Anthropic reasons for nonzero flatness and Λ

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In some cosmological theories with varying constants there are anthropic reasons why the expansion of the universe must not be too *close* to flatness or the cosmological constant too close to zero. Using exact theories which incorporate time variations in α and in G we show how the presence of negative spatial curvature and a positive cosmological constant play an essential role in bringing to an end variations in the scalar fields that drive time changes in these "constants" during any dust-dominated era of a universe's expansion. In spatially flat universes with $\Lambda=0$ the fine structure constant grows to a value which makes the existence of atoms impossible.

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I. INTRODUCTION

The collection of considerations now known as the anthropic principles emerged from attempts by Whitrow [1] to understand why it is unsurprising that we find space to have three dimensions, and by Dicke [2] to understand the inevitability of Dirac "large number" coincidences in cosmology. Dicke recognized that it was unnecessary to introduce the idea of a time-varying gravitational constant in order to understand why we could not fail to observe that the number of protons in the observable universe is of the order at the square of the ratio of electromagnetic to gravitational force strengths. Subsequently, Dicke inspired a detailed observational and theoretical investigation of gravity theories in which the Newtonian gravitational constant becomes a space-time variable. He was partly motivated by apparent discrepancies between the predictions of standard general relativity and observations of the perihelion precession of Mercury. These discrepancies were subsequently ascribed to errors in the measurements of the shape and diameter of the Sun created by solar surface activity [3].

There have been many investigations of the apparent coincidences that allow complexity to exist in the universe (see [4–7]). Typically, they examine the stability of life-supporting conditions to small (or large) perturbations to the values of constants of nature or to quantities fixed by cosmological "initial" conditions at $t=0$ or $t=-\infty$. These in turn divide into studies of two sorts: first, those in which the hypothetical changes introduced to the "constants" are self-consistently permitted by the cosmological or physical theory employed; and second, those in which they are not. An investigation of the first kind might be one in which the cosmological initial conditions were enlarged to allow anisotropies or the possibility of a significant deviation from flatness. An investigation of the second type might note that a change in the observed value of the electron to proton mass ratio to another fixed value would make it difficult to produce ordered molecular structures. Studies of universes in which traditional "constants" of nature are changed are restricted by the lack of self-consistent theories which allow all these possible changes to be accommodated. Without them, it is impossible to determine the possible knock-on effects of varying one constant on others.

There are some exceptions. Varying gravitation "constant," G (or dimensionless constants formed with it like Gm^2/hc for any mass m), can be studied using scalar-tensor gravity theories [8]. A varying fine structure "constant" can be studied using the theory of Bekenstein and Sandvik, Barrow and Magueijo (BSBM) [9,10]. Moreover, the formulation of physical theories whose true constants inhabit more than three space dimensions provides a framework for the rigorous study of the simultaneous variation of their three-dimensional counterparts [11–13]. Recently there has also been much interest in theories where a variation in the fine structure constant is due to a change in the light propagation speed [14–16]. In another paper we propose various methods for experimentally distinguishing between these different theories [17].

Observational evidence for a variation in a traditional constant can be found without the need for a self-consistent theory of its variation simply by demonstrating incompatibility with the predictions of the standard theory. The most observationally sensitive "constant" is the electromagnetic fine structure constant, $\alpha\equiv e^2/\hbar c$, and recent observations motivate the formulation of varying-α theories. The new many-multiplet technique of Webb *et al.*, [18,19], exploits the extra sensitivity gained by studying relativistic transitions to different ground states using absorption lines in quasar (QSO) spectra at medium redshift. It maximizes the information extracted from the data set and has provided the first evidence that the fine structure constant might change with cosmological time [18–20]. The trend of these results is that the value of α was lower in the past, with $\Delta\alpha/\alpha=-0.72\pm0.18\times10^{-5}$ for $z\approx0.5$–3.5. Other investigations [21–23] have claimed preferred nonzero values of $\Delta\alpha<0$ to best fit the cosmic microwave background (CMB) and big bang nucleosynthesis (BBN) data at $z\approx10^3$ and $z\approx10^{10}$ respectively but appeal to much larger variations. We have shown that the simplest theory which joins varying α to general relativity via the propagation of a scalar field can explain these observations together with the lack of evidence for a similar level of variation locally, 2 billion years ago, or at very high redshifts, $z\geqslant 10^3$. In this paper we will show how this theory also provides some novel anthropic perspectives on the evolution of our universe or others.

