Perturbative study of a general class of lattice Dirac operators

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A perturbative study of a general class of lattice Dirac operators is reported, which is based on an algebraic realization of the Ginsparg-Wilson relation in the form $\gamma_5(\gamma_5 D)+(\gamma_5 D)\gamma_5=2a^{2k+1}(\gamma_5 D)^{2k+2}$ where k stands for a non-negative integer. The choice $k=0$ corresponds to the commonly discussed Ginsparg-Wilson relation and thus to the overlap operator. We study one-loop fermion contributions to the self-energy of the gauge field, which are related to the fermion contributions to the one-loop β function and to the Weyl anomaly. We first explicitly demonstrate that the Ward identity is satisfied by the self-energy tensor. By performing careful analyses, we then obtain the correct self-energy tensor free of infrared divergences, as a general consideration of the Weyl anomaly indicates. This demonstrates that our general operators give correct chiral and Weyl anomalies. In general, however, the Wilsonian effective action, which is supposed to be free of infrared complications, is expected to be essential in the analyses of our general class of Dirac operators for the dynamical gauge field.

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I. INTRODUCTION

Recent developments in the treatment of fermions in lattice gauge theory are based on a Hermitian lattice Dirac operator $\gamma_5 D$ which satisfies the Ginsparg-Wilson relation [1]

$$\gamma_5 D+D\gamma_5=2aD\gamma_5 D, \tag{1.1}$$

where the lattice spacing a is utilized to make a dimensional consideration transparent, and γ_5 is a Hermitian chiral Dirac matrix. An explicit example of the operator satisfying Eq. (1.1) and free of species doubling has been given by Neuberger [2]. The relation (1.1) led to an interesting analysis of the notion of index in lattice gauge theory [3]. This index theorem in turn led to a new form of chiral symmetry, and the chiral anomaly is obtained as a non-trivial Jacobian factor under this modified chiral transformation [4]. This chiral Jacobian is regarded as a lattice realization of that in the continuum path integral [5]. See Refs. [6] for reviews of these developments.

We have recently studied a specific generalization of the algebra (1.1) [7]:

$$\gamma_5(\gamma_5 D)+(\gamma_5 D)\gamma_5=2a^{2k+1}(\gamma_5 D)^{2k+2}, \tag{1.2}$$

where k stands for a non-negative integer and $k=0$ corresponds to the ordinary Ginsparg-Wilson relation. When one defines

$$H\equiv\gamma_5 aD, \tag{1.3}$$

Eq. (1.2) is rewritten as

$$\gamma_5 H+H\gamma_5=2H^{2k+2}. \tag{1.4}$$

The algebra (1.4) is equivalent to a set of equations

$$H^{2k+1}\gamma_5+\gamma_5 H^{2k+1}=2H^{2(2k+1)},$$
$$H^2\gamma_5-\gamma_5 H^2=0, \tag{1.5}$$

where the second relation is shown by using the defining relation (1.4), and the first relation in Eq. (1.5) becomes identical to the ordinary Ginsparg-Wilson relation (1.1) if one defines $H_{(2k+1)}=H^{2k+1}$. One can thus construct a solution to Eq. (1.5) by following the prescription used by Neuberger [2]:

$$H_{(2k+1)}=\frac{1}{2}\gamma_5\left[1+D_W^{(2k+1)}\frac{1}{\sqrt{(D_W^{(2k+1)})^\dagger D_W^{(2k+1)}}}\right] \tag{1.6}$$

where

$$D_W^{(2k+1)}\equiv i(\mathcal{C})^{2k+1}+B^{2k+1}-\left(\frac{m_0}{a}\right)^{2k+1}. \tag{1.7}$$

Here we note that the conventional Wilson fermion operator D_W (with a nonzero mass term) is given by

$$D_W(x,y)\equiv i\gamma^\mu C_\mu(x,y)+B(x,y)-\frac{1}{a}m_0\delta_{x,y},$$

$$C_\mu(x,y)=\frac{1}{2a}[\delta_{x+\hat{\mu}a,y}U_\mu(y)-\delta_{x,y+\hat{\mu}a}U_\mu^\dagger(x)],$$

$$B(x,y)=\frac{r}{2a}\sum_\mu[2\delta_{x,y}-\delta_{y+\hat{\mu}a,x}U_\mu^\dagger(x)-\delta_{y,x+\hat{\mu}a}U_\mu(y)],$$

$$U_\mu(y)=\exp[iagA_\mu(y)]. \tag{1.8}$$

The parameter r stands for the Wilson parameter. Our matrix convention is that γ^μ are anti-Hermitian, $(\gamma^\mu)^\dagger=-\gamma^\mu$, and thus $\mathcal{C}\equiv\gamma^\mu C_\mu(n,m)$ is Hermitian:

$$\mathcal{C}^\dagger=\mathcal{C}. \tag{1.9}$$

The Hermitian operator H itself is then finally defined by (in the representation where $H_{(2k+1)}$ is diagonal)

$$H=(H_{(2k+1)})^{1/2k+1} \tag{1.10}$$

in such a manner that the second relation of Eq. (1.5) is satisfied, which is in fact confirmed in the representation where $H_{(2k+1)}$ is diagonal [7]. Also the conditions $0<m_0<2r=2$ and

$$2m_0^{2k+1}=1 \tag{1.11}$$

ensure the absence of species doublers and a proper normalization of the Dirac operator H.

The locality properties are crucial in any construction of the lattice Dirac operator, and the locality of the standard overlap operator with $k=0$ has been established by Hernandez, Jansen and Lüscher [10], and by Neuberger [11].

As for the direct proof of locality of the operator D for general k, it is shown for the vanishing gauge field by using the explicit solution for the operator H in momentum representation [12,9]

$$\begin{aligned} H(ap_\mu)&=\gamma_5\left(\frac{1}{2}\right)^{(k+1)/(2k+1)}\left(\frac{1}{\sqrt{H_W^2}}\right)^{(k+1)/(2k+1)}\Bigg\{(\sqrt{H_W^2}\\ &\quad+M_k)^{(k+1)/(2k+1)}-(\sqrt{H_W^2}-M_k)^{k/(2k+1)}\frac{\not{s}}{a}\Bigg\}\\ &=\gamma_5\left(\frac{1}{2}\right)^{(k+1)/(2k+1)}\left(\frac{1}{\sqrt{F_{(k)}}}\right)^{(k+1)/(2k+1)}\{(\sqrt{F_{(k)}}\\ &\quad+\tilde{M}_k)^{(k+1)/(2k+1)}-(\sqrt{F_{(k)}}-\tilde{M}_k)^{k/(2k+1)}\not{s}\}, \end{aligned} \tag{1.12}$$

where

$$F_{(k)}=(s^2)^{2k+1}+\tilde{M}_k^2,$$
$$\tilde{M}_k=\left[\sum_\mu (1-c_\mu)\right]^{2k+1}-m_0^{2k+1} \tag{1.13}$$

and

$$s_\mu=\sin ap_\mu,$$
$$c_\mu=\cos ap_\mu,$$
$$\not{s}=\gamma^\mu \sin ap_\mu. \tag{1.14}$$

For $k=0$, this operator is reduced to Neuberger's overlap operator [2]. Here the inner product is defined to be $s^2\geqslant 0$. This operator for an infinitesimal p_μ, i.e., for $|ap_\mu|\ll 1$, gives rise to

$$H\simeq-\gamma_5 a\not{p}(1+O(ap)^2)+\gamma_5(\gamma_5 a\not{p})^{2k+2} \tag{1.15}$$

to be consistent with $H=\gamma_5 aD$; the last term in the right-hand side is the leading term of chiral symmetry breaking terms. The locality of this explicit construction (1.12) has been shown by examining the analytic properties in the Brillouin zone [12].

