Natural quintessence and large extra dimensions

A. Albrecht,[1] C. P. Burgess,[2,3] F. Ravndal,[4] and C. Skordis[1]
[1]*Department of Physics, University of California Davis, 1 Shields Avenue, Davis, California 95616*
[2]*School of Natural Sciences, Institute for Advanced Study, Princeton, New Jersey 08540*
[3]*Physics Department, McGill University, 3600 University Street, Montréal, Québec, Canada H3A 2T8*
[4]*Institute of Physics, University of Oslo, N-0316 Oslo, Norway*
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We examine the late-time (nucleosynthesis and later) cosmological implications of brane-world scenarios having large (millimeter sized) extra dimensions. In particular, recent proposals for understanding why the extra dimensions are so large in these models indicate that moduli such as the radion appear (to four-dimensional observers) to be extremely light, with a mass of order 10^{-33} eV, allowing them to play the role of the light scalar of quintessence models. The radion-as-quintessence solves a long-standing problem since its small mass is technically natural, in that it is stable against radiative corrections. Its challenges are to explain why such a light particle has not been seen in precision tests of gravity, and why Newton's constant has not appreciably evolved since nucleosynthesis. We find the couplings suggested by stabilization models can provide explanations for both of these questions. We identify the features which must be required of any earlier epochs of cosmology in order for these explanations to hold.

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I. INTRODUCTION

If current experimental indications are to be believed, the universe is composed of islands of luminous material embedded among darker baryonic matter, wrapped in the riddle of nonbaryonic dark matter inside the enigma of dark energy. Best fits to cosmological models give the last three of these in the rough relative proportions of few%:30%:70% [1]. It is staggering that essentially nothing is known about *either* of the two dominant components of the universe's present energy density.

The very existence of dark energy presents serious puzzles. The simplest account of the cosmic acceleration seems to be a nonzero cosmological constant Λ. However the energy scale of the cosmological constant is problematic, requiring an energy density $\rho_\Lambda \approx v^4$ with $v \sim 10^{-3}$ eV. The long-standing cosmological constant problem [2] stems from the fact that vacuum fluctuations contribute to the cosmological constant, and simple estimates calculated by assuming a Planck-mass cutoff give Λ a value 120 orders of magnitude greater than is consistent with observations. This discrepancy is reduced somewhat in models where the natural cutoff is the supersymmetry breaking scale, but even in that case there is a very serious problem.

Many physicists are attracted by the idea that the cosmological constant problem could eventually be solved by some symmetry argument that would set Λ precisely to zero (although very interesting alternative views do exist [3]). The observed cosmic acceleration creates problems with that line of reasoning, since it appears to indicate that Λ actually is nonzero. One way out is to assume that Λ really is zero, and that something else is causing the cosmic acceleration. This ''something else'' is usually called ''dark energy'' or ''quintessence'' [4,5].

For quintessence, typically one proposes the existence of a scalar field whose mass is small enough for it to be evolving in a cosmologically interesting way at present. So far, essentially all quintessence models present the following challenges or puzzles [6]. Problem 1: Why do radiative corrections not destabilize the fantastic hierarchy between v, the scalar masses which are required ($m \sim v^2/M_p \sim 10^{-33}$ eV) and the other known scales of physics? (Only the first quintessence model attempts to address this problem [4].) Problem 2: Why is the very long-range force mediated by the new light scalar not detected in precision tests of gravity within the solar system?

We here report on a class of quintessence models that is motivated by recent efforts to understand the hierarchy problem, in which all observed particles except gravitons (and their supersymmetric partners) are trapped on a $(3+1)$-dimensional surface, or brane, in a higher-dimensional ''bulk'' space [7,8]. It turns out that within this framework the above two puzzles are resolved in a natural way. We believe ours is the only quintessence model to successfully address *both* these particular challenges.[1]

A great virtue of the class of models which we explore is that they generally arise in the low-energy limit of theories which were originally proposed to solve the gauge hierarchy problem, without any cosmological applications in mind. As we shall see, the generic low-energy limit of these models looks like scalar-tensor gravity [10], with the four-dimensional scalars of relevance to cosmology coming from some of the extra-dimensional modes of the six-dimensional metric. We show here that although these models were not proposed with cosmology in mind, the couplings they predict for these scalars have the following properties.

[1]Reference [9] discusses another type of quantum correction which corrects classical rolling of the quintessence field off a local maximum. Those calculations do not apply to the type of potentials considered here.

(1) *Good news*. They can predict a late-time cosmology that provides a quintessence-style description of the dark energy. And they do so with scalars whose masses are extremely small, yet technically natural.

(2) *Good news*. They can evade the constraints on the existence of very long-ranged forces because the couplings of ordinary matter to the the very light scalars are predicted to be *field dependent*, and so evolve over cosmological time scales. As has been observed elsewhere [11], once free to evolve, such couplings often like to evolve toward zero, implying acceptably small deviations from general relativity within the solar system at the present epoch.

(3) *Good news*. The expectation values of the scalar fields whose cosmology we explore determine the value of Newton's constant, and the success of big-bang nucleosynthesis (BBN) implies strong limits on how much this can have changed between then and now. We find the evolution of Newton's constant changes very little over cosmologically long times, for a reasonably wide range of initial conditions when we enter the BBN epoch: Newton's constant tends not to change once the motion of the scalar field is either kinetic or potential dominated.

(4) *Bad news*. The previous two cosmological successes are *not* completely generic, in the following sense. First, they depend on dimensionless couplings taking values which are of order $\epsilon \sim 1/50$ or so. (Although couplings of this order of magnitude naturally do arise in the models of interest, we require some modest coincidences in their values to ensure sufficiently small evolution in Newton's constant.) Second, although many initial conditions do produce an acceptable cosmology, this success is not completely generic. Attractor solutions exist that would cause too much variation in Newton's constant if they described the cosmology between BBN and today. Furthermore, obtaining the present-day value of Newton's constant requires adjusting the features of the scalar potential.

We present our results here as an example of a viable cosmology, based on using the brane-world's extra-dimensional ''radion'' mode as a quintessence field. Our ability to do so may itself be considered something of a surprise, since extra-dimensional aficionados had earlier examined the radion as a quintessence candidate, and found it wanting. Our conclusion here differs, because of our recognition that the radion couplings tend to be field dependent, in a way we make precise below.