There have been several studies, following Carter [24] and Tryon [25], of the need for life-supporting universes to expand close to the "flat" Einstein de Sitter trajectory for long periods of time. This ensures that the universe cannot collapse back to high density before galaxies, stars, and biochemical elements can form by gravitational instability, or expand too fast for stars and galaxies to form by gravitational instability (see also [26,27] and [5]). Likewise, it was pointed out by Barrow and Tipler [5] that there are similar anthropic restrictions on the magnitude of any cosmological constant, Λ. If it is too large in magnitude it will either precipitate premature collapse back to high density (if $\Lambda<0$) or prevent the gravitational condensation of any stars and galaxies (if $\Lambda>0$). Thus existing studies provide anthropic reasons why we can expect to live in an old universe that is neither too far from flatness nor dominated by a much stronger cosmological constant than observed ($|\Lambda|\leqslant 10|\Lambda_{obs}|$).

Inflationary universe models provide a possible theoretical explanation for proximity to flatness but no explanation for the smallness of the cosmological constant. Varying speed of light theories [14–16,28,29] offer possible explanations for proximity to flatness and smallness of a classical cosmological constant (but not necessarily for one induced by vacuum corrections in the early universe). Here, we shall show that if we enlarge our cosmological theory to accommodate variations in some traditional constants then *it appears to be anthropically disadvantageous for a universe to lie too close to flatness or for the cosmological constant to be too close to zero*. This conclusion arises because of the coupling between time variations in constants like α and the curvature or Λ, which control the expansion of the universe. The onset of a period of Λ or curvature domination has the property of dynamically stabilizing the constants, thereby creating favorable conditions for the emergence of structures. This point has been missed in previous studies because they have never combined the issues of Λ and flatness and the issue of the values of constants. By coupling these two types of anthropic considerations we find that too little Λ or curvature can be as poisonous for life as too much.

II. TIME VARIATION OF α

First, consider a simple theory with varying $\alpha\equiv e^{2\psi}/\hbar c$ where ψ is a scalar field that can vary in space and time. A generalization of the scalar theory proposed by Bekenstein [9] described in Ref. [10] to include the gravitational effects of ψ gives the field equations

$$G_{\mu\nu}=8\pi G(T_{\mu\nu}^{matter}+T_{\mu\nu}^{\psi}+T_{\mu\nu}^{em}e^{-2\psi}), \tag{1}$$

and the ψ field obeys the equation of motion

$$\Box\psi=\frac{2}{\omega}e^{-2\psi}\mathcal{L}_{em}. \tag{2}$$

We have defined the coupling constant $\omega=(\hbar c)/l^2$, where l is the length scale down to which the theory is accurately Coulombic. It is clear that $\mathcal{L}_{em}$ vanishes for a sea of pure radiation since then $\mathcal{L}_{em}=(E^2-B^2)/2=0$. We therefore expect the variation in α to be driven by electrostatic and magnetostatic energy components rather than electromagnetic radiation. In order to make quantitative predictions we need to know how much of the nonrelativistic matter contributes to the right-hand side (RHS) of Eq. (2). This is parametrized by $\zeta\equiv\mathcal{L}_{em}/\rho$, where ρ is the energy density , and for baryonic matter $\mathcal{L}_{em}=E^2/2$. In previous papers [10,30] we showed how the cosmological value of ζ (denoted ζ_m) is largely determined by the nature of dark matter. To accommodate for a lower α in the past, as preferred by the data, the dark matter constituents need to have high magnetostatic energy content (one possible contender would be superconducting cosmic strings which have $\zeta_m\sim-1$). In line with our recent work and the observational data we will in this paper confine ourselves to negative values of ζ_m.

