

# Bounds on broken $R$ -parity from leptonic meson decays

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Investigating leptonic decays of  $\pi^-$ ,  $K^-$ ,  $B^-$ ,  $\pi^0$ ,  $K_L^0$ , and  $B_S^0$  we present new bounds on some products of two  $R$ -parity violating coupling constants. For mesons of a similar structure but with poor experimental data we give the corresponding formulas, to be used in the future.

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## I. INTRODUCTION

The minimal supersymmetric standard model (MSSM) plus  $R$ -parity violation ( $\mathcal{R}_p$ ) is obtained from the MSSM by adding the following terms to the superpotential (cf. Ref. [1]):

$$\Delta\mathcal{W}_{\mathcal{R}_p} = \frac{1}{2}\varepsilon^{ab}\lambda_{ijk}L_a^iL_b^jE^{kC} + \varepsilon^{ab}\delta^{xy}\lambda'_{ijk}L_a^iQ_{bx}^jD_y^{kC} + \frac{1}{2}\varepsilon^{xyz}\lambda''_{ijk}U_x^iC^jD_y^kD_z^{kC} + \varepsilon^{ab}\kappa_iL_a^iH_b^U. \quad (1)$$

$H$ ,  $Q$ , and  $L$  represent the left chiral  $SU(2)_W$ -doublet superfields of the Higgs bosons, the quarks, and the leptons;  $U$ ,  $D$ , and  $E$  represent the right chiral superfields of the  $u$ -type quarks,  $d$ -type quarks and electron-type leptons, respectively; a superscript  $C$  denotes charge conjugation;  $a, b$  and  $x, y, z$  are  $SU(2)_W$  and  $SU(3)_C$  indices,  $i, j, k$  and later also  $f, g, l, n$  are generational indices (summation over repeated indices is implied);  $\delta^{xy}$  is the Kronecker symbol, and  $\varepsilon^{\dots}$  symbolizes any tensor that is totally antisymmetric with respect to the exchange of any two indices, with  $\varepsilon^{12\dots} = 1$ . The coupling constants  $\lambda_{ijk}$  and  $\lambda''_{ijk}$  are antisymmetric with respect to the exchange of the first two and last two indices. The last term in Eq. (1) can be rotated away by utilizing a unitary field redefinition.

Good agreement between SM theory and experiment gives stringent upper bounds on the extra 45 coupling constants  $\lambda_{ijk}$ ,  $\lambda'_{ijk}$ , and  $\lambda''_{ijk}$ , as well as on products thereof. For a list of references and the processes dealt with, see, e.g.,

Refs. [2–4]. In particular, in  $\mathcal{R}_p$  there are new operators for leptonic meson decays. The SM theoretical predictions for the decay widths of mesons and the measured values match up within the experimental uncertainty. We can thus determine yet further tight constraints on several products of coupling constants:  $\lambda'^*\lambda'$  and  $\lambda'^*\lambda$ . This was first done in Ref. [5] for single coupling constants and later in Ref. [6] for some products, but treating only charged pions decaying via either  $d$ -type squark or slepton exchange, respectively. Reference [7] treated general leptoquark reactions of several particles, one of them the  $K_L^0$ ; this result was quoted in terms of  $\mathcal{R}_p$  by Ref. [8]; the same result was reached in Ref. [9]. Reference [10] among other things dealt with the decay of  $K_L^0$ , but with only  $u$ -squark exchange contributing to SM-allowed processes. The decays of neutral and charged  $B$  mesons were treated in Ref. [11] and Ref. [12], respectively. We generalize these calculations, focusing on products of two coupling constants, and stress where we obtain new or stricter bounds.

## II. $\mathcal{R}_p$ -DECAY OF CHARGED MESONS

### A. Calculation of the decay rate

Consider a negatively charged meson  $\pi^{ij}$  at rest made of a  $d$ -type quark  $d^i$  and a  $u$ -type antiquark  $u^{jC}$  which decays into an antineutrino  $\nu^{nC}$  and a charged lepton  $l^f$ , i.e.,

$$|\pi^{ij}(p_1)\rangle \rightarrow |\nu^{nC}(p_2); l^f(p_3)\rangle, \quad (2)$$

the  $p_{1,2,3}$  being four-momenta. We now calculate the partial decay rate of this process. Focusing on the Yukawa couplings of the first two terms in Eq. (1), again with summation over repeated indices implied, leads to

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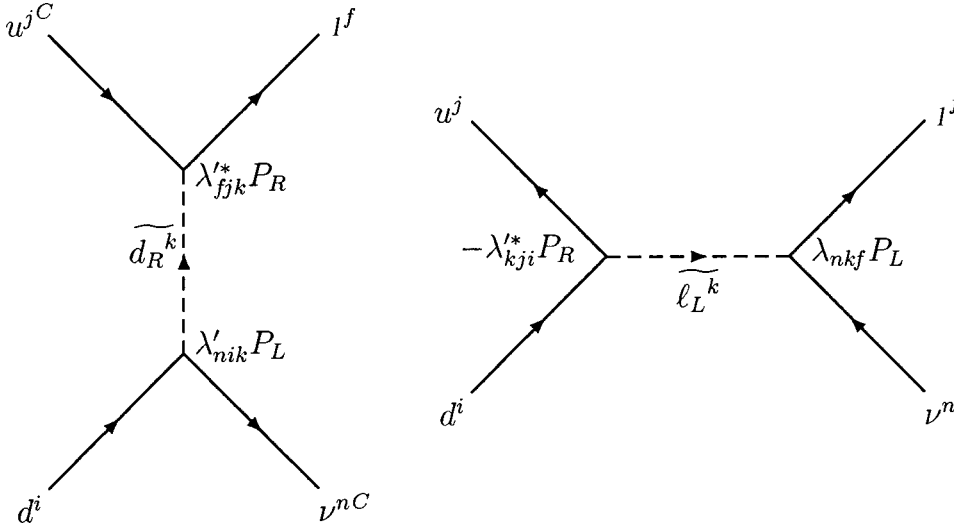


FIG. 1. The tree-level MSSM +  $\mathcal{R}_p$  processes contributing to the decay of the charged mesons.

$$\begin{aligned} \mathcal{L}_{\mathcal{R}_p} \supset & \lambda_{ijk} (\overline{\nu^{iC}} P_L l^j \widetilde{l}_R^{k*} + \overline{l^k} P_L l^j \widetilde{\nu}_L^i + \overline{l^k} P_L \nu^i \widetilde{l}_L^j) + \lambda'_{ijk} (\overline{\nu^{iC}} P_L d^j \widetilde{d}_R^{k*} + \overline{d^k} P_L d^j \widetilde{\nu}_L^i + \overline{d^k} P_L \nu^i \widetilde{d}_L^j) \\ & - \lambda'_{ijk} (\overline{u^{iC}} P_L l^j \widetilde{d}_R^{k*} + \overline{d^k} P_L u^j \widetilde{l}_L^i + \overline{d^k} P_L l^i \widetilde{u}_L^j) + \text{c.c.} \end{aligned} \quad (3)$$

