Constraints on neutrino degeneracy from the cosmic microwave background and primordial nucleosynthesis

M. Orito,[1] T. Kajino,[1] G. J. Mathews,[2] and Y. Wang[3]
[1]*National Astronomical Observatory, Mitaka, Tokyo 181-8588, Japan*
[2]*University of Notre Dame, Center for Astrophysics, Notre Dame, Indiana 46556*
[3]*Department of Physics & Astronomy, University of Oklahoma, Norman, Oklahoma 73019*
(Received 18 March 2001; revised manuscript received 14 March 2002; published 28 May 2002)

We reanalyze the cosmological constraints on the existence of a net universal lepton asymmetry and neutrino degeneracy based upon the latest high resolution CMB sky maps from BOOMERANG, DASI, and MAXIMA-1. We generate likelihood functions by marginalizing over $(\Omega_b h^2, \xi_{\nu_{\mu,\tau}}, \xi_{\nu_e}, \Omega_\Lambda, h, n)$ plus the calibration uncertainties. We consider flat $\Omega_M+\Omega_\Lambda=1$ cosmological models with two identical degenerate neutrino species, $\xi_{\nu_{\mu,\tau}}\equiv|\xi_{\nu_\mu}|=|\xi_{\nu_\tau}|$ and a small ξ_{ν_e}. We assign weak top-hat priors on the electron-neutrino degeneracy parameter ξ_{ν_e} and $\Omega_b h^2$ based upon allowed values consistent with the nucleosynthesis constraints as a function of $\xi_{\nu_{\mu,\tau}}$. The change in the background neutrino temperature with degeneracy is also explicitly included, and Gaussian priors for $h=0.72\pm0.08$ and the experimental calibration uncertainties are adopted. The marginalized likelihood functions show a slight (0.5σ) preference for neutrino degeneracy. Optimum values with two equally degenerate μ and τ neutrinos imply $\xi_{\nu_{\mu,\tau}}=1.0^{+0.8(1\sigma)}_{-1.0(0.5\sigma)}$, from which we deduce $\xi_{\nu_e}=0.09^{+0.15}_{-0.09}$, and $\Omega_b h^2=0.021^{+0.06}_{-0.002}$. The 2σ upper limit becomes $\xi_{\nu_{\mu,\tau}}\leqslant 2.1$, which implies $\xi_{\nu_e}\leqslant 0.30$, and $\Omega_b h^2\leqslant 0.030$. For only a single large-degeneracy species the optimal value is $|\xi_{\nu_\mu}|$ or $|\xi_{\nu_\tau}|=1.4$ with a 2σ upper limit of $|\xi_{\nu_\mu}|$ or $|\xi_{\nu_\tau}|\leqslant 2.5$.

DOI: 10.1103/PhysRevD.65.123504 PACS number(s): 98.80.Cq, 11.30.Fs, 14.60.St, 98.80.Es

I. INTRODUCTION

The present relic neutrino number asymmetry is not directly observable. Hence, there is no firm experimental basis for postulating that the lepton number for each species is zero. Charge neutrality, however, demands that any universal net lepton number beyond the net baryon number must reside entirely within the neutrino sector. It has been suggested that the total lepton number could be large in the context of the $SU(5)$ and $SO(10)$ grand unified theories [1–4], or supersymmetric baryogenesis [5–7] based upon the Affleck-Dine scenario [8]. Such mechanisms might generate lepton number asymmetry up to ten orders of magnitude larger than the baryon number asymmetry. Furthermore, even if one demands that $B-L\approx 0$, it is possible for the lepton numbers L_l of individual neutrino species to be large compared to the baryon number of the universe, B, as long as the net total lepton number is small.

Moreover, there presently exists at least some marginal cosmological evidence for neutrino asymmetry. For example, neutrinos with large lepton asymmetry and masses ~ 0.07 eV might be required to explain the existence of cosmic rays with energies in excess of the Greisen-Zatsepin-Kuzmin cutoff [9,10]. Also, degenerate, massive (2.4 eV) neutrinos might be required [11] to provide a good fit to the power spectrum of large scale structure in mixed dark matter models. It is thus important to carefully scrutinize the limits which cosmology places on the allowed values of a possible universal lepton asymmetry. Indeed, a number of recent papers [12–23] have addressed this issue with varying degrees of complexity. The present work differs from those in several details as summarized below. It represents an independent examination of this issue.

II. PRESENT APPROACH

In a recent paper [24] we considered new constraints imposed on neutrino degeneracy from primordial nucleosynthesis. Particular attention was paid to the neutrino decoupling temperatures before the nucleosynthesis epoch. Of relevance to the present work is that we have shown that neutrinos can decouple at a higher temperature than estimated in earlier studies [25]. This means that more particle degrees of freedom could be present at neutrino decoupling. This causes the relic neutrino temperature to be lower by simple entropy considerations. A smaller relic neutrino energy density implies that larger neutrino degeneracies may be allowed. For example, we have shown that interesting regions of the model parameters for big-bang nucleosynthesis (BBN) are allowed such that substantial lepton asymmetry and baryon density (even $\Omega_b h^2\approx 1$ where h is the present value of the Hubble constant in units of 100 km sec^{-1} Mpc^{-1}) are possible while still satisfying the adopted abundance constraints from primordial nucleosynthesis.