Assuming a homogeneous and isotropic Friedmann metric with expansion scale factor $a(t)$ and curvature parameter k we obtain the field equations ($c\equiv 1$)

$$\left(\frac{\dot a}{a}\right)^2=\frac{8\pi G}{3}\left(\rho_m(1+|\zeta_m|e^{-2\psi})+\rho_r e^{-2\psi}+\frac{\omega}{2}\dot\psi^2+\rho_\Lambda\right)-\frac{k}{a^2}, \tag{3}$$

where the cosmological vacuum energy ρ_Λ is a constant that is proportional to the cosmological constant $\Lambda\equiv 8\pi G\rho_\Lambda$. For the scalar field we have

$$\ddot\psi+3H\dot\psi=-\frac{2}{\omega}e^{-2\psi}\zeta_m\rho_m \tag{4}$$

where $H\equiv\dot a/a$. The conservation equations give for the noninteracting radiation, and matter densities $\rho_r\propto e^{2\psi}a^{-4}$ and $\rho_m\propto a^{-3}$, respectively. This theory enables the cosmological consequences of varying α to be analyzed self-consistently rather than by changing the constant value of α in the standard theory, as in the original proposals made in response to the large numbers coincidences [31].

The cosmological behavior of the solutions to these equations was studied by us [10,30] for the $k=0$ case and is shown in Fig. 1. The evolution of α is summarized as follows:

(1) During the radiation era α is constant and $a(t)\sim t^{1/2}$. It increases in the dust era, where $a(t)\sim t^{2/3}$, until the cosmological constant starts to accelerate the universe, $a(t)\sim\exp[\Lambda t/3]$, after which α asymptotes rapidly to a constant; see Fig. 1.

(2) If we set the cosmological constant equal to zero then, during the dust era, α will increase indefinitely. The increase however, is very slow with a late-time solution for ψ propor-

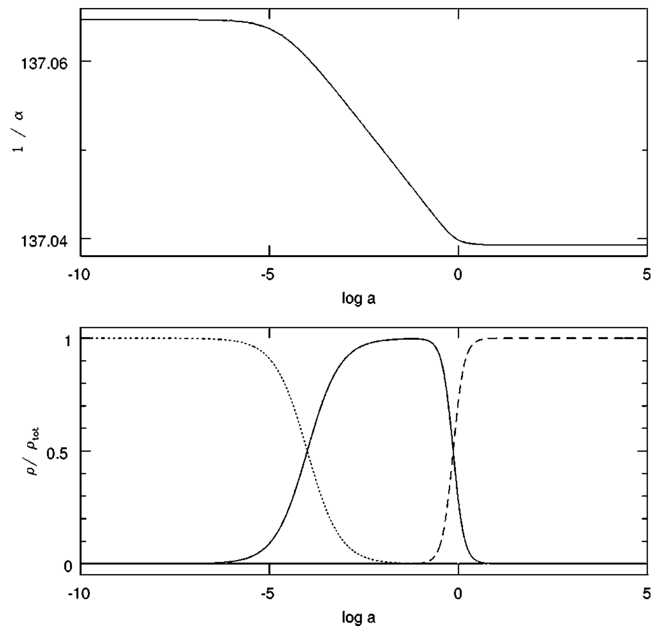

FIG. 1. The top plot shows the change in alpha throughout the dust epoch. This ends as lambda takes over the expansion. The lower plot shows the radiation (dotted), dust (solid) and lambda (dashed) densities as fractions of the total energy density. The present epoch is $a=1$.

tional to $\log(2N\log(t))$; see Fig. 2. N is defined as $N\equiv -2\zeta_m/\rho_m a^3$, a positive constant since we have confined ourselves to $\zeta_m<0$.

(3) If we set the cosmological constant equal to zero and introduce a negative spatial curvature ($k<0$) then α increases only during the dust-dominated phase, where $a(t)\sim t^{2/3}$, but tends to a constant after the expansion becomes curvature dominated, with $a(t)\sim t$. This is illustrated in Fig. 3.