All spinors are Dirac spinors, the overbar denotes the Dirac adjoint, and  $P_{L,R}$  are the projection operators on the left- and right-handed parts. *The fermions are mass-eigenstates.* A tilde denotes a scalar; the scalars' subscripts  $L,R$  indicate the chirality of the corresponding Weyl spinor. The fourth term in Eq. (3) together with the complex conjugate of the seventh term, and the third term together with the complex conjugate of the eighth term lead to the meson decay processes depicted in Fig. 1, which give the effective Hamiltonians

$$\begin{aligned} \mathcal{H}^{\widetilde{d}_R} &= \frac{1}{2} \sum_k \frac{\lambda'_{fjk} \lambda'_{nik}}{m_{\widetilde{d}_R^k}^2} \overline{l^f} \gamma_\nu P_L \nu^n \overline{u^j} \gamma^\nu P_L d^i, \\ \mathcal{H}^{\widetilde{l}_L} &= - \sum_k \frac{\lambda'_{kji} \lambda_{nkf}}{m_{\widetilde{l}_L^k}^2} \overline{l^f} P_L \nu^n \overline{u^j} P_R d^i, \end{aligned} \quad (4)$$

where  $m$  is the mass of a particle. To obtain the first equation we employed a Fierz identity. These two Hamiltonians have to be added to the effective Hamiltonian for the SM process:

$$\mathcal{H}^W = \frac{4G_F V_{ji}}{\sqrt{2}} \overline{l^f} \gamma_\nu P_L \nu^j \overline{u^i} \gamma^\nu P_L d^i. \quad (5)$$

Here  $G_F$  is the Fermi constant and  $V_{ji}$  is an element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. We obtain, for the transition amplitude  $\mathcal{M}_{ijfn}$ ,

$$\begin{aligned} \mathcal{M}_{ijfn} \delta^4(p_1 - p_2 - p_3) &= \frac{1}{2\pi i} \int \langle l^f; \nu^n | \\ & \times (\mathcal{H}^W + \mathcal{H}^{\widetilde{d}_R} + \mathcal{H}^{\widetilde{l}_L}) | \pi^{ij} \rangle d^4x. \end{aligned} \quad (6)$$

We expand the fields in the initial and final states, perform the integrations, and use

$$\begin{aligned} \langle 0 | \overline{u^j}(y) \gamma^\nu P_{L,R} d^i(y) | \pi^{ij}(p_1) \rangle \\ &= \pm \frac{1}{\sqrt{2}} f_{\pi^{ij}} p_1^\nu e^{-ip_1 y}, \\ \langle 0 | \overline{u^j}(y) P_{L,R} d^i(y) | \pi^{ij}(p_1) \rangle \\ &= \mp \frac{1}{\sqrt{2}} \frac{m_{\pi^{ij}}^2}{m_{u^j} + m_{d^i}} f_{\pi^{ij}} e^{-ip_1 y}; \end{aligned} \quad (7)$$

$f_{\pi^{ij}}$  is the meson decay constant.<sup>1</sup> Thus

$$\begin{aligned} \mathcal{M}_{ijfn} &= \frac{f_{\pi^{ij}}}{2\sqrt{2}} \overline{U^f}(\vec{p}_3) \sum_k \left\{ \left( \frac{\delta_{fn} 8G_F V_{ji}}{3\sqrt{2}} + \frac{\lambda'_{fjk} \lambda'_{nik}}{m_{\widetilde{d}_R^k}^2} \right) \right. \\ & \left. \times \not{p}_1 - 2 \frac{m_{\pi^{ij}}^2}{m_{u^j} + m_{d^i}} \frac{\lambda'_{kji} \lambda_{nkf}}{m_{\widetilde{l}_L^k}^2} \right\} P_L \mathcal{V}^n(\vec{p}_2). \end{aligned} \quad (8)$$

<sup>1</sup>There are several ways of defining the meson decay constant, differing by factors of  $\sqrt{2}$ ; in the convention we use  $f_\pi = 92.4 \pm 0.3$  MeV (see Ref. [15]).

$\overline{\mathcal{U}}^f, \mathcal{V}^n$  are the Fourier coefficient functions of  $l^f, \nu^{nC}$ , respectively. Next we take the absolute value squared, average over the spins, and use the trace theorems. Then we sum over  $n$ , because the experiments that measured the partial decay widths did not determine the flavor of the antineutrinos,<sup>2</sup> resulting in

$$\begin{aligned} \sum_n \langle |\mathcal{M}_{ijfn}|^2 \rangle &= 4G_F^2 f_{\pi ij}^2 |V_{ji}|^2 (m_{\pi ij}^2 - m_{lf}^2) m_{lf}^2 \\ &\times \sum_n (\delta_{fn} + 2\delta_{fn} \text{Re}[K_{ijfn}] + |K_{ijfn}|^2), \end{aligned} \quad (9)$$

with

$$\begin{aligned} K_{ijfn} &= \frac{\sqrt{2}}{8G_F |V_{ji}|} \sum_k \left( \frac{\lambda'_{fjk} \lambda_{nik}}{m_{dR}^2} \right. \\ &\left. - 2 \frac{m_{\pi ij}^2}{m_{lf}(m_{uj} + m_{di})} \frac{\lambda'_{kji} \lambda_{nkf}}{m_{lL}^2} \right), \end{aligned} \quad (10)$$

containing all  $\mathcal{R}_p$  contributions;  $2\text{Re}[K_{ijfn}]$  in Eq. (9) is due to the interference between SM and  $\mathcal{R}_p$  amplitudes. For simplicity we neglect the phase of the CKM matrix. The partial decay rate is then

$$\begin{aligned} \Gamma_{\pi ij \rightarrow lf \nu^C}^{\text{SM} + \mathcal{R}_p} &= \Gamma_{\pi ij \rightarrow lf \nu^C}^{\text{SM}} \\ &\times \left( 1 + 2\text{Re}[K_{ijff}] + \sum_n |K_{ijfn}|^2 \right), \end{aligned} \quad (11)$$

with  $\nu^C$  being an *arbitrary* antineutrino, and

$$\Gamma_{\pi ij \rightarrow lf \nu^C}^{\text{SM}} = C_{ijf} G_F^2 f_{\pi ij}^2 |V_{ji}|^2 \frac{[(m_{\pi ij}^2 - m_{lf}^2) m_{lf}^2]}{4\pi m_{\pi ij}^3}; \quad (12)$$

the correction factor  $C_{ijf}$  of  $\mathcal{O}(1)$  is due to higher order electroweak leading logarithms, short distance QCD corrections, and structure dependent effects (see Ref. [13] and also Ref. [14]).