It is important to recognize that this operator is not ultralocal but exponentially local [13]; the operator $H(x,y)$ in Eq. (1.12) decays exponentially for large separation in coordinate representation as [12]

$$H(x,y)\sim\exp[-|x-y|/(2.5ka)]. \tag{1.16}$$

An explicit analysis of the locality of the operator $H_{(2k+1)}=H^{2k+1}$ (*not* H itself) in the presence of gauge field, in particular, the locality domain for the gauge field strength $\|F_{\mu\nu}\|$ has been performed. The locality domain for $\|F_{\mu\nu}\|$ becomes smaller for larger k, but a definite nonzero domain has been established [12]. The remaining task is to show the locality of the operator $H=(H_{(2k+1)})^{1/(2k+1)}$ itself in the presence of gauge field. Due to the operation of taking the $(2k+1)$th root, an explicit analysis has not been performed yet, though a supporting argument has been given in Ref. [12].

It has been shown that all the good chiral properties of the overlap operator [2] are retained in the generalization in Eq. (1.4) [8,9]. The practical applications of this generalized operator D are not known at this moment. We however mention the characteristic properties of this generalization: The spectrum near the continuum configuration is closer to that of continuum theory and the chiral symmetry breaking terms become more irrelevent in the continuum limit for $k>0$. The operator however spreads over more lattice points for larger k, as is indicated in Eq. (1.16).

In this paper we study a perturbative aspect of the general class of Dirac operators. To be specific, we study the one-loop fermion contribution to the gauge field self-energy, which is related to the β function and to the Weyl anomaly.

II. SELF-ENERGY TENSOR, β-FUNCTION AND WEYL ANOMALY

The lattice perturbation theory is very tedious in general [14–22], and it is more so in our generalization. For this reason, we study the simplest diagrams related to the one-loop self-energy correction to gauge fields. This effect is also related to the fermion contribution to the lowest order β-function and to the Weyl anomaly [23,24]. A rather general analysis of Weyl anomaly is possible, and we first briefly summarize it.

In the standard continuum formulation, one starts with the path integral defined in a background curved space [25]

$$\int d\mu \exp\left[\int d^4x\sqrt{g}\,\bar{\psi}i\not{D}\psi\right]. \tag{2.1}$$

The general coordinate invariant path integral measure is defined by

$$d\mu=\mathcal{D}\tilde{\bar{\psi}}\mathcal{D}\tilde{\psi} \tag{2.2}$$

and the Weyl transformation laws are given by

$$e_a^\mu(x)\to\exp[\alpha(x)]e_a^\mu(x),$$

$$\tilde{\psi}(x)\equiv(g)^{1/4}\psi(x)\to\exp\left[-\frac{1}{2}\alpha(x)\right]\tilde{\psi}(x),$$

$$\tilde{\bar{\psi}}(x)\equiv(g)^{1/4}\bar{\psi}(x)\to\exp\left[-\frac{1}{2}\alpha(x)\right]\tilde{\bar{\psi}}(x), \tag{2.3}$$

where $e_a^\mu(x)$ stands for the vierbein. This transformation law is fixed by the invariance of the action in the above path integral under a global (i.e., constant) α, and the Weyl weight factor of fermionic variables is essentially defined by the vierbein in $\not{D}=e_a^\mu(x)\gamma^a D_\mu$.

The Jacobian for this transformation of fermionic variables is given by

$$\ln J(\alpha)=\lim_{M\to\infty}\mathrm{Tr}\,\alpha(x)\exp[-(\not{D}/M)^2]$$

$$=\mathrm{Tr}\,\alpha(x)\frac{g^2}{24\pi^2}F_{\mu\nu}F^{\mu\nu}, \tag{2.4}$$

where the mode cutoff of $\not{D}$ is provided by $e^{-(\lambda_n/M)^2}$ in terms of the eigenvalues of $\not{D}$. See Ref. [25] for further details.

When one analyzes the higher derivative theory

$$\mathcal{L}_{2k+1}=\int d^4x\sqrt{g}\bar{\psi}_i(\not{D})^{2k+1}\psi, \tag{2.5}$$

the Weyl transformation laws are given by

$$e_a^\mu(x)\to\exp[\alpha(x)]e_a^\mu(x),$$

$$\tilde{\psi}(x)=(g)^{1/4}\psi(x)\to\exp\left[-\frac{2k+1}{2}\alpha(x)\right]\tilde{\psi}(x),$$

$$\tilde{\bar{\psi}}(x)=(g)^{1/4}\bar{\psi}(x)\to\exp\left[-\frac{2k+1}{2}\alpha(x)\right]\tilde{\bar{\psi}}(x) \tag{2.6}$$

and the Weyl anomaly is given by

$$\ln J_{2k+1}(\alpha)=\lim_{M\to\infty}\mathrm{Tr}(2k+1)\alpha(x)$$

$$\times\exp[-((\not{D})^{2k+1}/M^{2k+1})^2]. \tag{2.7}$$

Since the Weyl anomaly is independent of the regulator function [25], we have

$$\ln J_{2k+1}(\alpha)=(2k+1)\ln J(\alpha). \tag{2.8}$$

This relation (2.8) is also understood from a viewpoint of the self-energy correction to the gauge field as follows:

$$\det(\not{D})^{2k+1}$$

$$=\exp[(2k+1)\mathrm{Tr}\ln\not{D}]$$

$$=\exp[(2k+1)\mathrm{Tr}\ln(\not{\partial}-ig\not{A})]$$

$$=\exp\left[(2k+1)\mathrm{Tr}\ln\not{\partial}-\frac{2k+1}{2}(ig)^2\mathrm{Tr}\frac{1}{\not{\partial}}\not{A}\frac{1}{\not{\partial}}\not{A}+\cdots\right]. \tag{2.9}$$

The term quadratic in the gauge field A_μ gives the self-energy correction, which is $2k+1$ times larger than the self-energy correction generated by $\det\not{D}$.

This analysis of the self-energy correction is applicable to the present lattice operator. By our definition in Eq. (1.10) we have

$$\exp[\mathrm{Tr}\ln H]=\exp[\mathrm{Tr}\ln H_{(2k+1)}^{1/(2k+1)}]$$

$$=\exp\left[\frac{1}{2k+1}\mathrm{Tr}\ln H_{(2k+1)}\right]. \tag{2.10}$$

For a sufficiently small coupling constant g, we have

$$\exp[\mathrm{Tr}\ln H]=\exp[\mathrm{Tr}\ln H^{(0)}+\mathrm{Tr}\,g^2A_\mu(x)O(x,y)_{\mu\nu}A_\nu(y)$$

$$+O(g^3)], \tag{2.11}$$

where $H^{(0)}$ stands for the free Dirac operator given in Eq. (1.12), and the second term stands for the lowest order term in the effective potential and thus for the lowest order self-energy correction to the gauge field. Similarly, we have

$$\exp\left[\frac{1}{2k+1}\mathrm{Tr}\ln H_{(2k+1)}\right]$$

$$=\exp\left[\frac{1}{2k+1}\mathrm{Tr}\ln H_{(2k+1)}^{(0)}+\frac{1}{2k+1}\mathrm{Tr}\,g^2A_\mu(x)\right.$$

$$\left.\times\tilde{O}(x,y)_{\mu\nu}A_\nu(y)+O(g^3)\right], \tag{2.12}$$

where $\mathrm{Tr}\ln H_{(2k+1)}^{(0)}$ stands for the free part of $H_{(2k+1)}$. Those zeroth order terms satisfy the relation

$$\mathrm{Tr}\ln H^{(0)}=\frac{1}{2k+1}\mathrm{Tr}\ln H_{(2k+1)}^{(0)} \tag{2.13}$$

if one uses the explicit form of the operator in Eq. (1.12).

We thus conclude

$$\mathrm{Tr}\,g^2A_\mu(x)O(x,y)_{\mu\nu}A_\nu(y)$$

$$=\frac{1}{2k+1}\mathrm{Tr}\,g^2A_\mu(x)\tilde{O}(x,y)_{\mu\nu}A_\nu(y) \tag{2.14}$$

for a sufficiently small coupling constant g, which shows that the lowest order self-energy correction in the left-hand side for the operator H is evaluated by the self-energy correction in terms of $H_{(2k+1)}$. We use this relation for the evaluation of the lowest order self-energy correction for any $k\geqslant 1$. Note that the operator $H_{(2k+1)}=H^{2k+1}$ is much better understood than H itself in our construction. We also confirm that this relation (2.14) is in fact valid by evaluating the left-hand side directly for the simplest case $k=1$.