A stronger constraint, however, on lepton asymmetry comes from the power spectrum of fluctuations in the cosmic microwave background (CMB) which we now address in the present paper. We apply a likelihood analysis of neutrino-degenerate models to the combined latest BOOMERANG [26], DASI [27] and MAXIMA-1 [28] results. We note, however, that a recent analysis [29] of the implications of neutrino oscillations derived from a combination of the atmospheric, and solar neutrino constraints implies much tighter limits on degeneracy for all neutrino flavors. If neutrino oscillation parameters are in the range of the large mixing angle solution then an upper limit of $|\xi_{\nu_i}|\lesssim 0.07$ applies to all

 ©2002 The American Physical Society

neutrino flavors. The limits derived here do not assume any particular model for neutrino mixing, and should be taken as independent of, and complementary to, those constraints.

The implications of the CMB data for neutrino-degenerate cosmologies have been noted in a number of recent papers [16–23]. The constraints on the effective number of relativistic particles can also arise in other contexts, such as cosmic quintessence [30,31]. The present work, however, differs from those in several respects. For one, we consider the most recent combined data sets, not just the first year BOOMERANG data as in [16,17] and generate a marginalized likelihood function for the neutrino degeneracy and other cosmological parameters. Many of the existing studies have marginalized over a more limited set of cosmological parameters. For example, in [19] only $\Omega_b h^2$ and and neutrino degeneracy were marginalized to set limits while other cosmological parameters were set to various fixed values. In [17], for example, no likelihood analysis was made. In [16] a likelihood analysis was made but without window functions.

For the present work we use the updated RADPACK package [32] described below which includes all relevant window functions. Another difference between our analysis and other works is that our marginalization utilizes a global minimum search algorithm [33] rather than a discrete grid of cosmological parameters. Marginalizing parameters for each fixed value of one parameter requires at least 1000 model calculations to get 10^{-4} accuracy for the χ^2 minima even by using this algorithm. Nevertheless, in this way we are sure to identify the true marginalized likelihood functions.

The most similar recent likelihood analysis to that described here is in the work of [21]. Our analysis differs from [21] in several respects. In the present work we make use of our deduced new family of baryon densities and lepton asymmetries allowed by BBN to assign weak top-hat priors on the derived likelihood functions. This differs from that in [21] in which separate Gaussian likelihood functions were evaluated for the nucleosynthesis constraints and the CMB. A total likelihood function was then defined by marginalizing over the product of these two functions. We prefer our method because the uncertainties in the BBN constraints are dominated by systematic errors. Systematic errors are not equivalent to random Gaussian errors. We thus prefer weak top-hat priors as a more realistic representation of the systematic errors in the BBN constraints.

One other important difference is that we adopt a strong Gaussian prior of $h=0.72\pm0.08$ based upon the Hubble Key Project results [34]. In [21] a weak top-hat prior of $h=0.65\pm0.20$ was adopted. As noted above, another difference between the present work and all previous results is that we consider carefully the change in background neutrino temperatures as a function of degeneracy. Although this is a small effect for low degeneracies, it can slightly affect the upper limits.

III. NEUTRINO-DEGENERATE BBN

Neutrino degeneracy affects BBN in two ways. The inclusion of a small amount of electron ν_e degeneracy resets the equilibrium neutron to proton ratio at weak-reaction freezeout to $n/p=\exp\{-\Delta m/T-\xi_{\nu_e}\}$. This can cause a reduction in the primordial helium abundance. Indeed, it has been argued [35,36] that the apparent conflict between the low helium abundance inferred from HII regions of metal poor galaxies and the low Lyman-α deuterium abundance may even require ν_e degeneracy for its resolution. The present deuterium-absorption limits on $\Omega_b h^2 \approx 0.020\pm0.002$ (2σ) requires a large primordial helium abundance of $Y_p \gtrsim 0.25$ and substantial destruction of primordial ^{7}Li in stars. These conditions tax even the most generous adopted limits from observed light-element abundances [37]. Thus, a modification of BBN which allows for large values of $\Omega_b h^2$ while still satisfying the constraints from light-element abundances is worth investigating. Such conditions are easily satisfied by neutrino-degenerate models.

The inclusion of either ν_μ or ν_τ degeneracy on the other hand only enhances the background energy density and therefore the universal expansion rate. During the radiation dominated epoch, relativistic neutrinos contribute a large fraction of the mass energy. Thus, even a small modification of the neutrino energy density can significantly affect the expansion.

The energy density ρ_ν due to degenerate neutrinos (or any other fermions) are described by the usual Fermi-Dirac distribution functions $f_\nu=[\exp(E/T_\nu-\xi_\nu)+1]^{-1}$, where the neutrino degeneracy parameter is defined by $\xi_\nu \equiv \mu_\nu/T_\nu$, and μ_ν is the neutrino chemical potential. Thus, we have

$$\rho_\nu+\rho_{\bar{\nu}}=\frac{1}{2\pi^2}\int_0^\infty dp\, p^2 E_\nu (f_\nu(p)+f_{\bar{\nu}}(p)), \tag{1}$$

where p denotes the magnitude of the 3-momentum, and $E_\nu=\sqrt{p^2+m_\nu^2}$, with m_ν the neutrino mass. Here and throughout the paper we use natural units ($\hbar=c=k_B=1$).

For the present discussion it is sufficient to only consider massless neutrinos. (Possible limits on neutrino-degenerate models with massive neutrinos are considered in [18].) The energy density in massless neutrinos becomes

$$\rho_\nu+\rho_{\bar{\nu}}=\frac{7}{8}\frac{\pi^2}{15}\sum_i T_{\nu_i}^4\left[1+\frac{15}{7}\left(\frac{\xi_{\nu_i}}{\pi}\right)^4+\frac{30}{7}\left(\frac{\xi_{\nu_i}}{\pi}\right)^2\right], \tag{2}$$

from which it is clear that degeneracy in any neutrino species tends to increase the energy density. The associated increased expansion rate tends to increase the neutrino decoupling temperature. This causes an increase in the primordial helium and other light-element abundances.