### B. Calculation of the bounds

We prefer not to compare the experimental data directly with Eq. (11), since  $f_{\pi ij}$  has quite a large error. This leads to very weak bounds on  $K_{ijfn}$ . To avoid this, we introduce

<sup>2</sup>The upper experimental bounds on  $\pi^- \rightarrow \mu \nu_e^C$  and  $K^- \rightarrow \mu \nu_e^C$  (see Ref. [15]) come from a different type of experiment, compared to the one used to determine the branching ratios for  $\pi^- \rightarrow \mu \nu^C$  and  $K^- \rightarrow \mu \nu^C$ . They do not lead to better bounds on the coupling constants.

$$\mathcal{R}_{\pi ij} := \frac{\Gamma_{\pi ij \rightarrow lf \nu^C}}{\Gamma_{\pi ij \rightarrow l^g \nu^C}}, \quad (13)$$

with  $m_{l^g} > m_{lf}$ . If the experimental and SM-theoretical decay rates agree well we have  $|2\text{Re}[K_{ijff}] + \sum_n |K_{ijfn}|^2| \ll 1$  [see Eq. (11)]. Putting Eq. (11) into Eq. (13) one gets

$$\begin{aligned} \frac{\mathcal{R}_{\pi ij}^{\text{SM} + \mathcal{R}_p}}{\mathcal{R}_{\pi ij}^{\text{SM}}} &:= 1 + \epsilon_{\pi ij} \\ &\approx 1 + 2\text{Re}[K_{ijff} - K_{ijgg}] \\ &+ \sum_n |K_{ijfn}|^2 - \sum_n |K_{ijgn}|^2. \end{aligned} \quad (14)$$

Let  $\Delta \dots$  symbolize the theoretical or experimental uncertainty. If the theoretical prediction  $\mathcal{R}_{\pi ij}^{\text{SM}} \pm \Delta \mathcal{R}_{\pi ij}^{\text{SM}}$  lies within the experimental range  $\mathcal{R}_{\pi ij}^{\text{expt}} \pm \Delta \mathcal{R}_{\pi ij}^{\text{expt}}$ , one has

$$\begin{aligned} \epsilon_{\pi ij}^{\text{min}} &:= \frac{\mathcal{R}_{\pi ij}^{\text{expt}}}{\mathcal{R}_{\pi ij}^{\text{SM}}} - \Delta \left( \frac{\mathcal{R}_{\pi ij}^{\text{expt}}}{\mathcal{R}_{\pi ij}^{\text{SM}}} \right) - 1 \\ &\leq \epsilon_{\pi ij} \leq \frac{\mathcal{R}_{\pi ij}^{\text{expt}}}{\mathcal{R}_{\pi ij}^{\text{SM}}} + \Delta \left( \frac{\mathcal{R}_{\pi ij}^{\text{expt}}}{\mathcal{R}_{\pi ij}^{\text{SM}}} \right) - 1 \\ &=: \epsilon_{\pi ij}^{\text{max}}. \end{aligned} \quad (15)$$

We could use this to determine a bound on this general combination of  $\mathcal{R}_p$  coupling constants; however, the bounds on individual coupling constants are typically of the order  $\mathcal{O}(10^{-2})$  (see Ref. [2]), and thus we limit ourselves to at most two nonzero coupling constants at a time, and in each case suppose the other 34  $\lambda, \lambda'$  coupling constants vanish [Eq. (16), Eq. (17), and Eq. (18) are also valid for  $f \rightarrow g$  with  $\epsilon_{\pi ij}^{\text{max}} \leftrightarrow -\epsilon_{\pi ij}^{\text{min}}$ ]:

$$\begin{aligned} \epsilon_{\pi ij}^{\text{min}} &\leq 2\text{Re}[K_{ijff}] + |K_{ijff}|^2 \leq \epsilon_{\pi ij}^{\text{max}} \\ &\text{and for } n \neq f \quad |K_{ijfn}|^2 \leq \epsilon_{\pi ij}^{\text{max}}. \end{aligned} \quad (16)$$

We assume that the imaginary parts of the coupling constants are approximately the same as the corresponding real parts.<sup>3</sup> With  $G_F = (0.116639 \pm 0.000001) \times (100\text{GeV})^{-2}$  (see Ref. [15]), we obtain

$$\begin{aligned} &-0.330 |V_{ji}| (\sqrt{1 + 2\epsilon_{\pi ij}^{\text{min}}} + 1) \\ &\leq \frac{\text{Re}[\lambda'_{fjk} \lambda_{fik}]}{(m_{dR}^k / 100 \text{ GeV})^2}, \end{aligned}$$

<sup>3</sup>If the imaginary part vanishes the bounds are weaker by a factor of  $\mathcal{O}(1)$ .

$$\frac{-2m_{\pi ij}^2}{m_{f^i}(m_{u^j}+m_{d^i})} \frac{\text{Re}[\lambda'_{kji} \lambda_{fkf}]}{(m_{\widetilde{L}^k}/100 \text{ GeV})^2} \leq 0.330 |V_{ji}| (\sqrt{1+2\epsilon_{\pi ij}^{\max}} - 1) \quad (17)$$

and for  $n \neq f$

$$\frac{|\lambda'_{fjk} \lambda'_{nik}|}{(m_{\widetilde{d}_R^k}/100 \text{ GeV})^2} \frac{2m_{\pi ij}^2}{m_{f^i}(m_{u^j}+m_{d^i})} \frac{|\lambda'_{kji} \lambda_{nkf}|}{(m_{\widetilde{L}^k}/100 \text{ GeV})^2} \leq 0.66 |V_{ji}| \sqrt{\epsilon_{\pi ij}^{\max}}. \quad (18)$$

The prefactor  $2m_{\pi ij}^2/[m_{f^i}(m_{u^j}+m_{d^i})]$  results in much tighter bounds for  $\lambda'_{kji} \lambda_{nkf}$ . We will apply these results only to processes with sufficiently small experimental error bars.