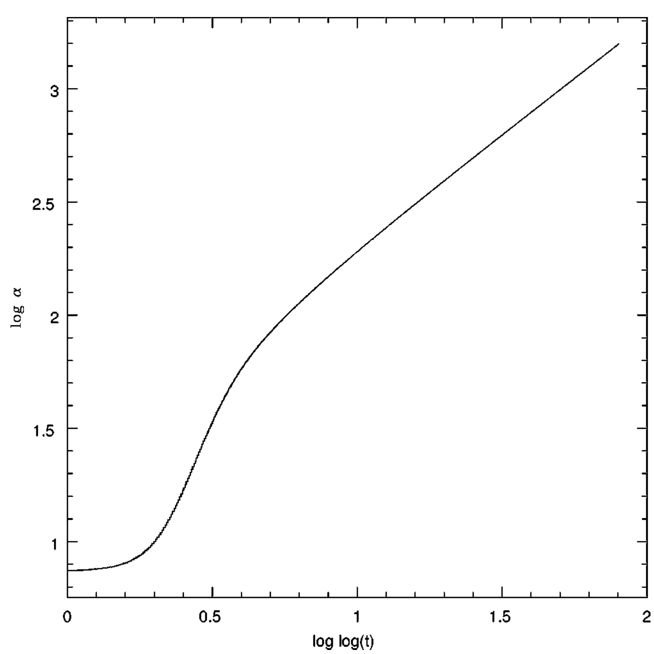

FIG. 2. $\psi\propto\ln\alpha$ changes as $\log(2N\log t)$ in the dust era.

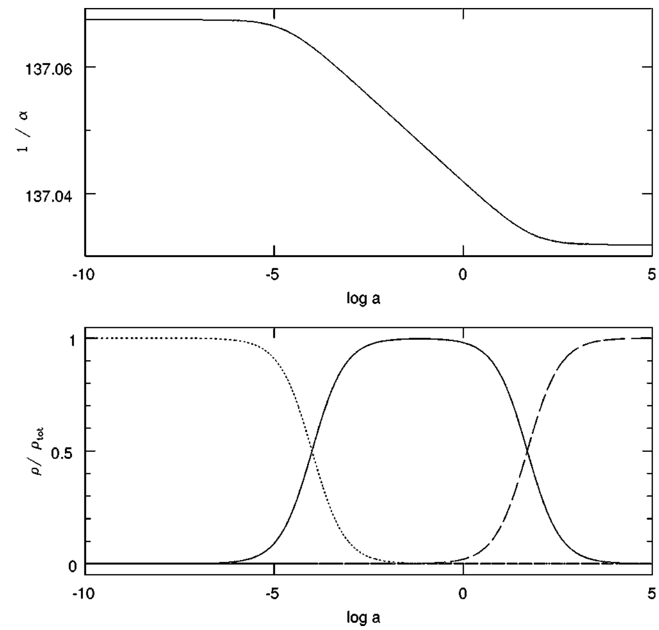

FIG. 3. Top: The change in alpha comes to an end as curvature takes over the expansion. The bottom graph again shows the different constituents of the universe as a function of the scale factor.

From these results it is evident that nonzero curvature or cosmological constant brings to an end the increase in the value of α that occurs during the dust-dominated era.[1] Hence, if the spatial curvature and Λ are too small it is possible for the fine structure constant to grow too large for biologically important atoms and nuclei to exist in the universe. There will be a time in the future when α reaches too large a value for life to emerge or persist. The closer a universe is to flatness or the closer Λ is to zero so the longer the monotonic increase in α will continue, and the more likely it becomes that life will be extinguished. Conversely, a nonzero positive Λ or a nonzero negative curvature will stop the increase of α earlier and allow life to persist for longer. If life can survive into the curvature or Λ-dominated phases of the universe's history then it will not be threatened by the steady cosmological increase in α unless the universe collapses back to high density.

III. ANTHROPIC LIMITS ON α

We have seen that varying-α cosmologies with zero curvature and Λ lead to a monotonic increase in α with time. Here we summarize the principal upper limits on α that are needed for atomic complexity and stars to exist. There are a variety of constraints on the maximum value of the fine

[1]In some Friedmann universes with initial conditions unlike our own there can be power-law growth of α during the radiation era [30]. In such universes the same general effects of negative curvature and positive Λ are seen. They still halt any growth in $\alpha(t)$. Our initial conditions are chosen so as to give a present day value of $\alpha\approx 1/137$. The initial value of alpha would have to be several orders of magnitude lower in order to obtain the power-law growth.

structure compatible with the existence of nucleons, nuclei, atoms and stars under the assumption that the forms of the laws of nature remain the same. The running of the fine structure constant with energy due to vacuum polarization effects leads to an exponential sensitivity of the proton lifetime with respect to the low-energy value of α with $t_{pr} \sim \alpha^{-2}\exp(\alpha^{-1})m_{pr}^{-1} \sim 10^{32}$ yr. In order that the lifetime be less than the main sequence lifetime of stars we have $t_{pr} < (Gm_{pr}^2)^{-1}m_{pr}^{-1}$ which implies that α is bounded above by $\alpha < 1/80$ approximately [32].