From a viewpoint of Weyl anomaly, one may tentatively take

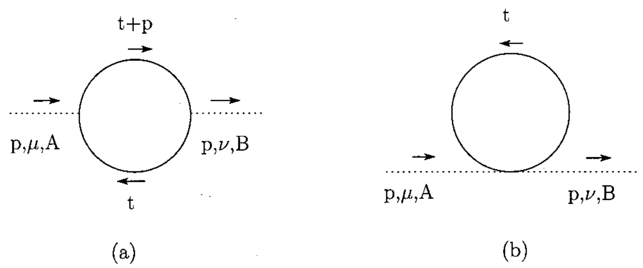

FIG. 1. Feynman diagrams for the vacuum polarization.

$$\lim_{M\to\infty}\mathrm{Tr}(2k+1)\alpha(x)\exp[-(H_{(2k+1)}/(aM)^{2k+1})^2]$$
$$\to\lim_{M\to\infty}\mathrm{Tr}(2k+1)\alpha(x)\times\exp[-((\not{D})^{2k+1}/M^{2k+1})^2] \tag{2.15}$$

as a lattice version of the Weyl anomaly. We then obtain the same result as the self-energy correction in the limit $a\to 0$, although no systematic formulation of Weyl anomaly on the lattice is known.

In lattice perturbative calculations, however, we should be careful of the possible appearance of infrared divergences, which should cancel in the final result. We show that a careful analysis gives the correct result of continuum theory free of infrared divergences for $a\to 0$.

III. THE VACUUM POLARIZATION TENSOR BY $H_{(2k+1)}$

In this section, we calculate the one-loop fermion contribution to the vacuum polarization $\Pi_{\mu\nu}$ on the basis of the operator $H_{(2k+1)}$ (and not H itself) following the analyses in Sec. II. We first show that the Ward identity is satisfied to be consistent with gauge invariance and that there appear no divergences except for the logarithmic divergence for $a\to 0$. We then discuss the gauge field wave function renormalization factor.

Feynman diagrams for the vacuum polarization with fermion one loop are shown in Fig. 1, and the necessary Feynman rules are given in Appendix A.

The amplitude corresponding to Fig. 1(a) is given in terms of the notation in Appendix A by [by using $\mathrm{tr}(T^AT^B)=1/2\delta^{AB}$ and N_f flavors in QCD]

$$\Pi^{(a)}_{\mu\nu}(p)=\frac{-g^2}{4a^{4k+2}}\frac{N_f}{2}\delta^{AB}\int_t\frac{1}{\{w(t)+w(t+p)\}^2}\mathrm{tr}\Big[D_0^{-1}(t)\gamma_5\Big\{X_{1\mu}(t,t+p,-p)-\frac{X_0(t)}{w(t)}X^{\dagger}_{1\mu}(t,t+p,-p)\frac{X_0(t+p)}{w(t+p)}\Big\}$$
$$\times D_0^{-1}(t+p)\gamma_5\Big\{X_{1\nu}(t+p,t,p)-\frac{X_0(t+p)}{w(t+p)}X^{\dagger}_{1\nu}(t+p,t,p)\frac{X_0(t)}{w(t)}\Big\}\Big]. \tag{3.1}$$

We omit the factor δ^{AB} from now on.

The amplitude corresponding to Fig. 1(b) is similarly given by

$$\Pi^{(b)}_{\mu\nu}(p)=\frac{g^2}{2a^{2k+1}}\frac{N_f}{2}\int_t\mathrm{tr}\Big[D_0^{-1}(t)\gamma_5\frac{1}{2w(t)}\Big[X_{2\mu\nu}(t,t,-p,p)-\frac{X_0(t)}{w(t)}X^{\dagger}_{2\mu\nu}(t,t,-p,p)\frac{X_0(t)}{w(t)}-\frac{1}{(w(t)+w(t+p))^2}$$
$$\times\Big\{X_{1\mu}(t,p+t,-p)X^{\dagger}_{1\nu}(p+t,t,p)X_0(t)+X_{1\mu}(t,p+t,-p)X^{\dagger}_0(p+t)X_{1\nu}(p+t,t,p)$$
$$+X_0(t)X^{\dagger}_{1\mu}(t,p+t,-p)X_{1\nu}(p+t,t,p)-\frac{2w(t)+w(p+t)}{w(t)^2w(p+t)}X_0(t)X^{\dagger}_{1\mu}(t,p+t,-p)X_0(p+t)X^{\dagger}_{1\nu}(p+t,t,p)X_0(t)\Big\}$$
$$+X_{2\nu\mu}(t,t,p,-p)-\frac{X_0(t)}{w(t)}X^{\dagger}_{2\nu\mu}(t,t,p,-p)\frac{X_0(t)}{w(t)}-\frac{1}{(w(t)+w(t-p))^2}$$
$$\times\Big\{X_{1\nu}(t,t-p,p)X^{\dagger}_{1\mu}(t-p,t,-p)X_0(t)+X_{1\nu}(t,t-p,p)X^{\dagger}_0(t-p)X_{1\mu}(t-p,t,-p)+X_0(t)$$
$$\times X^{\dagger}_{1\nu}(t,t-p,p)X_{1\nu}(t-p,t,-p)-\frac{2w(t)+w(t-p)}{w(t)^2w(t-p)}X_0(t)X^{\dagger}_{1\nu}(t,t-p,p)X_0(t-p)X^{\dagger}_{1\mu}(t-p,t,-p)X_0(t)\Big\}\Big]\Big]. \tag{3.2}$$

A. Ward identity

We first show that the Ward identity for $\Pi_{\mu\nu}$ as a manifestation of gauge invariance holds as follows,

$$\sum_{\nu} \tilde{p}_{\nu}(\Pi^{(a)}_{\mu\nu}(p)+\Pi^{(b)}_{\mu\nu}(p))=0, \tag{3.3}$$

where $\tilde{p}_{\nu}=(2/a)\sin ap_{\nu}/2$.[1] For this purpose we first calculate $\Sigma_{\nu}\tilde{p}_{\nu}X_{1\nu}$ and $\Sigma_{\nu}\tilde{p}_{\nu}X_{2\mu\nu}$. For $\Sigma_{\nu}\tilde{p}_{\nu}X_{1\nu}$ we have

$$\begin{aligned}
a^{2k+1}\sum_{\nu} \tilde{p}_{\nu}X_{1\nu}(t+p,t,p)=&\sum_{l+m=2k}\Bigg[i(i\not{s}_{t+p})^{l}(i\not{s}_{t+p}-i\not{s}_{t})i(i\not{s}_{t})^{m}+\left(r\sum_{\rho}(1-\cos(t+p)_{\rho}a)\right)^{l}\\
&\times\left\{\left(r\sum_{\rho}(1-\cos(t+p)_{\rho}a)\right)-\left(r\sum_{\rho}(1-\cos t_{\rho}a)\right)\right\}\left(r\sum_{\rho}(1-\cos t_{\rho}a)\right)^{m}\Bigg]\\
=&\,i(i\not{s}_{t+p})^{2k+1}+\left(r\sum_{\rho}(1-\cos(t+p)_{\rho}a)\right)^{2k+1}-m_0^{2k+1}\\
&-\left\{i(i\not{s}_{t})^{2k+1}+\left(r\sum_{\rho}(1-\cos t_{\rho}a)\right)^{2k+1}-m_0^{2k+1}\right\}\\
=&\,a^{2k+1}\{X_0(t+p)-X_0(t)\},
\end{aligned} \tag{3.4}$$

and further we have

$$\sum_{\nu} \tilde{p}_{\nu}\left(X_{1\nu}(t+p,t,p)-\frac{X_0(t+p)}{w(t+p)}X^{\dagger}_{1\nu}(t+p,t,p)\frac{X_0(t)}{w(t)}\right)=2\gamma_5 a^{2k+1}(w(t+p)+w(t))(D_0(p+t)-D_0(t)), \tag{3.5}$$

where we used the following relations:

$$\sum_{\nu} \tilde{p}_{\nu}\gamma_{\nu}\cos(t+p/2)_{\nu}a=\frac{1}{a}(\not{s}_{t+p}-\not{s}_{t}), \tag{3.6}$$

$$\sum_{\nu} \tilde{p}_{\nu}r\sin(t+p/2)_{\nu}a=\frac{1}{a}\left\{\left(r\sum_{\rho}(1-\cos(t+p)_{\rho}a)\right)-\left(r\sum_{\rho}(1-\cos t_{\rho}a)\right)\right\}. \tag{3.7}$$