### C. $\pi^- \rightarrow f^i g + \nu^C$

As a first application, we consider pion decay with  $f, i, j = 1, g = 2$ . The SM gives the  $2\sigma$  theoretical value  $\mathcal{R}_{\pi^-}^{\text{SM}} = (1.2354 \pm 0.0004) \times 10^{-4}$  (see Ref. [14]; the uncertainty mainly derives from  $C_{111}$  and  $C_{112}$ ). From the partial decay widths at the  $2\sigma$  level in Ref. [15], namely,  $\Gamma_{\pi^- \rightarrow e\nu}^{\text{expt}}/\Gamma_{\pi^-}^{\text{expt}} = (1.230 \pm 0.008) \times 10^{-4}$  and  $\Gamma_{\pi^- \rightarrow \mu\nu}^{\text{expt}}/\Gamma_{\pi^-}^{\text{expt}} = 0.9998770 \pm 8 \times 10^{-7}$ , one calculates  $\mathcal{R}_{\pi^-}^{\text{expt}} = (1.230 \pm 0.008) \times 10^{-4}$ . Hence,  $\epsilon_{\pi^-}^{\min} = -0.0107$  and  $\epsilon_{\pi^-}^{\max} = 0.0022$ . With  $|V_{11}| = 0.9750 \pm 0.0008$ , Ref. [5] obtained bounds on a single coupling constant; this was updated in Ref. [16]. We have reproduced their results. The experimental data have only marginally changed and the new bounds are  $|\lambda'_{11k}| \leq 0.027 m_{\widetilde{d}_R^k}/100 \text{ GeV}$  and  $|\lambda'_{21k}| \leq 0.059 m_{\widetilde{d}_R^k}/100 \text{ GeV}$ . We obtain bounds for the products of couplings  $|\lambda'_{11k} \lambda'_{21k}| \leq 0.03 (m_{\widetilde{d}_R^k}/100 \text{ GeV})^2$ ,  $|\lambda'_{11k} \lambda'_{31k}| \leq 0.03 (m_{\widetilde{d}_R^k}/100 \text{ GeV})^2$  and  $|\lambda'_{21k} \lambda'_{31k}| \leq 0.066 (m_{\widetilde{d}_R^k}/100 \text{ GeV})^2$ . The first bound is redundant since the product of the single bounds is stronger; the second and the third bounds are almost the same as the single bound on  $|\lambda'_{11k}|$  and  $|\lambda'_{21k}|$ . Furthermore, we obtain the following new bounds (see Ref. [15]) using  $m_e = (0.510998902 \pm 2.1 \times 10^{-8}) \text{ MeV}$ ,  $m_\mu = (105.6583568 \pm 5.2 \times 10^{-6}) \text{ MeV}$ ,  $m_{\pi^-} = (139.57018 \pm 0.00035) \text{ MeV}$ , and  $m_u + m_d = (8.5 \pm 3.5) \text{ MeV}$ .<sup>4</sup>

$$\begin{aligned} -7.9 \times 10^{-8} \left( \frac{m_{\widetilde{L}^k}}{100 \text{ GeV}} \right)^2 &\leq \text{Re}[\lambda'_{k11} \lambda_{1k1}] \\ &\leq 7.1 \times 10^{-5} \left( \frac{m_{\widetilde{L}^k}}{100 \text{ GeV}} \right)^2, \\ -7.9 \times 10^{-5} \left( \frac{m_{\widetilde{L}^k}}{100 \text{ GeV}} \right)^2 &\leq \text{Re}[\lambda'_{311} \lambda_{232}], \end{aligned}$$

<sup>4</sup>This is the biggest source of inaccuracy, going linearly into the bounds on  $\lambda' \lambda$ . The same applies to  $m_s$ .

$$|\lambda'_{k11} \lambda_{3k1}| \leq 3.4 \times 10^{-6} \left( \frac{m_{\widetilde{L}^k}}{100 \text{ GeV}} \right)^2,$$

$$|\lambda'_{211} \lambda_{322}| \leq 1.5 \times 10^{-3} \left( \frac{m_{\widetilde{L}^k}}{100 \text{ GeV}} \right)^2,$$

$$|\lambda'_{111} \lambda_{211}| \leq 3.4 \times 10^{-6} \left( \frac{m_{\widetilde{L}^k}}{100 \text{ GeV}} \right)^2. \quad (19)$$

The upper bound (bounds) we obtained for  $\text{Re}[\lambda'_{311} \lambda_{232}]$  ( $\text{Re}[\lambda'_{111} \lambda_{212}]$ ,  $|\lambda'_{111} \lambda_{312}|$ ) are weaker than the products of the two bounds on the single coupling constants (see Ref. [4]); there also much stricter bounds were stated for  $|\lambda'_{k11} \lambda_{1k2}|$  as well as for  $|\lambda'_{311} \lambda_{231}|$ .

### D. $K^- \rightarrow f^i g + \nu^C$

Next we consider charged kaon decay with  $f, j = 1, g, i = 2$ . According to Ref. [14],  $\mathcal{R}_{K^-}^{\text{SM}} = (2.472 \pm 0.002) \times 10^{-5}$  at the  $2\sigma$  level. Experimentally  $\Gamma_{K^- \rightarrow e\nu}^{\text{expt}}/\Gamma_{K^-}^{\text{expt}} = (1.55 \pm 0.14) \times 10^{-5}$  and  $\Gamma_{K^- \rightarrow \mu\nu}^{\text{expt}}/\Gamma_{K^-}^{\text{expt}} = 0.6351 \pm 0.0036$ , at the  $2\sigma$  level [15]. Therefore  $\mathcal{R}_{K^-}^{\text{expt}} = (2.44 \pm 0.22) \times 10^{-5}$ ,  $\epsilon_{K^-}^{\min} = -0.10$  and  $\epsilon_{K^-}^{\max} = 0.076$ . Using  $|V_{12}| = 0.222 \pm 0.004$  we obtain  $|\lambda'_{11k} \lambda'_{32k}| \leq 0.04 (m_{\widetilde{d}_R^k}/100 \text{ GeV})^2$  and  $|\lambda'_{21k} \lambda'_{32k}| \leq 0.046 (m_{\widetilde{d}_R^k}/100 \text{ GeV})^2$ . As for the pion these bounds are almost the same as the ones on  $|\lambda'_{11k}|, |\lambda'_{21k}|$ . Our bounds on  $\text{Re}[\lambda'_{11k} \lambda'_{12k}]$  and  $\text{Re}[\lambda'_{21k} \lambda'_{22k}]$  are much weaker than the bounds on  $|\lambda'_{i1k} \lambda'_{i2k}|$  (see Ref. [4]), and we do not list them. Similarly, the existing bounds on  $|\lambda'_{11k} \lambda'_{22k}|$  and  $|\lambda'_{21k} \lambda'_{12k}|$  are much stronger than ours. Furthermore, with  $m_{K^-} = (493.677 \pm 0.016) \text{ MeV}$ ,  $m_s = (122.5 \pm 47.5) \text{ MeV}$ , and  $m_s = (21 \pm 4) m_d$  (see Ref. [15]) we have the following new bounds:

$$\begin{aligned} -7.0 \times 10^{-7} \left( \frac{m_{\widetilde{L}^k}}{100 \text{ GeV}} \right)^2 &\leq \text{Re}[\lambda'_{k12} \lambda_{1k1}] \\ &\leq 1.8 \times 10^{-5} \left( \frac{m_{\widetilde{L}^k}}{100 \text{ GeV}} \right)^2, \\ -1.8 \times 10^{-4} \left( \frac{m_{\widetilde{L}^k}}{100 \text{ GeV}} \right)^2 &\leq \text{Re}[\lambda'_{k12} \lambda_{2k2}] \\ &\leq 3.8 \times 10^{-3} \left( \frac{m_{\widetilde{L}^k}}{100 \text{ GeV}} \right)^2 \\ &(k=3), \end{aligned}$$

$$|\lambda'_{k12} \lambda_{2k1}| \leq 5.4 \times 10^{-6} \left( \frac{m_{\widetilde{L}^k}}{100 \text{ GeV}} \right)^2,$$

$$|\lambda'_{k12} \lambda_{3k1}| \leq 5.4 \times 10^{-6} \left( \frac{m_{\widetilde{L}^k}}{100 \text{ GeV}} \right)^2,$$

$$\begin{aligned}
 |\lambda'_{k12} \lambda_{1k2}| &\leq 1.3 \times 10^{-3} \left( \frac{m_{\widetilde{L}^k}}{100 \text{ GeV}} \right)^2, \\
 |\lambda'_{k12} \lambda_{3k2}| &\leq 1.3 \times 10^{-3} \left( \frac{m_{\widetilde{L}^k}}{100 \text{ GeV}} \right)^2.
 \end{aligned} \tag{20}$$