The stability of nuclei is controlled by the balance between nuclear binding and electromagnetic surface forces [33]. A nucleus (Z,A) will be stable if $Z^2/A < 49(\alpha_s/0.1)^2(1/137\alpha)$. In order for carbon $(Z=6)$ to be stable we require $\alpha < 16(\alpha_s/0.1)^2$. Detailed investigations of the nucleosynthesis processes in stars have shown that a change in the value of α by 4% shifts the key resonance level energies in the carbon and oxygen nuclei which are needed for the production of a mixture of carbon and oxygen from beryllium plus helium-4 and carbon-12 plus helium-4 reactions in stars [34,35]. These upper bounds on α are model independent and were considered in more detail in Refs. [5,4,6]. However, sharper limits can be found by using our knowledge of the stability of matter derived from analysis of the Schrödinger equation. Stability of matter with Coulomb forces has been proved for nonrelativistic dynamics, including arbitrarily large magnetic fields, and for relativistic dynamics without magnetic fields. In both cases stability requires that the fine structure constant be not too large.

The value of α controls atomic stability.[2] If α increases in value then the innermost Bohr orbital contracts and electrons will eventually fall into the nucleus when $\alpha > Z^{-1}m_{pr}/m_e$. As α increases, atoms all become relativistic and unstable to pair production. In order that the electromagnetic repulsion between protons does not exceed nuclear strong binding $e^2/r_n < \alpha m_\pi$ is needed and so we require $\alpha < 1/20$. It is also known that atomic instability of atoms with atomic number Z occurs in the relativistic Schrödinger equation when the fine structure constant is increased in value to $\alpha = 2/\pi Z$. However, when the many-electron and many-nucleon problem is examined with the relativistic Schrödinger theory there is a bound on α for stability that is independent of Z [36]. If $\alpha < 1/94$ then stability occurs all the way up to the critical value $\alpha = 2/\pi Z$, whereas if $\alpha > 128/15\pi$ the "atomic" system is unstable for all values of Z. In the presence of arbitrarily large magnetic fields, which aid binding by creating a two-dimensional form for the potential, matter composed of electrons and nuclei is known to be unstable if α or Z is too large: matter is stable if $\alpha < 0.06$ and $\alpha < 0.026(6/Z)^{1/2}$ [37,38].

If stars are to exist, their centers must be hot enough for thermonuclear reactions to occur. This requires α to be bounded above by $\alpha^2 < 20m_e/m_{pr}$. Carter has also pointed out the existence of a very sensitive condition, $\alpha^{12} \sim (m_e/m_{pr})^4 Gm_{pr}^2$, that must be met if stars are to undergo a convective phase, although this stringent condition no longer seems to be essential for planetary formation [24].

The results collected above show that there are a number of general *upper limits* on the value of α if atoms, molecules, and biochemistry are to exist. These bounds do not involve the gravitation constant explicitly. Other astrophysical upper bounds on α exist in order that stars are able to form but these involve the gravitational constant.

IV. TIME VARIATION OF *G*

A similar trend can be found in relativistic cosmologies in scalar-tensor gravity theories. Consider the paradigmatic case of Brans-Dicke (BD) theory to fix ideas. The form of the general solutions to the Friedmann metric in BD theories are fully understood [39,40]. The general solutions begin at high density dominated by the BD scalar field $\phi \sim G^{-1}$ and approximated by the vacuum solution. At late times they approach particular exact power-law solutions for $a(t)$ and $\phi(t)$ and the evolution is "Machian" in the sense that the cosmological evolution is driven by the matter content rather than by the kinetic energy of the free ϕ field. There are three essential field equations for the evolution of ϕ and $a(t)$ in a BD universe

$$3\frac{\dot{a}^2}{a^2} = \frac{8\pi\rho}{\phi} - 3\frac{\dot{a}\dot{\phi}}{a\phi} + \frac{\omega_{BD}}{2}\frac{\dot{\phi}^2}{\phi^2} - \frac{k}{a^2}$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = \frac{8\pi}{3+2\omega_{BD}}(\rho - 3p)$$

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0.$$

Here, ω_{BD} is the BD constant parameter and the theory reduces to general relativity in the limit $\omega_{BD} \to \infty$ and $\phi = G^{-1} \to \text{const}$.

