that $D\to-\infty$ from Eq. (3.11). The solution exhibits

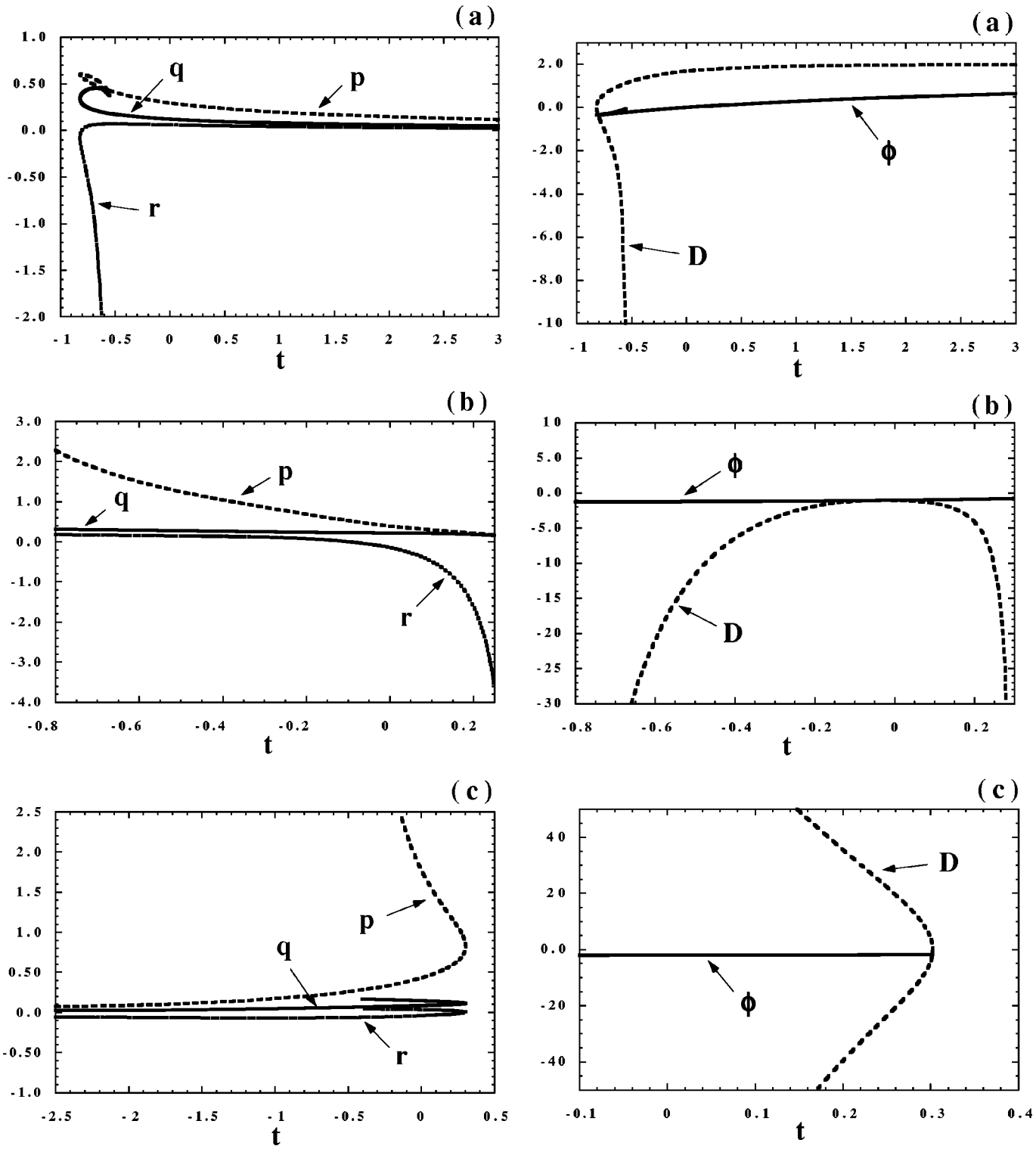

FIG. 1. The evolution of p, q, r, ϕ, and D in the dilaton-driven case for the plus sign in Eq. (2.16). Each figure corresponds to initial conditions with $h=0.16$ and (a) $\phi=0$, $\alpha=0.05$, $\beta=0.05$, (b) $\phi=-1$, $\alpha=0.15$, $\beta=0.05$, (c) $\phi=-2$, $\alpha=0.1$, $\beta=0.1$, respectively. The determinant singularity ($D=0$) can be passed through by introducing a new time parameter $\tau=\int dt/D$.

the same kind of singularity ($D\rightarrow-\infty$) in the future. In this case we have numerically found the determinant is always negative ($D=-1.02$ at $t=0$) and the solution never crosses $D=0$ in both the past and future [see the right panel of Fig. 1(b)]. In Fig. 2(b) the asymptotic property of singularities is presented for $h=0.16$ and $\phi=-1$ at $t=0$. There exists a region III, where the determinant is singular ($D\rightarrow-\infty$) in both the past and future. We also find that the region IV with $D\rightarrow+\infty$ appears in the past when anisotropy parameters are large. This corresponds to asymptotic past nonsingular solu-

