Spin-dependent forces of quarks in a baryon

Yu. A. Simonov
Jefferson Laboratory, Newport News, Virginia 23606
and State Research Center, Institute of Theoretical and Experimental Physics, Moscow 117218, Russia
(Received 7 March 2002; published 13 June 2002)

Nonperturbative spin-dependent forces of quarks in a baryon are calculated directly from the QCD Lagrangian in the framework of the field correlator method both for heavy and light quarks. The resulting forces contain terms of five different structures, only one being known before in asymptotic form. The perturbative terms obtained by the same method are standard and have different signs and structures with respect to the corresponding nonperturbative ones, implying possible cancellations for some baryonic states.

DOI: 10.1103/PhysRevD.65.116004 PACS number(s): 11.15.Tk, 12.38.Aw, 12.38.Lg

I. INTRODUCTION

The spin structure of baryons presents a still unsolved problem, both on the partonic and quark model levels. For an excited baryon spectrum the apparent small spin-orbit splitting of some baryonic states is a topic of vivid discussions [1–3]. Some baryonic states, such as Roper resonance $N(1440)$ or $\Lambda(1405)$, are not yet explained in the traditional framework of the relativistic quark model (RQM) [2,4]. More detailed information about the spin structure of baryons comes from the polarization experiments on electroproduction of excited resonances [5,6], which effectively measure the convolution of the baryon wave function, and is very sensitive to its structure.

Meanwhile the theoretical knowledge of the quark spin forces in the baryon is limited to the perturbative expressions calculated decades ago [7], and the nonperturbative spin-orbit Thomas term, written in the framework of the RQM [8].

In applying these results to light baryons, the notion of constituent quark masses is introduced in the RQM, which appear in spin-dependent (SD) forces and play the role of fitting parameters.

It is the purpose of the present paper to derive SD forces in a baryon in a most straightforward way from the QCD Lagrangian with the nonperturbative vacuum described by vacuum field correlators [9,10]. Limiting ourselves to the lowest (Gaussian) field correlators, we can express all the terms of SD forces through the scalar functions D and D_1 representing this Gaussian correlator [11]. The high accuracy of such a procedure is supported by recent lattice data [12] and the contribution of higher correlators can be estimated to be of the order of a few percent [13].

The functions D,D_1 are themselves measured on the lattice [14] and also found in analytic approaches [15,16].

An essential element of the present approach is that it is not connected to the heavy mass expansion, and can be applied also to light quarks in a baryon. In this case an effective Hamiltonian is constructed from first principles, which contains einbein (auxiliary) fields. It was shown previously that the stationary point of these einbein fields yields exactly the constituent quark masses which can be expressed unambiguously through the only parameter of this approach—the string tension σ.

The decisive check of this procedure is the calculation of baryon magnetic moments, since they are inversely proportional to the quark constituent masses. That was done in [17] and results agreed with all known experimental data within $\sim 10\%$.

The SD forces derived below are computed as a series in field correlators (cumulants) with growing powers of fields.

The lowest (Gaussian) term yields SD forces inversely proportional to the square of constituent quark masses. Having in mind the high accuracy of the Gaussian approximation [13] and baryon magnetic moments [17], one should expect that SD forces found below have an accuracy of the order of 10%.

Analogous expressions for heavy quarkonia [18,19] and light mesons [20] have been reported earlier and estimated for a realistic meson system, respectively, in [21] and [22].

The plan of the paper is as follows. In Sec. II, a general expression for the $3q$ baryon Green's function is introduced, and the Fock-Feynman-Schwinger representation (FFSR) is used to reveal the dependence on gauge fields with spin operators explicitly written. Averaging over those with the help of the field correlator method (FCM), one finally obtains an expression for the Green's function written in terms of field correlators.

In Sec. III, a special case of heavy quark masses is considered and all SD forces are obtained in closed form, expressed in terms of correlator functions D and D_1.

In Sec. IV, the perturbative contribution to SD forces is written down. In Sec. V, a general case is considered when current quark masses can also be vanishingly small, and SD forces are written again in terms of integrals over functions D and D_1 with constituent (dynamical) masses entering in the denominator.

Section VI is devoted to the discussion of the relativistic structure of SD forces in the excited baryon spectrum. Comparison to other results in the literature is also made and possible extension of the method is suggested. The main points of the paper are summarized in the Conclusions.

II. $3q$ GREEN'S FUNCTION WITH SPIN INSERTIONS

Following [9,10,23], we consider the $3q$ Green's function, which can be written as

$$G_{3q}(x,y)=\mathrm{tr}_L\left[\Gamma_{\mathrm{out}}\prod_{i=1}^{3}(m_i-\hat{D})\int D\mu_i Dz^{(i)}\times e^{-K_i}\langle W_3 \exp g\,\sigma F\rangle\Gamma_{\mathrm{in}}\right], \tag{1}$$

where tr_L is the trace over Dirac matrix indices, $\Gamma_{\mathrm{out}}(\Gamma_{\mathrm{in}})$ are final (initial) state operators created given J^{PC} assignment to the $3q$ state, and we have also denoted as in [17,20]

$$K_i=\int_0^T dt\left[\frac{m_i^2}{2\mu_i(t)}+\frac{\mu_i(t)}{2}[\dot{\mathbf{z}}^2(t)+1]\right], \tag{2}$$

$$\langle W_3 \exp(g\sigma F)\rangle=\mathrm{tr}_Y \exp\left[\sum_{n=0}^{\infty}\frac{(ig)^n}{n!}\int\langle\langle F(1)\cdots F(n)\rangle\rangle\times d\rho(1)\cdots d\rho(n)\right]. \tag{3}$$

In Eq. (2), m_i is the current(pole)quark mass renormalized at the typical scale of 1 GeV, and $\mu(t)$ is the einbein field discussed in the Introduction, yielding constituent quark mass when taken at the stationary point of the effective Hamiltonian. Here and in what follows, $F(1)$ is always implied to be gauge-transported to one point x_0, namely $F(1)\equiv F(z(1),x_0)=\Phi(x_0,z(1))F(z(1))\Phi(z(1),x_0)$, where $\Phi(x,y)$ is defined in the Appendix, and finally, $d\rho(n)=\Sigma_{i=1}^{3}d\rho^{(i)}(n)$, with

$$d\rho^{(i)}(n)\equiv ds^{(i)}_{\mu_n\nu_n}(u^{(n)})+\frac{1}{i}\sigma^{(i)}_{\mu_n\nu_n}\frac{dt_n}{2\mu_i(t_n)}. \tag{4}$$

The integration in Eq. (3) extends over all three lobes of the minimal area surface $(S_1+S_2+S_3)$ inside the quark trajectories $z^{(i)}(t)$ and the string-junction trajectory $z^{(Y)}(t)$. We have also denoted $\sigma_{\mu\nu}=1/4i(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)$ and everywhere Euclidean space-time is used (until the last moment when the resulting Hamiltonian is obtained in Minkowski space-time) with γ-matrices

$$\gamma_4=\gamma_0\equiv\beta;\quad \gamma_i=-i\beta\alpha_i,\quad \gamma_\mu\gamma_\nu+\gamma_\nu\gamma_\mu=2\delta_{\mu\nu}.$$

Note also that notation tr_Y means

$$\mathrm{tr}_Y P\equiv\frac{1}{6}e_{abc}e_{a'b'c'}P_{abc/a'b'c'}. \tag{5}$$

In the combination $F(k)d\rho(k)$ in Eq. (3) one can write

$$F_{\mu\nu}\sigma^{(i)}_{\mu\nu}=\begin{pmatrix}\boldsymbol{\sigma}^{(i)}\mathbf{B}, & \boldsymbol{\sigma}^{(i)}\mathbf{E}\\ \boldsymbol{\sigma}^{(i)}\mathbf{E}, & \boldsymbol{\sigma}^{(i)}\mathbf{B}\end{pmatrix},\quad i=1,2,3. \tag{6}$$

As it was shown in [10], the spin-independent part of Eq. (3) which is obtained neglecting the Σ term in Eq. (4) yields at large quark separations, $|\mathbf{z}^{(i)}-\mathbf{z}^{(Y)}|\gg T_g$, $i=1,2,3$, the familiar area-law asymptotics

$$\langle W_3\rangle=\exp[-\sigma(S_1+S_2+S_3)] \tag{7}$$

implying linear confinement for each quark. In what follows, we shall use the general expression (1) to derive the spin-dependent part of the interaction both for heavy quarks (expansion in inverse powers of mass) and for light quarks.

