On the dark energy clustering properties

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We highlight a viable mechanism leading to the formation of dark energy structures on subhorizon cosmological scales, starting from linear perturbations in scalar-tensor cosmologies. We show that the coupling of the dark energy scalar field, or quintessence, to the Ricci scalar induces a ''dragging'' of its density perturbations through the general relativistic gravitational potentials. We discuss, in particular, how this process forces dark energy to behave as a pressureless component if the cosmic evolution is dominated by nonrelativistic matter. This property is also analyzed in terms of the effective sound speed of the dark energy, which correspondingly approaches the behavior of the dominant cosmological component, being effectively vanishing after matter-radiation equality. To illustrate this effect, we consider extended quintessence scenarios involving a quadratic coupling between the field and the Ricci scalar. We show that quintessence density perturbations reach non-linearity at scales and redshifts relevant for the structure formation process, respecting all the existing constraints on scalar-tensor theories of gravity. This study opens new perspectives on the standard picture of structure formation in dark energy cosmologies, since the quintessence field itself, if nonminimally coupled to gravity, may undergo clustering processes, eventually forming density perturbations exiting from the linear regime. A non-linear approach is then required to further investigate the evolution of these structures, and in particular their role in the dark halos surrounding galaxies and clusters.

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I. INTRODUCTION

The role of vacuum energy in cosmology is receiving renewed interest since at least three cosmological observables indicate that a relevant fraction of the energy density in the Universe is presently in the form of a sort of vacuum energy, which is commonly known as dark energy. Type Ia supernovae observations suggest that cosmic expansion has been accelerating [1,2]; recently it also has been noticed that data indicate that acceleration is a relatively recent occurrence in cosmological evolution [3]. Moreover, present data on cosmic microwave background (CMB) anisotropies favor a total energy density which is very close to the critical value and which is made at roughly 70% by a vacuum energy component [4]. Finally, large scale structure observations suggest a universe with a subcritical matter density [5].

This evidence represents a great stimulus for theoretical work, because understanding why the vacuum energy is at the level of or less than the critical density today represents one of the main mysteries in modern fundamental physics [6]. At the quantum level, a ''fine-tuning'' mechanism is needed to explain why the vacuum energy is so low with respect to any natural scale of vacuum expectations of fundamental fields: this discrepancy reaches 120 orders of magnitude if the present cosmological critical density is compared with the Planck scale. Moreover, if the mentioned cosmological observables are interpreted correctly, it is necessary to justify the ''coincidence'' with which vacuum energy is starting to dominate the cosmic expansion right now. Describing dark energy as a scalar field ϕ, or quintessence, first considered in [7,8], has the attractive feature of alleviating these problems, at least at the classical level; ''tracking'' [7,9] and ''scaling'' [8,10] solutions existing for quintessence (Q) with inverse power law and exponential potentials, respectively, allow dark energy density to be at the level of other cosmological components in the very early universe. *k*-essence models involving a generalized form of the kinetic energy of the scalar field can provide a mechanism to justify the present level of dark energy [11,12].

Dark energy cosmologies have been considered in the general framework of scalar-tensor theories of gravity. These ''extended quintessence'' (EQ) scenarios [13], in which the scalar field responsible for cosmic acceleration possesses an explicit coupling with the Ricci scalar, have been studied through several perspectives [14–19], including a detailed study of tracking trajectories and of their effects on cosmological perturbations [20]; in particular, both for Q and for EQ models, characteristic signatures have been accurately predicted on the CMB spectrum of anisotropies and compared with existing data; see [21] and references therein.

Elevating the cosmological vacuum energy to a dynamical role through its representation as a scalar field introduces the natural question about its relation with the other main dark component which is currently under study in cosmology, i.e., the cold dark matter (CDM). The latter is currently thought to be made of nonrelativistic particles, possibly generated during the process of supersymmetry breaking in the early Universe, see, e.g. [22], which are supposed to form the ha-

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los around galaxies via gravitational collapse of primordial linear perturbations. It is natural to ask if the formation of matter clumps in the Universe can have some effect on dark energy, or even if dark energy clumps can form starting from primordial linear perturbations. In the past this intriguing issue has been faced from different points of view, either by investigating the structure and stability of nonlinear spherically symmetric scalar field overdensities [23], or by describing dark matter and dark energy as two different smoothly distributed classical fields [24,25]; in [26], clumps of an extremely light classical scalar field have been proposed as a candidate for dark matter in galactic halos. In [27], the formation of matter structures has been faced in a background filled by matter and a homogeneous quintessence, which still does not undergo structure formation processes.

In any case, the occurrence of eventual dark energy clumps arising from perturbations in the linear regime did not receive any explanation. Even more, in Ref. [28], an effective fluid with negative equation of state has been considered as ''generalized dark matter'' in the general context of cosmological linear perturbation theory, and the main consequences for structure formation theory have been obtained. In particular, it has been shown that perturbations of a minimally coupled scalar field playing the role of quintessence behave as scalar radiation on subhorizon cosmological scales, relativistically dissipating scalar field density fluctuations, so that ordinary quintessence rapidly becomes a smooth component. In the present work we investigate the subhorizon dark energy perturbation behavior in scalar-tensor cosmologies, and we show that the conclusions can be much different. In particular, our aim is to give an answer to the following question: in which conditions it is possible to form growing dark energy density perturbations on subhorizon scales, in a reference-frame independent manner? We consider this problem in the general context of linear perturbation theory in scalar-tensor cosmology. We study the perturbation properties of the nonminimally coupled dark energy field, focusing on the influence that metric fluctuations can have on its subhorizon behavior; we establish in particular in which conditions such influence is effectively dominant, resulting in a ''gravitational dragging'' of the dark energy itself. Moreover, we give a worked example of this phenomenology considering a typical extended quintessence model and giving numerical results on the subhorizon behavior of its energy density fluctuations at redshifts relevant for structure formation.

The results presented here are complementary with respect to our previous works on the same topic [13,20]: indeed, in [13] we wrote the basic perturbations equations, analyzing the main cosmological features of EQ models. In [20], we focused on the impact of tracking EQ trajectories on the CMB anisotropies and on the matter power spectrum. Here we deal with the clustering properties of the dark energy itself, therefore completing the picture of linear perturbation theory in EQ cosmologies; this will require working in a setting in which the stress-energy tensor of the quintessence field is conserved, opposite to the formalism adopted in the previous works.

The paper is organized as follows. In Sec. II we review the general formalism for scalar-tensor cosmologies. In Sec. III we study the motion equations, both for background and perturbations, giving emphasis to the role of the nonminimal coupling in the dynamics of the scalar field density fluctuations. The resulting dark energy clustering properties are illustrated in Sec. IV, where we expose some examples of these effects as the result of numerical integrations in typical EQ cosmologies. In Sec. V we give an equivalent interpretation of these results in terms of the effective dark energy sound speed. Finally, Sec. VI contains the concluding remarks.

II. SCALAR-TENSOR COSMOLOGIES

In this section we give general definitions and formalism for describing general scalar-tensor cosmologies, both for background and linear perturbations. We follow, as much as possible, the notation adopted in original works [29,30].

Scalar-tensor theories of gravity are generally represented by the action

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa}f(\phi,R)-\frac{\omega(\phi)}{2}\nabla^{\mu}\phi\nabla_{\mu}\phi-V(\phi)+L_{fluid}\right], \tag{1}$$

where R is the Ricci scalar and ϕ is a scalar field which is supposed to be coupled only with gravity through the function $f(\phi,R)$, while the functions $\omega(\phi)$ and $V(\phi)$ specify the kinetic and potential scalar field energies, respectively; the Lagrangian L_{fluid} includes all the components but ϕ, and the constant κ plays the role of the ''bare'' gravitational constant G_*, which in scalar-tensor theories can differ from the Newton's constant G as it is measured by Cavendish-type experiments [31]; without loss of generality, we choose the relation between κ and G_* defined in [31] to be $\kappa=8\pi G_*$. We also pose the light velocity c equal to 1. Einstein equations from the general action (1) can be written in the familiar form

$$G_{\mu\nu}=\kappa T^{total}_{\mu\nu} \tag{2}$$

with the stress-energy tensor $T^{total}_{\mu\nu}$ being made of the scalar field and the other components, indicated with the subscript *fluid*:

$$T^{total}_{\mu\nu}=T^{fluid}_{\mu\nu}+T_{\mu\nu}[\phi]. \tag{3}$$

As a consequence of the contracted Bianchi identities $T^{total}_{\mu\nu}$ is conserved; moreover, $T^{fluid}_{\mu\nu}$ and $T_{\mu\nu}[\phi]$ are separately conserved since no explicit coupling is assumed between fluid and ϕ:

$$\nabla^{\mu}T^{fluid}_{\mu\nu}=\nabla^{\mu}T_{\mu\nu}[\phi]=0. \tag{4}$$

By defining

$$F=\frac{1}{\kappa}\frac{\partial f}{\partial R}, \tag{5}$$

the conserved scalar field contribution assumes the form

$$T_{\mu\nu}[\phi]=\omega\left[\nabla_\mu\phi\nabla_\nu\phi-\frac{1}{2}g_{\mu\nu}\nabla^\lambda\phi\nabla_\lambda\phi\right]-Vg_{\mu\nu}+\frac{f/\kappa-RF}{2}g_{\mu\nu}+\nabla_\mu\nabla_\nu F-g_{\mu\nu}\Box F+\left(\frac{1}{\kappa}-F\right)G_{\mu\nu}, \tag{6}$$

where we can recognize the origin of the different terms composing the scalar field stress-energy tensor (6): the minimal coupling (including a "kinetic" and a "potential" part), the nonminimal coupling including f, F and R, and the gravitational term, proportional to $(\kappa^{-1}-F)$, containing the Einstein tensor itself. We can define them as

$$T^{mc}_{\mu\nu}[\phi]=\omega\left[\nabla_\mu\phi\nabla_\nu\phi-\frac{1}{2}g_{\mu\nu}\nabla^\lambda\phi\nabla_\lambda\phi\right]-Vg_{\mu\nu}, \tag{7}$$

$$T^{nmc}_{\mu\nu}[\phi]=\frac{f/\kappa-RF}{2}g_{\mu\nu}+\nabla_\mu\nabla_\nu F-g_{\mu\nu}\Box F, \tag{8}$$

$$T^{grav}_{\mu\nu}[\phi]=\left(\frac{1}{\kappa}-F\right)G_{\mu\nu}. \tag{9}$$

It is relevant to note that, as extensively discussed in [32], the gravitational term may also be taken to the left-hand side of Eq. (2):

$$FG_{\mu\nu}=\tilde{T}^{total}_{\mu\nu}\equiv T^{fluid}_{\mu\nu}+T^{mc}[\phi]+T^{nmc}[\phi]. \tag{10}$$

With such an approach, one would be left with a nonconserved total stress-energy tensor $\tilde{T}^{total}_{\mu\nu}$, which would differ from Eq. (3) because of the absence of the merely "gravitational" term in Eq. (6). Including the gravitational sector in the stress-energy tensor of the scalar field is not only a dichotomy, as we will discuss in this paper. First of all, in typical nonminimal coupling models in which F is κ^{-1} plus a function depending explicitly on ϕ, the latter function describes the energy transfer between field and metric, acting in particular at a quantum level; see e.g., [33]. Second, the Bianchi identities allow us to write down conserved quantities for the scalar field. Third, the linear perturbation theory describes the evolution of small perturbation in the scalar field energy density: if the latter is drawn from a conserved stress-energy tensor, perturbations in the scalar field energy density exhibit a behavior which is hidden with the approach (10).

Assuming a Friedmann-Robertson-Walker (FRW) background, the metric tensor assumes the form

$$g_{\mu\nu}=a^2(\eta)(\gamma_{\mu\nu}+h_{\mu\nu}), \tag{11}$$

where η is the conformal time, $a(\eta)$ is the scale factor with conformal time derivative $\dot{a}$ expressed through the Hubble parameter $\mathcal{H}=\dot{a}/a$, and $\gamma_{\mu\nu}=\mathrm{diag}[-1,(1-Kr^2)^{-1},r^2,r^2\sin^2\theta]$ is the background metric with spatial curvature K in spherical coordinates (η,r,θ,ϕ); $h_{\mu\nu}\ll 1$ represents linear metric cosmological perturbations, and is conveniently expressed in the Fourier space: for scalar perturbations, indicating with Y the solution of the Laplace equation $\nabla^{s\,i}\nabla^s_i Y=-|\vec{k}|^2 Y$, where ∇^s means covariant derivative with respect to the spatial metric γ_{ij}, the amplitude at wave vector $\vec{k}$ of the most general scalar metric perturbation, see e.g. [29], can be written as

$$h_{00}=-2AY,\quad h_{0j}=-BY_j,\quad h_{ij}=2H_L Y\gamma_{ij}+2H_T Y_{ij}, \tag{12}$$

where A,B,H_L and H_T represent the amplitude in the Fourier space at wave vector $\vec{k}$; Y_j and the traceless Y_{ij}, with $i,j=1,2,3$, are defined as $Y_j=-(1/k)\nabla^s_j Y$, $Y_{ij}=(1/k^2)\nabla^s_i\nabla^s_j Y+\gamma_{ij}Y/3$. Note that, as customary, we intentionally do not write the argument $\vec{k}$ explicitly in the amplitude of the perturbation quantities in the Fourier space. As it is well known [29], a gauge freedom exists because of the linearization of the problem, so that two of the four quantities in Eq. (12) can be set to zero, or, equivalently, two independent gauge invariant combinations can be built out of A,B,H_L and H_T. Correspondingly, the stress energy tensor $T^{\nu}_{\mu\, x}$ relative to any cosmological component x splits in a background component $T^{\nu}_{\mu\, x}=\mathrm{diag}[-\rho_x,p_x,p_x,p_x]$ and perturbations $\delta T^{\nu}_{\mu\, x}$ represented as

$$\delta T^0_{0\,x}=-\rho_x\,\delta_x Y,\quad \delta T^0_{j\,x}=(\rho_x+p_x)(v_x-B)Y_j,$$
$$\delta T^i_{j\,x}=p_x(\pi_{L\,x}\delta^i_j+\pi_{T\,x}Y^i_j), \tag{13}$$

where in particular δ_x represent the density contrast fluctuation at wave vector $\vec{k}$. It is also useful to introduce the fluctuations in the expectation value of the scalar field at wave vector $\vec{k}$, which will be indicated as $\delta\phi\,Y$.

Einstein and conservation equations (2), (4) split into two separate sets describing the evolution of background and perturbations. In the next section we will write them explicitly, focusing on the role of the different quantities composing the scalar field stress energy tensor $T_{\mu\nu}[\phi]$ defined in Eq. (6).

III. GRAVITATIONAL DRAGGING

Let us then concentrate on the conserved tensor (6), first considering background quantities and their evolution equations. Conservation laws (4) for the unperturbed scalar field reduce to

$$\dot{\rho}_\phi=-3\mathcal{H}(1+w_\phi)\rho_\phi, \tag{14}$$

having defined the dark energy field equation of state as $w_\phi=p_\phi/\rho_\phi$. The background evolution equations will be completely determined by the Friedmann-Robertson-Walker equations

$$\mathcal{H}^2=\frac{a^2\kappa}{3}(\rho_{fluid}+\rho_\phi)+K,$$
$$\dot{\mathcal{H}}=-\frac{a^2\kappa}{6}(\rho_{fluid}+\rho_\phi+3p_{fluid}+3p_\phi), \tag{15}$$

and by the conservation equations for each component x: $\dot{\rho}_x=-3\mathcal{H}(\rho_x+p_x)$. As it can be easily seen from Eq. (6), the expressions for the scalar field energy density and pressure which satisfy Eq. (14) are given by

$$\rho_\phi=\omega\frac{\dot{\phi}^2}{2a^2}+V(\phi)+\frac{RF-f/\kappa}{2a^2}-\frac{3}{a^2}\mathcal{H}\dot{F}+3\frac{\mathcal{H}^2+K}{a^2}\left(\frac{1}{\kappa}-F\right), \quad (16)$$

$$p_\phi=\omega\frac{\dot{\phi}^2}{2a^2}-V(\phi)-\frac{RF-f/\kappa}{2a^2}+\frac{1}{a^2}(\mathcal{H}\dot{F}+\ddot{F})-\frac{2\dot{\mathcal{H}}+\mathcal{H}^2+K}{a^2}\left(\frac{1}{\kappa}-F\right). \quad (17)$$

It is useful to mention that these expressions combine in the continuity equation (14) to give the Klein-Gordon equation:

$$\ddot{\phi}+2\mathcal{H}\dot{\phi}+\frac{1}{2\omega}\left(\omega_{,\phi}\dot{\phi}^2-\frac{a^2}{\kappa}f_{,\phi}+2a^2V_{,\phi}\right)=0. \quad (18)$$

Before considering perturbed quantities, it is relevant to comment briefly on the role of the gravitational component of the scalar field stress-energy tensor (9), since as we will see in a moment the same arguments hold for perturbations. As first noted in [18,19], under conditions in which F differs from κ^{-1}, even by a small amount due to a nonzero value of ϕ, the gravitational term appearing in the expression (16) switches on, feeding the scalar field energy density itself with a term proportional to the square of the real time Hubble parameter $H=\mathcal{H}/a$, which in turn is proportional to the *total* cosmological energy density through the Einstein equations. Since the latter is made of matter and radiation scaling as $1/a^3$ and $1/a^4$, respectively, it is straightforward that at sufficiently early times the gravitational term dominates the dynamics of ρ_ϕ. As we will see, this process, which can be meaningfully named "gravitational dragging," is also very important for the dynamics of the dark energy perturbations.