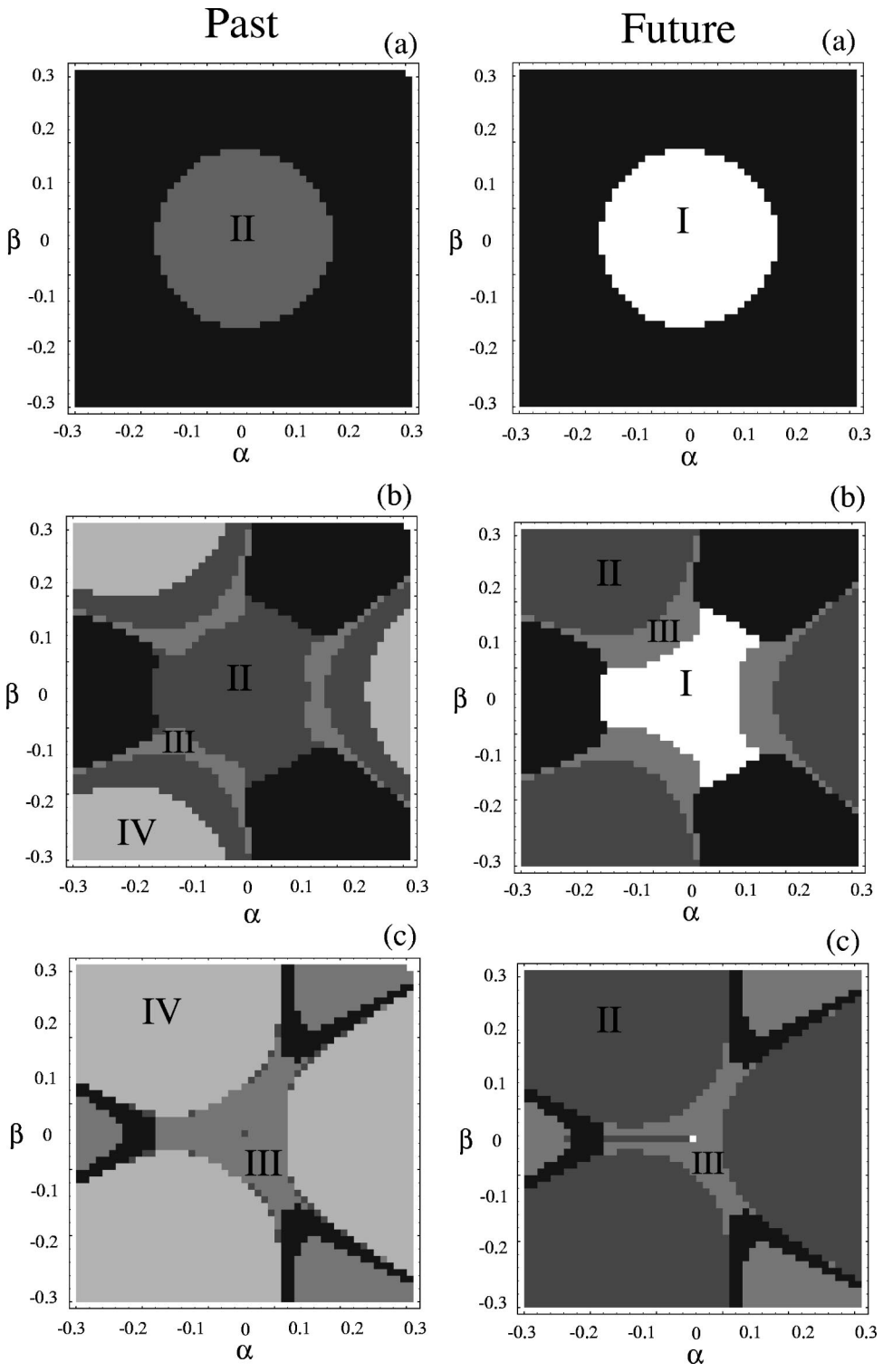

FIG. 2. Nature of singularities in the (α,β) plane with initial conditions (a) $\phi=0$, (b) $\phi=-1$, and (c) $\phi=-2$ for the plus sign in Eq. (2.16). The left figures correspond to past solutions while right figures correspond to future solutions. Each region corresponds to (I) nonsingular solutions with $D\to 2$, (II) singular solutions with determinant singularity ($D=0$), (III) singular solutions with $D\to-\infty$, (IV) nonsingular solutions with $D\to+\infty$, respectively. The black color indicates prohibited regions in the initial condition space.

tions given by Eq. (3.5) where quadratic curvature corrections are dominant. It is important to separate this case from the ordinary singularity with $D=-\infty$, although this classification was not done in Ref. [15]. From Fig. 2(b) the solutions with $D=0$ in the future correspond to, in the past, the determinant singularity or the regular solutions with $D\to+\infty$. Notice that the parameter range of future nonsingular solutions are smaller compared to the case of Fig. 2(a).

In Fig. 2(c) we show the density plot for $h=0.16$ and $\phi=-2$ at $t=0$. This suggests that future nonsingular trajectories are restricted to be very narrow near the isotropic point $\alpha=\beta=0$. We also find that the determinant singularity in the future corresponds to the regular solutions with $D\to+\infty$ in the past. As one example we plot in Fig. 1(c) the evolution of p,q,r for $h=0.16$ and $\phi=-2$ at $t=0$ with anisotropy parameters $\alpha=0.1$ and $\beta=0.1$. In this case the past asymptotic solution is categorized in the nonsingular solution given by Eq. (3.5). Figure 2(c) shows that two expansion rates p,q are

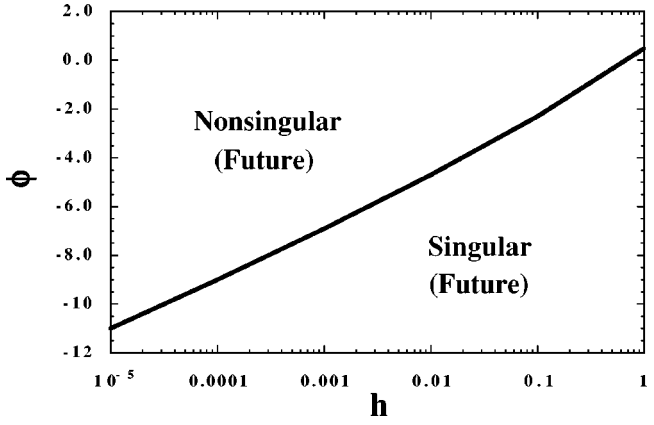

FIG. 3. The minimal initial values of ϕ which allow solutions nonsingular in future in the dilaton-driven case for the plus sign in Eq. (2.16).

positive while r is negative for $t<0$, implying $c_1c_2c_3<0$ in Eq. (3.5). The solution comes regularly from the asymptotic past with D being decreased toward the future [see the right panel of Fig. 1(c)]. It crosses the determinant singularity around $t=0.3$, after which the determinant continues to decrease until the solution falls into the ordinary singularity with $D\to-\infty$. Note that p diverges as $t-t_s\to+0$, while q and r approach positive constant values, in which case one has $D\to-\infty$ by Eq. (3.11). This trajectory can be regarded as the pair annihilation of two branches with $D>0$ and $D<0$. From Figs. 2(b) and 2(c), the solutions nonsingular in the past ($D\to+\infty$) meet the determinant singularity in the future. In this case the solutions do not go forwards beyond the determinant singularity. In Fig. 2(c) the parameter regions with ordinary singularity in the future ($D=-\infty$) typically correspond to those with the same singularity in the past, in which case the asymptotic behavior of the expansion rates are the same as in Fig. 1(b). Notice that in Fig. 2(c) there exist some parameter ranges where the solutions meet the determinant singularity in both the past and future. Although this case is very rare, it is still possible to cross the determinant singularity twice.

From Fig. 2 we find that nonsingular solutions in the future are determinant type in the past. This can be understood that the determinant evolves from $D=2$ to $D=0$, if we solve the equations of motions from the asymptotic future to the past [see the right panel of Fig. 1(a)]. We also analyzed other cases varying the values of h and ϕ for $10^{-5}<h<1$ and $-15<\phi<15$. For a fixed value of h there exists a minimal value of ϕ which leads to nonsingular solutions in the future (see Fig. 3). The allowed range of the dilaton for future nonsingular solutions gets wider with the decrease of h as shown in Fig. 3. In all cases analyzed in our numerical simulations, the past singularity for future nonsingular solutions corresponds to the determinant type in an anisotropic background. We also found that trajectories with nonsingular past asymptotic meet the determinant singularity in the future. In addition we showed that the ordinary singularity with Eq. (3.8) appears, in which case the determinant is divergent as $D\to-\infty$. This is different from the case $D\to+\infty$ where the solutions are nonsingular in the asymptotic past. In the next subsection we will analyze how the behavior of the determinant is altered for different signs in Eq. (2.16).