The upper bound on  $\text{Re}[\lambda'_{k12} \lambda_{212}]$  obtained from the two bounds on the single coupling constants is stricter than the one we obtained.

### E. $B^- \rightarrow l' + \nu^C$

For the charged  $B$ -meson decay the procedure is slightly different since it has not been directly measured. Unlike the two previous cases one only has an experimental upper bound on the branching ratio  $\mathcal{B}$  (see Ref. [15]) and thus has to go back to Eq. (11). This was done in Ref. [12]. We go beyond their work with a more conservative account of the experimental errors and obtain weaker bounds. We also work from the beginning in the mass eigenstate basis to avoid model dependent results (see Ref. [18]).

First  $f=3$ . The theoretical predictions are limited by  $\Gamma_{B \rightarrow \tau \nu^C}^{\text{SM}+\mathcal{R}_p} / \Gamma_{B \text{ total}}^{\text{SM}+\mathcal{R}_p} \leq 5.7 \times 10^{-4}$ . As the total widths  $\Gamma_{B \text{ total}}^{\text{SM}+\mathcal{R}_p}$  and  $\Gamma_{B \text{ total}}^{\text{SM}}$  agree fairly well one has  $\Gamma_{B \text{ total}}^{\text{SM}+\mathcal{R}_p} \approx \Gamma_{B \text{ total}}^{\text{SM}}$ , so that, utilizing Eq. (11), we obtain for the branching ratio

$$\frac{\Gamma_{B \rightarrow \tau \nu^C}^{\text{SM}+\mathcal{R}_p}}{\Gamma_{B \text{ total}}^{\text{SM}+\mathcal{R}_p}} \approx \left( 1 + 2\text{Re}[K_{3133}] + \sum_n |K_{313n}|^2 \right) \frac{\Gamma_{B \rightarrow \tau \nu^C}^{\text{SM}}}{\Gamma_{B \text{ total}}^{\text{SM}}}. \tag{21}$$

To keep the combined uncertainties of  $|V_{13}|$  and  $f_B$  as small as possible we use the theoretical prediction (see Ref. [17])

$$\frac{\Gamma_{B \rightarrow \tau \nu^C}^{\text{SM}}}{\Gamma_{B \text{ total}}^{\text{SM}}} = (4.08 \pm 0.24) \times 10^{-4} \left| \frac{V_{13}}{V_{31}} \right|^2. \tag{22}$$

In order to take into account the correlated uncertainties in  $V_{13}/V_{31}$  we use the Wolfenstein parametrization (see, e.g., Ref. [19]):

$$\frac{V_{13}}{V_{31}} = \frac{\bar{\rho} - i\bar{\eta}}{1 - \lambda^2/2 - \bar{\rho} - i\bar{\eta}}. \tag{23}$$

The Wolfenstein parameters are given by (see Ref. [20])  $\bar{\rho} = 0.21 \pm 0.12$ ,  $\bar{\eta} = 0.38 \pm 0.11$ , and  $\lambda = 0.222 \pm 0.004$ , all at 95% C.L. We thus obtain for the theoretical prediction  $\Gamma_{B \rightarrow \tau \nu^C}^{\text{SM}} / \Gamma_{B \text{ total}}^{\text{SM}} = (1.05 \pm 0.65) \times 10^{-4}$ . The lower value should be used in Eq. (21), to be compared with the experimental upper bound. Thus

$$2\text{Re}[K_{3133}] + \sum_n |K_{313n}|^2 \leq 13.3. \tag{24}$$

In the following, we again assume that only two coupling constants are nonzero. Thus we have  $|K_{3131}|, |K_{3132}| \leq \sqrt{13.3}$ . Furthermore, the imaginary part is again taken to be about the same as the real part; hence  $-\sqrt{1/4 + 13.3/2} - 1/2 \leq \text{Re}[K_{3133}] \leq \sqrt{1/4 + 13.3/2} - 1/2$ . Thus, with  $|V_{13}| = 0.0035 \pm 0.0015$  (see Ref. [15]),  $m_{B^-} = (5279.0 \pm 0.5) \text{ MeV}$ ,  $m_b = (4200 \pm 200) \text{ MeV}$ , and  $m_\tau = (1776.99 \pm 0.29) \text{ MeV}$  (see Ref. [15]) we obtain

$$\begin{aligned}
 |\lambda'_{313} \lambda_{233}| &\leq 2 \times 10^{-3} \left( \frac{m_{\widetilde{L}^k}}{100 \text{ GeV}} \right)^2, \\
 -6 \times 10^{-4} \left( \frac{m_{\widetilde{L}^k}}{100 \text{ GeV}} \right)^2 &\leq \text{Re}[\lambda'_{213} \lambda_{323}] \\
 &\leq 1 \times 10^{-3} \left( \frac{m_{\widetilde{L}^k}}{100 \text{ GeV}} \right)^2.
 \end{aligned} \tag{25}$$

According to Ref. [12] the bounds on  $|\lambda'_{k13} \lambda_{1k3}|$ ,  $|\lambda'_{31k} \lambda'_{13k}|$ ,  $|\lambda'_{31k} \lambda'_{23k}|$ ,  $\text{Re}[\lambda'_{31k} \lambda'_{33k}]$ , and  $\text{Re}[\lambda'_{113} \lambda'_{313}]$  are not better than the previous ones; furthermore, the bound on  $|\lambda'_{113} \lambda_{213}|$  is weaker than the product of the two bounds on the single coupling constants (see Ref. [4]).