Equations $\nabla_\mu\delta T^\mu_0[\phi]=0$ and $\nabla_\mu\delta T^\mu_j[\phi]=0$ correspond respectively to the continuity and Euler equations

$$\left(\frac{\delta_\phi}{1+w_\phi}\right)^{\cdot}+kv_\phi+3\dot{H}_L+3\mathcal{H}\frac{w_\phi}{1+w_\phi}\Gamma_\phi=0, \quad (19)$$

$$(\dot{v}_\phi-\dot{B})+\mathcal{H}(v_\phi-B)(1-3c^2_{s\,\phi})-k\times\frac{1}{1+w_\phi}\pi_{L\,\phi}+\frac{2}{3}\frac{1}{1+w_\phi}\frac{k^2-3K}{k}\pi_{T\,\phi}-kA=0, \quad (20)$$

where we have defined the scalar field entropy perturbation

$$\Gamma_\phi=\pi_{L\,\phi}-\frac{c^2_{s\,\phi}}{w_\phi}\delta_\phi, \quad (21)$$

and its sound velocity

$$c^2_{s\,\phi}=\frac{\dot{p}_\phi}{\dot{\rho}_\phi}. \quad (22)$$

Equations (19), (20) hold formally for any cosmological component. As for the background, they combine in the perturbed Klein-Gordon equation

$$\ddot{\delta\phi}+\left(2\mathcal{H}+\frac{\omega_{,\phi}}{\omega}\dot{\phi}\right)\dot{\delta\phi}+\left[k^2+\left(\frac{\omega_{,\phi}}{\omega}\right)_{,\phi}\frac{\dot{\phi}^2}{2}+\left(\frac{-a^2f_{,\phi}/\kappa+2a^2V_{,\phi}}{2\omega}\right)_{,\phi}\right]\delta\phi=\dot{\phi}\dot{A}-\left(3\mathcal{H}\dot{\phi}+\frac{-a^2f_{,\phi}/\kappa+2a^2V_{,\phi}}{2\omega}\right)\times A+\dot{\phi}(3\mathcal{H}A-3\dot{H}_L-kB)+\frac{1}{2\omega\kappa}f_{,\phi R}\delta R, \quad (23)$$

with the variation of the Ricci scalar given by

$$\delta R=-\frac{2}{a^2}(3\mathcal{H}A-3\dot{H}_L-kB)^{\cdot}Y-\frac{6}{a^2}\mathcal{H}(3\mathcal{H}A-3\dot{H}_L-kB)Y+\frac{2}{a^2}(k^2-3\dot{\mathcal{H}}+3\mathcal{H}^2)AY+\frac{4}{a^2}(k^2-3K)\left(H_L+\frac{1}{3}H_T\right)Y. \quad (24)$$

In order to gain insight into the behavior of the scalar field perturbations specifically, let us write explicitly the formal solutions to the above equations. The variation of the stress-energy tensor (6) yields contributions which we classify in mc, nmc, and $grav$ as we did in Eqs. (7), (8), (9). Let us start from the energy density perturbations $\delta T^0_0[\phi]=-\rho_\phi\delta_\phi Y=\delta T^{0\;mc}_0[\phi]+\delta T^{0\;nmc}_0[\phi]+\delta T^{0\;grav}_0[\phi]$; the different contributions are given by

$$\delta T^{0\;mc}_0[\phi]=\left[-\omega_{,\phi}\frac{\dot{\phi}^2}{2a^2}\delta\phi+\frac{\omega}{a^2}(A\dot{\phi}^2-\dot{\phi}\dot{\delta\phi})-V_{,\phi}\delta\phi\right]Y, \quad (25)$$

$$\delta T^{0\;nmc}_0[\phi]=\left[-\frac{3}{a^2}A\mathcal{H}\dot{F}+\frac{3}{a^2}\mathcal{H}\dot{\delta F}+\frac{1}{2\kappa}f_{,\phi}\delta\phi+\left(-\frac{R}{2}+\frac{k^2}{a^2}\right)\delta F+\frac{3}{a^2}\mathcal{H}\mathcal{K}_g\dot{F}\right]Y, \quad (26)$$

$$\delta T^{0\;grav}_0[\phi]=\frac{3}{a^2}(\mathcal{H}^2+K)\delta FY+\left(\frac{1}{\kappa}-F\right)\frac{2}{a^2}\left[3\mathcal{H}^2A-\mathcal{H}kB-3\mathcal{H}\dot{H}_L-(k^2-3K)\left(H_L+\frac{H_T}{3}\right)\right]Y, \quad (27)$$

where we have defined $\mathcal{K}_g \equiv -A+\mathcal{H}^{-1}B/3+\mathcal{H}^{-1}\dot{H}_L$; note also that in general $\delta F = F_{,\phi}\delta\phi + F_{,R}\delta R$. The momentum perturbation $\delta T^0_j[\phi]$ is composed by

$$\delta T_j^{0\ mc} = \frac{k}{a^2}(\omega\dot{\phi}\delta\phi)Y_j\,, \tag{28}$$

$$\delta T_j^{0\ nmc} = \frac{k}{a^2}(\delta\dot{F} - \mathcal{H}\delta F - A\dot{F})Y_j\,, \tag{29}$$

$$\delta T_j^{0\ grav} = \frac{2}{a^2}\left(\frac{1}{\kappa} - F\right)\left(k\mathcal{H}A - k\dot{H}_L - \left(\frac{k^2-3K}{3k}\right)\dot{H}_T\right)Y_j\,. \tag{30}$$

Let us consider now δT^i_j. In general, it will have both trace and traceless components, $\pi_{L\ \phi}$ and $\pi_{T\ \phi}$, respectively, as in Eq. (13). The different contributions are given by

$$\delta T_j^{i\ mc}[\phi] = \frac{1}{a^2}\left[\omega\dot{\phi}\delta\dot{\phi} + \frac{\omega_{,\phi}}{2}\dot{\phi}^2\delta\phi - a^2V_{,\phi}\delta\phi - A\omega\dot{\phi}^2\right]Y\delta^i_j\,, \tag{31}$$

$$\delta T_j^{i\ nmc}[\phi] = \frac{1}{a^2}\left[\frac{1}{2\kappa}f_{,\phi}\delta\phi + 2\dot{F}\mathcal{H}\mathcal{K}_g + \left(\frac{R}{2}+\frac{2}{3}k^2\right)\delta F + \mathcal{H}\delta\dot{F} + \delta\ddot{F} - \dot{F}\dot{A} - 2\ddot{F}A\right]Y\delta^i_j + \frac{k^2}{a^2}\left[\delta F + (kB - \dot{H}_T)\frac{\dot{F}}{k^2}\right]Y^i_j\,, \tag{32}$$

$$\begin{aligned}\delta T_j^{i\ grav}[\phi] = {} & \frac{1}{a^2}(2\dot{\mathcal{H}}+\mathcal{H}^2+2K)\delta F Y\delta^i_j + \frac{2}{a^2}\left(\frac{1}{\kappa}-F\right) \times\left[(2\dot{\mathcal{H}}+\mathcal{H}^2)A + \mathcal{H}\dot{A} - \frac{k^2}{3}A - \frac{k}{3}\dot{B} - \frac{2}{3}k\mathcal{H}B - \ddot{H}_L - 2\mathcal{H}\dot{H}_L - \frac{k^2-3K}{3}\right.\\ & \left.\times\left(H_L + \frac{H_T}{3}\right)\right]Y\delta^i_j + \frac{1}{a^2}\left(\frac{1}{\kappa}-F\right)\left[-k^2A - k(\dot{B}+\mathcal{H}B) + \ddot{H}_T + \mathcal{H}(2\dot{H}_T - kB) - k^2\left(H_L + \frac{H_T}{3}\right)\right]Y^i_j\,,\end{aligned} \tag{33}$$

where $\pi_{L\ \phi}$ and $\pi_{T\ \phi}$ are given by the terms proportional to δ^i_j and Y^i_j, respectively. It is worth noting that an interesting feature of nonminimally coupled scalar fields is the presence of the gauge invariant anisotropic stress π_T: as shown in [28] and [34], stress perturbations have a role in the structure formation; we will return to this in Sec. V.