B. Minus sign of Eq. (2.16)

In the previous subsection, solutions which are nonsingular in the future meet the determinant singularity in the past, due to the fact that the determinant evolves from the asymptotic value $D=2$ toward the determinant singularity. For the minus sign of Eq. (2.16), the determinant can increase toward the past, implying that the trajectories may not cross the determinant singularity. In fact we will show that the past singularity is not necessarily of the determinant type.

Let us first analyze the asymptotic behavior of the determinant when all expansion rates diverge as Eq. (3.10). In Eq. (2.16) when the $2(pq+qr+rp)$ term is positive and much larger than $(24pqrf')^2$, one has $\dot\phi\sim-\sqrt{2(pq+qr+rp)}\sim-\sqrt{6}p_0/(t-t_s)$. Therefore the dilaton grows as $\phi\sim-\sqrt{6}p_0\ln(t-t_s)$ for $t-t_s\to+0$, in which case the asymptotic form of the determinant yields

$$D\sim 2+6\sqrt{6}\lambda p_0^2(t-t_s)^{2(\sqrt{6}p_0-1)}+39\lambda^2p_0^4(t-t_s)^{4(\sqrt{6}p_0-1)}+18\sqrt{6}\lambda^3p_0^6(t-t_s)^{6(\sqrt{6}p_0-1)}+18\lambda^4p_0^6(t-t_s)^{8(\sqrt{6}p_0-1)}. \tag{3.16}$$

This indicates that the determinant grows infinitely ($D\to+\infty$) for $p_0<1/\sqrt{6}$, while it decreases toward a finite value $D=2$ for $p_0>1/\sqrt{6}$. The latter case corresponds to the one where the average expansion rates are large initially.

When the condition $(24pqrf')^2\gg|2(pq+qr+rp)|$ is satisfied, the asymptotic form of $\dot\phi$ is given by $\dot\phi\sim-6\lambda p_0^3e^{-2\phi}/(t-t_s)^3$, which yields $e^{2\phi}\sim6\lambda p_0^3/(t-t_s)^2$. Then one has the following positive finite value of the determinant in the limit of $t-t_s\to+0$:

$$D\sim 2+\frac{13}{12p_0^2}+\frac{7}{36p_0^4}+\frac{5}{432p_0^6}. \tag{3.17}$$

Whether the asymptotic form is given by Eq. (3.16) or (3.17) depends on the initial values of the expansion rates and the dilaton. In the latter case ϕ is generally small so that the condition $(24pqrf')^2\gg2(pq+qr+rp)$ is fulfilled. In both cases we can expect that the trajectories do not cross the determinant singularity in the past.

We show in Fig. 5 the density plot of the asymptotic behavior of the determinant for $h=0.16$ at $t=0$. When $\phi=0$ at $t=0$ all trajectories satisfying the constraint (2.15) are nonsingular in the future with $D=2$. In this case the past solutions are dominated by the singularity with asymptotic positive determinant as found from the left panel of Fig. 5(a). As one example we plot in Fig. 4(a) the evolution of p, q, r, ϕ, and D for $\phi=0$ at $t=0$ with anisotropy parameters $\alpha=0.05$ and $\beta=0.05$. This belongs to the class (3.16) with $\alpha<1/\sqrt{6}$ where all expansion rates diverge with determinant $D\to+\infty$. The determinant is always positive and continues to grow backwards. The left panel of Fig. 5(a) shows that there exist some past solutions which meet the determinant

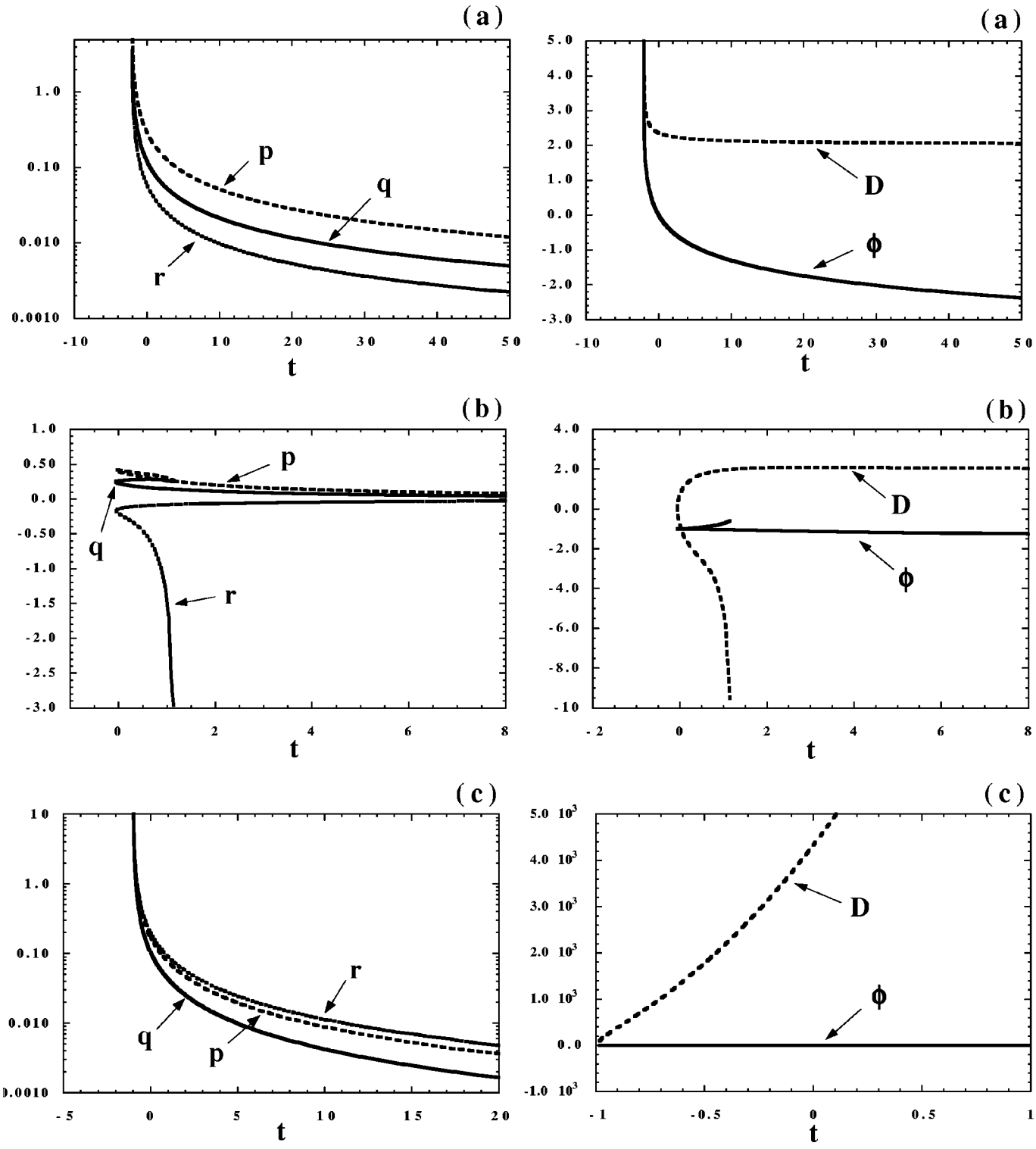

FIG. 4. The evolution of p, q, r, ϕ, and D in the dilaton-driven case for the minus sign in Eq. (2.16). Each figure corresponds to initial conditions with $h=0.16$ and (a) $\phi=0$, $\alpha=0.05$, $\beta=0.05$; (b) $\phi=-1$, $\alpha=0.15$, $\beta=0.05$; (c) $\phi=-2$, $\alpha=-0.02$, $\beta=0.02$, respectively.