Analogously, for  $f=1, 2$ ,

$$\begin{aligned}
 &\left( 1 + 2\text{Re}[K_{31ff}] + \sum_n |K_{31fn}|^2 \right) \Gamma_{B \rightarrow l' \nu^C}^{\text{SM}} \times \tau_{B^-} \\
 &< 5.7 \times 10^{-4},
 \end{aligned} \tag{26}$$

where  $\tau_{B^-}$  is the  $B$ -meson lifetime. Instead of arguing that the error on  $\Gamma_{B \rightarrow l' \nu^C}^{\text{SM}}$  is  $\pm (0.65/1.05) \Gamma_{B \rightarrow l' \nu^C}^{\text{SM}}$ , we are going to be as conservative as possible. Due to isospin invariance  $B^0$  and  $B^-$  have the same decay constant. From Ref. [21],  $f_{B^0} = (200 \pm 30) \text{ MeV}$ , and thus with our convention  $f_B = (141 \pm 21) \text{ MeV}$  (c.f. footnote II A). Therefore,  $f_B^2 |V_{13}|^2 = (0.24 \pm 0.22) \text{ MeV}^2$ , and with  $\tau_{B^-} = 1.655 \times 10^{-12} \text{ s}$  (see Ref. [15]), we obtain for  $f=1$  that  $(1 + \dots)(9.0 \pm 8.3) \times 10^{-12} \leq 1.5 \times 10^{-5}$  and for  $f=2$  that  $(1 + \dots)(3.8 \pm 3.5) \times 10^{-7} \leq 2.1 \times 10^{-5}$ . Working with the lower value, for  $f=1$  we get

$$|\lambda'_{k13} \lambda_{3k1}| \leq 6 \times 10^{-4} \left( \frac{m_{\widetilde{L}^k}}{100 \text{ GeV}} \right)^2. \tag{27}$$

The bounds on  $|\lambda'_{k13} \lambda_{2k1}|$  and  $\text{Re}[\lambda'_{k13} \lambda_{1k1}]$  are not improved compared to the previous ones (see Ref. [12]), and the ones on  $|\lambda'_{11k} \lambda'_{23k}|$ ,  $|\lambda'_{11k} \lambda'_{33k}|$  and  $\text{Re}[\lambda'_{11k} \lambda'_{13k}]$  are too poor to be listed.  $f=2$  yields

$$|\lambda'_{k13} \lambda_{3k2}| \leq 7 \times 10^{-4} \left( \frac{m_{\widetilde{L}^k}}{100 \text{ GeV}} \right)^2. \tag{28}$$

Reference [12] states that there exist better bounds on  $|\lambda'_{k13}\lambda_{1k2}|$  and  $\text{Re}[\lambda'_{k13}\lambda_{2k2}]$ . The bounds on  $|\lambda'_{21k}\lambda'_{13k}|$ ,  $|\lambda'_{21k}\lambda'_{33k}|$ , and  $\text{Re}[\lambda'_{21k}\lambda'_{23k}]$  are almost the same as the single bound on  $|\lambda'_{21k}|$ .

### III. $R_p$ -DECAY OF NEUTRAL MESONS

#### A. Calculation of the bounds

Now we deal with the bound state  $(d^j C d^i)$  decaying into  $l^f$  and  $l^n C$ , with momenta  $p_1, p_3, p_2$ , respectively. We consider only  $n \neq f$ , in which case the process does not occur in the SM and therefore no contributions from loop diagrams have to be taken into account. We proceed as in the previous section. In Eq. (3) the ninth term together with its complex conjugate contributes to the decay, in analogy to the  $\widetilde{d}_R^k$ -exchange in the last section. Furthermore, the second term together with the complex conjugate of the fifth and the fifth term together with the complex conjugate of the second contribute, both in analogy to the  $\widetilde{l}_L^k$  exchange. The Hamiltonian is given by

$$\mathcal{H}^{\widetilde{u}_L} = \frac{1}{2} \sum_k \frac{\lambda'_{nkj}\lambda'_{fki}}{m_{\widetilde{u}_L}^2} \bar{l}^f \gamma^\nu P_L l^n \overline{d^j} \gamma_\nu P_R d^i,$$

$$\begin{aligned} \mathcal{H}^{\widetilde{\nu}_L} = & \sum_k \frac{\lambda'_{kji}\lambda_{knf}}{m_{\widetilde{\nu}_L}^2} \bar{l}^f P_L l^n \overline{d^j} P_R d^i \\ & + \sum_k \frac{\lambda'_{kij}\lambda_{kfn}}{m_{\widetilde{\nu}_L}^2} \bar{l}^f P_R l^n \overline{d^j} P_L d^i. \end{aligned} \quad (29)$$

Using the results corresponding to Eq. (7) we obtain

$$\begin{aligned} \mathcal{M}_{ijfn} = & -\frac{f_{(d^j C d^i)}}{2\sqrt{2}} \overline{\mathcal{U}}^f(\vec{p}_3) \{A_{ijfn} \not{p}_1 P_L \\ & + B_{ijfn} P_L - B_{jinf}^* P_R\} \mathcal{V}^n(\vec{p}_2), \end{aligned} \quad (30)$$

where

$$\begin{aligned} A_{ijfn} = & \sum_k \frac{\lambda'_{nkj}\lambda'_{fki}}{m_{\widetilde{u}_L^k}^2} = A_{jinf}^*, \\ B_{ijfn} = & 2 \frac{m_{(d^j C d^i)}^2}{(m_{d^j} + m_{d^i})} \sum_k \frac{\lambda'_{kji}\lambda_{knf}}{m_{\widetilde{\nu}_L^k}^2}. \end{aligned} \quad (31)$$

Hence

$$\Gamma_{(d^j C d^i) \rightarrow l^f + l^n C}^{\text{SM} + R_p} = \sqrt{m_{(d^j C d^i)}^4 + m_{l^f}^4 + m_{l^n}^4 - 2(m_{(d^j C d^i)}^2 m_{l^f}^2 + m_{l^f}^2 m_{l^n}^2 + m_{l^n}^2 m_{(d^j C d^i)}^2)} \frac{f_{(d^j C d^i)}^2}{128\pi m_{(d^j C d^i)}^3} Y_{ijfn}, \quad (32)$$

where

$$\begin{aligned} Y_{ijfn} = & (m_{(d^j C d^i)}^2 - m_{l^f}^2) |A_{ijfn} m_{l^f} + B_{ijfn}|^2 + (m_{(d^j C d^i)}^2 - m_{l^n}^2) |A_{ijfn} m_{l^n} + B_{jinf}^*|^2 - |B_{ijfn} m_{l^n} - B_{jinf}^* m_{l^f}|^2 + m_{l^f} m_{l^n} \{ |B_{ijfn} \\ & + B_{jinf}^*|^2 - |A_{ijfn} m_{l^n} - B_{ijfn}|^2 - |A_{ijfn} m_{l^f} - B_{jinf}^*|^2 + |(m_{l^f} + m_{l^n}) A_{ijfn}|^2 \}. \end{aligned} \quad (33)$$