Although the expressions above appear complicated, it is quite simple to highlight the point we are interested in. In the quantities (27), (30), (33), the terms multiplying $(1/\kappa - F)$ are $\delta G^0_0, \delta G^0_j, \delta G^i_j$, respectively. Focusing on the gravitational part of the scalar field energy density perturbation, as we noted above for the case of the background quantities, if F differs from $1/\kappa$ the scalar field density perturbation is fed by the *total* density fluctuation, since $\delta G^0_0 = \kappa\delta T^0_0$. Therefore, if this term dominates over the others (mc,nmc), the gravitational dragging is active on the density perturbations and forces the scalar field density fluctuations to behave as the dominant cosmological component.

As we will see in the next section, this process becomes crucially important in dark energy cosmologies, where the scalar field plays an important role in the cosmic evolution, determining the cosmic acceleration today. In the next section we will give a worked example of this issue. We integrate Einstein and conservation equations to get the cosmological evolution in a typical extended quintessence scenario where $1/\kappa - F \propto \phi^2$, focusing on the behavior of the dark energy density fluctuations at the relevant redshifts for structure formation.

IV. DARK ENERGY CLUSTERING

Let us focus on the nonminimally coupled scalar field fluctuations; combining Eqs. (25), (26), (27) and the expression for the background energy density (16), one gets the following expression for the scalar field energy density fluctuation:

$$\delta_\phi = \delta_\phi^{mc} + \delta_\phi^{nmc} + \delta_\phi^{grav}\,, \tag{34}$$

where

$$\delta_\phi^{mc} = \frac{\omega_{,\phi}\dot{\phi}^2\delta\phi - 2\omega(A\dot{\phi}^2 - \dot{\phi}\delta\dot{\phi}) + 2a^2V_{,\phi}\delta\phi}{\omega\dot{\phi}^2 + 2a^2V + RF - f/\kappa - 6\mathcal{H}\dot{F} - 2a^2(1/\kappa - F)G^0_0}\,, \tag{35}$$

$$\delta_\phi^{nmc} = \frac{6A\mathcal{H}\dot{F} - 6\mathcal{H}\delta\dot{F} - a^2(f_{,\phi}/\kappa)\delta\phi - (-a^2R + 2k^2)\delta F - 6\mathcal{H}\mathcal{K}_g\dot{F}}{\omega\dot{\phi}^2 + 2a^2V + RF - f/\kappa - 6\mathcal{H}\dot{F} - 2a^2(1/\kappa - F)G^0_0}\,, \tag{36}$$

$$\delta_\phi^{grav} = \frac{-6\delta F\mathcal{H}^2 - 2a^2(1/\kappa - F)\delta G^0_0}{\omega\dot{\phi}^2 + 2a^2V + RF - f/\kappa - 6\mathcal{H}\dot{F} - 2a^2(1/\kappa - F)G^0_0}\,. \tag{37}$$

Again, we point out that this is precisely the form obtained perturbing the field energy density, whenever the latter is drawn from a conserved energy-momentum tensor: only in this case, we are allowed to use Eq. (19) for the field energy density evolution. Most importantly, the use of conserved quantities allows a more direct interpretation of the interchange between different species. The gauge invariant total density perturbation Δ and the gravitational potential Φ, defined by

$$\rho\Delta=\sum_x\left[\rho_x\delta_x+3\frac{\mathcal{H}}{k}(\rho_x+p_x)v_x\right]-(\rho+p)B, \tag{38}$$

$$\Phi=H_L+\frac{H_T}{3}+\frac{\mathcal{H}}{k}\left(B-\frac{\dot{H}_T}{k}\right), \tag{39}$$

are related through the Einstein equation [35,29]:

$$\Phi=\frac{\kappa a^2}{2k^2}\rho\Delta. \tag{40}$$

In such a way, since Δ sums up perturbations in all the fluid components, a ''potential well'' may be generated by each of them (in particular, by a perturbation in the scalar field energy density), affecting the behavior of density perturbations in all the other species (in particular, matter perturbations). Vice versa, perturbations in the matter component will perturb the gravitational potential to drive the scalar field energy density perturbations: such a kind of back reaction is precisely what we expect by looking at Eq. (37), due to the presence of the δG^0_0 term. When the energy density perturbations of the total fluid are dominated by perturbations in the matter component (i.e., at sufficiently high redshifts in typical dark energy cosmologies) for some scale k^{-1}, the term $\delta G^0_0=\kappa\delta T^0_0$ in Eq. (34) is in turn dominated by matter energy density perturbations, which then act as a source of the scalar field density perturbations.

The very interesting feature here is that nonvanishing energy density perturbations of a nonminimally coupled scalar field can even be associated with a homogeneous scalar field as long as a nonzero value of ϕ makes $\kappa^{-1}\neq F$: we can easily see that $\delta\rho_\phi$ in Eq. (34) survives even in the limit $\delta\phi\rightarrow 0$, because of the gravitational dragging. In other words, perturbations of a nonminimally coupled scalar field are sourced by two complementary mechanisms: proper scalar field perturbations, and metric induced perturbations, related to the Ricci scalar coupled with the field itself.

Focusing now on dark energy cosmologies, the described process introduces genuine new features with respect to ''ordinary'' (minimally coupled) quintessence scenarios: the growth in the matter perturbations may drag EQ density perturbations to a nonlinear regime, opening the possibility of the formation of quintessence clumps.

To give a concrete example, we numerically evolve linear perturbations in typical EQ scenarios [20]. The coupling of quintessence with the Ricci scalar is chosen to have the structure

$$\frac{1}{\kappa}f(\phi,R)=F(\phi)R. \tag{41}$$

The measured gravitational constant G, in scalar-tensor theories (1) with the choice (41), is related to the various quantities in the Lagrangian as follows [31]:

$$G=\frac{G_*}{\kappa F}\left(\frac{2\omega F+4F^2_{,\phi}}{2\omega F+3F^2_{,\phi}}\right). \tag{42}$$

As in [13,20], we model F as a constant plus a term yielding a quadratic coupling between the field and the Ricci scalar, so that

$$F(\phi)=\frac{1}{\kappa}+\xi\phi^2, \tag{43}$$

where ξ is the nonminimal coupling constant, with the constraint that today F satisfy the relation (42). Note also that ξ can in principle assume both positive and negative values. Moreover, its magnitude is not arbitrary, due to constraints from local and solar-system experiments on the time variation of the gravitational constant and from effects induced on photon trajectories [36]. The time derivative G_t of the observed gravitational constant G as defined in Eq. (42) must satisfy $G_t/G\lesssim 10^{-12}$ yr^{-1} at present. Moreover, the Jordan-Brans-Dicke parameter $\omega_{JBD}=\omega F/F^2_{,\phi}$ must be greater than about 2500 at present; to be conservative, we choose $\omega_{JBD}=3000$, with a negative sign of the coupling constant (in our specific model this corresponds to $\xi\simeq-1.78\times 10^{-2}$, $\phi_0\simeq 1/\sqrt{G}$), which yields $G_t/G\simeq 10^{-14}$ yr^{-1}. The quintessence potential, responsible for cosmic acceleration today, has an inverse power law form $V\propto\phi^{-\alpha}$; moreover, we fix $\omega(\phi)=1$.

The cosmological model is specified as follows. The Hubble parameter at present is fixed at $H_0=100h$ km/sec/Mpc with $h=0.7$ and the spatial metric is taken to be flat, $K=0$. The fraction of critical density in dark energy is $\Omega_\phi=0.70$. The equation of state of quintessence at present $w_{\phi 0}$ is chosen to be -0.9, yielding cosmic acceleration. Baryon abundance is set to $\Omega_b h^2=0.022$, CDM represents the remaining matter component, $\Omega_{CDM}=1-\Omega_\phi-\Omega_b$, and three massless neutrino families are assumed. Perturbations are taken to be Gaussian with an initially scale invariant adiabatic spectrum [37]. The evolution of background and perturbations has been determined by numerically solving equations in the synchronous gauge $A=B=0$. Their expressions are reported in the Appendix.