III. SPIN-DEPENDENT INTERACTION IN 1/*m* EXPANSION

To illustrate the method, we shall start with the derivation of spin-dependent (SD) forces via a $1/m$ expansion. Defining the SD potential as V_{SD}, one can write $G_{3q}\sim e^{-TV_{\mathrm{SD}}}\sim 1-TV_{\mathrm{SD}}$, and for V_{SD} the following general form will be obtained below, similar (but not identical) to the corresponding form for heavy quarkonia [24,18]:

$$V_{\mathrm{SD}}(\mathbf{R}^{(1)},\mathbf{R}^{(2)},\mathbf{R}^{(3)})=\sum_{i=1}^{3}\frac{\boldsymbol{\sigma}^{(i)}\mathbf{L}^{(i)}}{2m_i^2}\left(\frac{1}{R^{(i)}}\frac{dV_1}{dR^{(i)}}+\frac{1}{2R^{(i)}}\frac{d\varepsilon}{dR^{(i)}}\right)+\frac{1}{N_c-1}\sum_{i<j}\frac{(\boldsymbol{\sigma}^{(i)}\mathbf{L}^{(j)}+\boldsymbol{\sigma}^{(j)}\mathbf{L}^{(i)})}{2m_im_j}\frac{1}{R^{(j)}}\frac{dV_2(R^{(i)},R^{(j)})}{dR^{(j)}}$$
$$+\sum_{i<j}\left[\frac{(\boldsymbol{\sigma}^{(i)}\boldsymbol{\sigma}^{(j)})V_4(R_{ij})}{12m_im_j(N_c-1)}+\frac{3(\boldsymbol{\sigma}^{(i)}\mathbf{n})(\boldsymbol{\sigma}^{(j)}\mathbf{n})-(\boldsymbol{\sigma}^{(i)}\boldsymbol{\sigma}^{(j)})}{12m_im_j(N_c-1)}V_3(R_{ij})\right]+V_5, \tag{8}$$

where $\mathbf{n}=\mathbf{R}_{ij}/R_{ij}$, $\mathbf{R}_{ij}=\mathbf{R}_i-\mathbf{R}_j$. We assume that current quark masses are large, $m_i\gg\sqrt{\sigma}$, $i=1,2,3$, and hence also μ_i are large, since the latter are defined through m_i and σ in the stationary point analysis [10,17] and always satisfy $\mu_i=m_i+O(1/m_i)$. Hence for simplicity we keep in the following $\mu_i=m_i\gg\sqrt{\sigma}$ and expand in inverse powers of $1/m_i$.

As was observed in [18,19], the SD terms of the lowest order $(1/m_i^2,\ 1/m_im_j)$ come from three different sources.

(A) Diagonal terms in Eq. (6) are kept together with diagonal terms in $\Lambda_i\equiv(m_i-D_\mu\gamma_\mu)$, yielding one power of $1/m_i$. An additional power of $1/m_i$ or $1/m_j$ then comes from the expansion of $\langle W_3\rangle$. This yields spin-orbit terms V_1',V_2', and V_5.

(B) The off-diagonal terms are kept both in Eq. (6) and in Λ_i. This gives a spin-orbit potential $d\varepsilon/dR$.

(C) Diagonal terms from two matrices (6) with $i\neq j$ are retained. This yields spin-spin potentials V_3 and V_4. We now calculate the SD contributions from (A)–(C) point by point.

(A) From Eqs. (3) and (4) one gets for $i=1$

$$\left\langle \mathrm{tr}_Y\left[\left(1+\frac{g}{2m_1}\sigma_k^{(1)}\int_0^T B_k(z^{(1)},t^{(1)})dt_1\right)W_3\right]\right\rangle \approx 1-TV_{\mathrm{SD}}^{(1)}. \tag{9}$$

Using the relation $igF_{\mu\nu}W=[\delta/\delta\sigma_{\mu\nu}(z)]W$ which obtains easily with non-abelian Stokes representation for W, one has

$$\langle \mathrm{tr}_Y F_{\lambda\sigma}(x,z_0)W(C)\rangle = \left\langle \mathrm{tr}_Y\left\{ ig\int ds_{\mu\nu}(z)F_{\mu\nu}(z,z_0)F_{\lambda\sigma}(x,z_0)W_3(C)\right\}\right\rangle \tag{10}$$

and one can rewrite the left-hand side of Eq. (9) as

$$\mathrm{tr}_Y W+\frac{ig^2}{2m_1}\sigma_k^{(1)}\int_0^T dt_1\left\langle \mathrm{tr}_Y B_k(z^{(1)},t^{(1)}) \times\int_{S(C')}ds_{\mu\nu}(u)F_{\mu\nu}(u,x_0)W_3(C')\right\rangle. \tag{11}$$

In Eq. (10) and (11), the common reference point z_0 is chosen to make both expressions gauge invariant; as will be seen, this point will not appear in the final equations.

In Eq. (11), the contour C' is deformed due to orbital momentum of quarks as compared to the zeroth-order contour C_0 consisting of straight lines. This is essential since otherwise the vacuum average $\langle B_k W_3(C_0)\rangle$ vanishes because it is odd with respect to reflection $z_i\to -z_i,\ i\neq k$. Therefore, all nonzero contribution in Eq. (11) is due to deflection of the quark path in C' from the straight line in C_0.

At this point we shall describe the quark trajectory $z_\mu^{(i)}(t)$ and the corresponding string piece $W_\mu^{(i)}$ from the quark position to the string junction (which for simplicity we take at the origin):

$$w_\mu^{(i)}(t,\beta)=z_\mu^{(i)}(t)\beta, \quad 1\geqslant\beta\geqslant 0, \tag{12}$$

$$ds_{ik}^{(i)}=d\beta^{(i)}dt e_{ikm}\frac{\beta L_m^{(i)}}{im_i} \tag{13}$$

where $L_m^{(i)}$ is the (Minkowskian) angular momentum of the ith quark,

$$L_s^{(i)}=im_i e_{skm}R_k^{(i)}\dot{z}_m^{(i)}, \quad \mathbf{R}^{(i)}=\mathbf{z}^{(i)}-\mathbf{z}^{(Y)}=\mathbf{z}^{(i)}. \tag{14}$$

Similarly $d\sigma_{k4}^{(i)}=R_k^{(i)}d\beta^{(i)}du_4$, and one arrives at the result

$$\langle B_k(z^{(i)},t_i)W_3(C')\rangle = ig\int d\beta^{(i)}du_4\frac{\beta^{(i)}L_n^{(i)}}{im_i}\langle B_k B_n(u_4,\beta)W\rangle + ig\int d\beta^{(i)}du_4 R_l^{(i)}(u_4)\langle B_k E_l(u_4,\beta^{(i)})W\rangle. \tag{15}$$

Denoting

$$\langle B_k(z^{(1)},t_1)E_i(\mathbf{u},u_4)W_3\rangle \equiv e_{\mathrm{kin}}(u_n-z_n^{(1)})\frac{\partial\Lambda_0}{\partial u_4} \tag{16}$$

one obtains

$$\boldsymbol{\sigma}^{(i)}\mathbf{L}^{(i)}\left(\frac{1}{R}\frac{dV_1}{dR}\right)^{(i)} = -g^2\int_0^1\beta d\beta\int_0^T du_4\frac{\sigma_k^{(i)}L_n^{(i)}}{\langle W_3\rangle}B_k(z^{(i)},t_i)B_n(\beta z^i,u_4) - \frac{\boldsymbol{\sigma}^{(i)}\mathbf{L}^{(i)}}{\langle W_3\rangle}\int_0^T du_4(u_4-t_1)d\beta\frac{\partial\Lambda_0}{\partial u_4}, \tag{17}$$

where we have used the relation

$$u_n(u_4)-z_n^{(1)}(t_1)\cong \dot{z}_n^{(1)}(u_4)(u_4-t_1).$$

.