In the radiation era the scale factor approaches the standard general relativistic behavior for large times:

$$a(t) \sim t^{1/2}, \quad G = \text{const}. \tag{5}$$

After the dust density dominates the dynamics the expansion approaches a simple exact solution with

$$a(t) \propto t^{(2-n)/3}, \quad G \propto t^{-n}, \tag{6}$$

which continues until the curvature term takes over the expansion. Here, n is related to the constant Brans-Dicke ω_{BD} parameter by

$$n \equiv \frac{2}{4+3\omega_{BD}} \tag{7}$$

and the usual general relativistic Einstein de Sitter universe is obtained as $\omega_{BD} \to \infty$ and $n \to 0$. If the universe is open

[2]Note that if the electron mass and velocity of light are varied along with the value of α then the eigenvalues of the nonrelativistic Schrödinger equation can remain invariant and atomic structure is unchanged [5]. Here, we break the scale invariance by varying only α.

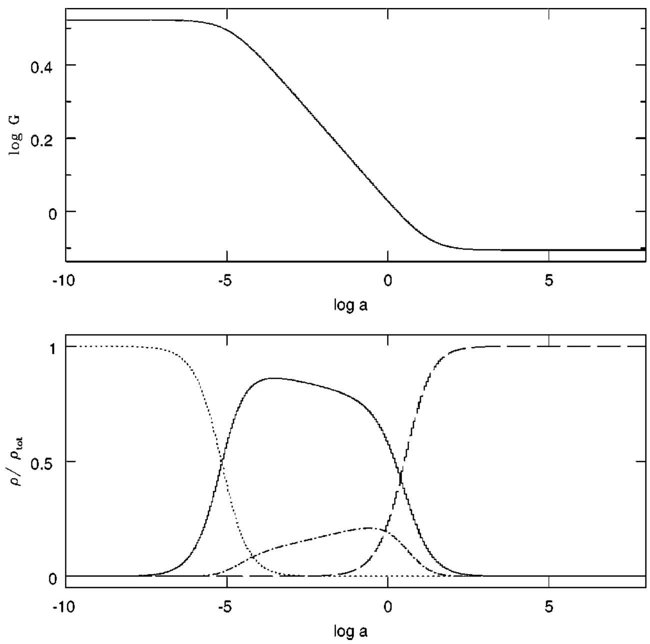

FIG. 4. Top plot shows cosmological evolution of G for Brans-Dicke theory, with $\omega_{BD}=10$, from radiation domination into dust domination and through to curvature driven expansion. Lower plot shows radiation (dotted), dust (solid) and curvature (dashed) energies, as well as the scalar field energy (combined), as a fraction of the total energy density.

$(k=-1)$, then the negative curvature will eventually dominate the gravitational effects of the dust and then the BD model approaches the general relativistic Milne model with constant G:

$$a(t)\propto t, \quad G=\text{const.} \tag{8}$$

Again, we see the same pattern of behavior seen for the evolution of α in the BSBM theory. The smaller the curvature term, so the longer the dust-dominated era lasts, and the greater the fall in the value of G, and the smaller its ultimate asymptotic value when the curvature intervenes to turn off the variation. In general, in such cosmologies, if there exists a critical value of G below which living complexity cannot be sustained, then a universe that is too close to flatness will have a smaller interval of cosmic history during which it can support life.

So far, we have discussed only the independent variation of α and G. What happens if they both vary at the same time? Previous studies of varying constants have only examined the time variation of a single ''constant.'' We have produced a unified theory [41], which incorporates the BSBM varying α and BD varying G theories discussed above. When both α and G are allowed to vary simultaneously in this theory we find [41] that our general conclusions still hold, although the quantitative details are changed. During the dust era of a flat Friedmann universe with varying $\alpha(t)$ and $G(t)$, their time evolution approaches an attractor in which the product αG is a constant and