singularity when anisotropies parameters are large. In this case the determinant grows until some moment of time in the past, after which it begins to decrease toward the determinant singularity. This behavior can be understood by the fact that large anisotropies prevent all expansion rates from evolving almost as in Eq. (3.10). However, this region is typically small for the positive values of ϕ at $t=0$.

With the decrease of the initial ϕ, the constraint (2.15) gives wider allowed parameter ranges in the (α,β) plane. Let us consider the case $\phi=-1$ and $h=0.16$ at $t=0$. As is found from Fig. 5(b) we have additional regions with the $D=-\infty$ singularity in both the past and future. The nature of

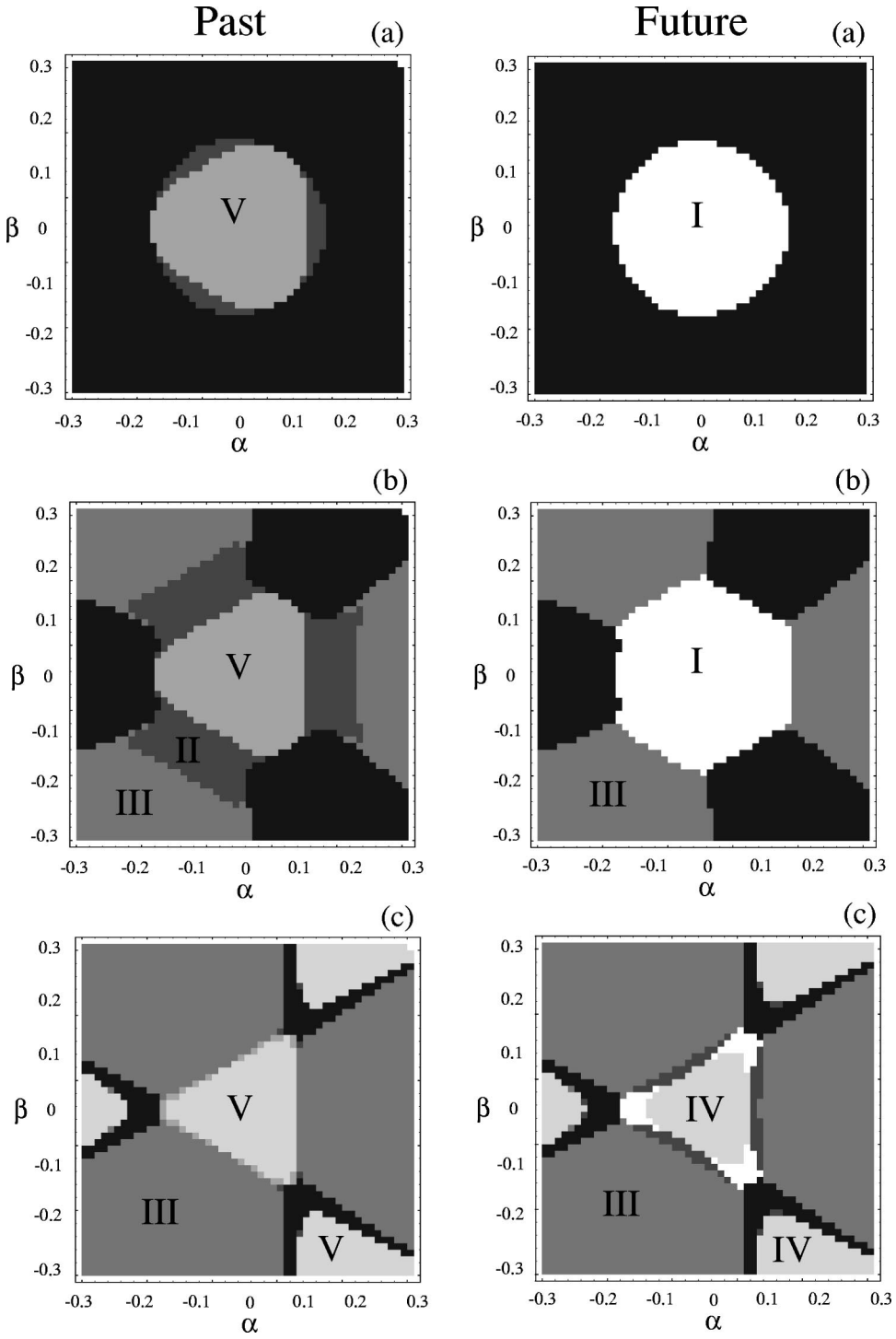

FIG. 5. Nature of singularities in the (α,β) plane with initial conditions, (a) $\phi=0$, (b) $\phi=-1$, and (c) $\phi=-2$ for the minus sign in Eq. (2.16). The left figures correspond to past solutions while right figures correspond to future solutions. Each region corresponds to (I) nonsingular solutions with $D\to 2$, (II) singular solutions with determinant singularity ($D=0$), (III) singular solutions with $D\to-\infty$, (IV) nonsingular solutions with $D\to\infty$, (V) singular solutions with positive determinant, respectively. The black color indicates prohibited regions in the initial condition space.

singularities around the isotropic point is similar to the $\phi=0$ case explained above. For the solutions nonsingular in the future, the past singularity is either type $D>0$ or $D=0$. Figure 4(b) is the latter case where the trajectory comes regularly from the asymptotic future ($D=2$) and meets the determinant singularity in the past. This evolution is similar to the case of large anisotropy parameters in Fig. 5(a). From Fig. 5(b) the solutions singular in the future ($D=-\infty$) correspond to, in the past, the singularity with $D=-\infty$ or the determinant singularity. The difference between the two cases is whether the determinant is always negative [as in Fig. 1(b)] or it passes through $D=0$.