Until now we have not used the Gaussian dominance of the vacuum, i.e., the fact that $\langle W_3\rangle$ is saturated by the lowest cumulant $\langle FF\rangle$, which is found to be an accurate approximation [13]. Using it one can write $\langle FFW_3\rangle\to\langle FF\rangle\langle W_3\rangle$, and introduce scalar functions D,D_1 for tensor $\langle FF\rangle$, as it was done in [11].

Referring the reader to the Appendix for the corresponding relations, one finally obtains

$$\left(\frac{1}{R}\frac{dV_1}{dR}\right)^{(i)} = -\int_0^R\frac{d\lambda}{R}\left(1-\frac{\lambda}{R}\right) \times\int_{-\infty}^{\infty}d\nu\left[D(\lambda,\nu)+D_1(\lambda,\nu)+\lambda^2\frac{\partial D_1}{\partial\lambda^2}\right] - \int_{-\infty}^{\infty}\nu^2 d\nu\int_0^R\frac{d\lambda}{R}\frac{\partial D_1}{\partial\lambda^2}. \tag{18}$$

Until now we have taken into account the interaction of the spin of the ith quark with the surface S_i, which yields the term $[(1/R)V_1']^{(i)}$ multiplied with $(m_i^2)^{-1}$. At this point we consider the interaction of the ith quark spin with the (deformed) surface S_j, which will give the term V_2' in Eq. (8). For this one needs to consider a vacuum average of two F's from two different surfaces S_i and S_j.

In general we have for two F's transported to the same point x ($\alpha,\dots,\eta$ are fundamental color indices)

$$\langle F(u,x)_{\alpha\xi}F(v,x)_{\gamma\eta}\rangle = \frac{\langle \mathrm{tr}FF\rangle}{N_c^2-1}\left(\delta_{\alpha\eta}\delta_{\xi\gamma}-\frac{1}{N_c}\delta_{\alpha\xi}\delta_{\gamma\eta}\right). \tag{19}$$

Taking into account Eq. (5) and the relation

$$\mathrm{tr}_Y\Phi_{\alpha\alpha'}(x,y)\Phi_{\beta\beta'}(x,y)\Phi_{\gamma\gamma'}(x,y)\equiv 1, \tag{20}$$

one obtains

$$\mathrm{tr}_Y\langle F_{\alpha\alpha'}(u,x)F_{\beta\beta'}(v,x)\rangle = \frac{\langle \mathrm{tr}F(u,x)F(v,x)\rangle}{N_c(N_c-1)}, \tag{21}$$

where we have also accounted for different orientation of plaquettes in S_i and S_j.

Now proceeding as in Eq. (15) one has

$$\langle B_k^{(i)} W_3(C'_j)\rangle = g\int_0^1 \beta^{(j)} d\beta^{(j)} du_4^{(j)} \frac{L_n^{(j)}}{m_j}\langle B_k^{(i)} B_n^{(j)}(u_4^{(j)},\beta^{(j)}R^{(j)})W_3\rangle + ig\int d\beta^{(j)} du_4^{(j)} R_l^{(j)}(u_4^{(j)})\langle B_k^{(i)} E_l^{(j)} W_3\rangle. \tag{22}$$

At this point one can use Gaussian dominance and relations (21) to obtain finally

$$\left(\frac{1}{R}V_2'(R)\right)_j = \int_0^1 d\beta^{(j)}\beta^{(j)}\int_{-\infty}^{\infty} d\nu\left[D(r^{(ij)},\nu)+D_1+[(r^{(ij)})^2+\nu^2]\frac{\partial D_1}{\partial (r^{(ij)})^2}\right]. \tag{23}$$

In a similar way one obtains from the first term on the right-hand side (r.h.s.) of Eq. (22), with the use of the last term on the r.h.s. of Eq. (A1),

$$V_5 = -\sum_{i>j}\int \frac{(\boldsymbol{\sigma}^{(i)}\mathbf{r}^{(ij)})(\mathbf{L}^{(j)}\mathbf{r}^{(ij)})}{2m_i m_j (N_c-1)} \times \beta^{(j)} d\beta^{(j)} d\nu \frac{\partial D_1(r^{(ij)},\nu)}{\partial (r^{(ij)})^2}. \tag{24}$$

Here we have defined $\mathbf{r}^{(ij)} = \mathbf{R}^{(i)} - \beta^{(j)}\mathbf{R}^{(j)}$, and one should take into account that D, D_1 depend on their arguments as

$$D(r,\nu) = D(\sqrt{r^2+\nu^2}).$$

This concludes derivation of terms with the procedure (A) and one goes over to the next point.

(B) Following Eq. (9) one can write for the corresponding term of a given quark (i)

$$V_{\rm SD}^{(\varepsilon)}T = -\frac{g}{(2m_i)^2}\left\langle\begin{pmatrix} m_i+\mu_i & -\boldsymbol{\sigma}^{(i)}\mathbf{p}^{(i)} \\ \boldsymbol{\sigma}^{(i)}\mathbf{p}^{(i)} & m_i-\mu_i\end{pmatrix} \times \begin{pmatrix} 0 & \boldsymbol{\sigma}^{(i)}\mathbf{E} \\ \boldsymbol{\sigma}^{(i)}\mathbf{E} & 0\end{pmatrix} W_3(C)\right\rangle. \tag{25}$$

Now one can use relation

$$\langle E_k(z^{(i)},t^{(i)})W_3(C_0)\rangle = \frac{\delta\langle W_3(C_0)\rangle}{ig\,\delta\sigma_{k4}(z^{(i)},t^{(i)})} = -\frac{1}{ig}\frac{\partial\varepsilon}{\partial R_k^{(i)}}\langle W_3(C_0)\rangle, \tag{26}$$

where the following notation was introduced for the spin-independent potential $\varepsilon(R^{(1)},R^{(2)},R^{(3)})$:

$$\langle W_3(C_0)\rangle = \exp[-\varepsilon(R^{(1)},R^{(2)},R^{(3)})T]. \tag{27}$$

Note that in Eqs. (25)–(27) one can keep in $W_3(C)$ the unperturbed (straight-line) contours for quark trajectories since the prefactor in Eq. (25) is already $O(1/m^2)$. Keeping in mind the relation

$$\sigma_k^{(i)} E_k^{(i)} \sigma_l^{(i)} p_l^{(i)} = E_k^{(i)} p_k^{(i)} + i e_{kln}\sigma_n^{(i)} E_k^{(i)} p_l^{(i)} \tag{28}$$

one recovers the second term on the r.h.s. of Eq. (8).