Since ξ_{ν_μ} and ξ_{ν_τ} primarily affect the expansion rate, they are roughly interchangeable as far as their effects on nucleosynthesis or the CMB are concerned. Furthermore, it now seems likely [38] that the mixing parameters for ν_μ and ν_τ involve a large mixing angle and small δm^2. In this case it is plausible that the muon and tau neutrinos were interconverted in the early universe and would therefore obtain nearly an identical degeneracy parameter if an asymmetry exists. Thus, we adopt a conservative model in which the μ and τ neutrinos are equally degenerate, $|\xi_{\nu_\mu}|=|\xi_{\nu_\tau}|\equiv\xi_{\nu_{\mu,\tau}}$.

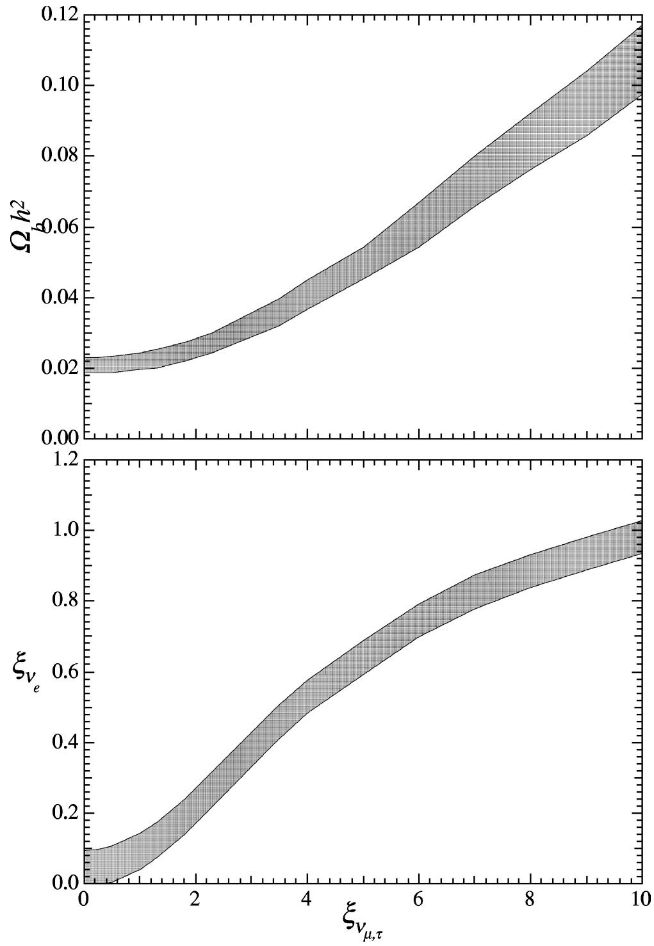

FIG. 1. Allowed values of $\Omega_b h^2$ and ξ_{ν_e} for which the constraints from light-element abundances are satisfied as a function of $\xi_{\nu_{\mu,\tau}}$.

As shown in [24], for each value of $\xi_{\nu_{\mu,\tau}}$ there is a unique range of ξ_{ν_e} and $\Omega_b h^2$ which satisfies the combined deuterium and primordial helium constraints. The allowed family of neutrino-degenerate models employed in this work is summarized in Fig. 1.

This figure differs slightly from the family of allowed solutions given in [24] in that we have adopted the newer D/H constraint from [39] [i.e., D/H$=3.0(\pm 0.4)\times 10^{-5}$] and slightly different limits on the primordial helium abundance ($0.228 \leqslant Y_p \leqslant 0.248$). In the limit of the standard nondegenerate big bang ($\xi_{\nu_{\mu,\tau}} = \xi_{\nu_e} = 0$) our limits on $\Omega_b h^2$ reduce to those of [40,41], i.e., $\Omega_b h^2 = 0.021 \pm 0.002$. The allowed shaded regions in Fig. 1 will be adopted as weak top-hat priors in the CMB likelihood analysis described below. These regions include both the uncertainties from the abundance constraints described above and the uncertainties in the BBN model predictions [42].

IV. CMB POWER SPECTRUM

Having defined the family of allowed priors from BBN we can now do a likelihood search for optimum cosmological parameters which fit the CMB data. Several recent works [12–16] have explained how neutrino degeneracy can dramatically alter the power spectrum of the CMB. For massless neutrinos it can be shown [15] that the only effect of neutrino degeneracy is to increase the background pressure and energy density of relativistic particles. The essence of this constraint is that degenerate neutrinos increase the energy density in radiation at the time of photon decoupling and delay the time of matter-radiation energy-density equality. This mainly causes an increase in the amplitude of the first acoustic peak in the CMB power spectrum at $l \approx 200$. For example, based upon a χ^2 analysis [13] of 19 experimental points and window functions, it was concluded in [15] that $\xi_\nu \leqslant 6$ for a single degenerate neutrino species with an $\Omega_\Lambda = 0$ cosmology.

However, in the existing CMB constraint calculations [12–23] only small degeneracy parameters with the standard relic neutrino temperatures were studied in the derived constraint. Hence, the possible effect of a diminished relic neutrino temperature at high degeneracy needs to be considered. To investigate this we have done calculations of the CMB power spectrum, $\Delta T^2 = l(l+1)C_l/2\pi$ based upon the CMBFAST code of Seljak and Zaldarriago [43]. We have explicitly modified this code to account for the contribution of massless degenerate neutrinos with varying relic neutrino temperatures T_{ν_i} for each species [24].