The situation becomes somewhat different when ϕ is decreased further. Figure 5(c) indicates that nonsingular trajectories with $D\to+\infty$ appear in the future around the isotropic point for $\phi\lesssim-2$ at $t=0$. This is the case where all expansion rates are finite as described by Eq. (3.5). However, these solutions are singular in the past with all expansion rates being infinite. In region V shown in Fig. 5(c), the condition $(24pqrf')^2>|2(pq+qr+rp)|$ is typically satisfied at $t=0$. Therefore the determinant tends to approach the finite value (3.17) in the past. We show in Fig. 4(c) the evolution of the

system for $\alpha=-0.02$, $\beta=0.02$, $\phi=-2$, and $h=0.16$. While the evolution of the expansion rates in Fig. 4(c) looks similar to that in Fig. 4(a), the behavior of the determinant is different in both the past and future. In Fig. 4(c) the determinant decreases from infinity to a finite positive value toward the past. We have numerically found that two terms $(24pqrf')^2$ and $2(pq+qr+rp)$ become comparable during the evolution. Therefore the asymptotic value of D is somewhat different than in Eq. (3.17), but it is still a finite positive value. We find from Fig. 5(c) that future nonsingular solutions with either $D=2$ or $D=+\infty$ correspond to the singularity (3.10) with a positive determinant in the past. Figures 5(b) and 5(c) also suggest that as ϕ decreases the ordinary singularity with $D\to-\infty$ appears in both the past and future for large anisotropy parameters.

Compared to the case of the previous subsection, the range of nonsingular solutions in the future is not so narrow due to the presence of nonsingular trajectories with $D\to+\infty$. However, this solution is not appropriate to lead to our present universe due to the dominance of the quadratic curvature term. In addition these solutions typically approach the ordinary singularity (3.10) with all expansion rates infinite in the past. This property is different from the plus sign of Eq. (2.16) where the past singularity for solutions nonsingular in the future corresponds to the determinant type. In both cases we have found that nonsingular cosmological solutions in both the past and future do not exist for the dilaton-driven case for wide ranges of the parameter space ($10^{-5}<h<1$ and $-15<\phi<15$). However, the situation is changed in the modulus-driven case as we will analyze in the next section.

IV. MODULUS-DRIVEN CASE

In the modulus-driven case the function $f(\phi)$ in Eq. (2.1) is expressed as [9,10,12–14]

$$f(\phi)=-\frac{1}{16}\delta\xi(\phi), \tag{4.1}$$

where the coefficient δ is determined by the 4-dimensional trace anomaly of the $N=2$ sector. Here the function $\xi(\phi)$ is defined by

$$\xi(\phi)=\ln[2e^{\phi}\eta^4(ie^{\phi})], \tag{4.2}$$

where $\eta(ie^{\phi})$ is the Dedekind η function. Since $\xi(\phi)$ is well approximated as $\xi(\phi)\simeq-(2\pi/3)\cosh\phi$ [14], the function f takes the form

$$f(\phi)\simeq\frac{\pi\delta}{48}(e^{\phi}+e^{-\phi}). \tag{4.3}$$

When ϕ is largely negative ($|\phi|\gg1$), Eq. (4.3) reduces to the form (3.1) by setting $\delta=3\lambda/\pi$. Therefore when $\delta>0$, solutions that are nonsingular in both the past and future do not exist. However, nonsingular cosmological solutions have been found for negative values of δ [9]. Hereafter we shall focus on the negative δ case (setting $\delta=-1$ for simplicity) using the function $f(\phi)$ given by Eq. (4.3).

When the Gauss-Bonnet term dominates in Eqs. (2.7)–(2.10), the asymptotic solution is similarly given as Eq. (3.5), i.e.,

$$p=c_1|t|^{-2},\quad q=c_2|t|^{-2},\quad r=c_3|t|^{-2},$$
$$\phi=\phi_0\pm5\ln|t|,\quad c_1c_2c_3=\mathrm{sgn}(t)\frac{5e^{2\phi_0}}{4\pi\delta}. \tag{4.4}$$

The past asymptotic solutions correspond to $c_1c_2c_3>0$ for negative δ. This sign is different from the dilaton-driven case with positive λ which we already analyzed in the previous section. Note that the determinant is divergent ($D\to+\infty$) for $|t|\to\infty$.

For another asymptotic solution where the effect of the Gauss-Bonnet term is negligible, the evolution of the background is given by

$$p=c_1|t|^{-1},\quad q=c_2|t|^{-1},\quad r=c_3|t|^{-1},\quad \phi=\phi_0+c_4\ln|t|,$$
$$c_1+c_2+c_3=\mathrm{sgn}(t),\quad c_1^2+c_2^2+c_3^2+c_4^2=1,$$
$$c_4^2=2(c_1c_2+c_2c_3+c_3c_1). \tag{4.5}$$

Therefore one has $f'\propto|t|^{|c_4|}$, $\dot\phi\propto|t|^{-1}$, and $p,q,r\propto|t|^{-1}$ with $|c_4|<1$ in Eq. (2.14), in which case the solutions are nonsingular with an asymptotic value of the determinant $D=2$.

For the nonsingular cosmological solutions found in Ref. [9], $\dot\phi$ does not change its sign [10]. Because of the symmetric structure of the function (4.3) with respect to $\phi=0$, we will consider the positive $\dot\phi$ case where the modulus continues to grow from past to future. For negative $\dot\phi$ the analysis is essentially the same by changing ϕ to $-\phi$. When the solutions are singular, they meet the determinant singularity [see Eqs. (3.6) and (3.7)] or the ordinary singularity [see Eqs. (3.8)–(3.10)]. Let us consider the asymptotic behavior of the determinant for the ordinary singularity. In the cases (3.8)–(3.10), possible asymptotic behavior of the determinant can be summarized as

$$\text{case (i)}\quad D\sim-\frac{256f_s'^2p_0^2}{3(t-t_s)^2}\left[\left(q_0-\frac{r_0}{2}\right)^2+\frac{3}{4}r_0^2\right]\to-\infty,$$

with

$$\dot\phi\sim-\frac{1}{24f_s'}\frac{q_0+r_0}{q_0r_0}\sim\text{const}, \tag{4.6}$$

$$\text{case (ii)}\quad D\sim-\frac{256f_s'^2p_0^4}{3(t-t_s)^4}\to-\infty,$$

with

$$\dot\phi\sim-\frac{1}{24r_0f_s'}\sim\text{const}, \tag{4.7}$$

$$\text{case (iii)}\quad D\to0,\quad \dot\phi\sim-\frac{t-t_s}{8p_0f_s'}, \tag{4.8}$$

where $f_s' \equiv (\pi\delta/48)(e^{\phi_s} - e^{-\phi_s})$ with ϕ_s being a constant. Although case (ii) is rare, we have numerically checked that this asymptotic solution certainly exists for the modulus-driven case. The asymptotic case (iii) corresponds to the isotropic case, as is similar to the dilaton-driven cosmology. It is easy to verify that this trajectory is possible for positive ϕ but impossible for negative ϕ. In what follows we shall analyze how the evolution of the expansion rates and the determinant are different from that of the dilaton-driven case paying particular attention for nonsingular trajectories.