We regard this work as a first exploration of the conditions that a successful cosmology must assume. We believe our results to be sufficiently promising to justify further exploration of this class of models, to see to what extent the unattractive features can be improved upon, and to ascertain what observations can test whether the universe indeed proceeds as we describe here.

Our presentation is organized as follows. In the next section we briefly describe the brane models we use, and their motivations and problems. The low-energy action relevant for cosmology is also presented in this section. Section III describes some of the cosmological solutions of the theory, focusing in particular on analytic expressions on which intuition can be based. More complicated, and realistic, solutions are then described numerically. We close in Sec. IV with a discussion of our results.

II. THE MODEL

Our point of departure is the brane-world scenario in which all observed particles except the graviton are trapped on a $(3+1)$-dimensional surface, or brane, in a higher-dimensional ''bulk'' space. In particular, we imagine the scale of physics on the brane is $M_b \sim 1$ TeV and the bulk space is approximately six dimensional, comprising the usual four, plus two which are compact but with radius as large as $1/r \sim 10^{-3}$ eV [7,8]. The large ratio $M_b r \sim 10^{15}$ is phenomenologically required to reproduce an acceptably weak gravitational coupling in four dimensions, $M_p = (8\pi G)^{-1/2} \sim M_b^2 r \sim 10^{18}$ GeV.

These kinds of models face two main difficulties, one theoretical and one phenomenological. Theoretically, these models trade the large hierarchy $M_b/M_p = 10^{-15}$ for the hierarchy $M_b r = 10^{15}$, but an explanation of this last value ultimately must be provided. Recently progress toward providing this explanation has been made, by proposing energetic reasons why the extra-dimensional radii might prefer to take large values [12,13]. Our quintessence model does not rely on the details of these mechanisms, but our choice for the scalar field couplings is strongly motivated by the mechanism of Ref. [13].

Phenomenologically, there are strong upper limits on how large $M_b r$ can be. These limits come from supernova [14,15] and cosmological observations [16] which preclude the existence of Kaluza-Klein (KK) partners of the bulk-space graviton and its superpartners. We believe the present-day observational constraints on KK modes can be accommodated if only two dimensions have r as large as $(10^{-3}$ eV$)^{-1}$, but prevent r from being taken any larger. Although the cosmological constraints generally impose stronger upper limits on r they depend more on the details of the assumed cosmology, and can be evaded under certain conditions (which we describe in more detail later).

A. The low-energy action

We start by sketching the brane world scenario, and its low energy action. Our discussion closely follows that of Ref. [13].

The couplings of the brane-world model relevant to cosmology are those of the six-dimensional metric, $\mathcal{G}_{MN}$. (Six dimensions are required in order to allow the extra dimensions to have radii as large as we need.) The leading terms in the derivative expansion for the six-dimensional action have the form $S = S_B + S_b$, where the bulk action describing the metric degrees of freedom is

$$S_B = -\frac{M_b^4}{2} \int d^4x d^2y \sqrt{\mathcal{G}}\, \mathcal{R} + \cdots, \tag{1}$$

where $\mathcal{R}$ denotes the scalar curvature built from the six-dimensional metric, and we write no cosmological constant for the bulk.[2]

The dependence of the brane action on its various matter fields is not crucial for our purposes, but what is important is that it depends only on the metric evaluated at the brane's position:

$$S_b = \int d^4x \sqrt{g} \mathcal{L}_b(g_{\mu\nu}, \ldots), \tag{2}$$

where the ellipses denote all of the other fields on which $\mathcal{L}_b$ could depend.

The tree-level dimensional reduction of this action using the metric

$$\mathcal{G}_{MN} = \begin{pmatrix} \hat{g}_{\mu\nu}(x) & 0 \\ 0 & \rho^2(x)\, h_{mn}(y) \end{pmatrix}, \tag{3}$$

with $\rho = M_b\, r$, gives the leading contribution to the effective four-dimensional Lagrangian:

$$\mathcal{L}_{\rm kin} = -\frac{M_b^4 r^2}{2} \sqrt{\hat{g}} \left[R(\hat{g}) - 2\left(\frac{\partial r}{r}\right)^2 + \frac{R(h)}{M_b^2 r^2} \right], \tag{4}$$

where we have adopted the conventional normalization $M_b^2 \int d^2y\, \sqrt{h} = 1$. Here $R(\hat{g})$ and $R(h)$ denote the curvature scalars computed from the metrics $\hat{g}_{\mu\nu}$ and h_{mn}, respectively.

Quantum corrections modify this action in several important ways. First, the kinetic terms can become renormalized, introducing the possibility that their coefficients can acquire a dependence on r. Second, terms not written in Eq. (4) can be generated, including in particular a no-derivative potential, $U(r)$. At large r, $U(r)$ can typically be expanded in powers of $1/r$, with the leading term falling off as $1/r^p$, for some $p \geqslant 0$. For instance, for toroidal compactifications, the leading contribution can arise due to the Casimir energy, which produces the potential $U(r) = U_0/r^4 + \cdots$.

We are in this way led to consider a scalar-tensor theory [10,11], for the couplings of r and the four-dimensional metric, which we write as follows:

$$\frac{\mathcal{L}}{\sqrt{-\hat{g}}} = -\frac{M_b^4 r^2}{2} \left[A(r) R(\hat{g}) - 2B(r) \left(\frac{\partial r}{r}\right)^2 \right] - U(r) - \frac{\mathcal{L}_m(\hat{g})}{\sqrt{-\hat{g}}}. \tag{5}$$

As just described, the tree-level dimensional reduction of the 6D Einstein-Hilbert action predicts $A = B = 1$ and $U = 0$, and with these choices Eq. (5) describes a Jordan-Brans-Dicke theory with coupling parameter $\omega = -1/2$ [18]. Notice that the matter Lagrangian, $\mathcal{L}_m$, describes the couplings of all of the known particles, and depends only on $\hat{g}_{\mu\nu}$ but not on r. (Ordinary matter does not couple directly to r in the brane-world picture because it is trapped on the brane, and does not carry stress energy in the extra dimensions.)