The experimental uncertainties are non-Gaussian, but can be well represented by an offset log-normal distribution [32]. As in [27] we have evaluated the χ^2 goodness of fit for a range of theoretical power spectra C_l as follows: We define the goodness of fit by

$$\chi^2 = \sum_{i,j} (Z_i^t - \overline{Z_i^d}) M_{ij}^Z (Z_j^t - Z_j^d) + \chi_{cal}^2, \tag{3}$$

where separate summation over the different data sets is implied. For each set of binned power data Z_i^d we utilize the published off-set log-normal data from the three data sets:

$$Z_i^d \equiv \ln(D_i + x_i) \tag{4}$$

where D_i is the measured band power. The corresponding binned theoretical power spectra are

$$Z_i^t \equiv \ln\left[\sum_l \epsilon_i (W_{il}/l)(\mathcal{C}_l + x_i) \right], \tag{5}$$

where the ϵ_i are the published calibration uncertainties taken to be 8%, for MAXIMA-1 and DASI, and 20% for BOOMERANG. Window functions W_{il} for the three data sets are available on the world wide web. The error matrix is simply

$$M_{ij}^Z = M_{ij}(D_i + x_i)(D_j + x_j) \tag{6}$$

where M_{ij} is the weight matrix for the band powers D_i. The effect of the calibration uncertainty on the goodness of fit is obtained from

$$\chi^2_{cal} = \sum_i \frac{(\epsilon_i - 1)^2}{\sigma_i^2}, \tag{7}$$

where σ_i is the experimental uncertainty. The total χ^2 evaluated in this way can be converted [27,46] into a likelihood function for each parameter x marginalized over the remaining parameter set $\vec{y}$:

$$\mathcal{L}(x) = \int P_{prior}(x,\vec{y}) \exp(-\chi^2/2) d\vec{y}. \tag{8}$$

In neutrino-degenerate models which satisfy the constraints from primordial nucleosynthesis [24], increasing the neutrino-degeneracy must be accompanied by a commensurate increase in baryon density. Fits to the CMB power spectrum for large degeneracy [24], therefore show a suppression of the second acoustic peak due to baryon drag [12].

Indeed, such suppression of the second acoustic peak seemed to be present in the first reported power spectra based upon the balloon-based CMB sky maps from the BOOMERANG [44] and MAXIMA-1 [28] Collaborations. This remains true for the likelihood analysis based upon MAXIMA-1 data [45] which indicates $\Omega_b h^2 = 0.030^{+0.018}_{-0.010}$ (2σ). However, in the most recent data sets from BOOMERANG [26] and DASI [27] the second peak has become much better defined. Both the BOOMERANG and DASI data sets now imply $\Omega_b h^2 = 0.022^{+0.004}_{-0.003}$ (1σ) $(\eta_{10} = 6.00^{+1.10}_{-0.81})$. This value is close to the value implied by the cosmic deuterium abundance in high-redshift Lyman-α clouds observed along the line of sight to background quasars [40,41] $\Omega_b h^2 = 0.020 \pm 0.001$ (1σ) $(\eta_{10} = 5.46 \pm 0.27)$. Hence, the newer data imply at most a marginal requirement for a larger baryon density or neutrino degeneracy. Indeed, these new data tighten constraints on the possibility of degenerate cosmological neutrinos. In the present paper we explore the new limits on possible neutrino degeneracy implied by the combined data sets and our BBN constraints.

V. RESULTS

We limit our consideration to flat $\Omega_{tot} = \Omega_M + \Omega_\Lambda = 1$ cosmological models with ionization parameter $\tau = 0$. This is sufficient for our purposes since the likelihood functions so deduced are not expected to be much different if Ω_{tot} or τ are varied (cf. [26]). This is because τ and the spectrum tilt n are nearly degenerate parameters, i.e., changing one is equivalent to changing the other. Moreover, Ω_{tot} is generally tightly constrained to be near unity anyway.

There are then nine parameters over which we marginalize. These are $(\Omega_b h^2, \xi_{\nu_{\mu,\tau}}, \xi_{\nu_e}, \Omega_\Lambda, h, n, \epsilon_i)$. We utilize a strong Gaussian prior for $h = 0.72 \pm 0.08$ and for the calibration uncertainties ϵ_i as listed above. Also, as noted above, we adopt weak top-hat priors when marginalizing over $\Omega_b h^2$ and ξ_{ν_e} designated by the shaded regions of Fig. 1 for each value of $\xi_{\nu_{\mu,\tau}}$. In [22] it has been argued that without some priors on Ω_M (through flatness, h, etc.) it is difficult to place bounds on the amount of relativistic matter. Hence, the model constraints adopted here are probably required to break the parameter degeneracy between relativistic and nonrelativistic matter. Ultimately, however, high resolution sky maps such as the Planck mission will be able to determine separately the amounts of relativistic and nonrelativistic matter.