Due to the large experimental error in  $f_{(d^j C d^i)}$ , we can neglect  $m_{l^n}$  compared to  $m_{l^f}$  (with  $f, n$  chosen correspondingly) and  $m_{l^f}$  compared to  $m_{(d^j C d^i)}$ . Thus, focusing again on the bounds on products of two coupling constants, with all other coupling constants vanishing,

$$\begin{aligned} & |A_{ijfn}|, \frac{|B_{ijfn}|}{m_{l^f}}, \frac{|B_{jinf}^*|}{m_{l^f}} \\ & \leq \frac{20}{f_{(d^j C d^i)} m_{l^f}} \sqrt{\frac{\Gamma_{(d^j C d^i) \rightarrow l^f + l^n C}^{\text{expt upper bound}} / \Gamma_{(d^j C d^i) \rightarrow \text{total}}^{\text{expt}}}{m_{(d^j C d^i)} \tau_{(d^j C d^i)}^{\text{expt}}}}. \end{aligned} \quad (34)$$

Here  $\tau$  is the mean lifetime. The same considerations apply to mesons that have wave functions of the form

$$\pi_{ij}^0 = \frac{1}{\sqrt{2}} [(d^j C d^i) \pm (d^i C d^j)]; \quad (35)$$

one replaces every  $A_{ijfn}$  by  $(1/\sqrt{2})(A_{ijfn} \pm A_{jifn})$ , and likewise for  $B_{ijfn}, B_{jinf}^*$ . As in the previous section, we will apply Eq. (34) only to processes with satisfactory experimental data, as was done in Ref. [11], treating among other processes  $B^0 \rightarrow l^f + l^n C$ ; we confirm their results.

#### B. $B_s^0 \rightarrow \mu + e^C$

We now consider  $B_s^0, B_s^{0C} \rightarrow \mu + e^C$ ,  $i, f=2, j=3, n=1$ , and  $i \leftrightarrow j$ . The relevant parameters are given by  $f_{B_s^0} = (1.16 \pm 0.04) f_{B^0}$  (see Ref. [21]),  $\mathcal{B}(B_s^0 \rightarrow \mu + e^C) < 6.1 \times 10^{-6}$ ,  $\tau_{B_s^0} = (1.464 \pm 0.057) \times 10^{-12}$  s, and  $m_{B_s^0} = (5369.6 \pm 2.4)$  MeV (see Ref. [15]). Thus

TABLE I. Bounds on products of  $\mathcal{R}_p$  coupling constants.

Lower limit/ ( $m_{\text{SUSY}}/100\text{GeV}$ ) <sup>2</sup>	Product of $\mathcal{R}_p$ coupling constants	Upper limit/ ( $m_{\text{SUSY}}/100\text{GeV}$ ) <sup>2</sup>	Exchanged sfermion
$-7.9 \times 10^{-8}$	$\text{Re}[\lambda'_{k11} \lambda_{1k1}^*]$	$7.1 \times 10^{-5}$	$\widetilde{l}_{L^k}$
$-7.9 \times 10^{-5}$	$\text{Re}[\lambda'_{311} \lambda_{232}^*]$	—	$\widetilde{l}_{L^k}$
0	$ \lambda'_{k11} \lambda_{3k1}^* $	$3.4 \times 10^{-6}$	$\widetilde{l}_{L^k}$
0	$ \lambda'_{211} \lambda_{322}^* $	$1.5 \times 10^{-3}$	$\widetilde{l}_{L^k}$
0	$ \lambda'_{111} \lambda_{211}^* $	$3.4 \times 10^{-6}$	$\widetilde{l}_{L^k}$
$-7.0 \times 10^{-7}$	$\text{Re}[\lambda'_{k12} \lambda_{1k1}^*]$	$1.8 \times 10^{-5}$	$\widetilde{l}_{L^k}$
$-1.8 \times 10^{-4}$	$\text{Re}[\lambda'_{k12} \lambda_{2k2}^*]$	$3.8 \times 10^{-3}$ , $k=3$	$\widetilde{l}_{L^k}$
0	$ \lambda'_{k12} \lambda_{2k1}^* $	$5.4 \times 10^{-6}$	$\widetilde{l}_{L^k}$
0	$ \lambda'_{k12} \lambda_{3k1}^* $	$5.4 \times 10^{-6}$	$\widetilde{l}_{L^k}$
0	$ \lambda'_{k12} \lambda_{1k2}^* $	$1.3 \times 10^{-3}$	$\widetilde{l}_{L^k}$
0	$ \lambda'_{k12} \lambda_{3k2}^* $	$1.3 \times 10^{-3}$	$\widetilde{l}_{L^k}$
0	$ \lambda'_{313} \lambda_{233}^* $	$2 \times 10^{-3}$	$\widetilde{l}_{L^k}$
$-6.4 \times 10^{-4}$	$\text{Re}[\lambda'_{213} \lambda_{323}^*]$	$1 \times 10^{-3}$	$\widetilde{l}_{L^k}$
0	$ \lambda'_{k13} \lambda_{3k1}^* $	$6 \times 10^{-4}$	$\widetilde{l}_{L^k}$
0	$ \lambda'_{k13} \lambda_{3k2}^* $	$7 \times 10^{-4}$	$\widetilde{l}_{L^k}$
0	$ \lambda'_{123} \lambda_{222}^* $	$8 \times 10^{-3}$	$u_{L^k}$
0	$ \lambda'_{1k2} \lambda_{2k3}^* $ , $k \neq 1$	$8 \times 10^{-3}$	$u_{L^k}$
0	$ \lambda'_{1k2} \lambda_{2k1}^* $	$3 \times 10^{-7}$	$u_{L^k}$
0	$ \lambda'_{1k1} \lambda_{2k2}^* $	$3 \times 10^{-7}$	$u_{L^k}$
0	$ \lambda'_{k32} \lambda_{k12}^* $	$7 \times 10^{-5}$	$\widetilde{\nu}_{L^k}^c$
0	$ \lambda'_{k23} \lambda_{k21}^* $	$7 \times 10^{-5}$	$\widetilde{\nu}_{L^k}^c$
0	$ \lambda'_{k32} \lambda_{k21}^* $	$7 \times 10^{-5}$	$\widetilde{\nu}_{L^k}^c$
0	$ \lambda'_{k23} \lambda_{k12}^* $	$7 \times 10^{-5}$	$\widetilde{\nu}_{L^k}^c$
0	$ \lambda'_{k21} \lambda_{k12}^* $	$6 \times 10^{-9}$	$\widetilde{\nu}_{L^k}^c$
0	$ \lambda'_{k12} \lambda_{k12}^* $	$6 \times 10^{-9}$	$\widetilde{\nu}_{L^k}^c$
0	$ \lambda'_{k12} \lambda_{k21}^* $	$6 \times 10^{-9}$	$\widetilde{\nu}_{L^k}^c$
0	$ \lambda'_{k21} \lambda_{k21}^* $	$6 \times 10^{-9}$	$\widetilde{\nu}_{L^k}^c$
0	$ \lambda'_{311} \lambda_{312}^* $	$3 \times 10^{-3}$	$\widetilde{\nu}_{L^k}^c$
0	$ \lambda'_{311} \lambda_{321}^* $	$3 \times 10^{-3}$	$\widetilde{\nu}_{L^k}^c$