Nonsingular asymptotic solutions for $\dot{\phi} > 0$ can be described as $\phi = \phi_0 - 5\ln|t|$ in the past [see Eq. (4.4)] and $\phi = \phi_0 + c_4 \ln|t|$ with $0 < c_4 < 1$ in the future [see Eq. (4.5)]. Therefore ϕ continues to grow from the asymptotic past starting from large negative values of ϕ toward the future. Such examples are plotted in Figs. 6(a) and 6(b). In these cases the determinant evolves from $D = +\infty$ (past) to $D = 2$ (future) without passing through the determinant singularity. In the dilaton-driven case when the past trajectories are nonsingular with $D = +\infty$ they inevitably meet the determinant singularity in the future [see Figs. 2(b) and 2(c)]. This is mainly due to the fact that f' is negative in the dilaton case while its sign is different in the modulus case for $\delta < 0$ and $\phi < 0$. Therefore the determinant (2.14) is dominated by positive terms in the modulus case, which provides a way not to pass through the determinant singularity. Namely negative δ is crucial for the existence of nonsingular solutions.

In Fig. 7 we show past and future asymptotic properties in three different cases. Note that we have defined $\tilde{\alpha} \equiv \alpha/h$ and $\tilde{\beta} \equiv \beta/h$ in order to compare the cases where the average expansion rate h is changed. When $h = 0.05$ and $\phi = -5$, nonsingular solutions in the past ($D = +\infty$) are not singular in the future with determinant $D \to 2$ [see Fig. 7(a)]. The universe exhibits superinflation with growing expansion rates until the graceful exit around $t = 0$ [see Fig. 6(a)]. Notice that we have $pqr > 0$ in the asymptotic past, as predicted by Eq. (4.4) for negative δ. The expansion rates begin to decrease after the graceful exit, whose asymptotic solutions in the future are given by Eq. (4.5). In Fig. 6(a) we find that the future solution corresponds to the Kasner-type where the universe is contracting in one direction.

Figure 7(a) indicates that some trajectories which are nonsingular in the future cross the determinant singularity in the past. The evolution of the background is similar to Fig. 1(a) which we already analyzed in the dilaton case. We also find from Fig. 7(a) that when anisotropy parameters are large the solutions meet the ordinary singularity with $D = -\infty$ in both the past and future. This is the case of Eq. (3.8) or (3.9) where at least one expansion rate diverges as plotted in Fig. 1(b). Although in Fig. 7 past nonsingular solutions ($D \to +\infty$) do not meet the determinant singularity in the future, we have checked that this singular behavior occurs for smaller values of ϕ as shown in Fig. 1(c). These results imply that the property of singularities is similar to the dilaton-driven case described in Sec. III A.

For larger initial values of ϕ, the allowed region can be approximately described as $\tilde{\alpha}^2 + \tilde{\beta}^2 \lesssim 1$. When $h = 0.05$ and $\phi = 2$ shown in Fig. 7(b), all future asymptotic solutions are nonsingular with determinant $D \to 2$. However, we find from the left panel of Fig. 7(b) that the region of past nonsingular solutions gets smaller relative to Fig. 7(a). For large anisotropy parameters, the determinant tends to decrease backwards from the asymptotic future with $D = 2$, thereby resulting in the singularity with $D = 0$. This is not the case for the nonsingular trajectories with small anisotropy parameters. One example is plotted in Fig. 6(b). Although the determinant decreases from $t = +\infty$ to $t \sim -6$, it begins to grow toward the asymptotic past before crossing the determinant singularity. From Fig. 6(b) we find that the trajectory comes regularly from the asymptotic past and connects to the Friedmann-type branch where the universe is expanding in all directions. When anisotropy parameters are small and belong to the region (2.6), the future solution is of Friedmann type.

When the average expansion rate h gets larger, the region of nonsingular cosmological solutions becomes smaller. We plot in Fig. 7(c) the density plot of the nature of the determinant for $h = 0.1$ and $\phi = 2$. The past solutions are dominated by the determinant singularity, whereas the future solutions are nonsingular which is similar to Fig. 7(b). In this case nonsingular trajectories in both the past and future exist in only small parameter ranges around the isotropic point.

It is also worth mentioning that the ordinary singularity (3.10) with asymptotic determinant (3.16) and (3.17) for the minus sign of Eq. (2.16) does not appear in the present case. When ϕ increases toward the past, $|f'|$ also grows in the modulus case. Therefore the condition $(24pqrf')^2 \ll |2(pq + qr + rp)|$ is not satisfied, implying that the case of Eq. (3.16) does not occur. When $(24pqrf')^2 \gg |2(pq + qr + rp)|$, there are two possibilities for the asymptotic form of $\dot{\phi}$, one of which is $\dot{\phi} \simeq \pi\delta(e^{\phi} - e^{-\phi})p_0^3/(t - t_s)^3$, and the other $\dot{\phi} \simeq C(t - t_s)$, where C is a constant. In the former case it is easy to show that asymptotic solutions do not exist by integrating $\dot{\phi} \simeq \pi\delta(e^{\phi} - e^{-\phi})p_0^3/(t - t_s)^3$. [Note that this is possible in the dilaton-driven case with determinant (3.17).] In the latter case the determinant approaches $D = 0$ as in Eq. (4.8).

We have done numerical simulations for other cases varying the values of h and ϕ. When h is large, ϕ is required to be small for the existence of nonsingular cosmological solutions. This property is found in Fig. 8 where we plot the regions of nonsingular and singular solutions in the (h, ϕ) plane. Nonsingular trajectories come regularly from the asymptotic past with $D = +\infty$ where the quadratic curvature term is dominant [see Eq. (4.4)], and smoothly connect another nonsingular branch with $D = 2$ where the Gauss-Bonnet term is negligible [see Eq. (4.5)]. For singular solutions nature of singularities is found to be similar to that of the dilaton-driven case discussed in Sec. III A.

V. SUMMARY AND DISCUSSIONS

In this paper we have analyzed past and future asymptotic regimes in Bianchi type-I string-inspired cosmological models in the presence of a Gauss-Bonnet curvature invariant. If the loop correction is not taken into account, one has the no-go result that the initial big-bang singularity cannot be