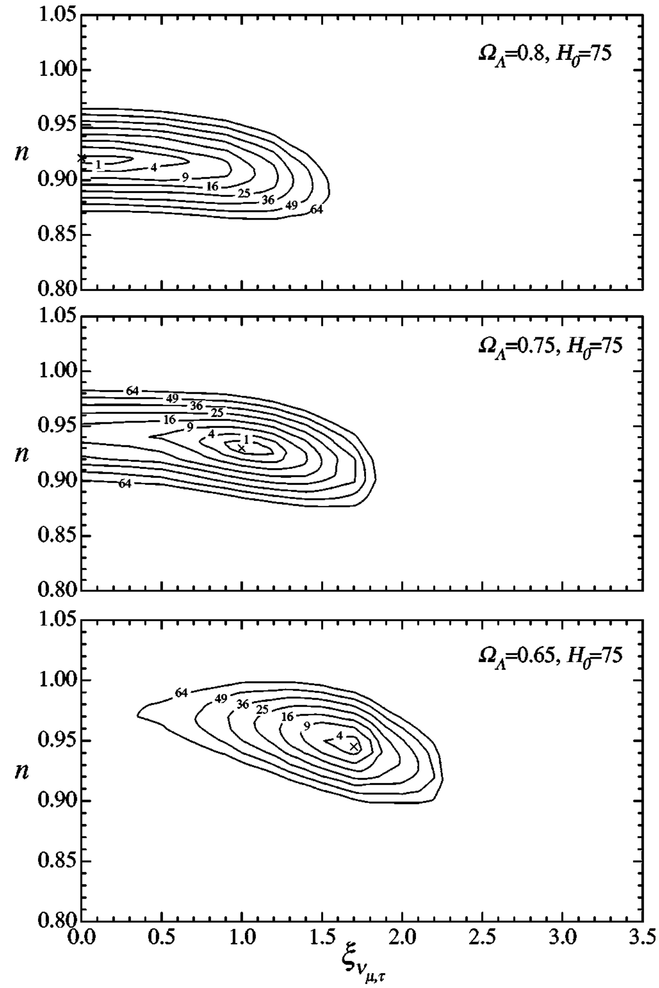

FIG. 2. Contours of constant goodness of fit $\Delta\chi^2$ in the $\xi_{\nu_{\mu,\tau}}$ vs n plane for three different Ω_Λ and $h = 0.75$ values as indicated. Note the well developed minimum for $\xi_{\nu_{\mu,\tau}} \approx 1\text{–}2$ and $\Omega_\Lambda \leqslant 0.75$.

Figure 2 illustrates one of the main results of this study. Shown are contours of constant $\Delta\chi^2$ in the $\xi_{\nu_{\mu,\tau}}$ vs n plane for three values of the cosmological constant (i.e., $\Omega_\Lambda = 0.65$, 0.75, and 0.8) and for fixed $h = 0.75$ as noted. For $\Omega_\Lambda \leqslant 0.75$, a minimum in χ^2 develops for values of $\xi_{\nu_{\mu,\tau}} \approx 1\text{–}2$. Indeed, for a simple 2 parameter search with fixed values, $\Omega_\Lambda = 0.75$ and $h = 0.75$, neutrino degeneracy is preferred at the level of more than 3σ over a nondegenerate model. For smaller values of Ω_Λ, this minimum for neutrino-degenerate models becomes even more pronounced.

A second minimum also develops for higher degeneracy $(\xi_{\nu_{\mu,\tau}} \approx 11.4)$ as noted in [24]. This is due to the large change in particle degrees of freedom for neutrinos which decouple just above the QCD transition. However, the goodness of fit is so poor $(\Delta\chi^2 \geqslant 500)$ that it would not be apparent in the contours drawn on Fig. 2. Hence, this large-degeneracy so-