(C) Here one considers spin-spin interaction and the corresponding term looks such as

$$1 - V_{\rm SD}T = \mathrm{tr}_Y \sum_{i>j}\left\langle\left(1+\frac{g}{2m_i}\int \boldsymbol{\sigma}^{(i)}\mathbf{B}^{(i)}(z^{(i)},t_i)dt_i\right) \times\left(1+\frac{g}{2m_j}\int \boldsymbol{\sigma}^{(j)}\mathbf{B}^{(j)}(z^{(j)},t_j)dt_j\right)W_3\right\rangle. \tag{29}$$

Identifying spin-spin terms in Eq. (29) and using Eq. (21) and relations for $\langle BBW_3\rangle$ in the Appendix one arrives at

$$V_{\rm SD}^{(\sigma\sigma)} = \sum_{i<j}\int_{-\infty}^{\infty}\frac{d\nu\,\sigma_k^{(i)}\sigma_{k'}^{(j)}}{4m_i m_j (N_c-1)}\left[\delta_{kk'}\left(D+D_1+(\mathbf{u})^2\frac{\partial D_1}{\partial(\mathbf{u})^2}\right) - u_k u_{k'}\frac{\partial D_1}{\partial (u)^2}\right], \tag{30}$$

where the following notations are used:

$$\mathbf{u} = \mathbf{R}^{(i)} - \mathbf{R}^{(j)}, \quad \nu = t_i - t_j. \tag{31}$$

Rewriting Eq. (30) as

$$V_{\rm SD}^{(\sigma\sigma)} = \sum_{i<j}\frac{\boldsymbol{\sigma}^{(i)}\boldsymbol{\sigma}^{(j)}V_4(u)+S_{ij}V_3(u)}{12m_i m_j (N_c-1)} \tag{32}$$

one has

$$V_4(u) = \int_{-\infty}^{\infty} d\nu\left(3D(u,\nu)+3D_1(u,\nu)+2u^2\frac{\partial D_1}{\partial u^2}\right) \tag{33}$$

$$V_3(u) = -\int_{-\infty}^{\infty} d\nu u^2 \frac{\partial D_1(u,\nu)}{\partial u^2}, \tag{34}$$

$$S_{ij}=3(\boldsymbol{\sigma}^{(i)}\mathbf{n})(\boldsymbol{\sigma}^{(j)}\mathbf{n})-\boldsymbol{\sigma}^{(i)}\boldsymbol{\sigma}^{(j)}, \quad \mathbf{n}=\frac{\mathbf{u}}{|\mathbf{u}|}. \tag{35}$$

This concludes the definition of all NP spin-dependent terms in Eq. (8) to the order $O(1/m^2)$ and in the approximation when only the lowest, $\langle FF\rangle$, correlator is retained in the Wilson loop.

Now comparing our expressions for $V_1', V_2', \varepsilon', V_3, V_4$ with the corresponding ones for the heavy $Q\bar{Q}$ case, given in [18,19], one can see that they coincide exactly, the only difference being that one should sum up over all three quarks for V' and ε', and take a double sum, $i<j$, for V_2', V_3, V_4. In addition, there is a term V_5 which is of three-body character and vanishes in two-body situation since in that situation $\mathbf{L}^{(j)}\mathbf{r}^{(ij)}=\mathbf{L}^{(j)}\mathbf{r}\equiv 0$.

IV. PERTURBATIVE SPIN-DEPENDENT FORCES

We are now in a position to consider also perturbative contributions to the SD potentials, which were calculated in [7]. The easiest way for us is to remember that to the lowest order, $O(\alpha_s)$, all perturbative terms are pairwise interactions of quarks, and they can be reconstructed from the expressions obtained above, Eqs. (18), (23), (24),and (32)–(34), using an $O(\alpha_s)$ contribution to $D_1(x)$, while $D(x)$ does not have contributions at this order [25],

$$D(x)=D^{\rm NP}(x), \quad D_1(x)=\frac{16\alpha_s}{3\pi x^4}+D_1^{\rm NP}(x). \tag{36}$$

It is rewarding to realize that D_1 does not enter into V_1' [terms containing D_1 in Eq. (18) cancel exactly], so that the perturbative contribution occurs only in the nondiagonal, $i\neq j$, terms in Eq. (8) and in ε'.

In the $Q\bar{Q}$ case, analogous calculations have been done and compared to standard ones in [18,19].

We start with the $d\varepsilon/dR$ term and rewrite it in the original form (26), not assuming that ε depends on $R^{(i)}$ only, but also on $\mathbf{R}^{(i)}-\mathbf{R}^{(j)}$, as it is for the Coulomb term which should be added to ε. In this way one obtains, from Eqs. (25), (26) and (28),

$$V_{\rm SD}^{(\varepsilon)}=\frac{e_{kln}p_l^{(i)}\sigma_n^{(i)}}{4m_i^2}\frac{\partial\varepsilon}{\partial R_k^{(i)}} \tag{37}$$

and for $\varepsilon\to\varepsilon+V_{\rm Coul}$, $V_{\rm Coul}=-(2\alpha_s/3)\Sigma_{i>j}(1/|R^{(i)}-R^{(j)}|)$ one obtains

$$V_{\rm SD}^{(\varepsilon,\rm pert)}=\frac{2\alpha_s}{3}\sum_{i>j}\left[\frac{(\mathbf{R}^{(ij)}\times\mathbf{p}^{(i)})\boldsymbol{\sigma}^{(i)}}{4m_i^2(R^{(ij)})^3}+\frac{(\mathbf{R}^{(ji)}\times\mathbf{p}^{(j)})\boldsymbol{\sigma}^{(j)}}{4m_j^2(R^{(ij)})^3}\right]. \tag{38}$$

This expression coincides with the corresponding one in [7]. Consider now a nondiagonal spin-orbit term, equivalent to V_2'. Instead of using replacement (36) and doing integrations in Eq. (23), we start from the more general expression (9) to derive

$$V_{\rm SD}^{(2,\rm pert)}=-\frac{2\alpha_s}{3(N_c-1)}\times\sum_{i>j}\frac{[\boldsymbol{\sigma}^{(i)}(\mathbf{R}^{(ij)}\times\mathbf{p}^{(j)})+\boldsymbol{\sigma}^{(j)}(\mathbf{R}^{(ji)}\times\mathbf{p}^{(i)})]}{m_im_j(R^{(ij)})^3}, \tag{39}$$

which is again in agreement with [7].

Next we consider the spin-spin interaction. Here it is straightforward to replace in V_4 (33) D_1 as in Eq. (36) $D_1\to D_1^{(\rm pert)}=16\alpha_s/3\pi x^4$ to obtain

$$\begin{aligned}V_4^{(\rm pert)}(r)&=\int_{-\infty}^{\infty}d\nu\left(3D_1^{(\rm pert)}(r,\nu)+2r^2\frac{\partial D_1^{(\rm pert)}}{\partial r^2}\right)\\&=-\frac{8\alpha_s}{3r}\left(\frac{\partial^2}{\partial r^2}+\frac{2}{r}\frac{\partial}{\partial r}\right)\frac{1}{r}\\&=\frac{32\pi\alpha_s}{3}\delta^{(3)}(\mathbf{r}).\end{aligned} \tag{40}$$

In a similar way one obtains for $V_3^{(\rm pert)}$,

$$V_3^{(\rm pert)}(r)=\frac{4\alpha_s}{r^3}. \tag{41}$$

One can also persuade oneself that V_5 has no pertubative counterpart since there $\mathbf{L}^{(j)}\to\mathbf{L}^{(ij)}$ and it is orthogonal to $\mathbf{r}^{(ij)}$ and therefore the total perturbative SD contribution to the order $O(\alpha_s)$ can be written as

$$V_{\rm SD}^{(\rm pert)}=V_{\rm SD}^{(\varepsilon,\rm pert)}+V_{\rm SD}^{(2,\rm pert)}+\sum_{i<j}\frac{\boldsymbol{\sigma}^{(i)}\boldsymbol{\sigma}^{(j)}V_4^{(\rm pert)}(R^{(ij)})+S_{ij}V_3^{(\rm pert)}(R^{(ij)})}{12m_im_j(N_c-1)}, \tag{42}$$

where the explicit form of four terms on the r.h.s. of Eq. (42) is given in Eqs. (38), (39), (40), and (41).