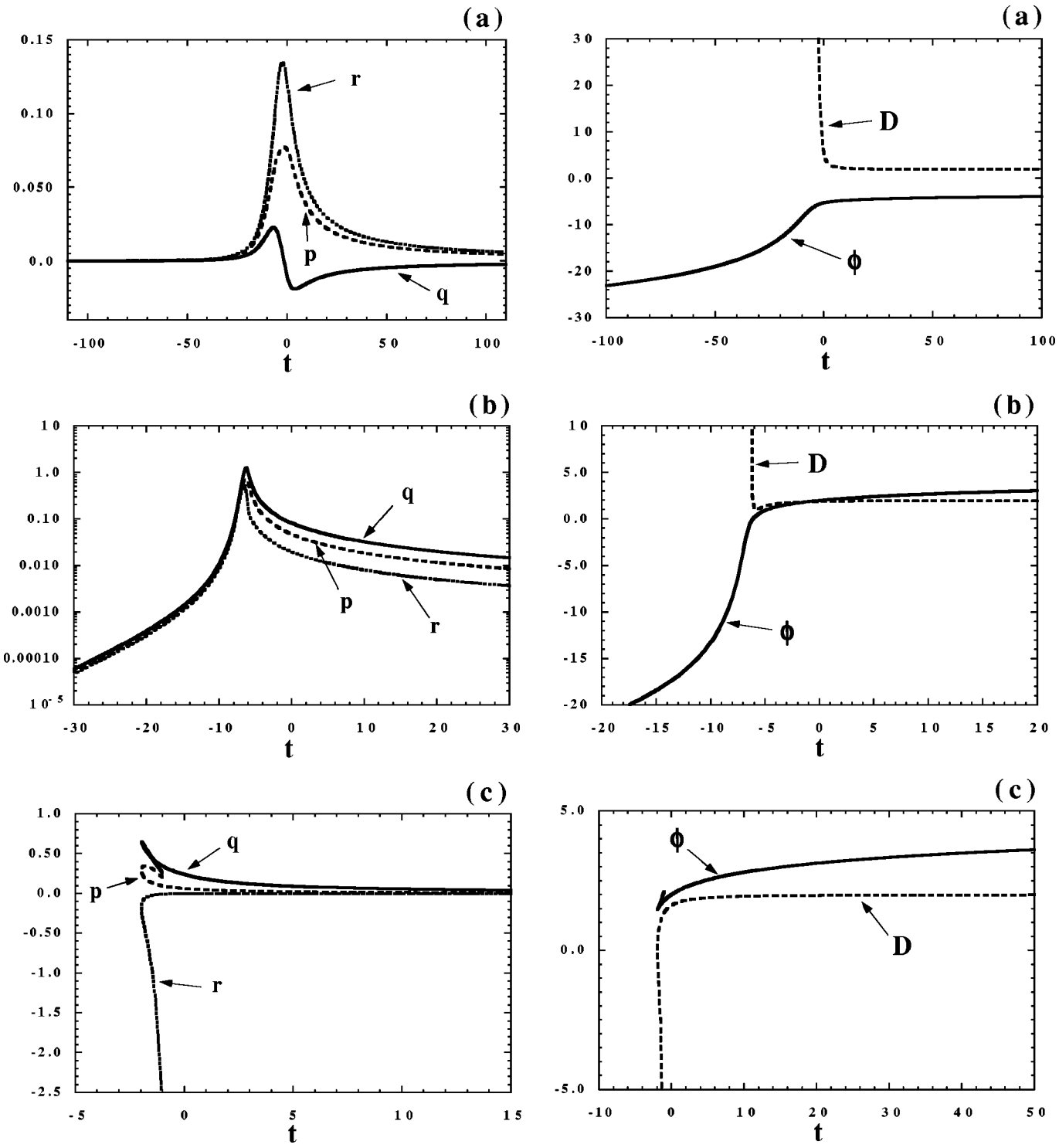

FIG. 6. The evolution of p, q, r, ϕ, and D in the modulus-driven case. Each figure corresponds to initial conditions with (a) $\phi=-5$, $h=0.05$, $\tilde{\alpha}\equiv\alpha/h=-0.5$, $\tilde{\beta}\equiv\beta/h=0.5$; (b) $\phi=2$, $h=0.05$, $\tilde{\alpha}=0.3$, $\tilde{\beta}=-0.2$; (c) $\phi=2$, $h=0.1$, $\tilde{\alpha}=0.5$, $\tilde{\beta}=-0.5$, respectively.

avoided. The Gauss-Bonnet term allows the existence of nonsingular cosmological solutions, depending on the theories we adopt. We investigated two gravity theories, viz., dilaton- and modulus-driven cosmologies. In the former case the dynamics appears to depend significantly on the sign on the right-hand side of Eq. (2.16). Hence, we treated possible signs in Eq. (2.16) separately and constructed three pictures of cosmological evolution: (a) dilaton-driven cosmology with plus sign in Eq. (2.16); (b) dilaton-driven cosmology with minus sign in Eq. (2.16); (c) modulus-driven cosmology with $\delta<0$.