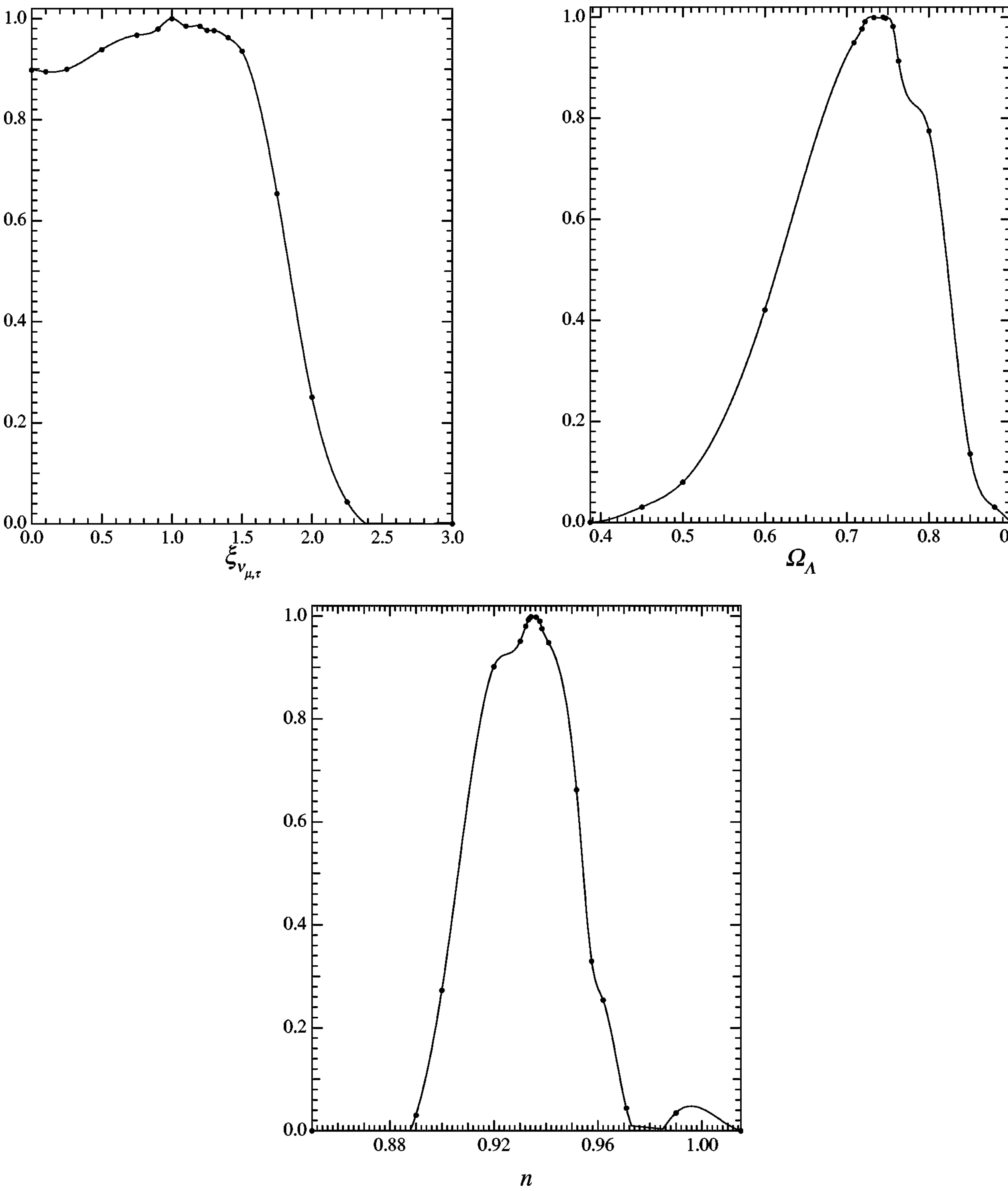

FIG. 3. Marginalized likelihood distribution functions for $\xi_{\nu_{\mu,\tau}}$, Ω_Λ, and the spectrum tilt n as labeled.

lution is definitely ruled out by the current CMB power spectrum.

Figure 3 shows the marginalized likelihood distributions for three of the cosmological parameters $(\xi_{\nu_{\mu,\tau}}, \Omega_\Lambda, n)$ considered here. For the present study the likelihood functions for ξ_{ν_e}, $\Omega_b h^2$ and Ω_M are related to these since ξ_{ν_e} and $\Omega_b h^2$ are functions of $\xi_{\nu_{\mu,\tau}}$ and $\Omega_M = 1 - \Omega_\Lambda$. From Fig. 3 we deduce optimum values of $\Omega_\Lambda = 0.74^{+0.08}_{-0.11}$ and $n = 0.93 \pm 0.02$. A slight preference for finite neutrino degeneracy is evident $\xi_{\nu_{\mu,\tau}} = 1.0^{+0.8(1\sigma)}_{-1.0(0.5\sigma)}$. This preference, however, is not particularly significant. For now, the data mainly imply (2σ) upper limits on neutrino degeneracy of $\xi_{\nu_{\mu,\tau}} \leqslant 2.1$. This value implies upper limits of $\xi_{\nu_e} \leqslant 0.30$ and $\Omega_b h^2 \leqslant 0.030$ from Fig. 1. For a single large-degeneracy neutrino species, these lim-

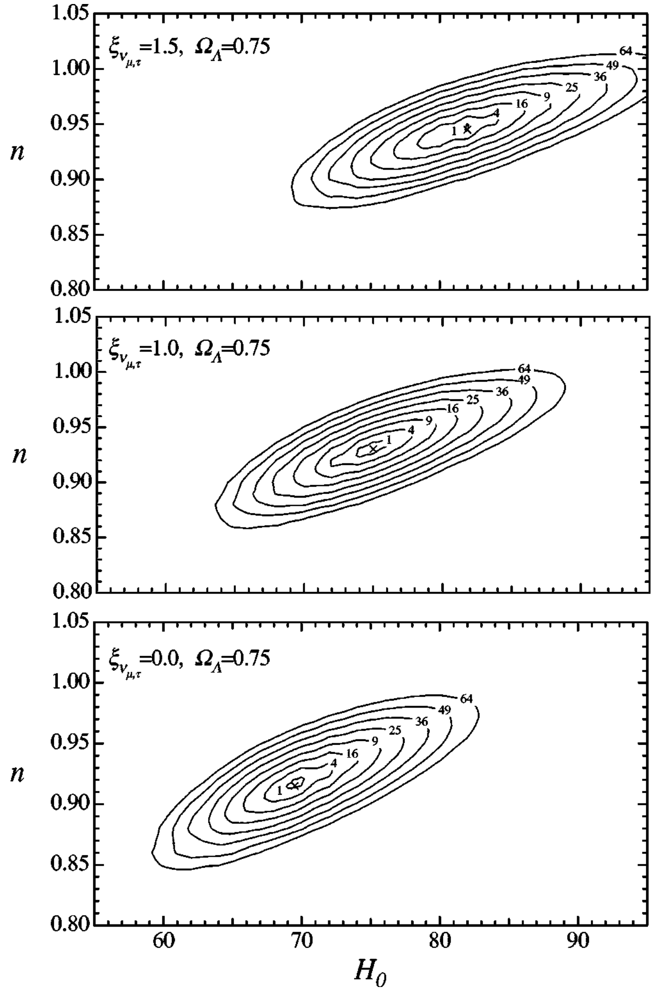

FIG. 4. Contours of constant goodness of fit $\Delta\chi^2$ in the H_0 vs n plane for $\Omega_\Lambda = 0.75$ and neutrino degeneracy parameters $\xi_{\nu_{\mu,\tau}} = 0.$, 1.0, and 1.5 as labeled.

its become $|\xi_{\nu_\mu}|$ or $|\xi_{\nu_\tau}| = 1.4^{+1.1(1\sigma)}_{-1.4(0.5\sigma)}$ with a 2σ upper limit of $\xi_{\nu_{\mu,\tau}} \leqslant 2.5$.