Notice that if the scale of $U(r)$ is completely set by r, as is true for the cases discussed in [13], then the mass which is predicted for this scalar is incredibly small: of order $m \sim v^2/M_p \sim 10^{-33}$ eV. It is the existence of such a small mass that allows r to potentially play a cosmologically interesting role right up to the present epoch, and so to be a candidate for a quintessence field.

Since the predictions for A, B and U depend crucially on the approximation made in their derivation, in general we expect them to be given by more complicated functions. This turns out to be true, in particular, within the hierarchy-generating mechanism described in Ref. [13]. Since we use this proposal to motivate our choices for these functions, we briefly pause here to summarize its implications.

The proposal supposes that the spectrum of bulk states includes a six-dimensional scalar, ϕ, which counts amongst its interactions a renormalizable cubic self-coupling, $g\phi^3$. Since the coupling g of this interaction is dimensionless, it can introduce corrections to the low-energy action that are logarithmic in r. Since all other couplings have negative mass dimension in 6D, their quantum contributions are suppressed by powers of $1/r$, implying the coupling g can dominate at large r.

The bottom line of this scenario is that the dominant corrections to A, B and U are of the form $1 + c_1\, \alpha(r) + c_2\, \alpha^2(r) + \cdots$, where the running loop-counting parameter is given by $\alpha(r) = \alpha_0 + a\, \alpha_0^2 \log(r/r_0) + \cdots$, with $\alpha_0 = g_0^2/(4\pi)^3$. In this scenario the potential $U(r)$ is plausibly minimized for $\alpha(r) \sim 1$, implying a minimum for which there is the desired large hierarchy $M_b r \sim \exp[+1/\alpha_0] \gg 1$. It is generic in this scenario that the quantum corrections to the other low-energy functions, A and B, are also likely to be large.

For illustrative purposes when exploring cosmology, we choose the functions A, B and U which are suggested by this picture in the perturbative regime, for which $A \approx 1 + a \log(M_b r)$, $B \approx 1 + b \log(M_b r)$ and $U \approx (U_0/r^4)[1 + c \log(M_b r)]$. Here a, b and c are small, since they are proportional to the dimensionless coupling constant, $\epsilon \sim g^2/(4\pi)^3$, and higher orders in ϵ give higher powers of $\log(M_b r)$ although these have not been written explicitly. In the spirit of this picture we suppose ϵ to be of order 1/50, as required in order to ensure that U is minimized when $M_b r \sim 10^{15}$. For cosmological purposes, we shall be interested in the regime for which $\epsilon \log(M_b r) \lesssim 1$ is negligible, but $\epsilon \log^2(M_b r)$ is not. We take U_0 to be generic in size: $U_0 \sim O(M_b^4)$.

B. Naturalness

Since naturalness is one of our motivations, we briefly summarize the arguments presented in Ref. [13] as to why

[2]This can be ensured by symmetries, such as in a supersymmetric compactification [17]. Like everyone else we assume the vanishing of the effective four-dimensional cosmological constant, and our ideas do not add to the understanding of why this should be so.

radiative corrections do not destabilize the scalar mass. The argument takes a different form for the contribution due to the integrating out of scales above, or below, $v \sim 1/r \sim 10^{-3}$ eV.

For scales below v, the effective theory consists of a four-dimensional scalar-tensor theory coupled to itself, and to ordinary matter with couplings, κ, of gravitational strength $\kappa \sim 1/M_p$. Radiative corrections to the scalar mass within this effective theory are then generically acceptably small, being of order $\delta m^2 \sim \kappa^2 \Lambda^4$, with ultraviolet cutoff $\Lambda \sim v$. These are therefore the same size as the tree-level mass, as is required if they are to be technically natural.

Radiative corrections to the masses coming from higher scales, between v and M_b, are the dangerous ones. Generically, these are too large and it is the integrating out of these scales that introduces the naturalness problem to most quintessence models. Within the framework of Ref. [13], the contributions from these scales are suppressed by 6D general covariance and by supersymmetry, the latter of which is proposed to be broken at the scale M_b on the brane on which observable particles exist. (This ensures acceptably large splittings between the masses of ordinary particles and their superpartners.)

These symmetries suppress corrections to the radion mass because they relate r to the four dimensional graviton, which is massless. Indeed, above the scale v the bulk space is really six dimensional, and the radion is simply a component of the six-dimensional metric. Since the metric and its superpartners only couple to the supersymmetry breaking with gravitational strength, supersymmetry-breaking effects in the bulk sector are suppressed to be of order $M_b^2/M_p \sim 1/r \sim v$.

Since the scalar potential for the radion cannot arise until these symmetries break, it is naturally of order $v^4 F(\varphi)$ where φ is the dimensionless component of the six-dimensional metric which defines the radion. φ's kinetic term, on the other hand, comes from the 6D Einstein-Hilbert action (which is allowed by unbroken supersymmetry). At tree level it is of order $M_p^2(\partial\varphi)^2$, and the coefficient M_p^2 is much larger than the contributions produced by integrating out physics at scales smaller than $M_b \ll M_p$. Again we are led on dimensional grounds to a mass that is of order $m \sim v^2/M_p$.

C. Cosmological equations

The cosmology of the radion is most easily described by passing to the Einstein frame, for which the radion and metric are canonically normalized. For the choices of $A = 1 + a \log(M_b r)$ and $B = 1 + b \log(M_b r)$ the canonically normalized variables are

$$\hat{g}_{\mu\nu} = g_{\mu\nu}/[(M_b r)^2 A^2] \quad \text{and} \quad \log(M_b r) = \hat{a}\hat{\chi}(1 - \hat{b}\hat{\chi}), \tag{6}$$

with $\hat{a} = (1 - 3a/4)/2$, $\hat{b} = (a - b)/16$ and $\hat{\chi} = \chi/M_b$. This choice of variables is called the Einstein frame, to distinguish it from the Jordan frame, as defined by Eq. (5).[3]

In the Einstein frame the action, Eq. (5), becomes

$$\frac{\mathcal{L}}{\sqrt{g}} = -\frac{M_b^2}{2} R - \frac{1}{2}(\partial\chi)^2 - V(\chi) - \frac{\mathcal{L}_m(g,\chi)}{\sqrt{g}}, \tag{7}$$

where

$$\begin{aligned} V(\chi) &= \frac{U[r(\chi)]}{(M_b r)^4 A^2} \\ &= V_0[1 + (v - 2a)\hat{a}\hat{\chi} + \cdots] \\ &\quad \times \exp[-\lambda_0 \hat{\chi}(1 - \hat{b}\hat{\chi})], \end{aligned} \tag{8}$$

and $\lambda_0 = 8\hat{a}$. [Even though $\hat{b} = O(\epsilon)$ we do not expand the second term in the exponent in this expression because such an expansion presupposes $\epsilon\hat{\chi}^2 \ll 1$, which is not true for our applications.]