Using the above relations, we also have

$$\begin{aligned}
a^{2k}\sum_{\nu} \tilde{p}_{\nu}X_{2\mu\nu}(t,t,-p,p)=&\sum_{l+m=2k}\Bigg[i(i\not{s}_{t})^{l}\left(i\gamma_{\mu}\cos\left(t+\frac{p}{2}\right)_{\mu}a\right)(i\not{s}_{t+p})^{m}-i(i\not{s}_{t})^{l}\left(i\gamma_{\mu}\cos\left(t+\frac{p}{2}\right)_{\mu}a\right)(i\not{s}_{t})^{m}\\
&+\left(r\sum_{\rho}(1-\cos t_{\rho}a)\right)^{l}\left(r\sin\left(t+\frac{p}{2}\right)_{\mu}a\right)\left(r\sum_{\rho}(1-\cos(t+p)_{\rho}a)\right)^{m}\\
&-\left(r\sum_{\rho}(1-\cos t_{\rho}a)\right)^{l}\left(r\sin\left(t+\frac{p}{2}\right)_{\mu}a\right)\left(r\sum_{\rho}(1-\cos t_{\rho}a)\right)^{m}\\
&+\frac{1}{2}\Bigg\{i(i\not{s}_{t})^{l}\left(i\gamma_{\mu}\cos\left(t+\frac{p}{2}\right)_{\mu}a-i\gamma_{\mu}\cos\left(t-\frac{p}{2}\right)_{\mu}a\right)(i\not{s}_{t})^{m}\\
&+\left(r\sum_{\rho}(1-\cos t_{\rho}a)\right)^{l}\left(r\sin\left(t+\frac{p}{2}\right)_{\mu}a-r\sin\left(t-\frac{p}{2}\right)_{\mu}a\right)\times\left(r\sum_{\rho}(1-\cos t_{\rho}a)\right)^{m}\Bigg\}\Bigg].
\end{aligned} \tag{3.8}$$

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[1]The Ward identity in the case of the overlap Dirac operator has been confirmed explicitly in Ref. [20].

Therefore we obtain

$$\sum_{\nu}\tilde{p}_{\nu}[X_{2\mu\nu}(t,t,-p,p)+X_{2\nu\mu}(t,t,p,-p)]=X_{1\mu}(t,t+p,-p)-X_{1\mu}(t-p,t,-p). \tag{3.9}$$

By using these relations, $\Sigma_{\nu}\tilde{p}_{\nu}\Pi^{(a)}_{\mu\nu}(p)$ is written as

$$\sum_{\nu}\tilde{p}_{\nu}\Pi^{(a)}_{\mu\nu}(p)=\frac{-g^2}{2a^{2k+1}}\frac{N_f}{2}\int_t\frac{1}{\{w(t)+w(t+p)\}}\mathrm{tr}\Bigg[\Big\{D_0^{-1}(t)-D_0^{-1}(t+p)\Big\}\gamma_5 \times\Bigg\{X_{1\mu}(t,t+p,-p)-\frac{X_0(t)}{w(t)}X^{\dagger}_{1\mu}(t,t+p,-p)\frac{X_0(t+p)}{w(t+p)}\Bigg\}\Bigg], \tag{3.10}$$

and similarly $\Sigma_{\nu}\tilde{p}_{\nu}\Pi^{(b)}_{\mu\nu}(p)$ is written as

$$\begin{aligned}\sum_{\nu}\tilde{p}_{\nu}\Pi^{(b)}_{\mu\nu}(p)=&\frac{g^2}{2a^{2k+1}}\frac{N_f}{2}\int_t\mathrm{tr}\Bigg[D_0^{-1}(t)\gamma_5\Bigg[\frac{1}{w(t)+w(t+p)}\Bigg\{X_{1\mu}(t,t+p,-p)-\frac{X_0(t)}{w(t)}X^{\dagger}_{1\mu}(t,t+p,-p)\frac{X_0(t+p)}{w(t+p)}\Bigg\}\\&-\frac{1}{w(t)+w(t-p)}\Bigg\{X_{1\mu}(t-p,t,-p)-\frac{X_0(t-p)}{w(t-p)}X^{\dagger}_{1\mu}(t-p,t-p)\frac{X_0(t)}{w(t)}\Bigg\}\Bigg]\Bigg]\\=&\frac{g^2}{2a^{2k+1}}\frac{N_f}{2}\int_t\mathrm{tr}\Bigg[\{D_0^{-1}(t)-D_0^{-1}(t+p)\}\gamma_5\\&\times\Bigg[\frac{1}{w(t)+w(t+p)}\Bigg\{X_{1\mu}(t,t+p,-p)-\frac{X_0(t)}{w(t)}X^{\dagger}_{1\mu}(t,t+p,-p)\frac{X_0(t+p)}{w(t+p)}\Bigg\}\Bigg]\Bigg].\end{aligned} \tag{3.11}$$

Combining these two expressions, the Ward identity for the vacuum polarization tensor holds as in Eq. (3.3). This Ward identity dictates the tensor structure of $\Pi_{\mu\nu}(p)$ for small p_{μ} to be

$$\Pi_{\mu\nu}(p)\simeq(p^2\delta_{\mu\nu}-p_{\mu}p_{\nu})\Pi(a^2p^2). \tag{3.12}$$

B. Structure of divergences

We next examine the structure of various divergences. To evaluate the divergent parts of $\Pi^{(a)}_{\mu\nu}(p)$ and $\Pi^{(b)}_{\mu\nu}(p)$, we rescale the integration momenta $t_{\mu}\to t_{\mu}/a$ in each amplitude (3.1) and (3.2). For QCD with N_f flavors, we obtain

$$\begin{aligned}\Pi^{(a)}_{\mu\nu}(p)=&\frac{-N_f g^2}{8a^2}\int_t\frac{1}{\{w(t)+w(t+pa)\}^2}\mathrm{tr}\Bigg[\Bigg\{\frac{(s_t^2)^k \not{s}_t}{w(t)+M(t)}+1\Bigg\}\\&\times\Bigg\{X_{1\mu}(t,t+pa,-pa)-\frac{X_0(t)}{w(t)}X^{\dagger}_{1\mu}(t,t+pa,-pa)\frac{X_0(t+pa)}{w(t+pa)}\Bigg\}\\&\times\Bigg\{\frac{(s_{t+pa}^2)^k \not{s}_{t+pa}}{w(t+pa)+M(t+pa)}+1\Bigg\}\Bigg\{X_{1\nu}(t+pa,t,pa)-\frac{X_0(t+pa)}{w(t+pa)}X^{\dagger}_{1\nu}(t+pa,t,pa)\frac{X_0(t)}{w(t)}\Bigg\}\Bigg]\end{aligned} \tag{3.13}$$