Let us consider the background evolution first. In Fig. 1 the energy densities of radiation (dashed line), matter (dotted), and dark energy (solid) are plotted as a function of the redshift $z=1/a-1$. At late times, $z\lesssim 5$, the dark energy density is dominated by the kinetic and potential energies. At higher redshift the effect of the gravitational dragging is evident: the last term in Eq. (16) actually dominates and forces dark energy to scale with redshift as the dominant cosmo-

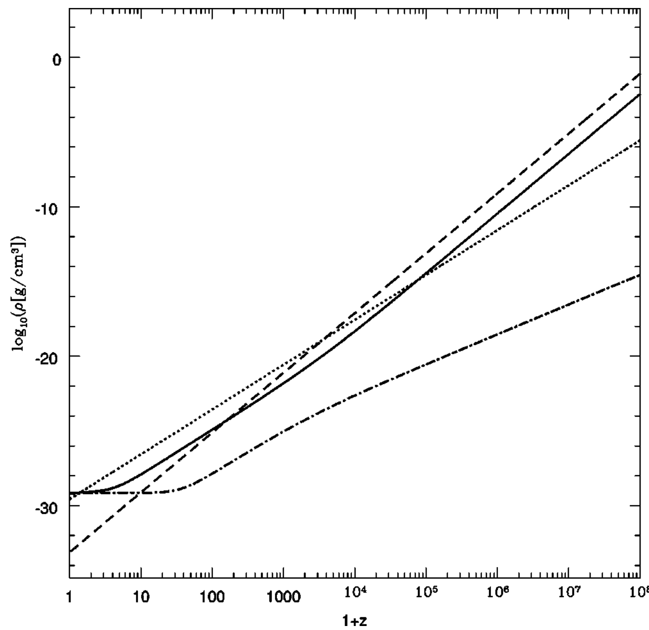

FIG. 1. Redshift scaling of cosmological components in tracking EQ scenario (see text): radiation (dashed), matter (dotted), total dark energy as from Eq. (16) (solid), kinetic and potential dark energy contributions [first two terms in Eq. (16)] (dotted dashed).

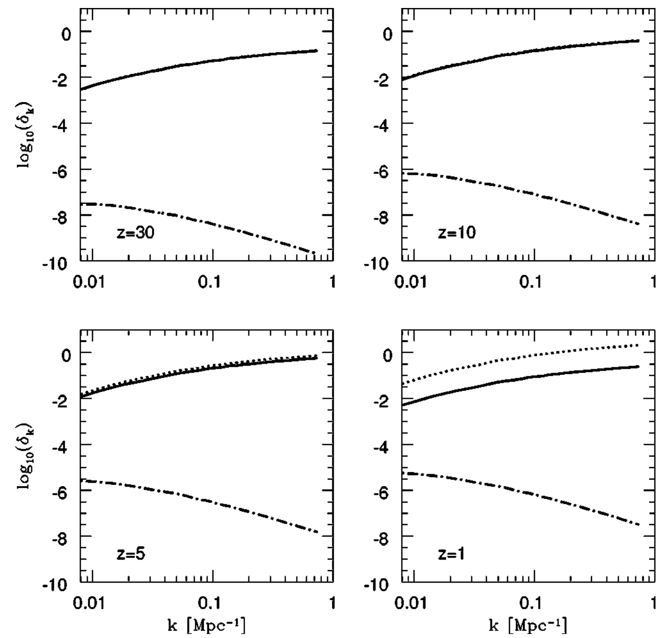

FIG. 2. Spectral power of density fluctuations for matter (dotted) and dark energy in tracking EQ (solid) and Q (dotted dashed) scenarios at different redshifts.

logical component. As it is easy to see, in this regime, the dark energy cosmological abundance is simply given by

$$\Omega_{\phi}(z) \simeq -\kappa\xi\phi^2(z), \tag{44}$$

where the minus is due to our sign conventions. Note that the quantity Ω_ϕ can be constrained by big bang nucleosynthesis (BBN), because a variation in the gravitational constant can be regarded as inducing a change into the effective number of massless neutrinos [38]; however, in the present case this value is at percent level, too small to produce observable effects. The dotted-dashed line represents indeed only the contribution from the mc terms in Eq. (16): the rising part of this curve at $z \gtrsim 1000$ is due to the R boost [20] induced by the effective gravitational potential in the Klein-Gordon equation (18). We stress that while the latter contribution comes from the kinetic energy of the field rolling on the effective gravitational potential, so that ultimately implies a change in time of the physical value of the scalar field ϕ, the gravitational dragging can be thought of as a power injection into the dark energy density coming from the total one, while it does not require directly a spatial dependence of the expectation value of ϕ. Note also that a condition in which dark energy scales as the dominant cosmological component can be achieved with an exponential potential [8,10]; however, in that case the field is minimally coupled and quintessence density perturbations vanish relativistically after horizon crossing [28].

Let us turn now to considering the perturbations. Since we are interested in the dark energy clustering during the formation of matter structures, we concentrate on the behavior in the matter dominated era. Moreover, we focus on the logarithmic power of density fluctuations at the scale k, defined by

$$\delta_{k\,x}^2 = 4\pi k^3 \delta_x^2, \tag{45}$$

where x represents a generic component. As is well known, a scale for which $\delta_{k\,x} \simeq 1$ has to be considered in a nonlinear regime. In Fig. 2 we plot δ_k for matter (dotted) and dark energy (solid) at relevant redshifts. As is expected, the gravitational dragging is active and forces quintessence perturbations to behave as nonrelativistic matter on subhorizon scales, when the gravitational terms dominate both ρ_ϕ and $\delta\rho_\phi$. Under these conditions, by using Eqs. (16) and (37) we see that we can write approximately

$$\delta_\phi \simeq \delta_m, \tag{46}$$

which is mostly satisfied, in our specific model, at $z \gtrsim 5$. It is in fact not a case that all the quantities specifying our specific model disappeared in the above relation. Indeed, Eq. (46) holds if three general conditions are satisfied in dark energy cosmologies, namely, (i) gravity deviates from general relativity, (ii) quintessence plays the role of the nonminimally coupled field, and (iii) gravitational terms dominate [Eqs. (34), (16)]. At present, when the kinetic and potential energies of the field dominate the cosmic expansion imprinting acceleration, the condition (46) is broken because matter is no longer the dominant component.

To fully understand the importance of this plot, we reported also $\delta_{k\phi}$ for a corresponding model in which the quintessence is minimally coupled, represented by the dashed-dotted curve. The latter is rising with time because, as noticed in [39], the inhomogeneous term of the perturbed

Klein-Gordon equation (23) is driving the evolution of $\delta\phi$. However, as is evident from the figure, in the absence of nonminimal coupling the dark energy density perturbations do not play any role in structure formation.

Therefore, maybe the most interesting consequence of the gravitational dragging in dark energy cosmologies is that the nonlinearity may arise for the quintessence component, at a redshift depending in particular on the coupling strength, opening the possibility of the formation of quintessence large overdensities and cavities on subhorizon scales. On the other hand, at a linear level, the effect produced by $\delta\rho_\phi$ on the total gravitational potential Φ resides in a fraction ρ_ϕ/ρ_m, which is small in the limit in which ϕ is subdominant. For example, in models in which the assumed scalar-tensor gravity theory is slightly different from general relativity, i.e., $|1-\kappa F|\ll 1$, in the gravitational dragging regime when the gravitational contribution dominate ρ_ϕ and δ_ϕ through the products $(\kappa^{-1}-F)G_0^0$ and $(\kappa^{-1}-F)\delta G_0^0$, it can be easily seen that the portion of gravitational potential which is sourced by the quintessence is given approximately by

$$\Phi_\phi \simeq (1-\kappa F)\Phi \ll \Phi, \tag{47}$$

which means that the bulk of the gravitational potential arising from the clustering process is still provided by matter. Note, however, that this is true for linear perturbations, which we are treating in this work; the effect of a nonminimally coupled dark energy on the gravitational potential associated with a nonlinear structure is still unknown, and actually this study could be an interesting development of the present work.