For cosmological applications we take $ds^2 = -dt^2 + a^2 d\vec{x}^2$ in the Einstein frame, in which case the Lagrangian (7) implies the usual Friedmann equations:

$$3H^2 M_b^2 = \rho, \quad 2\dot{H} M_b^2 = -(\rho + p), \tag{9}$$

where both the energy density, ρ, and pressure, p, receive contributions from χ and the radiation and matter sectors $\rho = \rho_r + \rho_m + \rho_\chi$ and $p = p_r + p_m + p_\chi$, where $p_r \approx \frac{1}{3}\rho_r$ and $p_m \approx 0$.

In the Einstein frame the matter contributions also depend explicitly on χ:

$$\rho_m = \frac{\rho_{m0}}{a^3 (M_b r)}, \quad \rho_r = \frac{\rho_{r0}}{a^4}, \tag{10}$$

where the quantities ρ_{m0} and ρ_{r0}, may be related to the density of matter and radiation at the present epoch (which we choose to be $a = 1$).

Equations (9) must be supplemented with the evolution equation for χ, which is

$$\ddot{\chi} + 3H\dot{\chi} + V'(\chi) + \frac{\eta(\chi)}{M_b} T^\mu{}_\mu = 0, \tag{11}$$

where $T_{\mu\nu}$ is the stress energy of the radiation and matter, and $\eta(\chi) = -\hat{a}(1 + a/2 - 2\hat{b}\hat{\chi})$. The successful comparison of general relativity with precision tests of gravity within the solar system requires $\eta^2(\chi_0) \lesssim 10^{-3}$ when evaluated at the present epoch, $\chi = \chi_0$ [11,19]. Notice that since radiation sat-

[3] A note on numerics: In the Jordan frame $M_p \sim M_b^2 r$, ordinary particle masses are $m_p \sim M_b$ and Kaluza-Klein masses are $m_{\rm KK} \sim 1/r$, while in the Einstein frame $M_p \sim M_b$, $m_p \sim 1/r$ and $m_{\rm KK} \sim 1/(M_b r^2)$. Clearly, ratios of masses like $m_{\rm KK}/m_p = 1/(M_b r)$ are the same in either frame, but $M_b = 1$ corresponds to using TeV units in the Jordan frame, but Planck units in the Einstein frame.

isfies $T^{\mu}{}_{\mu}=0$, only matter contributes to this last coupling, which is proportional to $T^{\mu}{}_{\mu}=\rho_m$.

III. COSMOLOGY

We now show that cosmologically reasonable solutions may be found to Eqs. (9) and (11), which satisfy all of the bounds to which the theory must be subject. Since our goal is to demonstrate the existence and illustrate some of the features of these solutions, our analysis is not systematic and should be regarded as only a preliminary exploration of models of this type.

A. Constraints

Our main assumption is that the universe enters into an effectively four-dimensional radiation-dominated evolution at some temperature T_* higher than a few MeV, and so before big bang nucleosynthesis. This assumption is a prerequisite for describing cosmology in terms of four-dimensional fields, and seems likely to be required if the success of standard BBN is not to be forsaken.

There is a limit to how high T_* may be chosen, since the four-dimensional picture requires the extra-dimensional KK modes to be cold. If T_* is too high, then these KK modes can be thermally excited too efficiently through their couplings with ordinary particles. If these modes are too abundant they can overclose the universe or decay too frequently into photons after recombination, and so contribute unacceptably to the diffuse gamma ray background [7,16]. If the KK modes all decay into observable daughters, then these constraints require T_* not to be larger than the TeV scale, and can preclude having $1/r \sim 10^{-3}$ eV.

As the authors of Refs. [7,16] themselves point out, these constraints may be evaded, depending on the cosmology that is assumed at earlier times in the universe's history. For instance, contributions to the diffuse gamma ray background are suppressed if the KK modes decay quickly enough into unobserved modes to be no longer present after the recombination epoch, when photons decouple from matter. Such fast decays can be arranged, for instance, if the KK modes decay more frequently into particles on other branes. Such decays may, in fact, be likely in cosmological scenarios for which the very early universe is dominated by a gas of branes and antibranes [20]. (Another escape is possible if the extra-dimensional space is hyperbolic instead of toroidal, with its volume much greater than the appropriate power of its radius of curvature: $V \gg r_c^2$ [21]. In this case KK modes can be very heavy, $m_{\rm KK} \sim 1/r_c$, even though the hierarchy problem is solved by having V large.) Although our present interest is for $T_* < 1$ TeV, we have no difficulty imagining some variation on these themes being invoked if scenarios requiring $T_* > 1$ TeV were of interest.

Our second major constraint is the requirement that the quintessence field, χ, not destroy the successes of BBN. This constraint arises because the success of BBN may be regarded as indicating that the hierarchy $M_w/M_p \propto (G_N/G_F)^{-1/2}$ is not much different at the epoch of nucleosynthesis than it is today. A sufficient condition for this to be true is to ensure that $M_b r$ has not evolved by more than about 10% of its present value between BBN and the present epoch. This constraint is much stronger than what is required by garden-variety quintessence models. It is very strong because it requires that the field r cannot have rolled significantly over cosmologically long times. A special property of the cosmological solutions we shall examine is that they often have the property that r becomes fixed over cosmologically large time scales. Motivated by the BBN constraints we are particularly interested in those cosmological solutions for which r does not strongly vary after BBN. We shall see that such solutions can arise for reasonably generic initial conditions for the functions $V(\chi)$ and $\eta(\chi)$ we are using.

Next, we must demand agreement with present-day observations, which come in several important types. First, precision measurements of gravitational forces indicate that the coupling $\eta(\chi)$ must be $\lesssim 0.03$ in order not to be in conflict with the success of general relativity. Second, if χ is to provide an explanation of the dark energy, then its energy density must now be beginning to dominate that of matter, with $w = p_\chi/\rho_\chi < -\frac{1}{3}$ if this energy density is to be currently accelerating the universe.