Our results for $V_{\rm SD}^{(2,\rm pert)}, V_3^{(\rm pert)}, V_4^{(\rm pert)}$ coincide with the corresponding expressions in [8], however our $V_{\rm SD}^{(\varepsilon,\rm pert)}$ is two times smaller than the corresponding term in [8]. In the next sections, we shall argue that for light quarks this term gets indeed twice as big, since there one should replace $m_i\to\mu_i$, and for $V_{\rm SD}^{(\varepsilon)}$ the coefficient appears to be two times larger.

V. SPIN-DEPENDENT FORCES FOR LIGHT QUARKS

In Secs. III and IV, the SD forces have been obtained as an expansion in $1/m_i, 1/m_j$ taking all three quark current masses large, $m_i\gg\sqrt{\sigma}$, $i=1,2,3$.

It was noticed before [20], however, that the general expressions (1)–(4) for Green's functions written in FFSR, with the einbein function $\mu_i(t)$ introduced as in [10], allow us to obtain expressions for SD forces also for light quarks without $1/m$ expansion, and the corresponding terms for the meson case have been written before [10,22]. Below we demonstrate in this section that the same procedure works also for the $3q$ case with light current quark masses as well.

We start again with the general form (3) and instead of an expansion in $1/m$ (or $1/\mu$, which is equivalent for heavy quarks) we shall do the only approximation, keeping in the sum in the exponent (3) the lowest (Gaussian) cumulant $\langle\langle F(1)F(2)\rangle\rangle$. This approximation was recently supported by lattice data for Casimir scaling [12], while higher cumulants provide (for the Wilson loop) less than 2% [13]. The spin-spin interaction is easily obtained keeping in Eq. (3), the bilocal term and in Eq. (4) only the σ-dependent term. In Eq. (3), one should take into account that $F(1)$ and $F(2)$ belong to different lobes S_1,S_2 of the S_{123} surface, hence $n=1$ for each of them; moreover one uses Eqs. (19) and (21), the latter with opposite sign, since the orientation of $S(1,3)$ and $S(2,3)$ is the same in our case. As a result, one obtains

$$V_{\rm SD}^{(\sigma\sigma)}=\sum_{i>j}\int_{-\infty}^{\infty}\frac{d(t_i-t_j)}{4\mu_i\mu_j(N_c-1)}\times\sigma^{(i)}_{\mu_i\nu_i}\sigma^{(j)}_{\mu_j\nu_j}\langle F_{\mu_i\nu_i}(i)F_{\mu_j\nu_j}(j)\rangle. \tag{43}$$

The combination $\sigma^{(i)}\sigma^{(j)}$ in Eq. (43) is a product of two 4×4 matrices, which can be split into the product of Pauli spin matrices σ_i and chiral 2×2 matrices $\hat{1}$ and $\hat{\rho}_1\equiv\binom{01}{10}$. Thus one can rewrite Eq. (43) as

$$V_{\rm SD}^{(\sigma\sigma)}=\sum_{i>j}\int_{-\infty}^{\infty}\frac{g^2d(t_i-t_j)}{4\mu_i\mu_j(N_c-1)}\sigma_m^{(i)}\sigma_n^{(j)}[\langle {\rm tr}B_m(i)B_n(j)\rangle(\hat{1}\times\hat{1})++\langle{\rm tr}E_m(i)E_n(j)\rangle(\hat{\rho}_1\times\hat{\rho}_1)+\langle{\rm tr}B_m(i)E_n(j)\rangle(\hat{1}\times\hat{\rho}_1)$$
$$\times\langle{\rm tr}E_m(i)B_n(j)\rangle(\hat{\rho}_1\times\hat{1})]\equiv V_{\rm SD}^{(\sigma\sigma)}(BB)+V_{\rm SD}^{(\sigma\sigma)}(EE)+V_{\rm SD}^{(\sigma\sigma)}(BE)+V_{\rm SD}^{(\sigma\sigma)}(EB). \tag{44}$$

Using formulas from the Appendix for correlators of B,E, one has

$$V_{\rm SD}^{(\sigma\sigma)}(B_1B)=\sum_{i>j}\frac{\boldsymbol{\sigma}^{(i)}\boldsymbol{\sigma}^{(j)}V_4(u)+S_{ij}V_3(u)}{12\mu_i\mu_j(N_c-1)}(\hat{1}\times\hat{1}), \tag{45}$$

where $\mathbf{u}\equiv\mathbf{R}^{(i)}-\mathbf{R}^{(j)}$.

One can see that Eq. (45) coincides with Eqs. (32)–(35) with substitution $m_i,m_j\to\mu_i,\mu_j$. For the EE term one obtains

$$V_{\rm SD}^{(\sigma\sigma)}(EE)=\sum_{i>j}\frac{\boldsymbol{\sigma}^{(i)}\boldsymbol{\sigma}^{(j)}\tilde{(}V_4(u)+S_{ij}\tilde{V}_3(u)}{12\mu_i\mu_j(N_C-1)}(\hat{\rho}_1\times\hat{\rho}_1), \tag{46}$$

where we have defined

$$\tilde{V}_4(u)=\int_{-\infty}^{\infty}d\nu\left(3D(u,\nu)+3D_1(u,\nu)+(3\nu^2+u^2)\frac{\partial D_1(u,\nu)}{\partial\nu^2}\right), \tag{47}$$

$$\tilde{V}_3(u)=\int_{-\infty}^{\infty}d\nu u^2\frac{\partial D_1(u,\nu)}{\partial u^2}=-V_3(u). \tag{48}$$

Finally for the last two terms in Eq. (44) one has

$$V_{\rm SD}^{(\sigma\sigma)}(BE)=-V_{\rm SD}^{(\sigma\sigma)}(EB)=\sum_{i>j}\frac{(\boldsymbol{\sigma}^{(i)}\times\boldsymbol{\sigma}^{(j)})\frac{1}{2i}\left(\frac{\mathbf{p}^{(i)}}{\mu_i}+\frac{\mathbf{p}^{(j)}}{\mu_j}\right)}{4\mu_i\mu_j(N_c-1)}\int_{-\infty}^{\infty}\frac{\partial D_1(u,\nu)}{\partial u^2}\nu^2d\nu. \tag{49}$$

This concludes the calculation of the spin-spin interaction.

We turn now to the calculation of spin-orbit terms. The corresponding expression in Eq. (3) can be written as

$$\langle W_3\exp(g\sigma F)\rangle_{\rm so}=\exp\left\{\sum_{i=1}^{3}i\int\frac{dt_i}{2\mu_i}\sigma^{(i)}_{\mu\nu}ds^{(i)}_{\rho\sigma}(u)D_{\mu\nu,\rho\sigma}(z,u)+\frac{i}{N_c-1}\sum_{i\neq j}\int\frac{dt_i}{2\mu_i}ds^{(j)}_{\rho\sigma}(u)\sigma^{(i)}_{\mu\nu}D_{\mu\nu,\rho\sigma}(z,u)\right\}. \tag{50}$$

In Eq. (50) we have defined as in the Appendix

$$D_{\mu\nu,\rho\sigma}(z,u)=\frac{g^2}{N_c}\langle{\rm tr}F_{\mu\nu}(z(t_i))F_{\rho\sigma}(u)\rangle=D(h)(\delta_{\mu\rho}\delta_{\nu\sigma}-\delta_{\mu\sigma}\delta_{\nu\rho})+\tfrac{1}{2}[\partial_\mu h_\rho\delta_{\nu\sigma}+\text{perm.}]D_1(h)$$

and $h_\mu = z_\mu(t_i) - u_\mu$.