and

$$\Pi^{(b)}_{\mu\nu}(p)=\frac{N_f g^2}{4a^2}\int_t \mathrm{tr}\Bigg[\Bigg\{\frac{(s_t^2)\not{s}_t}{w(t)+M(t)}+1\Bigg\}\frac{1}{2w(t)}\Bigg[X_{2\mu\nu}(t,t,-pa,pa)-\frac{X_0(t)}{w(t)}X^{\dagger}_{2\mu\nu}(t,t,-pa,pa)\frac{X_0(t)}{w(t)}$$
$$-\frac{1}{(w(t)+w(t+pa))^2}\Bigg\{X_{1\mu}(t,pa+t,-pa)X^{\dagger}_{1\nu}(pa+t,t,pa)X_0(t)$$
$$+X_{1\mu}(t,pa+t,-pa)X^{\dagger}_0(pa+t)X_{1\nu}(pa+t,t,pa)+X_0(t)X^{\dagger}_{1\mu}(t,pa+t,-pa)X_{1\nu}(pa+t,t,pa)$$
$$-\frac{2w(t)+w(pa+t)}{w(t)^2w(pa+t)}X_0(t)X^{\dagger}_{1\mu}(t,pa+t,-pa)X_0(pa+t)X^{\dagger}_{1\nu}(pa+t,t,pa)X_0(t)\Bigg\}$$
$$+X_{2\nu\mu}(t,t,pa,-pa)-\frac{X_0(t)}{w(t)}X^{\dagger}_{2\nu\mu}(t,t,pa-pa)\frac{X_0(t)}{w(t)}-\frac{1}{(w(t)+w(t-pa))^2}$$
$$\times\Bigg\{X_{1\nu}(t,t-pa,pa)X^{\dagger}_{1\mu}(t-pa,t,-pa)X_0(t)+X_{1\nu}(t,t-pa,pa)X^{\dagger}_0(t-pa)X_{1\mu}(t-pa,t,-pa)$$
$$+X_0(t)X^{\dagger}_{1\nu}(t,t-pa,pa)X_{1\mu}(t-pa,t,-pa)-\frac{2w(t)+w(t-pa)}{w(t)^2w(t-pa)}$$
$$\times X_0(t)X^{\dagger}_{1\nu}(t,t-pa,pa)X_0(t-pa)X^{\dagger}_{1\mu}(t-pa,t,-pa)X_0(t)\Bigg\}\Bigg]\Bigg],\tag{3.14}$$

where $\int_t \equiv \int_{-\pi}^{\pi} d^4t/(2\pi)^4$. In the above two equations w, X_0, $X_{1\mu}$, $X_{2\mu\nu}$ are appropriately redefined according to the rescaling of t_μ. For example,

$$w(t)=\sqrt{(s_t^2)^{2k+1}+\Bigg\{\Bigg(r\sum_{\rho}\,(1-\cos t_\rho)\Bigg)^{2k+1}-(m_0)^{2k+1}\Bigg\}^2},\tag{3.15}$$

where $s_t^2=\Sigma_\mu \sin^2 t_\mu$.

We first want to show that there are no nonlocal divergences of the forms $p^2/(a^2p^2)^n$ or $p_\mu p_\nu/(a^2p^2)^n$ $(n\geqslant 2)$. For this purpose we confirm that Eq. (3.13) and Eq. (3.14) are not singular for $p=0$. Setting $p=0$ in these equations, we have

$$\Pi^{(a)}_{\mu\nu}(0)=\frac{-N_f g^2}{8a^2}\int_t\frac{1}{4w(t)^2}\mathrm{tr}\Bigg[\Bigg\{\frac{(s_t^2)^k\not{s}_t}{w(t)+M(t)}+1\Bigg\}\Bigg\{X_{1\mu}(t,t,0)-\frac{X_0(t)}{w(t)}X^{\dagger}_{1\mu}(t,t,0)\frac{X_0(t)}{w(t)}\Bigg\}$$
$$\times\Bigg\{\frac{(s_t^2)^k\not{s}_t}{w(t)+M(t)}+1\Bigg\}\Bigg\{X_{1\nu}(t,t,0)-\frac{X_0(t)}{w(t)}X^{\dagger}_{1\nu}(t,t,0)\frac{X_0(t)}{w(t)}\Bigg\}\Bigg]\tag{3.16}$$

and

$$\Pi^{(b)}_{\mu\nu}(0)=\frac{N_f g^2}{4a^2}\int_t \mathrm{tr}\Bigg[\Bigg\{\frac{(s_t^2)^k\not{s}_t}{w(t)+M(t)}+1\Bigg\}\frac{1}{2w(t)}\Bigg[X_{2\mu\nu}(t,t,0,0)-\frac{X_0(t)}{w(t)}X^{\dagger}_{2\mu\nu}(t,t,0,0)\frac{X_0(t)}{w(t)}-\frac{1}{4w(t)^2}$$
$$\times\Bigg\{X_{1\mu}(t,t,0)X^{\dagger}_{1\nu}(t,t,0)X_0(t)+X_{1\mu}(t,t,0)X^{\dagger}_0(t)X_{1\nu}(t,t,0)+X_0(t)X^{\dagger}_{1\mu}(t,t,0)X_{1\nu}(t,t,0)$$
$$-\frac{3}{w(t)^2}X_0(t)X^{\dagger}_{1\mu}(t,t,0)X_0(t)X^{\dagger}_{1\nu}(t,t,0)X_0(t)\Bigg\}+X_{2\nu\mu}(t,t,0,0)-\frac{X_0(t)}{w(t)}X^{\dagger}_{2\nu\mu}(t,t,0,0)\frac{X_0(t)}{w(t)}-\frac{1}{4w(t)^2}$$
$$\times\Bigg\{X_{1\nu}(t,t,0)X^{\dagger}_{1\mu}(t,t,0)X_0(t)+X_{1\nu}(t,t,0)X^{\dagger}_0(t)X_{1\mu}(t,t,0)+X_0(t)X^{\dagger}_{1\nu}(t,t,0)X_{1\mu}(t,t,0)$$
$$-\frac{3}{w(t)^2}X_0(t)X^{\dagger}_{1\nu}(t,t,0)X_0(t)X^{\dagger}_{1\mu}(t,t,0)X_0(t)\Bigg\}\Bigg]\Bigg].\tag{3.17}$$

Now on the basis of the expressions of w, X_0, $X_{1\mu}$ and $X_{2\mu\nu}$ and the fact that there are no doublers, the possible singularity may occur only around the region $t\simeq 0$ in each integral. Only the fermion propagators can exhibit singular behavior for $t\simeq 0$. The leading singularity in t in $\Pi^{(a)}_{\mu\nu}(0)$ vanishes as

$$\Pi^{(a)}_{\mu\nu}(0)\approx\int_{t^2<\delta^2}\mathrm{tr}\left[\frac{\not t}{(t^2)^{k+1}}(t^2)^k\gamma_\mu\frac{\not t}{(t^2)^{k+1}}(t^2)^k\gamma_\nu\right]\sim 0 \quad (\delta\ll 1), \tag{3.18}$$

and similarly the leading singularity in $\Pi^{(b)}_{\mu\nu}(0)$ vanishes as

$$\Pi^{(b)}_{\mu\nu}(0)\approx\int_{t^2<\delta^2}\mathrm{tr}\left[\frac{\not t}{(t^2)^{k+1}}\gamma_\mu\gamma_\nu(t^2)^{k-1}\not t\right]\sim 0 \quad (\delta\ll 1). \tag{3.19}$$

Higher order terms in t are obviously nonsingular. Since both $\Pi^{(a)}_{\mu\nu}(0)$ and $\Pi^{(b)}_{\mu\nu}(0)$ are not singular, $\Pi_{\mu\nu}(p)$ does not have the nonlocal divergences of the forms $p^2/(a^2p^2)^n$ or $p_\mu p_\nu/(a^2p^2)^n$ $(n\geqslant 2)$.