Finally, we require χ not to ruin the success of scale-invariant fluctuations in describing present-day observations of the cosmic microwave background radiation (CMB). Since a detailed analysis of the CMB in these models is beyond the scope of the present paper, we instead apply a poor-man's bound by requiring cosmological properties not to be too different from the standard picture near the epochs of radiation/matter crossover and recombination.

B. Approximate cosmological solutions

Before laying out a cosmology that satisfies the above constraints, it is worth building intuition by outlining several approximate analytical solutions to the cosmological equations which turn out to control the features that are seen in the numerical results. Since the microphysics motivates using the potential $V(\chi) = V_p e^{-\lambda\chi}$, with $V_p(\chi)$, $\lambda(\chi)$ and the coupling $\eta(\chi)$ varying much more slowly as (polynomial) functions of χ than does the exponential $e^{\lambda\chi}$, we explore solutions to the equations when λ, V_p and η are all constants. The cosmological evolution of such fields in the limit $\eta \to 0$ has been much studied [22], and our results go over to these in the appropriate limits.

There are two types of solutions in this limit which are of principal interest. These solutions may be classified according to which terms dominate in the χ equation, Eq. (11).

1. Scaling solutions

The first class of approximate solutions we consider are the scaling, or tracker, solutions, of which there are two different types. These solutions are characterized by having all energy densities scaling as a power of the scale factor, and either (i) $\ddot{\chi} \sim 3H\dot{\chi} \sim V' \gg \eta\rho_m$ or (ii) $\ddot{\chi} \sim 3H\dot{\chi} \sim \eta\rho_m \gg V'$. Both types of solutions can arise regardless of whether it is the scalar, radiation or matter which is dominating the evolution of the universe.

Assume, therefore, that the scale factor scales with time as $a/a_0=(t/t_0)^k$, and so $H=\dot{a}/a=k/t$. From the Friedmann equation, $H^2\propto\rho$, we see that if the dominant energy density scales as $\rho\propto a^{-n}$, then we must have $k=2/n$.

(i) $\ddot{\chi}\sim 3H\dot{\chi}\sim V'\gg\eta\rho_m$. In this regime we assume

$$V\propto e^{-\lambda\hat{\chi}}\propto\frac{1}{r^8}\propto\frac{1}{a^s}, \tag{12}$$

and we find from the χ field equation, Eq. (11), that $s=n$.

If radiation or matter dominate the energy density, then $n=3$ or $n=4$. In this case we find solutions for which $K=\frac{1}{2}\dot{\chi}^2\propto V(\chi)\propto 1/a^n$, and so $r\propto a^{n/8}$. Equation (11) also implies that these scaling solutions are trackers in the sense that ρ_χ tracks at precisely the fraction n/λ^2 of the dominant energy density as the universe expands. Solutions of this type are possible only if $\lambda^2>n$. For $\lambda^2<n$ the scalar energy density tends toward scalar domination of the universal expansion.

If it is χ itself that dominates the energy density, then $k=2/\lambda^2$ and a scaling solution of the type we seek exists so long as $\lambda^2<6$. In this case both K and V scale as $1/a^{\lambda^2}$, and so $r\propto a^{\lambda^2/8}$.

These are the tracker solutions which are familiar from standard discussions of scalars evolving with exponential potentials [22,23]. Clearly these tracker solutions imply a relatively rapid variation of r as the universe evolves.

(ii) $\ddot{\chi}\sim 3H\dot{\chi}\sim\eta\rho_m\gg V'$. We next consider the case where the $\eta T^\mu{}_\mu$ term dominates the scalar potential term in Eq. (11). Notice that this can hold even if both ρ_r and ρ_χ are much larger than ρ_m, so the tracker solution we obtain here can be relevant even when nonrelativistic matter makes up only a very small part of the universe's total energy budget.

Under these assumptions, because the last term of Eq. (11) dominates the potential term, we ask ρ_m to scale as $\ddot{\chi}$ and $H\dot{\chi}$,

$$\rho_m\propto\frac{1}{a^3 r}\propto\frac{1}{a^s}, \tag{13}$$

which, when inserted into Eq. (11) implies $s=n$ and so $K\propto\rho_m\propto\rho\propto 1/a^n$. As usual, if the universe is radiation or matter dominated then $n=4$ or $n=3$. (Interestingly, both matter *and* radiation fall as $1/a^4$ within this kind of tracker solution in a radiation-dominated universe.) Alternatively, $K\propto\rho_m\propto\rho\propto 1/a^{\lambda^2}$, if the universe is χ dominated.

Although the scaling of scalar kinetic energy is the same for tracker solutions of type (i) and type (ii), they differ markedly in the scaling that they imply for r. In the present solution the scaling of r vs a follows from the known scaling of ρ_m. For radiation/matter domination, where $\rho\propto 1/a^n$ we find $ra^3\propto a^n$ and so $r\propto a^{n-3}$. Clearly $r\propto a$ for radiation domination, but r only varies logarithmically during matter domination. During a scalar-dominated phase, on the other hand, we have $r\propto a^{\lambda^2-3}$. Again, r does not vary if $\lambda^2=3$.

What is striking about these solutions where r changes so slowly is that this happens in spite of its not being at the minimum of the potential. It happens, even though the radion's kinetic energy dominates its potential energy, because the amount of roll is limited by the Hubble damping generated by the dominant energy component of the universe.

2. Transient solutions

What is perhaps even more striking than the trackers for which r slowly varies is the behavior of r during the transient evolution as the tracker—which is an attractor—is approached. Since the tracker solution may be approached from initial conditions for which $K\gg V$ or $K\ll V$, we consider each of these in turn.

$K\ll V$ (slow roll). If $K\ll V$ then it is a good approximation to neglect $\ddot{\chi}$ in Eq. (11) relative to the other terms. If $\eta\rho_m$ is also negligible relative to V', then the evolution is described by $3H\dot{\chi}+V'=0$, which has as solutions

$$V(a)=\frac{2V_{\rm exit}a_{\rm exit}^p}{a^p+a_{\rm exit}^p}, \tag{14}$$

where $p=n$ for radiation ($n=4$) and matter ($n=3$) domination, and $p=\lambda^2$ for scalar domination. Along this trajectory the kinetic energy grows as $K/V=\lambda^2 V/(6\rho)$.