Our results are slightly more stringent than the results from [21] who found an equivalent single species upper limit based upon the CMB data alone of ξ_{ν_μ} or $\xi_{\nu_\tau} < 2.9$. This is at first surprising given that we have adopted weak top-hat (instead of Gaussian) priors for the BBN constraint. We have traced the main reason for the more stringent upper limits derived here to our adoption of a strong Gaussian prior on h. A larger neutrino degeneracy is possible if larger values of h are permitted. Figure 4 shows contours of constant $\Delta\chi^2$ in the H_0 vs n plane for $\Omega_\Lambda = 0.75$ models with $\xi_{\nu_{\mu,\tau}} = 0$, 1.0 and 1.5 as labeled. This illustrates the sensitivity of the degenerate solution to the assumed prior for h. If weaker priors on h are adopted, or if new larger values of h in the upper range of the present Key-Project uncertainty are ever determined, the neutrino-degenerate models could become strongly preferred over the nondegenerate models. The $\xi_{\nu_{\mu,\tau}} = 0$ nondegenerate solution is only the preferred minimum, for all values of Ω_Λ, when $h \leqslant 0.70$. This is consistent with the results of [45,46].

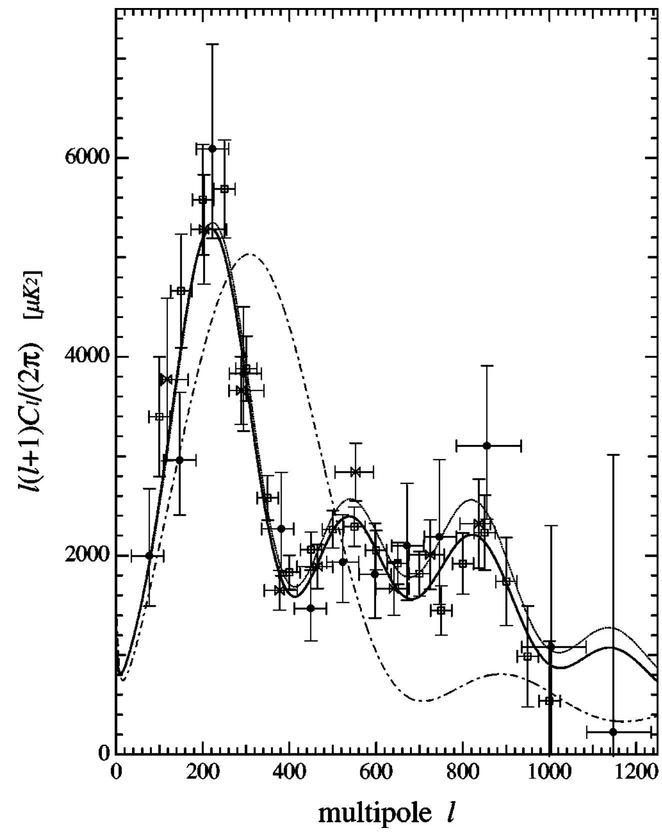

FIG. 5. Fits to the power spectrum of fluctuations in the CMB. The solid line shows the best neutrino-degenerate fit ($\xi_{\nu_{\mu,\tau}} = 1.0$). The dotted line shows a best nondegenerate ($\xi_{\nu_{\mu,\tau}} = \xi_{\nu_e} = 0$) model. For illustration, the dot-dashed line also shows the large-degeneracy minimum ($\xi_{\nu_{\mu,\tau}} = 11.4$).

Figure 5 shows some optimum model power spectra compared with the combined data sets. The solid line shows our optimum degenerate model for which $(\Omega_b h^2, \xi_{\nu_{\mu,\tau}}, \xi_{\nu_e}, \Omega_\Lambda, h, n) = (0.021, 1.0, 0.09, 0.74, 0.74, 0.93)$. For this parameter set we obtain a total $\chi^2 = 29.8$ for 29 degrees of freedom implying a nearly perfect fit. For comparison, the dotted line shows the best nondegenerate ($\xi_{\nu_{\mu,\tau}} = \xi_{\nu_e} = 0$) model $[(\Omega_b h^2, \Omega_\Lambda, h, n) = (0.021, 0.62, 0.62, 1.0)$ (dotted line)] from [26]. For illustration we also show the large-degeneracy minimum $[(\Omega_b h^2, \xi_{\nu_{\mu,\tau}}, \xi_{\nu_e}, \Omega_\Lambda, h, n) = (0.052, 11.4, 0.74, 0.45, 0.80, 0.72)$ (dot-dashed line)].

VI. CONCLUSIONS

In neutrino-degenerate models the larger baryon density associated with the observed low deuterium abundance can be more easily accommodated than in nondegenerate models. Moreover, neutrino-degenerate models provide a slightly improved goodness of fit for the latest CMB power spectra from BOOMERANG, DASI, and MAXIMA-1.

Using cosmological models consistent with the constraints from light-element abundances as a function of the neutrino degeneracy parameter $\xi_{\nu_{\mu,\tau}}$, we have shown that a slight maximum in the likelihood function forms for neutrino-degenerate models with $\xi_{\nu_{\mu,\tau}} \approx 1$. However, the improvement over the nondegenerate models is only at the level of about 0.5σ. Although this minimum is not particularly statistically significant for the present data set and assumed priors, it could become much more pronounced should larger values of h and/or smaller values of Ω_Λ ever be established near their current 1σ limits.

The present data place 2σ limits for two identical large-degeneracy neutrino species of $\xi_{\nu_{\mu,\tau}} \leqslant 2.1$, which implies $\xi_{\nu_e} \leqslant 0.30$. For only one species with large degeneracy, the limit becomes $|\xi_{\nu_\mu}|$ or $|\xi_{\nu_\tau}| \leqslant 2.5$. This is slightly more restrictive than the limits deduced in other studies.

Finally, we remark that, since neutrino degeneracy is now limited to such small values, the present work has established that the effects of the changing neutrino decoupling temperature with increased degeneracy has little effect. Hence, previous studies which neglected this effect are justified.