There may still exist the quadratic divergence in $\Pi_{\mu\nu}(p)$. From Eq. (3.3), the form of the quadratic divergence for small p_μ is

$$\frac{1}{a^2}\left(\delta_{\mu\nu}-\frac{\tilde p_\mu\tilde p_\nu}{\tilde p^2}\right)C, \tag{3.20}$$

with a constant C. We have already established that the $p_\mu\to 0$ limit of $\Pi_{\mu\nu}(p)$ is well-defined, which excludes the singular term $\tilde p_\mu\tilde p_\nu/\tilde p^2$; this term depends on the direction of the approach $p_\mu\to 0$. We thus conclude $C=0$, namely, the quadratic divergences cancel between diagrams (a) and (b).

Next we confirm that there are no divergences of the structure such as $a^2p^2\times\infty$, etc. which vanish in the naive continuum limit. These unusual divergences, which may be termed as infrared singularities, may occur in our treatment of $H_{(2k+1)}$ which corresponds to a higher derivative theory on the lattice. These divergences, if they should exist, could appear in the integration region around $t\simeq 0$ and could remain even for arbitrarily small p. Therefore we evaluate $\Pi^{(a)}_{\mu\nu}(p)$ (3.13) and $\Pi^{(b)}_{\mu\nu}(p)$ (3.14) for $t^2<\delta^2$ and $ap\sim 0$. After a straightforward calculation, we obtain

$$\begin{aligned}\Pi^{(a)}_{\mu\nu}(p)\simeq &-\frac{N_fg^2}{2a^2}\int_{t^2<\delta^2}\mathrm{tr}\Bigg[\frac{1}{(i\not t)^{2k+1}}\\ &\times\left(\sum_{l+m=2k}(i\not t)^l i\gamma_\mu(i(\not t+\not p a))^m\right)\\ &\times\frac{1}{(i(\not t+\not p a))^{2k+1}}\\ &\times\left(\sum_{l+m=2k}(i(\not t+\not p a))^l i\gamma_\nu(i\not t)^m\right)\Bigg],\end{aligned} \tag{3.21}$$

$$\begin{aligned}\Pi^{(b)}_{\mu\nu}(p)\simeq &\frac{N_fg^2}{2a^2}\int_{t^2<\delta^2}\mathrm{tr}\Bigg[\frac{1}{(i\not t)^{2k+1}}\\ &\times\Bigg\{\sum_{l+m+n=2k-1}(i\not t)^l i\gamma_\mu(i(\not t\\ &+\not p a))^m i\gamma_\nu(i\not t)^n\\ &+\sum_{l+m+n=2k-1}(i\not t)^l i\gamma_\nu(i(\not t\\ &-\not p a))^m i\gamma_\mu(i\not t)^n\Bigg\}\Bigg].\end{aligned} \tag{3.22}$$

These amplitudes (a) and (b) separately could contain infrared singularities. The cancellation between the amplitudes (a) and (b) further takes place as

$$\begin{aligned}&\Pi^{(a)}_{\mu\nu}(p)+\Pi^{(b)}_{\mu\nu}(p)\\ &\quad\simeq-\frac{N_fg^2}{2a^2}(2k+1)\int_{t^2<\delta^2}\mathrm{tr}\left[\frac{1}{i\not t}i\gamma_\mu\frac{1}{i(\not t+\not p a)}i\gamma_\nu\right].\end{aligned} \tag{3.23}$$

This final expression, which has the same structure as that in continuum theory, means that there are no divergences such as $a^2p^2\times\infty$, etc.

Finally, we investigate the logarithmic divergence. From the above analyses, we know that $\Pi_{\mu\nu}(p)$ does not have the divergences of the negative power in a. Therefore if there is the logarithmic divergence in $\Pi_{\mu\nu}(p)$, it appears from the singular part in the integral for $a\to 0$ and thus the singular part should appear in the integration region around $t\simeq 0$. We first evaluate $\Pi^{(a)}_{\mu\nu}(p)$. There are several ways to extract the logarithmic divergence [21,22]. Here we use the procedure discussed in the paper by Karsten and Smit [21]. First, the denominators of the propagator are combined using Feynman parameters and the integration variables are shifted $t_\mu\to t_\mu-p_\mu ax$ as follows:

$$
\begin{aligned}
\Pi_{\mu\nu}^{(a)}(p) = & \frac{-N_f g^2}{8a^2} \frac{\Gamma(4k+2)}{(\Gamma(2k+1))^2} \int_{-\pi+pax}^{\pi+pax} \frac{d^4t}{(2\pi)^4} \int_0^1 dx \frac{x^{2k}(1-x)^{2k}}{\{w(t-pax)+w(t+pa(1-x))\}^2} \\
& \times \frac{1}{[\alpha(1-x)+\beta x]^{4k+2}} \mathrm{tr}\Bigg[(s_{t-pax}^2)^k \not{s}_{t-pax} \Bigg\{ X_{1\mu}(t-pax, t+pa(1-x), -pa) \\
& - \frac{X_0(t-pax)}{w(t-pax)} X_{1\mu}^\dagger(t-pax, t+pa(1-x), -pa) \frac{X_0(t+pa(1-x))}{w(t+pa(1-x))} \Bigg\} \\
& \times (s_{t+pa(1-x)}^2)^k \not{s}_{t+pa(1-x)} \Bigg\{ X_{1\nu}(t+pa(1-x), t-pax, pa) \\
& - \frac{X_0(t+pa(1-x))}{w(t+pa(1-x))} X_{1\nu}^\dagger(t+pa(1-x), t-pax, pa) \frac{X_0(t-pax)}{w(t-pax)} \Bigg\} \Bigg],
\end{aligned} \tag{3.24}
$$

where[2]

$$\alpha \equiv \{w(t-pax)+M(t-pax)\}^{1/(2k+1)}, \tag{3.25}$$

$$\beta \equiv \{w(t+pa(1-x))+M(t+pa(1-x))\}^{1/(2k+1)}. \tag{3.26}$$

Then we split the integration domain into two regions as follows

$$\int_{t^2} = \int_{t^2<\delta^2} + \int_{t^2>\delta^2}, \quad \delta \ll 1, \tag{3.27}$$

and we evaluate the $t^2<\delta^2$ part in the continuum limit, ignoring the $t^2>\delta^2$ part which does not contain divergence. Equation (3.24) is the complicated integral including sines and cosines. However for $t^2<\delta^2$ and $a\to 0$ with fixed small p_μ we can expand both the denominator and the numerator of Eq. (3.24) separately in powers of t and a, and we have

$$
\begin{aligned}
\Pi_{\mu\nu}^{(a)}(p) \simeq & \frac{-N_f g^2}{2a^2} \frac{\Gamma(4k+2)}{(\Gamma(2k+1))^2} \int_{t^2<\delta^2} \frac{d^4t}{(2\pi)^4} \int_0^1 dx \frac{x^{2k}(1-x)^{2k}}{\{t^2+p^2a^2x(1-x)\}^{4k+2}} \\
& \times \mathrm{tr}\Bigg[i[i(\not{t}-\not{p}ax)]^{2k+1} \Bigg\{ \sum_{0\leqslant l\leqslant 2k} i[i(\not{t}-\not{p}ax)]^l i\gamma_\mu [i(\not{t}+\not{p}a(1-x))]^{2k-l} \Bigg\} \\
& \times i[i(\not{t}+\not{p}a(1-x))]^{2k+1} \Bigg\{ \sum_{0\leqslant m\leqslant 2k} i[i(\not{t}+\not{p}a(1-x))]^m i\gamma_\nu [i(\not{t}-\not{p}ax)]^{2k-m} \Bigg\} \Bigg].
\end{aligned} \tag{3.28}
$$