This solution implies $V\approx V_{\rm exit}$ is a constant (and so also must be r) until $a\sim a_{\rm exit}$, after which point the solution joins the tracker solution, for which $\rho_\chi\propto 1/a^p$ and $\rho_\chi/\rho=p/\lambda^2$.

$K\gg V$ (kinetic domination). If K dominates the scalar evolution, then V' and $\eta\rho_m$ may be dropped relative to $\ddot{\chi}$ and $3H\dot{\chi}$. In this case the χ equation may again be solved analytically, with solution $\chi=C_1-C_2(a_0/a)^{3-1/p}$, where C_1, C_2 are integration constants with C_2 related to the initial velocity by $C_2=(3-1/p)a_0(d\chi/da)_0$. Here, as usual, $p=2/n$ for radiation/matter domination, and $p=2/\lambda^2$ for scalar domination.

This solution implies that the kinetic energy, $K=\frac{1}{2}\dot{\chi}^2$ varies as

$$K(a)=K_0\left(\frac{a_0}{a}\right)^6, \tag{15}$$

regardless of the value of p.

The evolution of the potential energy is more interesting in these solutions. Consider first the case of radiation/matter domination, for which $p=2/n$ so

$$V(a)=V_0\exp\left\{c\left[\left(\frac{a_0}{a}\right)^{3-n/2}-1\right]\right\}, \tag{16}$$

with $c=(3-1/p)\lambda a_0(d\chi/da)_0$.

As the universe expands according to this solution, χ carries an ever-smaller fraction of the energy density since $K(a)\propto 1/a^6$ falls faster than does the dominant matter or radiation, $\rho\propto 1/a^n$. Eventually K becomes comparable in size to $V(a)$, which for large a asymptotes to a constant value $V(a)\to V_\infty=V_0e^{-c}$. At this point the approximation of dropping the scalar potential in Eq. (11) breaks down, and we match onto the slow-roll solution discussed above. The

amount of expansion that occurs before V and K are comparable is determined by $a/a_0=(e^c K_0/V_0)^{1/6}$.

What is remarkable about this solution is that the constant c cannot be made arbitrarily large. To see this, notice that c can be related to the initial kinetic energy by using $K=\frac{1}{2}H^2a^2(d\chi/da)^2$. Using the upper limit $K\leqslant 3H^2$, which is dictated by the inequality $K\leqslant\rho$ and the Friedmann equation, we see that $c\leqslant\sqrt{6}\lambda(3-1/p)$. For $\lambda=4$ and $p=1/2$ (radiation domination) we have $c\leqslant 4\sqrt{6}\sim 9.6$.

This upper limit on c implies that V cannot fall by more than a factor of $e^{-c}\leqslant e^{-\lambda\sqrt{6}}$ during a radiation dominated epoch, despite the fact that r's evolution is being dominated by its *kinetic* energy. Since $V\propto 1/r^8$ we see that $r/r_0<e^{\lambda\sqrt{6}/8}=e^{0.3\lambda}$ varies hardly at all during the entire kinetic roll. Physically the paradoxical upper limit on r during a kinetic dominated roll arises due to the Hubble damping, which increases with the kinetic energy, if the kinetic energy gets large. This observation has also been made in another context in Ref. [24].

A similar result holds for the kinetic roll in the scalar-dominated phase, although with results which differ strongly depending on the value of λ. If $\lambda^2<6$, the above analysis applies directly, with $p=2/\lambda^2$. As before $K\propto 1/a^6$ and

$$V(a)=V_0\exp\left\{c\left[\left(\frac{a_0}{a}\right)^{3-\lambda^2/2}-1\right]\right\}. \tag{17}$$

Provided $\lambda^2<6$, $V(a)$ again asymptotes to V_0e^{-c}, with c once more subject to a very general upper bound. Once K falls to be of order V, this solution matches onto a slow roll solution, and approaches the tracker solution, which exists for $\lambda^2<6$.

If $\lambda^2>6$, however, the large-a limit of $V(a)$ is very different. In this case $V(a)\propto\exp[-|c|(a/a_0)^{(\lambda^2-6)/2}]$, implying that $V(a)$ falls very quickly to zero as the roll proceeds. This solution describes the attractor for the case $\lambda^2>6$, which is a kinetic-dominated roll rather than a tracker solution.

3. Realistic complications

Although these solutions describe well most features of the numerical evolution we describe below, there are also important deviations. These deviations arise because η and the quantities λ and V_p (of the potential $V=V_p e^{-\lambda\chi}$) used in the numerical evolution are all polynomials in χ. Motivated by the microphysical discussion given in Sec. II we choose

$$\eta=\eta_0+\eta_1\hat{\chi},\quad \lambda(\chi)=\lambda_0+\lambda_1\hat{\chi}, \tag{18}$$

$$\text{and}\quad V_p=V_0+V_1\hat{\chi}+\frac{V_2}{2}\hat{\chi}^2, \tag{19}$$

with η_0, λ_0 and V_0 all taken to be $O(1)$, η_1, λ_1 and V_1 assumed to be $O(\epsilon)$ and V_2 taken $O(\epsilon^2)$, where $\epsilon\sim 1/50$. (The second-order quantity V_2 is included here in order to ensure that V does not change sign for any value of χ.)

Because these quantities vary more slowly with χ than does the exponential in V, their evolution over cosmological time scales is also slow, and the above approximate solutions provide a good description for most of the universe's history. We have found that their χ dependence can nevertheless play an important role when constructing viable cosmologies, by allowing important deviations from the approximate solutions, including the following:

The χ dependence of V_p becomes important for those χ satisfying $V_p'/V_p\approx\lambda$, since the potential V can acquire stationary points near such points. These kinds of stationary points can slow or trap the cosmological χ roll, in much the same way as was exploited in Ref. [25].

The χ dependence of λ can cause transitions between the domains of attraction of the various attractor solutions. For instance, in a scalar dominated era, the evolution of χ can drive λ across the dividing point, $\lambda^2=6$, forcing a transition between the scaling and kinetic-dominated attractor solutions.