As the quadratic curvature corrections may provide

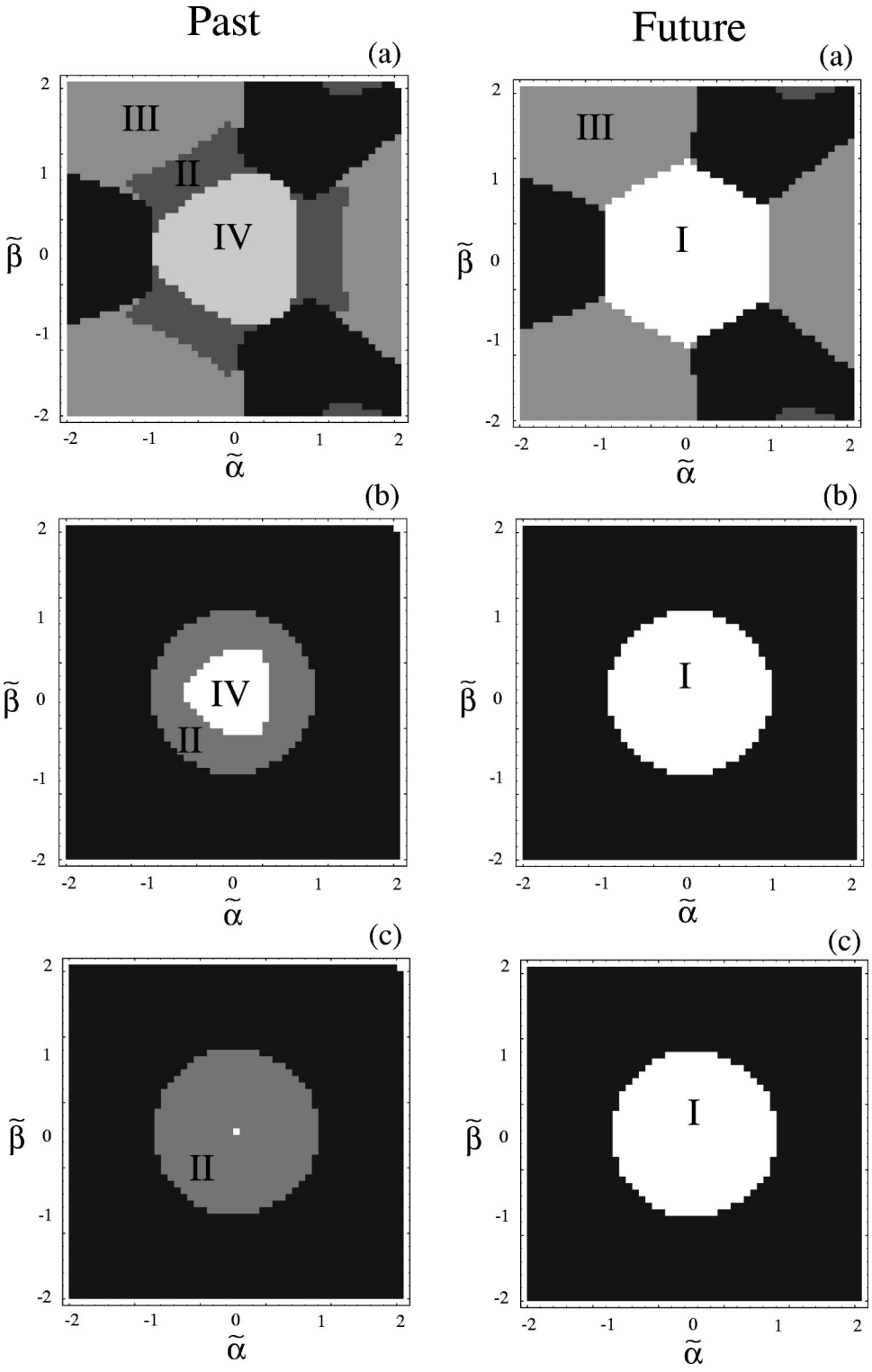

FIG. 7. Nature of nonsingular and singular solutions in the $(\tilde{\alpha},\tilde{\beta})$ plane with initial conditions, (a) $h=0.05$, $\phi=-5$, (b) $h=0.05$, $\phi=2$, and (c) $h=0.1$, $\phi=2$. The left figures correspond to past solutions while right figures correspond to future solutions. Each region corresponds to (I) nonsingular solutions with $D\to 2$, (II) singular solutions with determinant singularity ($D=0$), (III) singular solutions with $D\to-\infty$, (IV) nonsingular solutions with $D\to\infty$, respectively. The black color indicates prohibited regions in the initial condition space.

violations of strong and week energy conditions [13,15], the Bianchi type-I universe can recollapse in the high-curvature regime. This fact gives us a variety of possible types of trajectories with different past and future asymptotics. Some trajectories cannot leave the high-curvature regime, while some reach the low-curvature future attractor.

In the dilaton-driven cosmology the 4-dimensional string coupling λ is required to be positive. This forbids the existence of nonsingular cosmological solutions even when the Gauss-Bonnet term is taken into account. In case (a), nonsingular trajectories in the future correspond to the low-curvature solutions with determinant $D\to 2$ where the Gauss-Bonnet term is negligible. These trajectories meet the singularity where the determinant of the system vanishes at the finite past. At this determinant singularity, $\dot{p},\dot{q},\dot{r},\ddot{\phi}$ diverge in Eq. (2.12) while $p,q,r,\dot{\phi}$ stay finite. These kinds of singularities restrict the presence of nonsingular

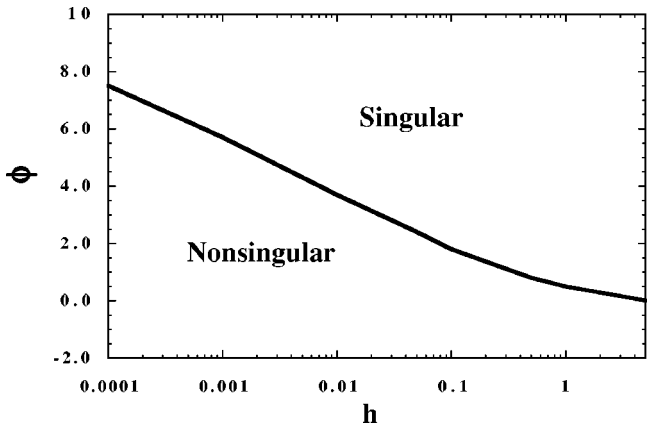

FIG. 8. The regions of nonsingular and singular solutions for the modulus-driven case in the initial condition space of h and ϕ.

solutions in an anisotropic background. Other kinds of singularities exist (we call them ordinary singularities) where at least one expansion rate diverges [see Eqs. (3.8)–(3.10)]. When the singularity (3.8) or (3.9) appears at the finite past or future, the determinant approaches $D=-\infty$. We also find some trajectories which are nonsingular in the asymptotic past with determinant $D\to+\infty$. These solutions appear for large anisotropy parameters, whose existence, to our knowledge, was not discovered previously. In case (a) these solutions are found to meet the determinant singularity in the future (see Fig. 2). In case (b) ordinary singularity of Eq. (3.10) appears where all expansion rates diverge. In this case the determinant is divergent or approaches a positive constant value, depending on the initial conditions of ϕ and h [see Eqs. (3.16) and (3.17)]. The solutions nonsingular in the future typically correspond to this type of singularity or the determinant singularity in the past as found by Fig. 5.

In the modulus-driven case the coupling δ can take either positive or negative value in Eq. (4.3). When $\delta>0$ solutions which are nonsingular in both the past and future are not found, similar to the dilaton-driven case. For negative values of δ, however, there exist nonsingular cosmological solutions where two branches of superinflation and decreasing curvature can be joined to each other. In this case the solutions come regularly from the asymptotic past ($D=+\infty$) with determinant being decreased. The determinant approaches the future asymptotic value $D=2$ without crossing the determinant singularity. This is the main difference from the dilaton-driven case where the past nonsingular trajectories inevitably meet the determinant singularity in the future. When the solutions are singular we find that nature of singularities is similar to the dilaton-driven case discussed in Sec. III A.

Our numerical investigations show that not all combinations of the past and future asymptotics are possible. We pay particular attention to nonsingular past and low-curvature nonsingular future regimes (the latter should correspond to our present Universe). For these regimes we get the following results: For the dilaton-driven case trajectories with a nonsingular past asymptotic meet the determinant singularity in the future; for cases (a) and (c) trajectories with low-curvature future asymptotic meet the determinant singularity in the past, when the past solutions are singular.

The first property tells us that the negative sign of the coupling constant is essentially important for constructing a purely nonsingular string cosmological models. This result is known for FRW universe [9,10] and, hence, is still valid in anisotropic Bianchi type-I case, despite the existence of a past nonsingular regime in the dilaton-driven cosmology. The second property indicates that the determinant singularity may play a crucial role when we try to trace back in time evolution of our present Universe in an anisotropic background.

Recently, string-inspired cosmological models which can avoid the big bang singularity have received much attention [20–24] together with the proposal of the ekpyrotic universe [25]. In those cases the quantum loop corrections or the higher-order derivatives play important roles in determining the dynamics before the graceful exit. It is certainly of interest to extend our analysis to more complicated models such as the multifield case in the presence of the Gauss-Bonnet term. In fact while nonsingular solutions exist in the single-field modulus-driven case considered in this work, density perturbations generated by the fluctuation of the modulus exhibits blue spectra with a spectral tilt $n=10/3$ [26,27]. This contradicts the observational supported flat spectra with $n\simeq 1$. In the multifield case, however, it may be possible to produce almost scale-invariant spectra if a light scalar field such as axion generates a flat isocurvature perturbation during superinflation [28–30] or if the axion is nonminimally coupled to the dilaton with some potential [31]. We leave to future work to construct such nonsingular cosmological models which are consistent with observations.

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