We next evaluate $\Pi_{\mu\nu}^{(b)}(p)$ in a similar way and we obtain

$$
\begin{aligned}
\Pi_{\mu\nu}^{(b)}(p) \simeq & \frac{-N_f g^2}{2a^2} \frac{\Gamma(4k+2)}{(\Gamma(2k+1))^2} \int_{t^2<\delta^2} \frac{d^4t}{(2\pi)^4} \int_0^1 dx \frac{x^{2k}(1-x)^{2k}}{\{t^2+p^2a^2x(1-x)\}^{4k+2}} \\
& \times \mathrm{tr}\Bigg[[i(\not{t}+\not{p}a(1-x))]^{4k+2} i[i(\not{t}-\not{p}ax)]^{2k+1} \\
& \times \Bigg\{ \sum_{0\leqslant l+m\leqslant 2k-1} i[i(\not{t}-\not{p}ax)]^l i\gamma_\mu [i(\not{t}+\not{p}a(1-x))]^m i\gamma_\nu [i(\not{t}-\not{p}ax)]^{2k-1-l-m} \Bigg\} \\
& + [i(\not{t}-\not{p}ax)]^{4k+2} i[i(\not{t}+\not{p}a(1-x))]^{2k+1} \\
& \times \Bigg\{ \sum_{0\leqslant l+m\leqslant 2k-1} i[i(\not{t}+\not{p}a(1-x))]^l i\gamma_\nu [i(\not{t}-\not{p}ax)]^m i\gamma_\mu [i(\not{t}+\not{p}a(1-x))]^{2k-1-l-m} \Bigg\} \Bigg].
\end{aligned} \tag{3.29}
$$

---

[2]We used the Feynman formula:

$$\frac{1}{\alpha^{2k+1}\beta^{2k+1}} = \frac{\Gamma(4k+2)}{\Gamma(2k+1)\Gamma(2k+1)} \int_0^1 dx \frac{x^{2k}(1-x)^{2k}}{[\alpha x+\beta(1-x)]^{4k+2}}.$$

In this way $\Pi_{\mu\nu}(p)\equiv\Pi^{(a)}_{\mu\nu}(p)+\Pi^{(b)}_{\mu\nu}(p)$ is written as

$$\Pi_{\mu\nu}(p)\simeq\frac{-N_f g^2}{2a^2}\frac{\Gamma(4k+2)}{(\Gamma(2k+1))^2}\int_{t^2<\delta^2}\frac{d^4t}{(2\pi)^4}\int_0^1 dx\,\frac{x^{2k}(1-x)^{2k}}{\{t^2+p^2a^2x(1-x)\}^{4k+2}}$$
$$\times(2k+1)(t-pax)^{4k}(t+pa(1-x))^{4k}\mathrm{tr}[(\not{t}-\not{p}ax)\gamma_\mu(\not{t}+\not{p}a(1-x))\gamma_\nu]. \tag{3.30}$$

The singular part corresponding to the logarithmic divergence is obtained from the leading part in t and a. Noting the spherical symmetry of the integral and dropping $\mathcal{O}(a^3)$ terms in the numerator, the singular part is given by

$$\Pi_{\mu\nu}(p)\simeq\frac{-N_f g^2}{2a^2}\int_{t^2<\delta^2}\frac{d^4t}{(2\pi)^4}\int_0^1 dx\,\frac{x^{2k}(1-x)^{2k}}{\{t^2+p^2a^2x(1-x)\}^{4k+2}}4(2k+1)\bigg[-\frac{1}{2}(t^2)^{4k+1}\delta_{\mu\nu}$$
$$+(t^2)^{4k}p^2a^2\delta_{\mu\nu}\bigg\{-(x^2+(1-x)^2)\left(\frac{4}{3}k^2+\frac{4}{3}k\right)+x(1-x)\left(\frac{8}{3}k^2+2k+1\right)\bigg\}$$
$$+(t^2)^{4k}p_\mu p_\nu a^2\bigg\{(x^2+(1-x)^2)\left(\frac{4}{3}k^2+\frac{4}{3}k\right)-x(1-x)\left(\frac{8}{3}k^2+4k+2\right)\bigg\}\bigg]. \tag{3.31}$$

After some calculations, the term proportional to $\log p^2a^2$ is obtained as (by restoring the factor δ^{AB})

$$(2k+1)\delta^{AB}\frac{N_f g^2}{24\pi^2}(p^2\delta_{\mu\nu}-p_\mu p_\nu)\log p^2a^2. \tag{3.32}$$

Combined with the general analysis (2.14) in the previous section, we conclude that the divergent part of the gauge field renormalization factor arising from fermion one-loop diagrams for the general Dirac operator $D=(\gamma_5/a)H$ is given by

$$Z_A=1+\frac{N_f g^2}{24\pi^2}\log\mu^2a^2, \tag{3.33}$$

where μ is the renormalization scale. This factor indeed reproduces the correct result for the QCD-type continuum theory [23,24].

Incidentally, the result (3.32) could also be directly obtained from Eq. (3.23), which corresponds to $2k+1$ times the vacuum polarization tensor generated by a conventional massless fermion.

IV. THE VACUUM POLARIZATION TENSOR FOR H WITH $k=1$

In this section we calculate the one-loop fermion contribution to the vacuum polarization tensor $\Pi_{\mu\nu}$ on the basis of H with the simplest case $k=1$. We perform essentially the same analysis as in the previous section.

Feynman diagrams for the vacuum polarization with a fermion loop are shown in Fig. 1, and the necessary Feynman rules are given in Appendix B.

The amplitude corresponding to Fig. 1(a) is given by (for QCD with N_f flavors)

$$\Pi^{(a)}_{\mu\nu}(p)=\frac{-g^2}{a^2}\frac{N_f}{2}\delta^{AB}\int_t\frac{1}{\alpha(t,t+p)^2}\mathrm{tr}[D_0^{-1}(t)\{D(t,t+p)\gamma_5H_{(3)1\mu}(t,t+p,-p)$$
$$-\gamma_5H_0(t)(\gamma_5H_{(3)1\mu}(t,t+p,-p))^\dagger\gamma_5H_0(t+p)\}D_0^{-1}(t+p)\{D(t+p,t)\gamma_5H_{(3)1\nu}(t+p,t,p)$$
$$-\gamma_5H_0(t+p)(\gamma_5H_{(3)1\nu}(t+p,t,p))^\dagger\gamma_5H_0(t)\}]. \tag{4.1}$$

The amplitude corresponding to Fig. 1(b) is

$$\Pi^{(b)}_{\mu\nu}(p)=\frac{g^2}{a}\frac{N_f}{2}\delta^{AB}\int_t\mathrm{tr}\bigg[D_0^{-1}(t)\frac{1}{\alpha(t,t)}\{D(t,t)\gamma_5H_{(3)2\mu\nu}(t,t,-p,p)-\gamma_5H_0(t)(\gamma_5H_{(3)2\mu\nu}(t,t,-p,p))^\dagger\gamma_5H_0(t)$$
$$-H_0(t)^2\gamma_5H_{1\mu}(t,t+p,-p)(\gamma_5H_{1\nu}(t+p,t,p))^\dagger\gamma_5H_0(t)-2H_0(t)^2\gamma_5H_{1\mu}(t,t+p,-p)$$
$$\times(\gamma_5H_0(t+p))^\dagger\gamma_5H_{1\nu}(t+p,t,p)-H_0(t)^2\gamma_5H_0(t)(\gamma_5H_{1\mu}(t,t+p,-p))^\dagger\gamma_5H_{1\nu}(t+p,t,p)$$
$$+\gamma_5H_0(t)(\gamma_5H_{1\mu}(t,t+p,-p))^\dagger\gamma_5H_0(t+p)(\gamma_5H_{1\nu}(t+p,t,p))^\dagger\gamma_5H_0(t)+(p,\mu\leftrightarrow-p,\nu)\}\bigg], \tag{4.2}$$

where

$$\gamma_5 H_{1\nu}(t+p,t,p)=\frac{1}{\alpha(t,t+p)}\{D(t+p,t)\gamma_5 H_{(3)1\nu}(t+p,t,p)-\gamma_5 H_0(t+p)(\gamma_5 H_{(3)1\nu}(t+p,t,p))^\dagger\gamma_5 H_0(t)\}. \tag{4.3}$$