The χ dependence of η is crucial for evading the bounds on long-range forces during the present epoch, because it allows η to vanish for some choices of χ. The bounds can be evaded provided the current value of χ is sufficiently close to such a zero of η. Furthermore, as described in detail in Ref. [11], in many circumstances χ's cosmological evolution is naturally attracted to the zeros of η as the universe expands.

Because of the natural hierarchy satisfied by the coefficients of these functions, the consequences of χ dependence generically occur for $\chi\sim O(1/\epsilon)$, corresponding to r close to its present value.

C. A realistic cosmological evolution

We next describe a realistic cosmology that exploits some of these properties. We present this model as an existence proof that cosmologically interesting evolution can really be found using the scalar properties which are suggested by the microphysics. We find that the main constraint which governs the construction of such solutions is the requirement that r not vary appreciably between BBN and the present epoch.

In order to fix ideas, we imagine starting the universe off at $T_*\gtrsim 1$ MeV, perhaps after an epoch of earlier inflation, in a kinetic-dominated roll. We note that such a state is the generic end point if we start with $\lambda\sim 4>\sqrt{6}$, as is motivated by the tree-level expression for the radion potential, and assume the scalar dominates the energy density of the universe. (As we will discuss briefly later, one might chose to make other assumptions about the initial conditions.)

With this assumption the scalar energy density must eventually fall below the density of radiation, thus initiating the radiation-dominated era. We assume that this radiation domination begins shortly before BBN, so that nucleosynthesis occurs during radiation domination.

Our second assumption is dictated by the requirement that r does not change appreciably between BBN and now. Although the kinetic-dominated roll before radiation domination will automatically ensure that $V\ll K$, we ask that V be within an order of magnitude of its present value as radiation domination begins. This ensures that r will have an initial value at the onset of radiation domination which is very close to its present-day value. We make this (fairly ugly)

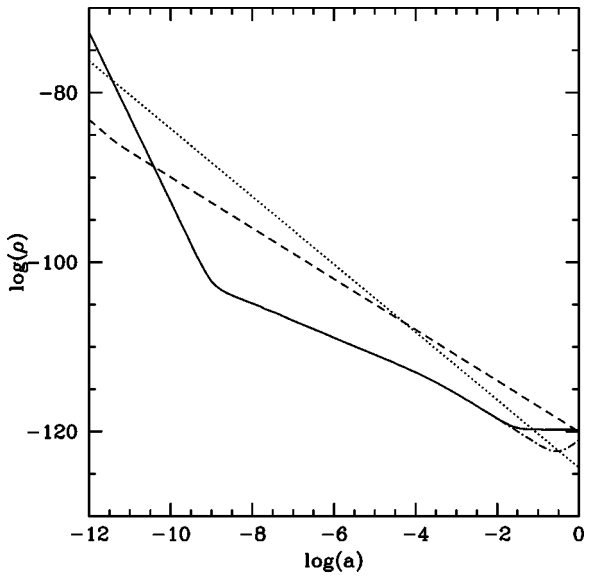

FIG. 1. The logarithm (base 10) of the energy density (in Planck units) of various components of the universe, plotted against the logarithm (base 10) of the Einstein frame scale factor, a (with $a=1$ at present). The various curves show the energy density of matter (dashed), radiation (dotted) and scalar energy (solid). The dot-dashed curve shows the scalar *kinetic* energy density, which coincides with the total scalar density except at late times.

assumption to demonstrate the feasibility of models along the lines we are presenting, but hope to be able to eventually identify a more natural framework in which such a condition could be ensured by the system's dynamics rather than as an initial condition.

For $\lambda \sim 4$, once the universe becomes radiation dominated the scalar is in the domain of validity of the tracker solutions

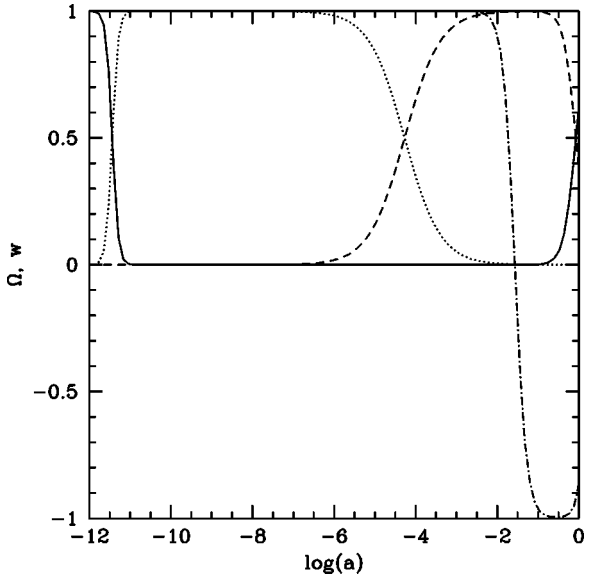

FIG. 2. The same plot as Fig. 1, but with the energy densities given as fractions of the critical density. The various curves show the energy density of matter Ω_m (dashed), radiation Ω_r (dotted) and total scalar energy Ω_χ (solid), while the dash-dotted curve gives the equation of state parameter, $w=p/\rho$, for the scalar field.

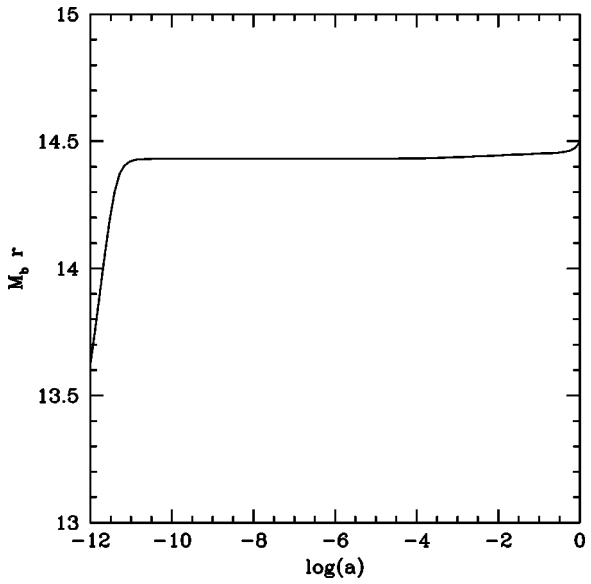

FIG. 3. The logarithm (base 10) of the dimensionless quantity $M_b r$, plotted against the logarithm (base 10) of the Einstein frame scale factor, a (with $a=1$ at present), showing how the radion has not evolved appreciably between nucleosynthesis and the present epoch.

described above. With the given initial conditions the scalar enters radiation domination in a kinetic-dominated roll, and so tries to approach a tracker solution by first damping its kinetic energy in a continued kinetic-dominated roll, and then performing a slow roll at fixed r. We have seen that the universe expands by an amount $(a/a_0)^2 \sim K_0/V_0$ during the transient phase, before the tracker solution is reached, so given the low values of V_0 assumed at the onset of radiation domination, K_0/V_0 is very large, and so the transient evolution to the tracker can easily take the entire time from BBN to the present epoch.