The Dirac structure of the exponent in Eq. (50) is a sum, which can be written with notations from Eq. (44) as

$$V^{(\mathrm{so})} = \sum_{i=1}^{3} V_i^{(\mathrm{so,diag})}(\hat{1}_i \times \hat{1}_{jk}) + V_i^{(\mathrm{so,nondiag})}(\hat{\rho}_1^{(i)} \times \hat{1}_{jk}), \quad i \neq j,k. \tag{51}$$

In the 4×4 matrix $\sigma_{\mu\nu}^{(i)}$ we first consider the diagonal part. Repeating all the steps leading to Eqs. (18), (23), and (24) one has the same expressions with the replacement $m_i \to \mu_i$, $i = 1,2,3$, namely

$$V_{\mathrm{SD}}^{(\mathrm{so,\ diag})} = \left\{ \sum_{i=1}^{3} \frac{\boldsymbol{\sigma}^{(i)}\mathbf{L}^{(i)}}{2\mu_i^2} \frac{1}{R^{(i)}} \frac{dV_1}{dR^{(i)}} + \frac{1}{N_c - 1} \sum_{i<j} \frac{\boldsymbol{\sigma}^{(i)}\mathbf{L}^{(i)} + \boldsymbol{\sigma}^{(j)}\mathbf{L}^{(i)}}{2\mu_i\mu_j} \frac{1}{R^{(j)}} \frac{dV_2}{dR^{(j)}} \right\}. \tag{52}$$

Let us now consider the nondiagonal part in Eq. (51), which can be written as

$$\langle W_3 \exp(g\sigma F)\rangle_{(\mathrm{so,nondiag})} = \exp\left\{ i\sum_{i=1}^{3} \int \frac{dt_i}{2\mu_i} \sigma_k^{(i)} [D_{k4,l4}^{(ii)}(z,u) ds_{l4}^{(i)}(u) + D_{k4,mn}(zu) ds_{nm}^{(i)}(u)] \right.$$
$$\left. + i\sum_{i\neq j} \int \frac{dt_i}{2\mu_i} \sigma_k^{(i)} [D_{k4,l4}^{(ij)}(z,u) ds_{l4}^{(j)}(u) + D_{k4nm}^{(ij)} ds_{nm}^{(j)}(u)] \right\}. \tag{53}$$

Now taking into account Eqs. (13) and (14), one can write Eq. (53) in the form

$$\langle W_3 \exp(g\sigma F)\rangle_{(\mathrm{so,nondiag})} = \exp\left[-T\left\{ \sum_{i=1}^{3} V_{(\mathrm{so,nondiag})}^{(ii)} + \sum_{i<j=1}^{3} V_{(\mathrm{so,nondiag})}^{(ij)} \right\} \right]. \tag{54}$$

One finds from Eq. (53), replacing $D_{\mu\nu,\rho\sigma}$ from Eq. (A1) and using Eqs. (13) and (14),

$$V_{(\mathrm{so,nondiag})}^{(ii)} = \Delta_{EE}^{(ii)} + \tilde{\Delta}_{EE}^{(ii)} + \Delta_{EB}^{(ii)} \tag{55}$$

and similarly for $V_{(\mathrm{so,nondiag})}^{(ij)}$. For terms on the r.h.s. of Eq. (55) one obtains

$$\begin{aligned}
\Delta_{EE}^{(ii)} &= -i \frac{\boldsymbol{\sigma}\mathbf{R}^{(i)}}{2\mu_i R^{(i)}} \Lambda^{(ii)}, \\
\Lambda^{(ii)} &= \int_0^{R^{(i)}} d\nu du \left(D(\nu,u) + D_1 + \nu^2 \frac{\partial D_1}{\partial \nu^2} \right), \\
\tilde{\Delta}_{EE}^{(ii)} &= -i \int (\boldsymbol{\sigma}\mathbf{u})(\mathbf{R}^{(i)}\mathbf{u}) \frac{\partial D_1}{\partial u^2}(\nu,u) d\nu d\beta R^{(i)}, \quad \mathbf{u} = \mathbf{R}^{(i)}\beta; \\
\Delta_{EB}^{(ii)} &= \frac{i}{2\mu_i^3} \int_{-\infty}^{\infty} \nu^2 d\nu \frac{\partial D_1(\nu_1, h)}{\partial \nu^2} \int_0^1 \beta^{(i)} d\beta^{(i)} (\mathbf{L}^{(i)} \times \boldsymbol{\sigma}^{(i)}) \mathbf{p}^{(i)}.
\end{aligned} \tag{56}$$

Similarly for $\Delta^{(ij)}$ one obtains from the second term on the r.h.s. of Eq. (53),

$$\begin{aligned}
\Delta_{EE}^{(ii)} &= -i \frac{\boldsymbol{\sigma}\mathbf{R}^{(j)}}{2\mu_i R^{(j)}} \frac{\Lambda^{(ij)}}{(N_c - 1)}, \\
\Lambda^{(ij)} &= R^{(i)} \int_0^1 d\beta^{(j)} \int_{-\infty}^{\infty} d\nu \left(D(\nu, r^{(ij)}) + D_1 + \nu^2 \frac{\partial D_1}{\partial \nu^2} \right), \\
\tilde{\Delta}_{EE}^{(ii)} &= -i \int \frac{(\boldsymbol{\sigma}\mathbf{r}^{(ij)})(\mathbf{R}^{(j)}\mathbf{r}^{ij)})}{2\mu_i(N_c - 1)} d\nu \int_0^1 d\beta^{(j)} \frac{\partial D_1(\nu, r^{(ij)})}{\partial (r^{(ij)})^2},
\end{aligned}$$

$$\Delta_{EB}^{(ij)} = \frac{i}{2\mu_i\mu_j} \int_{-\infty}^{\infty} \nu^2 d\nu \frac{\partial D_1(\nu, r^{(ij)})}{\partial \nu^2} \int_0^1 \beta^{(j)} d\beta^{(j)} (\mathbf{L}^{(j)} \times \boldsymbol{\sigma}^{(j)}) \left(\frac{\mathbf{p}^{(i)}}{\mu_i} + \beta^{(j)} \frac{\mathbf{p}^{(j)}}{\mu_j} \right). \tag{57}$$

Here we have defined $\mathbf{r}^{(ij)}=\mathbf{R}^{(i)}-\beta^{(j)}\mathbf{R}^{(j)}$.

VI. DISCUSSION

Let us now discuss the results obtained in the paper. For the heavy-quark case the nonperturbative spin-dependent potential is given in Eq. (8), and the perturbative part in Eq. (42), so that the total SD potential is

$$V_{\rm SD}^{(\rm total)}=V_{\rm SD}^{(\rm nonpert)}+V_{\rm SD}^{(\rm pert)}. \tag{58}$$

The perturbative part agrees with that obtained long ago in [7] and repeated in many subsequent papers. However, in [8] the term $V^{(\varepsilon,\rm pert)}$ is taken twice as big as Eq. (38) (or the corresponding term in [7]). A possible modification for light quarks, which can produce this increase, is discussed later.

The nonperturbative part $V_{\rm SD}^{(\rm nonpert)}$ (8) consists of six terms, which were never fully written before.

Only asymptotics at large distances of the first term in Eq. (8) has been written before in [26]. One can find it from Eq. (8),

$$V^{(\rm nonpert)}(R^{(i)}\to\infty)=-\frac{\boldsymbol{\sigma}^{(i)}\mathbf{L}^{(i)}\sigma}{4m_i^2R^{(i)}}. \tag{59}$$

In [8], the pairwise nonperturbative spin-orbit forces were postulated instead, which contradict expressions derived in this paper and in [26]. All other terms, proportional to V_2',V_3,V_4, and V_5, have never been written for the 3q case, while for the $q\bar{q}$ case the corresponding terms (except for V_5) have been written in [18,19]. The term V_5, which has no counterpart in the $q\bar{q}$ case, is completely new and its physical implication is still unclear.