The possibility of the presence of quintessence clumps in the Universe has been considered by several authors, mainly aimed at foreseeing their signatures on the galactic structure [23–25]; however, there is not, at present, a theory explaining how such ''vacuum energy clumps'' may form, and on which scales we expect eventually to find them at low redshifts. What we obtained is just a possible way to escape from the linear regime. In other models, as in [17], there can be even higher deviations from general relativity at high redshift than in the model considered here; the potential effect on the formation of nonlinear clumps may be even more important. Moreover, we have shown that the occurrence of dark energy nonlinear subhorizon structure does not require directly a space dependence of the expectation value of ϕ.

Obviously, this is just a first step towards a theory of ''vacuum energy clumps:'' the perturbation behavior out of the linear regime is still an open issue, and is strictly related to properties such as the sound speed of the dark energy component. In particular, we think that one of the key issues is the time needed for a quintessence primordial structure to collapse, and a fundamental role on this is played not only by the bare coupling constant ξ, but generally by the whole coupling function, through the effect they have on the value, and sign, of the quintessence sound speed. In the next section we will use a different approach to explain such properties, based on the effective sound speed introduced in [28,34].

V. DARK ENERGY SOUND SPEED

An important role in the perturbation growth is played by the nonadiabatic stress or entropy contribution, entering directly into the evolution equation for density perturbations; by defining $\delta p_x = p_x \pi_{L\,x}$ for a generic component x we can write the scalar field entropy contribution as

$$p_\phi \Gamma_\phi = \delta p_\phi - c_{s\,\phi}^2 \delta\rho_\phi, \tag{48}$$

where $c_{s,\phi}^2$ is the adiabatic sound speed of the scalar field, defined in Eq. (22), which can also be written as

$$c_\phi^2 = w_\phi - \frac{1}{3\mathcal{H}}\frac{\dot{w}_\phi}{1+w_\phi}. \tag{49}$$

In most quintessence scenarios, the field is modeled as a component with negative, and slowly varying, equation of state, so that from Eq. (49), $c_\phi^2 \simeq w_\phi$. As discussed in [28,34], looking at the continuity and Euler equations (19), (20), a fluid with negative sound speed without entropy and stress terms would make adiabatic fluctuations unable to give a pressure support against gravitational collapse of density perturbations: in other words, under these circumstances, the adiabatic pressure fluctuations would accelerate the collapse rather than oppose it, as can be derived from the density perturbations evolution in the limit of subhorizon scales. In this scenario, density perturbations would rapidly become nonlinear after entering the horizon, unless the entropic term in Eq. (19) acts as a stabilizing mechanism: this requires $w_\phi \Gamma_\phi > 0$. To check this possibility, Hu in [28] introduces the effective sound speed $c_{eff\,\phi}^2$ defined in the rest frame of the scalar field, where $\delta T^0_{j\,\phi}=0$ (in [28], these considerations are applied to a more general ''generalized dark matter'' component, which can recover the quintessence scalar field-case, as well as matter and radiation). Following this approach, we write the gauge invariant entropic term as

$$w_\phi \Gamma_\phi = (c_{eff}^2 - c_\phi^2)\,\delta_\phi^{(rest)}, \tag{50}$$

where $\delta_\phi^{(rest)}$ is the density contrast in the dark energy rest frame, which therefore coincides with the gauge invariant density perturbation Δ_ϕ as follows:

$$\delta_\phi^{(rest)} = \Delta_\phi = \delta_\phi + 3\frac{\mathcal{H}}{k}(1+w_\phi)(v_\phi - B). \tag{51}$$

By doing so, the stabilization mechanism of scalar field perturbations is expressed only in terms of gauge invariant quantities. In the mentioned case of $c_{s\,\phi}^2 < 0$, effective pressure support is obtained if the entropic term (50) is positive. The effective sound speed can be interpreted as a rest frame sound speed; importantly, it allows us to define a stabilization scale for a perturbation, given by the corresponding effective sound horizon. This formalism has been used in [28] to show that density perturbations in a minimally coupled scalar field of quintessence are damped out below the horizon, so that the quintessence rapidly becomes a smooth component: in this case, indeed, it can be verified that the effective sound speed is $c^2_{eff\ \phi}=1$, giving a relativistic behavior to the corresponding density fluctuations.

The situation in extended quintessence scenarios can however be much different, because the effective sound speed may be strongly affected by the presence of the nonminimal coupling term. Rewriting c^2_{eff} as

$$c^2_{eff\ \phi}=\frac{\delta p_\phi + c^2_{s\ \phi} 3\mathcal{H} h_\phi (v_\phi - B)/k}{\delta\rho_\phi + 3\mathcal{H} h_\phi (v_\phi - B)/k}, \qquad (52)$$

where $h_\phi \equiv \rho_\phi + p_\phi$, it is quite evident that on subhorizon scales $k \gg \mathcal{H}$, $c^2_{eff\ \phi} \simeq \delta p_\phi / \delta\rho_\phi$. As discussed in the previous section, in $\delta\rho_\phi$ lies the main difference between ordinary quintessence and extended quintessence: while $\delta p_\phi / \delta\rho_\phi \simeq 1$ for minimally coupled scalar fields, giving relativistic values to c^2_{eff} and damping field perturbations, this ratio may be much lower than unity whenever the energy density perturbations of the scalar field are enhanced by perturbations in the matter field, and this is a peculiar property of nonminimally coupled scalar fields.

Another genuine feature which is expected in EQ scenarios regards the viscosity of the dark energy component. As pointed out, again in [28], the anisotropic stress can also be a smoothing mechanism for the scalar field, damping density perturbations through its effects on velocity perturbations in Eq. (20). A viscosity parameter c^2_{vis} is introduced to relate velocity or metric shear and anisotropic stress; for the quintessence scalar field, we have

$$\dot{\pi}_{T\ \phi} + 3\mathcal{H}\pi_{T\ \phi} = \frac{4 c^2_{vis\ \phi}}{w_\phi}(k v_\phi - \dot{H}_T). \qquad (53)$$

In the limit of negligible $\dot{\pi}_{T\ \phi}$ and in shear-free frames ($H_T = 0$), $c^2_{vis} > 0$ determines a viscous damping of velocity perturbations, as can be seen through the Euler equation (20), which sums up with the viscosity effect arising from the cosmological expansion; thus, if $c^2_{vis} > 0$, viscosity can be an extra smoothing mechanism. On the other hand, $c^2_{vis} < 0$ results in a term which acts against the cosmological viscosity into the Euler equation.

The viscosity parameter turns out to be zero for minimally coupled quintessence, where anisotropic stress is not present: in that case, however, $c^2_{eff} = 1$, so that the adiabatic stress only is enough in smoothing the scalar field on subhorizon scales. On the other hand, for extended quintessence fields, we expect a nonzero contribution to c^2_{vis}, due to the traceless part of $\delta T^{i\ nmc}_j[\phi] + \delta T^{i\ grav}_j[\phi]$.

We plot the four quantities $c^2_{eff\ \phi}$, w_ϕ, $c^2_{s\ \phi}$, $c^2_{vis\ \phi}$ in Fig. 3, comparing results in our tracking EQ scenario (solid lines) and in an ordinary minimally coupled quintessence cosmology (dotted dashed).

The most striking effect is for $c^2_{eff\ \phi}$. For all the redshifts relevant for structure formation the effective dark energy sound speed is vanishing in the EQ case, allowing for a behavior of its density perturbations analogous to that of nonrelativistic matter. Correspondingly, minimally coupled quintessence has $c^2_{eff\ \phi} = 1$. This reproduces the same result as in Fig. 2 obtained with a different approach. The more the

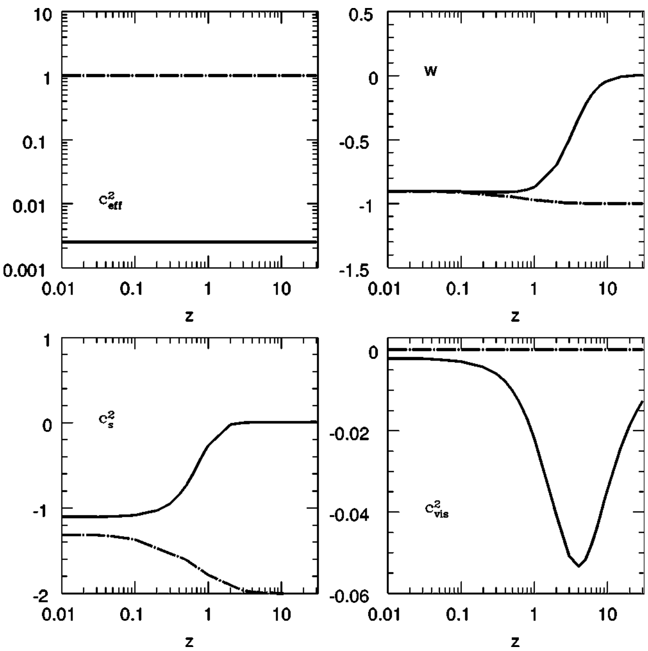

FIG. 3. Redshift behavior of effective sound speed, equation of state, sound speed and viscosity in tracking EQ (solid) and Q (dotted dashed) scenarios.

gravitational term in $\delta\rho_\phi$ dominates in the denominator of the expression (52), the larger the suppression of the dark energy pressure reaction to the density contrast growth. Even if the plotted results are strictly valid only in our model, we stress that this is an example of a general occurrence in scalar-tensor dark energy cosmologies. In addition, since the gravitational term in Eq. (27) is proportional to $(1/\kappa - F)$ which can in general assume both positive and negative values, even the sign of $c^2_{eff\ \phi}$ can be reversed realizing a scenario in which the sound speed accelerates the collapse on scales smaller than the sound horizon instead of opposing it.