$$|\lambda'_{123} \lambda_{222}^*| \leq 8 \times 10^{-3} \left( \frac{m_{u_L^k}}{100 \text{ GeV}} \right)^2, \quad |\lambda'_{k23} \lambda_{k12}^*| \leq 7 \times 10^{-5} \left( \frac{m_{\widetilde{\nu}_{L^k}^c}}{100 \text{ GeV}} \right)^2. \quad (36)$$

$$|\lambda'_{1k2} \lambda_{2k3}^*| \leq 8 \times 10^{-3} \left( \frac{m_{u_L^k}}{100 \text{ GeV}} \right)^2,$$

$$|\lambda'_{k32} \lambda_{k12}^*| \leq 7 \times 10^{-5} \left( \frac{m_{\widetilde{\nu}_{L^k}^c}}{100 \text{ GeV}} \right)^2,$$

$$|\lambda'_{k23} \lambda_{k21}^*| \leq 7 \times 10^{-5} \left( \frac{m_{\widetilde{\nu}_{L^k}^c}}{100 \text{ GeV}} \right)^2,$$

$$|\lambda'_{k32} \lambda_{k21}^*| \leq 7 \times 10^{-5} \left( \frac{m_{\widetilde{\nu}_{L^k}^c}}{100 \text{ GeV}} \right)^2,$$

Our results for  $|\lambda'_{1k3} \lambda_{2k2}^*|$  ( $k \neq 2$ ) and  $k=1$  for the second bound are weaker than the products of the bounds on single coupling constants (see Ref. [4]).

### C. $K_L^0 \rightarrow \mu + e^c$

$K_L^0$  is defined as  $[K_2^0 + \epsilon K_1^0]/\sqrt{1 + \epsilon^2}$ , with  $K_{1,2}^0 = [K^0 \pm K^{0C}]/\sqrt{2}$ .  $\epsilon$  parametrizes the  $CP$  violation. If we neglect  $\epsilon$ ,  $K_L^0 = [K^0 - K^{0C}]/\sqrt{2}$ , with  $K^0 = (s^c d)$ . From Ref. [15] one has  $m_{K_L^0} = (497.672 \pm 0.031) \text{ MeV}$ ,  $\tau_{K_L^0} = (5.17 \pm 0.04) \times 10^{-8} \text{ s}$ , and  $\mathcal{B}(K_L^0 \rightarrow \mu + e^c) < 4.7 \times 10^{-12}$ . Reference [15] gives  $f_K = (159 \pm 1.4 \pm 0.44) \text{ MeV}$ , which in the convention we use gives the central value  $112.4 \text{ MeV}$ . Hence, the first two bounds updating previous ones,

$$\begin{aligned}
|\lambda'_{1k2}\lambda'_{2k1*}| &\leq 3 \times 10^{-7} \left( \frac{m_{u_L^k}}{100 \text{ GeV}} \right)^2, & |\lambda'_{311}\lambda'_{321*}| &\leq 3 \times 10^{-3} \left( \frac{m_{\nu_L^k}}{100 \text{ GeV}} \right)^2. \\
|\lambda'_{1k1}\lambda'_{2k2*}| &\leq 3 \times 10^{-7} \left( \frac{m_{u_L^k}}{100 \text{ GeV}} \right)^2, & & \\
|\lambda'_{k21}\lambda'_{k12*}| &\leq 6 \times 10^{-9} \left( \frac{m_{\nu_L^k}}{100 \text{ GeV}} \right)^2, & & \\
|\lambda'_{k12}\lambda'_{k12*}| &\leq 6 \times 10^{-9} \left( \frac{m_{\nu_L^k}}{100 \text{ GeV}} \right)^2, & & \\
|\lambda'_{k12}\lambda'_{k21*}| &\leq 6 \times 10^{-9} \left( \frac{m_{\nu_L^k}}{100 \text{ GeV}} \right)^2, & & \\
|\lambda'_{k21}\lambda'_{k21*}| &\leq 6 \times 10^{-9} \left( \frac{m_{\nu_L^k}}{100 \text{ GeV}} \right)^2. & (37) &
\end{aligned}$$

#### D. $\pi^0 \rightarrow \mu + e^C$

With small modifications the result of Sec. III A can also be carried over to admixtures of  $(d^{jC}d^i)$  with  $(u^{jC}u^i)$ , as the latter term does not contribute to any decay because the up-type quarks do not couple together with the  $\mathcal{R}_p$  operators. However, we shall limit ourselves to the  $\pi^0$ :  $\eta$  and  $\eta'$  are more complicated (see Ref. [15]), and the experimental data do not suffice to extract satisfactory bounds.

The relevant parameters here are  $m_{\pi^0} = (134.9766 \pm 0.0006) \text{ MeV}$ ,  $\tau_{\pi^0} = (8.4 \pm 0.6) \times 10^{-17} \text{ s}$  and  $\mathcal{B}(\pi^0 \rightarrow \mu + e^C) < 3.8 \times 10^{-10}$  (see Ref. [15]). Thus

$$|\lambda'_{311*}\lambda'_{312}| \leq 3 \times 10^{-3} \left( \frac{m_{\nu_L^k}}{100 \text{ GeV}} \right)^2,$$

In Ref. [4] a much stronger bound is stated for  $|\lambda'_{1k1}\lambda'_{2k1*}|$ . Furthermore, the authors present a better bound for  $|\lambda'_{111}\lambda'_{121*}|$ ; and from Ref. [4] one finds a stricter bound on  $|\lambda'_{211*}\lambda'_{212}|$ , based on the bounds on single coupling constants.

#### IV. SUMMARY

We have determined the bounds on products of  $\mathcal{R}_p$  coupling constants from leptonic meson decays. In many cases these bounds are better than previous bounds. We have summarized the bounds in Table I at the end of this text. With the formulas given the bounds can easily be updated when the data improve. Furthermore, if additional decays are measured (e.g., from the  $B$  factories) one can determine additional bounds. Equations (17) and (18) can be used to consider 12 cases:  $D^- (i=1, j=2)$ ,  $D_s^- (i=2, j=2)$ ,  $B^- (i=3, j=1)$ ,  $B_c^- (i=3, j=2)$  decaying into  $e + \nu^C$  and  $\mu + \nu^C$  ( $f=1, g=2$ ),  $e + \nu^C$  and  $\tau + \nu^C$  ( $f=1, g=3$ ),  $\mu + \nu^C$  and  $\tau + \nu^C$  ( $f=2, g=3$ ); Eq. (34) can be applied to the decay of  $B_s^0 (i=2, j=3)$  to  $\tau + e^C$  ( $f=3, n=1$ ) or  $\tau + \mu^C$  ( $f=3, n=2$ ), and the decay of the  $Y (i=j=3)$  to  $\tau + e^C$  or  $\tau + \mu^C$  or  $\mu + e^C$  ( $f=2, n=1$ ).

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