We now turn to the light quark case. Here the total SD "potential" is in general a sum of products of (4×4) matrices, which can be written as

$$\hat{V}_{\rm SD}^{(\rm light\ \ quarks)}=\sum_{i,j}\ (V_{\rm diag}^{(ij)}\hat{1}_i\times\hat{1}_j+\hat{V}_{\rm nondiag}^{(ij)}), \tag{60}$$

where $\hat{V}_{\rm nondiag}^{(ij)}$ contains terms such as $\hat{1}_i\times\hat{\rho}_{1j},\hat{\rho}_{1i}\times\hat{1}_j,\hat{\rho}_{1i}\times\hat{\rho}_{1j}$, and $\rho_1=\left(\begin{smallmatrix}0&\hat{1}\\ \hat{1}&0\end{smallmatrix}\right)$, where each entry in ρ_1 is a 2×2 unit matrix.

Now for $V_{\rm diag}^{(ij)}$ one has

$$V_{diag}^{(ij)}=V_{\rm SD}^{(\sigma\sigma)}(BB)+V_{\rm SD}^{(so,diag)}, \tag{61}$$

where the first term on the r.h.s. of Eq. (61) is given in Eq. (45) and the second in Eq. (52). One can see in these expressions for $V_{\rm diag}^{(ij)}$ the same terms as in Eq. (8) with exchange $m_i\to\mu_i$ except for the spin-orbit term proportional to $d\varepsilon/dR$. Before discussing two different strategies for obtaining this last term, let us look at the general structure of Eq. (60). It has the described above matrix form and depends on einbein fields μ_i, $i=1,2,3$. The latter have been defined previously in [10,17,20,23,27,28] as scalars, $2\mu_i=dz_i(\tau)/d\tau$, and are assumed to be found from the stationary point equation in the path-integral form of the meson Green's function, or from the stationary point of the Hamiltonian. Now the spin-independent part of the Hamiltonian is a unit matrix and hence can produce scalar stationary values for μ_i. The situation changes, however, if one tries to incorporate also the SD part of the Hamiltonian in the stationary point equation for μ, since it would require μ to have a matrix form similar to that of $\hat{V}_{\rm SD}$.

This is possible in the generalized form of the FFSR, which is now under investigation, but in the present form the only possible way of treating the SD part of the Hamiltonian is to consider it as a perturbation. For light quarks it is not an expansion in $1/\mu_i$, and the whole expression (60) is obtained with the only and numerically good approximation—keeping the bilocal (Gaussian) correlator, neglecting all higher ones.

As was shown in the meson case [20,28], this perturbation procedure works well even for the lowest mesons, where SD corrections produce up to around 15% of the total mass (a similar situation holds true in earlier quark model calculations with fixed and prescribed constituent masses, see, e.g., [8]). For heavier meson and baryon states the masses μ_i grow rapidly with quantum numbers [20] and the validity of the perturbative treatment of SD terms becomes even better established. In what follows, we describe a perturbative procedure for treating the nondiagonal terms.

To this end we must remember [as in point (B) of the derivation in Sec. IV that nondiagonal terms are also present in the preexponential factor $(m-\hat{D})$ in Eq. (1). Consider the largest nondiagonal term $\Delta_{EE}^{(ii)}$ in Eq. (56), and take for simplicity its asymptotic form

$$V_{EE}=-i\sum_{i=1}^{3}\frac{\boldsymbol{\alpha}^{(i)}\mathbf{n}^{(i)}\sigma}{2\mu_i},\qquad \mathbf{n}^{(i)}=\mathbf{R}^{(i)}/R^{(i)}. \tag{62}$$

One has [omitting index (i) for simplicity]

$$\begin{aligned}&(m-\hat{D})\exp(-V_{EE}T)\\&\quad\cong\begin{pmatrix}m+\mu, & -\boldsymbol{\sigma}\mathbf{p}\\ \boldsymbol{\sigma}\mathbf{p}, & m-\mu\end{pmatrix}\left(1+i\frac{\sigma}{2\mu}\boldsymbol{\alpha}\mathbf{n}T+\cdots\right).\end{aligned} \tag{63}$$

Comparing with the leading term, given by the upper left corner, one normalizes $(m-\hat{D})$ by extracting the factor $(m+\mu)$ and thus obtains

$$\begin{aligned}&\text{Eq. }(63)=(1-V_{\rm SD}^{(\varepsilon)}T+\cdots),\\&V_{\rm SD}^{(\varepsilon)}=\sum_{i=1}^{3}\frac{\sigma\boldsymbol{\sigma}^{(i)}\mathbf{L}^{(i)}}{2\mu_i(m_i+\mu_i)R^{(i)}}.\end{aligned} \tag{64}$$

In the heavy quark limit, $\mu_i\approx m_i$ and Eq. (61) coincides with the term proportional to $(1/R)(d\varepsilon/dR)$ in Eq. (8). For light quarks , when $\mu_i\gg m_i$, however, one has a twice as large coefficient in (61), which coincides with the corresponding coefficient in [8], thus reconciling the heavy quark expansion and the light quark expression. Hence our total expression for the SD potential treated as perturbation is a 2×2 matrix

$$\tilde{V}_{\rm SD}^{(\rm light\ \ quarks)} = \sum_{i,j} V_{\rm diag}^{(ij)} + V_{\rm SD}^{(\varepsilon)}, \tag{65}$$

where $V_{diag}^{(ij)}$ is given in Eqs. (61), (45) and (52) and $V_{\rm SD}^{(\varepsilon)}$ is given in Eq. (64), with a general form obtained by replacing $\sigma \to \partial\varepsilon/\partial R$. Now one can see from these expressions that we have a full correspondence between terms in Eq. (8) and in Eq. (65), where each term in Eq. (65) is obtained from the corresponding one in Eq. (8) by the replacement $m_i \to \mu_i$, except for the term with $d\varepsilon/dR$, where one replaces $2m_i^2 \to \mu_i(\mu_i + m_i)$.

VII. CONCLUDING REMARKS

We have obtained all perturbative and nonperturbative spin-dependent terms in the $3q$ system in the approximation when the lowest (bilocal) field correlator is retained in the Wilson loop. The analogous procedure for mesons in [18,19] yielded SD potentials satisfying Gromes relation [24], with correct asymptotics at large distances of Thomas precession type. For the $3q$ system we also get this asymptotics for spin-orbit terms in the form of a sum of one-body Thomas terms, in agreement with earlier results in [26]. All other nonperturbative terms and the exact nonasymptotic form of Thomas terms are new. The signs of perturbative and nonperturbative spin-orbit terms are different and one may expect some cancellation, which should be checked in exact calculations of baryon spectra with spin splittings. All nonperturbative SD terms in Eq. (8) except for V_5 have a structure similar to that of the $Q\bar{Q}$ case, considered in [19], except that spin-orbit terms are of one-body rather than two-body character. The new term V_5 (24) does not have a $Q\bar{Q}$ analog, and after averaging over coordinates has a structure similar to two-body spin-orbit force.

The large N_c structure of SD interaction can be clearly seen from explicit expressions and may be represented as leading $[O(N_c^0)]$ terms of one-body spin-orbit interaction, when both fields in the field correlator are on the same sheet of the three-sheet surface, and suppressed $[O(N_c^{-1})]$ terms of spin-spin interactions and spin orbit from two different sheets. Hence the $3q$ dynamics in the large N_c limit reduces to the uncorrelated motion of N_c quarks around a common center (string junction), which can be taken as infinitely heavy.