An analogous behavior can be found by looking at the plots of w_ϕ and $c^2_{s\ \phi}$, in that they are severely depressed at relevant redshifts in EQ models. Also, the difference between $c^2_{s\ \phi}$ and w_ϕ is due to the time derivative of the equation of state through Eq. (49). Notice in particular that in minimally coupled quintessence we have $c^2_{s\ \phi}, w_\phi < 0$ while $c_{eff\ \phi} = 1$; as we already stressed, the latter quantity is indeed the appropriate one to explain the behavior of a minimally coupled scalar field, resulting in the relativistic damping of subhorizon perturbations. Finally it is interesting to check whether viscosity can be sufficiently effective in damping out density and velocity perturbations, even when adiabatic pressure fluctuations are not. As in the minimally coupled case, it turns out that this is not the case; first, the amplitude of $c^2_{vis\ \phi}$ is much lower than unity, and second the negative sign yields an enhancement of velocity perturbations, instead of a damping, as we stressed above; the peak at $z \simeq 1$ is due to the onset of cosmic acceleration, i.e., the sign change into the cosmic equation of state.

In practice, there are no mechanisms to slow or decrease the amplitude of extended quintessence density perturbations in the gravitational dragging regime. This analysis confirms, and better clarifies, the results of the previous section concerning the scalar field density fluctuations power spectrum.

VI. CONCLUDING REMARKS

We studied the behavior of linear perturbations in scalar-tensor cosmologies, focusing on the density fluctuations of the scalar field ϕ coupled with the Ricci scalar R. We found that such coupling can activate a "gravitational dragging" of the scalar field density fluctuations by the cosmological metric perturbations, which in turn are powered by the whole cosmological stress-energy tensor through the Einstein equations. That is, as the nonminimal coupling represents a power exchange channel between the scalar field component and the general relativistic cosmological gravitational potentials, we studied in particular how such a channel acts at the level of linear density perturbations in the scalar field, represented in particular by the density contrast $\delta_\phi = \delta\rho_\phi/\rho_\phi$. In conditions in which ϕ is not the dominant cosmological component, the power injection coming from gravity can largely dominate δ_ϕ forcing its dynamics to be similar to that of the dominant component itself. On the other hand, in the same conditions, the scalar field contributes to the cosmological gravitational potentials by a fraction given by the ratio between the scalar field and total energy densities.

This phenomenology has important consequences on the dark energy clustering properties in extended quintessence scenarios, where the nonminimally coupled scalar field is assumed to be responsible for the cosmic acceleration today. Namely, the dark energy assumes the features of a pressureless fluid when nonrelativistic matter m dominates, i.e., after matter radiation equality and in the preaccelerating stage of the cosmic expansion. In other words, the scalar field density perturbations can grow on subhorizon scales, tracing those in the matter component; this fact is depicted in the scalar field density contrast δ_ϕ, as well as in the properties of its effective sound speed $c_{eff\,\phi}$:

$$\delta_\phi \simeq \delta_m, \quad c^2_{eff\,\phi} \ll 1. \tag{54}$$

As we already mentioned, the reason for this behavior lies in the gravitational coupling to the Ricci scalar, contributing a gravitational term in the scalar field energy density which gets the dominant contribution from the perturbation in the matter component. The latter perturbations are therefore able to feed the dark energy density fluctuations up to a large amount even if the nonminimal coupling is small enough to respect all the existing constraints on scalar-tensor theories of gravity. We stress also that the behavior (54) does not depend on the particular form assumed to describe the nonminimal coupling; indeed, such a gravitational dragging regime holds whenever the contributions due to the nonminimal coupling dominate both in ρ_ϕ and $\delta\rho_\phi$, so that their ratio is rather insensitive to the detailed shape of such coupling. Moreover, it should be noticed that the behavior (54) is not related to variations of the expectation value of the scalar field ϕ; indeed, our study shows that density perturbations of a nonminimally coupled scalar field are sourced both by fluctuations of expectation value as well as by perturbations of the Ricci tensor. In particular, the dragging effect emphasized here is generated even in the limiting case of a homogeneous scalar field, being induced by the coupling with R.

We have provided a worked example of the above phenomenology in the extended quintessence scenario, involving a quadratic coupling between the field and R. Numerical integrations of the cosmological equation system shows that the dynamical condition (54) is satisfied at redshifts relevant for the structure formation process, respecting all the existing constraints on scalar-tensor gravity theories.

We believe that these results open new perspectives on the standard picture of structure formation in dark energy cosmologies, since the gravitational dragging expressed by Eq. (54) implies that both dark energy and matter exit the linear regime on subhorizon cosmological scales at relevant redshifts. This immediately poses the problem of their evolution afterwards, i.e., the gravitational clustering of large overdensities and deep cavities composed by matter and scalar energy tangled by a nonminimal gravitational interaction; while as we already stressed the gravitational dragging is rather insensitive to the detailed shape of the nonminimal coupling, the same could be untrue at a nonlinear level. In particular, for a given model, it would be interesting to look at the appearance of the resulting density profile after virialization, since this aspect could be constrained by observed rotational curves in nearby galaxies.

ACKNOWLEDGMENTS

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APPENDIX: SCALAR FIELD PERTURBATIONS IN SYNCHRONOUS GAUGE

Numerical integration and analytic treatment of the perturbation equations get simplified when developed in the synchronous gauge [40]. To obtain our numerical results, we used a modified version of CMBFAST [41] which covers extended quintessence cosmologies.

A scalar-type metric perturbation in the synchronous gauge is parametrized as

$$ds^2 = a^2[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j], \tag{A1}$$

$$h_{ij}(\mathbf{x},\tau) = \int d^3k e^{i\mathbf{k}\cdot\mathbf{x}}\left[\hat{\mathbf{k}}_i\hat{\mathbf{k}}_j h(\mathbf{k},\tau) + \left(\hat{\mathbf{k}}_i\hat{\mathbf{k}}_j - \frac{1}{3}\delta_{ij}\right) 6\,\eta(\mathbf{k},\tau)\right], \tag{A2}$$

h being the trace of h_{ij}. By choosing $A = B = 0$ in Eq. (12), the metric perturbations H_L and H_T are related to h and η by the following relations:

$$H_T = -\frac{h}{2} - 3\,\eta, \quad H_L = \frac{h}{6}. \tag{A3}$$

Defining the shear perturbation σ by the relation $\sigma = a^2/k^2 p \pi_T Y$, respectively, one has, for the scalar field, the following quantities derived from the conserved $T^{\mu}_{\nu}[\phi]$ [Eq. (3)], in synchronous gauge:

$$\delta\rho = \delta\rho_{fluid} + \omega\frac{\dot\phi\delta\dot\phi}{a^2} + \frac{1}{2}\left(\frac{\dot\phi^2\omega_{,\phi}}{a^2} - \frac{1}{\kappa}f_{,\phi} + 2V_{,\phi}\right)\delta\phi - 3\frac{\mathcal{H}\delta\dot F}{a^2} - \left(-\frac{R}{2} + \frac{k^2}{a^2}\right)\delta F - \frac{\dot F\dot h}{2a^2} - 3\frac{\mathcal{H}^2}{a^2}\delta F + \left(\frac{1}{\kappa} - F\right)\frac{2}{a^2}\left[-\frac{\mathcal{H}\dot h}{2} + k^2\eta\right], \tag{A4}$$