$$\alpha\propto G^{-1}\propto t^{n} \tag{9}$$

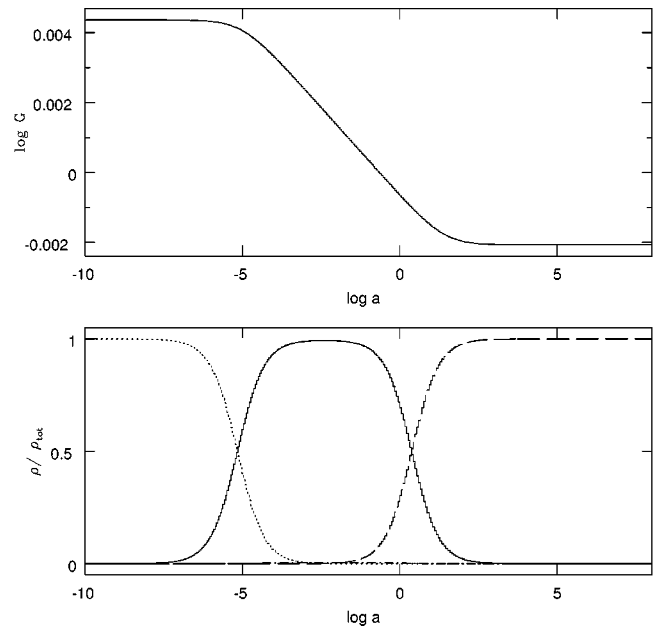

FIG. 5. Similar evolution for Brans-Dicke theory with $\omega_{BD}=1000$.

where n is given by Eq. (7). Thus we see that the G evolution is left unchanged by the effects of varying α, but variation of G changes the time evolution of $\alpha(t)$ from a logarithm to a power law in time. As before, the longer the dust era lasts before it is ended by deviation from flatness or zero cosmological constant, the longer the time increase of α continues, inevitably leading to values that make any atom-based complexity impossible. (See Figs. 4 and 5.)

V. DISCUSSION

We have shown that some theories which include the time variation of traditional constants like α and G introduce significant new anthropic considerations. A theory which self-consistently introduces the space-time variation of a traditional constant scalar quantity is strongly constrained in form by the requirements of causality and second-order propagation equations [9]. Typically, this requirement leads to equations for the driving scalar, φ, that have the form $\Box\varphi$ proportional to linear combinations of the energy-momentum components. Explicit examples are provided by the Bekenstein-Sandvik-Barrow-Magueijo and Brans-Dicke theories. This structure ensures that the evolution of the ''constant'' whose variations are derived from those of φ is strongly dependent upon the material or geometrical source governing the background expansion dynamics. In the case of varying α we have shown elsewhere [30,10] that this ties the epoch after which time variations in α become very small to the time when the cosmological constant starts to accelerate the expansion of the universe. In these theories there is therefore the possibility of a habitable time zone of finite duration during which a constant like α or G falls within a biologically acceptable range.

Surprisingly, there has been almost no consideration of habitability in cosmologies with time-varying constants since Haldane's discussions [42] of the biological consequences of

Milne's bimetric theory of gravity with two time scales, one for atomic phenomena, another for gravitational phenomena [43]. Since then attention has focused upon the consequences of universes in which the constants are different but still constants. Those cosmologies with varying constants that have been studied have not considered the effects of curvature or Λ domination on the variation of constants and have generally considered power-law variation to hold for all times. The examples described here show that this restriction has prevented a full appreciation of the coupling between the expansion dynamics of the universe and the values of the constants that define the course of local physical processes within it. Our discussion of a theory with varying α shows for the first time a possible reason why the 3-curvature of universes and the value of any cosmological constant may need to be bounded *below* in order that the universe permit atomic life to exist for a significant period. Previous anthropic arguments have shown that the spatial curvature of the universe and the value of the cosmological constant must be bounded *above* in order for life-supporting environments (stars) to develop. We note that the lower bounds discussed here are more fundamental than these upper bounds because they derive from changes in α which have direct consequences for biochemistry whereas the upper bounds just constrain the formation of astrophysical environments by gravitational instability (for alternative scenarios see Ref. [44]). Taken together, these arguments suggest that within an ensemble of all possible worlds where α and G are time variables, there might only be a finite interval of *nonzero* values of the curvature and cosmological constant contributions to the dynamics that both allow galaxies and stars to form and their biochemical products to persist.

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