ACKNOWLEDGMENTS

One of the authors (G.J.M.) wishes to acknowledge the hospitality of the National Astronomical Observatory of Japan where some of this work was done. This work has been supported in part by the Grant-in-Aid for Scientific Research (10640236,10044103,11127220,12047233, 13002001,13640313) of the Ministry of Education, Science, Sports, and Culture of Japan. Work at Univ. Notre Dame supported by DOE Nuclear Theory Grant (DE-FG02-95-ER40934). Work at Univ. Oklahoma supported by NSF CAREER grant AST-0094335.

[1] J.A. Harvey and E.W. Kolb, Phys. Rev. D **24**, 2090 (1981).
[2] J.N. Fry and C.J. Hogan, Phys. Rev. Lett. **49**, 1873 (1982).
[3] A. Dolgov and D.P.J. Kirilova, Moscow Phys. Soc. **1**, 217 (1991).
[4] A. Dolgov, Phys. Rep. **222**, 309 (1992).
[5] A. Casas, W.Y. Cheng, and G. Gelmini, Nucl. Phys. **B538**, 297 (1999).
[6] J. McDonald, Phys. Rev. Lett. **84**, 4798 (2000).
[7] M. Bastero-Gil and S.F. King, Phys. Rev. D **63**, 123509 (2001); Nucl. Phys. **B538**, 297 (1999).
[8] I. Affleck and M. Dine, Nucl. Phys. **B249**, 361 (1985).
[9] K. Greisen, Phys. Rev. Lett. **16**, 748 (1966).
[10] G. Gelmini and A. Kusenko, Phys. Rev. Lett. **82**, 5202 (1999).
[11] G.B. Larsen and J. Madsen, Phys. Rev. D **52**, 4282 (1995).
[12] W. Hu *et al.*, Phys. Rev. D **52**, 5498 (1995).
[13] C.H. Lineweaver and D. Barbosa, Astron. Astrophys. **319**, 799 (1998); Astrophys. J. **496**, 624 (1998).
[14] W.H. Kinney and A. Riotto, Phys. Rev. Lett. **83**, 3366 (1999).
[15] J. Lesgourgues and S. Pastor, Phys. Rev. D **60**, 103521 (1999).
[16] S. Hannestad, Phys. Rev. Lett. **85**, 4203 (2000).
[17] J. Lesgourgues and M. Peloso, Phys. Rev. D **62**, 081301 (2000).
[18] J. Lesgourgues and A.R. Liddle, Mon. Not. R. Astron. Soc. **327**, 1307 (2001).
[19] J.P. Kneller *et al.*, Phys. Rev. D **64**, 123506 (2001).
[20] S. Esposito *et al.*, Phys. Rev. D **63**, 043004 (2001).
[21] S.H. Hansen *et al.*, Phys. Rev. D **65**, 023511 (2002).
[22] R. Bowen *et al.*, astro-ph/0110636.
[23] A.D. Dolgov, Phys. Rep. (to be published).
[24] M. Orito, T. Kajino, G.J. Mathews, and R.N. Boyd, astro-ph/0005446; Nucl. Phys. **A688**, 17c (2001).
[25] H. Kang and G. Steigman, Nucl. Phys. **B372**, 494 (1992).
[26] BOOMERANG Collaboration, C.B. Netterfield *et al.*, astro-ph/0104460.
[27] DASI Collaboration, C. Pryke *et al.*, astro-ph/0104490.
[28] MAXIMA-1 Collaboration, S. Hanany *et al.*, Astrophys. J. Lett. **536**, L59 (2000).
[29] A.D. Dolgov *et al.*, hep-ph/0201287; Nucl. Phys. B (to be published); P. DiBari, Phys. Rev. D **65**, 043509 (2002); K.N. Abazajian, J.F. Beacom, and N.F. Bell, astro-ph/0203442.
[30] R. Bean, S.H. Hansen, and A. Melchiorri, Phys. Rev. D **64**, 103508 (2001).
[31] M. Yahiro *et al.*, Phys. Rev. D **65**, 063502 (2002).
[32] J.R. Bond, A.H. Jaffe, and L. Knox, Astrophys. J. **533**, 19 (2000).
[33] D.R. Jones, in *Encyclopedia of Optimization*, edited by C.A. Floudas and P.M. Pardalos (Kluwer Academic, Dordrecht, 2001), Vol. 1, pp. 431–440.
[34] W.L. Freedman *et al.*, Astrophys. J. **553**, 47 (2001).
[35] K. Kohri, M. Kawasaki, and K. Sato, Astrophys. J. **490**, 72 (1997).
[36] T. Kajino and M. Orito, Nucl. Phys. **A629**, 538c (1998).
[37] K.A. Olive, G. Steigman, and T.P. Walker, Phys. Rep. **333**, 389 (1999).
[38] S.M. Bilenky, hep-ph/0110306.
[39] J.M. O'Meara *et al.*, Astrophys. J. **552**, 718 (2001).
[40] K.M. Nollett and S. Burles, Phys. Rev. D **61**, 123505 (2000); S. Burles, K.M. Nollett, and M.S. Turner, *ibid.* **63**, 063512 (2001).
[41] D. Tytler, J.M. O'Meara, N. Suzuki, and D. Lubin, Phys. Scr. **85**, 12 (2000).
[42] R.H. Cyburt, B.D. Fields, and K.A. Olive, New Astron. **6**, 215 (2001).
[43] U. Seljak and M. Zaldarriago, Astrophys. J. **469**, 437 (1996).
[44] BOOMERANG Collaboration, P. de Bernardls *et al.*, Nature (London) **404**, 955 (2000); Astrophys. J. **564**, 559 (2002); astro-ph/0105296.
[45] A. Balbi *et al.*, Astrophys. J. Lett. **545**, L1 (2000); **558**, L145(E) (2001).
[46] A.E. Lange *et al.*, Phys. Rev. D **63**, 042001 (2001).