The general structure of the SD potential (8) at large N_c is in agreement with the classification done in [29] where the terms unsuppressed at large N_c are one-body spin-orbit potentials, while two-body spin-dependent terms are $1/N_c$ suppressed. In addition, in [29] there appear also spin-flavor terms which can be associated with with pion and kaon exchange forces. The latter were not considered in the present paper, but can be easily included in the same formalism, using the new chiral Lagrangian derived in [30]. It is shown there that in the $q - string - \bar{q}$ system pions are emitted by quarks with the known amplitude, so that the pion-exchange force can be predicted unambiguously and added to those obtained in the present work. This would complete the overall picture of SD forces in a baryon.

ACKNOWLEDGMENTS

This work was supported by DOE Contract No. DE-ACOS-84ER 40150 under which SURA operates the Thomas Jefferson National Accelerator Facility. The author is grateful to J. Goity for useful discussion, remarks and suggestions.

APPENDIX A: FIELD CORRELATORS

From general definitions of $g^2\langle F_{\mu\nu}F_{\lambda\sigma}\rangle$ through D, D_1 in [11] one gets

$$\frac{g^2}{N_c}{\rm tr}\langle F_{\mu\nu}(x)\Phi(x,y)F_{\rho\sigma}(y)\Phi(y,x)\rangle$$
$$= (\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho})D(z) + \frac{1}{2}[\partial_\mu z_\rho \delta_{\nu\rho} + \text{perm.}]D_1(z), \tag{A1}$$

$$\frac{g^2}{N_c}{\rm tr}\langle B_i(x)\Phi(x,y)B_j(y)\Phi(y,x)\rangle$$
$$= \delta_{ij}\left(D(z) + D_1(z) + \mathbf{z}^2\frac{\partial D_1}{\partial z^2}\right) - z_i z_j\frac{\partial D_1}{\partial z^2}, \tag{A2}$$

$$\frac{g^2}{N_c}{\rm tr}\langle E_i(x)\Phi(x,y)E_j(y)\Phi(y,x)\rangle$$
$$= \delta_{ij}\left(D(z) + D_1(z) + z_4^2\frac{\partial D_1}{\partial z^2}\right) + z_i z_j\frac{\partial D_1}{\partial z^2}, \tag{A3}$$

$$\frac{g^2}{N_c}{\rm tr}\langle B_i(x)\Phi(x,y)E_j(y)\Phi(y,x)\rangle = e_{ijk}z_4 z_k\frac{\partial D_1}{\partial z^2}, \tag{A4}$$

where we have defined

$$z_\mu = x_\mu - y_\mu, \quad \mu = 1,2,3,4,$$

$$\Phi(x,y) = P\exp ig\int_y^x A_\mu(u)du_\mu.$$

[1] N. Isgur, Phys. Rev. D **62**, 054026 (2000).

[2] N. Isgur, nucl-th/0007008.

[3] L.Ya. Glozman, Phys. Lett. B **475**, 329 (2000); L.Ya. Glozman and D.O. Riska, Phys. Rep. **268**, 263 (1996).

[4] V. Burkert, in *Perspectives in the Structure of Hadronic Systems*, edited by M.N. Harakeh *et al.* (Plenum Press, New York, 1994).

[5] V. Burkert, hep-ph/0106143

[6] CLAS Collaboration, V. Burkert, Nucl. Phys. **A684**, 16 (2001).
[7] A. De Rujula, H. Georgi, and S.L. Glashow, Phys. Rev. D **12**, 147 (1975); T. De Grand, R.L. Jaffe, K. Johnson, and J. Kiskis, *ibid.* **D12**, 2060 (1975).
[8] S. Capstick and N. Isgur, Phys. Rev. D **34**, 2809 (1996).
[9] A. Di Giacomo, H.G. Dosch, V.I. Shevchenko, and Yu.A. Simonov, hep-ph/0007223; Yu.A. Simonov, Nucl. Phys. **B307**, 512 (1988).
[10] Yu.A. Simonov, Phys. Lett. B **228**, 413 (1989); M. Fabre de la Ripelle, and Yu.A. Simonov, Ann. Phys. (N.Y.) **212**, 235 (1991).
[11] H.G. Dosch and Yu.A. Simonov, Phys. Lett. B **205**, 339 (1988).
[12] G.S. Bali, Nucl. Phys. B (Proc. Suppl.) **82**, 422 (2000); Phys. Rev. D **62**, 114503 (2000); S. Deldar, *ibid.* **62**, 034509 (2000).
[13] Yu.A. Simonov, JETP Lett. **71**, 187 (2000); V.I. Shevchenko and Yu.A. Simonov, Phys. Rev. Lett. **85**, 1811 (2000); V.I. Shevchenko and Yu.A. Simonov, hep-ph/0104135.
[14] M. Campostrini, A. Di Giacomo, and G. Mussardo, Z. Phys. C **25**, 173 (1984); A. Di Giacomo, and H. Panagopoulos, Phys. Lett. B **285**, 133 (1992); M. D'Elia, A. Di Giacomo, and E. Meggiolaro, *ibid.* **408**, 315 (1997); A. Di Giacomo, E. Meggiolaro, and H. Panagopoulos, Nucl. Phys. **B483**, 371 (1997).
[15] M. Eidemueller, H.G. Dosch, and M. Jamin, Nucl. Phys. B (Proc. Suppl.) **86**, 421 (2000).
[16] Yu.A. Simonov, Nucl. Phys. **B592**, 350 (2001).
[17] B.O. Kerbikov and Yu.A. Simonov, Phys. Rev. D **62**, 093016 (2000).
[18] Yu.A. Simonov, Nucl. Phys. **B324**, 67 (1989); M. Schiestl and H.G. Dosch, Phys. Lett. B **209**, 85 (1988).
[19] A.M. Badalian and Yu.A. Simonov, Phys. At. Nucl. **59**, 2164 (1996).
[20] Yu.A. Simonov, hep-ph/9911237.
[21] A.M. Badalian and V.l. Morgunov, Phys. Rev. D **60**, 116008 (1999); A.M. Badalian, V.l. Morgunov, and B.L.G. Bakker, Phys. At. Nucl. **63**, 1635 (2000); A.M. Badalian and B.L.G. Bakker, Phys. Rev. D **62**, 094031 (2000).
[22] A.M. Badalian and B.L.G. Bakker, Phys. Rev. D **64**, 114010 (2001).
[23] Yu.A. Simonov (in preparation).
[24] E. Eichten and F. Feinberg, Phys. Rev. D **23**, 2724 (1981); D. Gromes, Z. Phys. C **26**, 401 (1984).
[25] M. Eidemueller and M. Jamin, Phys. Lett. B **416**, 415 (1998); V.I. Shevchenko, hep-ph/9802274; V.I. Shevchenko and Yu.A. Simonov, Phys. Lett. B **437**, 131 (1998).
[26] C. Ford, J. Phys. G **15**, 1641 (1989); N. Brambilla, P. Consoli, and G.M. Prosperi, Phys. Rev. D **50**, 5878 (1994); N. Brambilla, P. Consoli, and G.M. Prosperi, hep-th/9401051.
[27] A.Yu. Dubin, A.B. Kaidalov, and Yu.A. Simonov, Yad. Fiz. **56**, 213 (1993).
[28] A.M. Badalian and B.L.G. Bakker, hep-ph/0202246.
[29] J.L. Goity, Phys. Lett. B **414**, 40 (1997); C.L. Schat, J.L. Goity, and N.N. Scoccola, Phys. Rev. Lett. **88**, 102002 (2002).
[30] Yu.A. Simonov, Phys. Rev. D **65**, 094018 (2002).