We have seen on general grounds that the value of r does not change by more than a factor of $O(1)$ during this transient phase, including both the transient's kinetic-dominated and slow-rolling parts. The challenge in making this scenario realistic is to have this transient evolution survive right up to the present day, without first being intercepted by a tracker solution.

We have been able to find initial conditions and plausible couplings for which this is accomplished. One such is illustrated in Figs. 1–4, which plot the evolution of various quantities from the onset of radiation domination to the present. The evolution described by this simulation uses the values suggested by the lowest-order microscopic action, $\eta_0=1$, $\lambda_0=4$ and $V_0=1$, plus the following correction terms: $\eta_1=-0.015$, $\lambda_1=0$, $V_1=-0.030$ and $V_2=0.00046$.

Figure 1 shows how the energy density in the universe is distributed between radiation, matter and the kinetic and potential energy of the scalar field, as a function of the universal scale factor, a, in the Einstein frame. Figure 2 plots the same information, but normalized as a fraction of the total universal energy density, and also plotting the scalar equation-of-state parameter, $w=p_\chi/\rho_\chi$. Figure 3 gives a plot of r vs the scale factor, showing that r does not vary

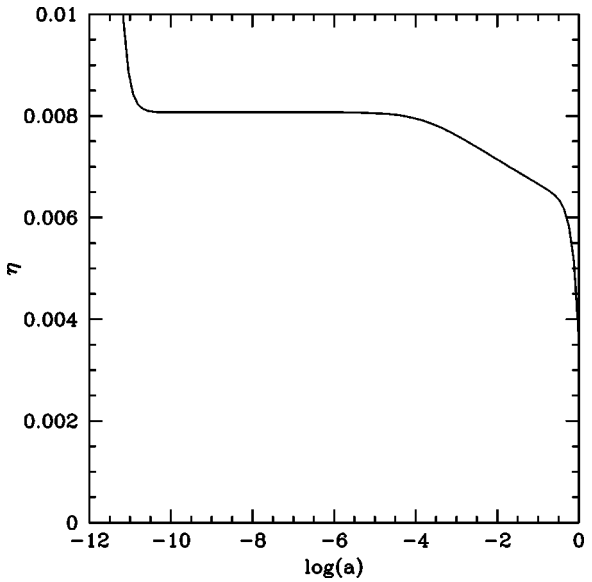

FIG. 4. The scalar coupling function, η, plotted against the logarithm (base 10) of the Einstein frame scale factor, a (with $a=1$ at present), showing how η is small enough to satisfy the present-day bounds on the existence of long-range scalar forces.

appreciably between BBN and the present epoch. Finally, Fig. 4 shows the evolution of η against the scale factor, showing that η evolves to sufficiently small values at the present epoch.

As is evident from the figures, this solution satisfies all of the cosmological and present-day bounds we listed earlier. This was accomplished by choosing appropriately the constants governing the scalar interactions, by ensuring V to have a minimum relatively near to a zero of η. This was required because although r naturally does not evolve significantly for most of the universe's history (after the onset of radiation domination), for pure exponential potentials it generically tends to join a tracker solution near the onset of matter domination. Because r evolves too far once in this tracker solution, it must be avoided. The solution illustrated in the figures does so by having χ become snagged by the minimum of V before entering into the matter-dominated phase.

IV. DISCUSSION

We have shown that viable cosmologies may be built using the radion as the quintessence field. We use radion interactions which were suggested by a microphysical model that was recently proposed to naturally solve the hierarchy problem within a brane-world framework. The radion in this model is naturally light enough to play a role in late-time cosmology, and this small mass is technically natural in that it is not destabilized by radiative corrections. The model also satisfies all current bounds that constrain the existence of long-range scalar-mediated forces [26].

The success of the radion in filling the quintessence role is somewhat of a surprise, since radion models have long been believed to have phenomenologically unacceptable couplings. The model evades these problems because the couplings are predicted to be weakly field dependent, allowing the scalar couplings to evolve over cosmological timescales. The model evades all present-day bounds on new forces because the relevant couplings evolve toward small values at late times.

Although most of the parameters and initial conditions chosen were natural in size, the model does have two features that we believe need further improvement. First, although all scalar couplings were chosen with natural sizes, the precise values chosen were adjusted to arrange the scalar potential to have a minimum close to a zero of η. We would prefer to find an attractor solution, like that of Damour and Nordtvedt [11], which more naturally draws η to small values in the present epoch.

A second unsatisfactory feature of the cosmology presented is its reliance on the entering of radiation domination with r close to its present value. Although large values of r are naturally generated by kinetic-dominated rolls during scalar dominated epochs, there is no natural reason why they should have precisely the current value. This problem might be resolved by a more developed understanding of radion initial conditions. Perhaps one could argue that there are no *a priori* constraints on the initial conditions for r. In that picture, it may be the case that the largest amount of phase space that matches the observation that Newton's constant is constant and nonzero today would be one where r has the current value throughout the cosmic evolution. (In that reasoning, the constancy of G is not a prediction, but an input.)

A natural way to alleviate both of these problems would be to consider more moduli than just the radion, since other moduli would be less constrained by the requirement that Newton's constant not appreciably change since nucleosynthesis. We believe work along these lines to be worthwhile, given the model's successful addressing of the naturalness issues of quintessence models, and the promising cosmology to which it leads despite its being devised to solve purely microphysical problems such as the hierarchy problem.

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