$$\delta p = \delta p_{fluid} + \omega\frac{\dot\phi\delta\dot\phi}{a^2} + \frac{1}{2}\left(\frac{\dot\phi^2\omega_{,\phi}}{a^2} + \frac{1}{\kappa}f_{,\phi} - 2V_{,\phi}\right)\delta\phi + \frac{\delta\ddot F}{a^2} + \frac{\mathcal{H}\delta\dot F}{a^2} + \left(-\frac{R}{2} + \frac{2k^2}{3a^2}\right)\delta F + \frac{1}{3}\frac{\dot F\dot h}{a^2} + \frac{\delta F}{a^2}(2\dot{\mathcal{H}} + \mathcal{H}^2) + \frac{2}{3a^2}\left(\frac{1}{\kappa} - F\right) \times\left[-\mathcal{H}\dot h - \frac{\ddot h}{2} + k^2\eta\right], \tag{A5}$$

$$(p+\rho)v = (p_{fluid} + \rho_{fluid})v_{fluid} + \frac{k^2}{a^2}\left[\omega\dot\phi\delta\phi + \delta\dot F - \mathcal{H}\delta F + 2\left(\frac{1}{\kappa} - F\right)\dot\eta\right], \tag{A6}$$

$$(p+\rho)\sigma = (p_{fluid} + \rho_{fluid})\sigma_{fluid} + \frac{2k^2}{3a^2}\left[\delta F + 3\frac{\dot F}{k^2}\left(\dot\eta + \frac{\dot h}{6}\right) + \left(\frac{1}{\kappa} - F\right)\left(-\frac{\mathcal{H}}{k^2}\dot h - \frac{6\mathcal{H}}{k^2}\dot\eta - \frac{2}{k^2}\ddot h + \frac{12}{k^2}\ddot\eta + \eta\right)\right], \tag{A7}$$

where $\delta p_\phi \equiv p_\phi \pi_{L\,\phi}$ is the isotropic pressure perturbation. The perturbed Klein-Gordon equation reads

$$\delta\ddot\phi + \left(2\mathcal{H} + \frac{\omega_{,\phi}}{\omega}\dot\phi\right)\delta\dot\phi + \left[k^2 + \left(\frac{\omega_{,\phi}}{\omega}\right)_{,\phi}\frac{\dot\phi^2}{2} + a^2\left(\frac{-f_{,\phi}/k + 2V_{,\phi}}{2\omega}\right)_{,\phi}\right]\delta\phi = -\frac{\dot\phi\dot h}{2} + \frac{a^2}{2\omega}\frac{f_{,\phi R}}{k}\delta R. \tag{A8}$$

These perturbations enter in the perturbed Einstein equations, easy to solve in this gauge:

$$k^2\eta - \frac{1}{2}\mathcal{H}\dot h = -\frac{a^2\kappa\delta\rho}{2}, \tag{A9}$$

$$k^2\dot\eta = \frac{a^2\kappa(p+\rho)v}{2}, \tag{A10}$$

$$\ddot h + 2\mathcal{H}\dot h - 2k^2\eta = -3a^2\kappa\delta p, \tag{A11}$$

$$\ddot h + 6\ddot\eta + 2\mathcal{H}(\dot h + 6\dot\eta) - 2k^2\eta = -3a^2\kappa(p+\rho)\sigma. \tag{A12}$$

This set of differential equations requires initial conditions on the metric and fluid perturbations; we adopt adiabatic initial conditions [37].

[1] S. Perlmutter *et al.*, Astrophys. J. **517**, 565 (1999).
[2] A. Riess *et al.*, Astron. J. **116**, 1009 (1998); Astrophys. J. **536**, 62 (2000).
[3] M.S. Turner and A. Riess, astro-ph/0106051; A.G. Riess *et al.*, Astrophys. J. **560**, 49 (2001).
[4] C.B. Netterfield *et al.*, astro-ph/0104460; P. De Bernardis *et al.*, Astrophys. J. **564**, 559 (2002); A.T. Lee *et al.*, Astrophys. J. Lett. **561**, L1 (2001); R. Stompor *et al.*, *ibid.* **561**, L7 (2001); N.W. Halverson *et al.*, astro-ph/0104489; C. Pryke *et al.*, Astrophys. J. **568**, 46 (2002).
[5] J.A. Peacock *et al.*, Nature (London) **410**, 169 (2001).
[6] S.M. Carroll, Living Rev. Relativ. **4**, 1 (2001).
[7] B. Ratra and P.J. Peebles, Phys. Rev. D **37**, 3406 (1988).
[8] C. Wetterich, Nucl. Phys. **B302**, 645 (1988).
[9] P.J. Steinhardt, L. Wang, and I. Zlatev, Phys. Rev. D **59**, 123504 (1999).
[10] A.R. Liddle and R.J. Scherrer, Phys. Rev. D **59**, 023509 (1999).
[11] T. Chiba, T. Okabe, and M. Yamaguchi, Phys. Rev. D **62**, 023511 (2000).
[12] C. Armendariz-Picon, V. Mukhanov, and P.J. Steinhardt, Phys. Rev. D **63**, 103510 (2001).
[13] F. Perrotta, C. Baccigalupi, and S. Matarrese, Phys. Rev. D **61**, 023507 (2000).
[14] J.P. Uzan, Phys. Rev. D **59**, 123510 (1999).
[15] L. Amendola, Phys. Rev. D **60**, 043501 (1999).
[16] T. Chiba, Phys. Rev. D **60**, 083508 (1999).
[17] N. Bartolo and M. Pietroni, Phys. Rev. D **61**, 023518 (2000).

[18] A. Riazuelo and J.P. Uzan, Phys. Rev. D **62**, 083506 (2000).
[19] T. Chiba, Phys. Rev. D **64**, 103503 (2001).
[20] C. Baccigalupi, S. Matarrese, and F. Perrotta, Phys. Rev. D **62**, 123510 (2000).
[21] C. Baccigalupi, A. Balbi, S. Matarrese, F. Perrotta, and N. Vittorio, Phys. Rev. D **65**, 063520 (2002).
[22] D.H. Lyth and A. Riotto, Phys. Rep. **314**, 1 (1999).
[23] C. Wetterich, Phys. Lett. B **522**, 5 (2001).
[24] F.S. Guzman and T. Matos, Class. Quantum Grav. **17**, L9 (2000).
[25] T. Matos, F.S. Guzman, and D. Nunez, Phys. Rev. D **62**, 061301(R) (2000).
[26] A. Arbey, J. Lesgourgues, and P. Salati, Phys. Rev. D **64**, 123528 (2001).
[27] E.L. Lokas and Y. Hoffman, astro-ph/0108283.
[28] W. Hu, Astrophys. J. **506**, 485 (1998).
[29] H. Kodama and M. Sasaki, Prog. Theor. Phys. Suppl. **78**, 1 (1984).
[30] J.C. Hwang, Astrophys. J. **375**, 443 (1991); Class. Quantum Grav. **7**, 1613 (1990).
[31] G. Esposito-Farese and D. Polarski, Phys. Rev. D **63**, 063504 (2001).
[32] V. Faraoni, Phys. Rev. D **62**, 023504 (2000).
[33] N. D. Birrel and P. C. W. Davies, *Quantum Fields in Curved Spacetime* (Cambridge University Press, Cambridge, England, 1982).
[34] W. Hu and D. Eisenstein, Phys. Rev. D **59**, 083509 (1999).
[35] J.M. Bardeen, Phys. Rev. D **22**, 1882 (1980).
[36] J.O. Dickey *et al.*, Science **265**, 482 (1994); J.G. Williams, X.X. Newhall, and J.O. Dickey, Phys. Rev. D **53**, 6730 (1996); C.M. Will, Phys. Today **52 (10)**, 38 (1999).
[37] F. Perrotta and C. Baccigalupi, Phys. Rev. D **59**, 123508 (1999).
[38] X. Chen, R.J. Scherrer, and G. Steigman, Phys. Rev. D **63**, 123504 (2001).
[39] P. Brax, J. Martin, and A. Riazuelo, Phys. Rev. D **62**, 103505 (2000).
[40] C.P. Ma and E. Bertschinger, Astrophys. J. **455**, 7 (1995).
[41] U. Seljak and M. Zaldarriaga, Astrophys. J. **469**, 437 (1996).