We first show that the Ward identity for $\Pi_{\mu\nu}$ holds in this case also. From the analysis in Sec. III A, we obtain

$$\sum_\nu \tilde{p}_\nu(\gamma_5 H_{(3)2\mu\nu}(t,t,-p,p)+\gamma_5 H_{(3)2\mu\nu}(t,t,p,-p))=\gamma_5 H_{(3)1\mu}(t,t+p,-p)-\gamma_5 H_{(3)1\mu}(t-p,t,-p). \tag{4.4}$$

Using these relations $\Sigma_\nu \tilde{p}_\nu \Pi^{(b)}_{\mu\nu}(p)$ is written as

$$\begin{aligned}\sum_\nu \tilde{p}_\nu \Pi^{(b)}_{\mu\nu}(p)=\frac{g^2}{a}\frac{N_f}{2}\delta^{AB}\int_t \mathrm{tr}\Big[D_0^{-1}(t)\frac{1}{\alpha(t,t)}\{&D(t,t)(\gamma_5 H_{(3)1\mu}(t,t+p,-p)-\gamma_5 H_{(3)1\mu}(t-p,t,-p))\\ &-\gamma_5 H_0(t)(\gamma_5 H_{(3)1\mu}(t,t+p,-p)-\gamma_5 H_{(3)1\mu}(t-p,t,-p))^\dagger\gamma_5 H_0(t)\\ &+H_0(t)^2\gamma_5 H_{1\mu}H_0(t+p)H_0(t)-I\times\gamma_5 H_{1\mu}-2H_0(t)^2\gamma_5 H_0(t)H^\dagger_{1\mu}H_0(t+p)\\ &+D(t,t+p)\gamma_5 H_0(t)H_{1\mu}H_0(t)-H_0(t)^2\gamma_5 H'_{1\mu}H_0(t-p)H_0(t)+I'\times\gamma_5 H'_{1\mu}\\ &+2H_0(t)^2\gamma_5 H_0(t)H^{\dagger\prime}_{1\mu}H_0(t-p)-D(t,t-p)\gamma_5 H_0(t)H'_{1\mu}H_0(t)\}\Big],\end{aligned} \tag{4.5}$$

where $'$ means $p\to -p$ and

$$I=-3H_0(t)^4+2H_0(t)^2 D(t,t+p). \tag{4.6}$$

Noting Eq. (B5), $\Sigma_\nu \tilde{p}_\nu \Pi^{(b)}_{\mu\nu}(p)$ is rewritten as follows:

$$\begin{aligned}\sum_\nu \tilde{p}_\nu \Pi^{(b)}_{\mu\nu}(p)=\frac{g^2}{a}\frac{N_f}{2}\delta^{AB}\int_t \mathrm{tr}\Big[D_0^{-1}(t)\frac{1}{\alpha(t,t)}\gamma_5\{&2H_0(t)^2(H_{(3)1\mu}(t,t+p,-p)-H_{(3)1\mu}(t-p,t,-p))\\ &-H_0(t)(H_{(3)1\mu}(t,t+p,-p)-H_{(3)1\mu}(t-p,t,-p))H_0(t)+3H_0(t)^4 H_{1\mu}(t,t+p,-p)\\ &-2H_0(t)^2 H_{(3)1\mu}(t,t+p,-p)+H_0(t)H_{(3)1\mu}(t,t+p,-p)H_0(t)\\ &-3H_0(t)^4 H_{1\mu}(t-p,t,-p)+2H_0(t)^2 H_{(3)1\mu}(t-p,t,-p)-H_0(t)H_{(3)1\mu}(t-p,t,-p)H_0(t)\}\Big]\\ =\frac{g^2}{a}\frac{N_f}{2}\delta^{AB}\int_t \mathrm{tr}[D_0^{-1}(t)\gamma_5(H_{1\mu}(t,t+p,-p)-H_{1\mu}(t-p,t,-p))].&\end{aligned} \tag{4.7}$$

We next calculate $\Sigma_\nu \tilde{p}_\nu \Pi^{(a)}_{\mu\nu}(p)$

$$\begin{aligned}\sum_\nu \tilde{p}_\nu \Pi^{(a)}_{\mu\nu}(p)&=\frac{-g^2}{a^2}\frac{N_f}{2}\delta^{AB}\int_t \mathrm{tr}[D_0^{-1}(t)\gamma_5 H_{1\mu}(t,t+p,-p)D_0^{-1}(t+p)\gamma_5(H_0(t+p)-H_0(t))]\\ &=\frac{-g^2}{a}\frac{N_f}{2}\delta^{AB}\int_t \mathrm{tr}[(D_0^{-1}(t)-D_0^{-1}(t+p))\gamma_5 H_{1\mu}(t,t+p,-p)]\\ &=\frac{-g^2}{a}\frac{N_f}{2}\delta^{AB}\int_t \mathrm{tr}[D_0^{-1}(t)\gamma_5(H_{1\mu}(t,t+p,-p)-H_{1\mu}(t-p,t,-p))].\end{aligned} \tag{4.8}$$

From Eq. (4.8) and Eq. (4.7), one can see that the Ward identity for the vacuum polarization tensor holds.

We next examine the structure of various divergences. Rescaling the integration momenta $t_\mu \to t_\mu/a$ in each amplitude, we obtain (by omitting the factor δ^{AB} from now on)

$$\Pi^{(a)}_{\mu\nu}(p)=\frac{-N_f g^2}{2a^2}\int_t \frac{1}{\alpha(t,t+pa)^2}\mathrm{tr}[D_0^{-1}(t)\{D(t,t+pa)\gamma_5 H_{(3)1\mu}(t,t+pa,-pa)$$
$$-\gamma_5 H_0(t)(\gamma_5 H_{(3)1\mu}(t,t+pa,-pa))^\dagger\gamma_5 H_0(t+pa)\}D_0^{-1}(t+pa)$$
$$\times\{D(t+pa,t)\gamma_5 H_{(3)1\nu}(t+pa,t,pa)$$
$$-\gamma_5 H_0(t+pa)(\gamma_5 H_{(3)1\nu}(t+pa,t,pa))^\dagger\gamma_5 H_0(t)\}] \tag{4.9}$$

and

$$\Pi^{(b)}_{\mu\nu}(p)=\frac{N_f g^2}{2a^2}\int_t \mathrm{tr}\Big[D_0^{-1}(t)\frac{1}{\alpha(t,t)}\{D(t,t)\gamma_5 H_{(3)2\mu\nu}(t,t,-pa,pa)-\gamma_5 H_0(t)(\gamma_5 H_{(3)2\mu\nu}(t,t,-pa,pa))^\dagger\gamma_5 H_0(t)$$
$$-H_0(t)^2\gamma_5 H_{1\mu}(t,t+pa,-pa)(\gamma_5 H_{1\nu}(t+pa,t,pa))^\dagger\gamma_5 H_0(t)$$
$$-2H_0(t)^2\gamma_5 H_{1\mu}(t,t+pa,-pa)(\gamma_5 H_0(t+pa))^\dagger\gamma_5 H_{1\nu}(t+pa,t,pa)$$
$$-H_0(t)^2\gamma_5 H_0(t)(\gamma_5 H_{1\mu}(t,t+pa,-pa))^\dagger\gamma_5 H_{1\nu}(t+pa,t,pa)$$
$$+\gamma_5 H_0(t)(\gamma_5 H_{1\mu}(t,t+pa,-pa))^\dagger\gamma_5 H_0(t+pa)(\gamma_5 H_{1\nu}(t+pa,t,pa))^\dagger\gamma_5 H_0(t)+(p,\mu\leftrightarrow -p,\nu)\}\Big]. \tag{4.10}$$

First, we want to show that $\Pi^{(a)}_{\mu\nu}(p)$ and $\Pi^{(b)}_{\mu\nu}(p)$ are finite and well defined for $p=0$, and thus the divergent terms of the forms $p^2/(a^2p^2)^{n+1}\delta_{\mu\nu}$ and $p_\mu p_\nu/(a^2p^2)^{n+1}$ with $n\geqslant 0$ do not appear. Setting $p=0$ in these expressions, we have

$$\Pi^{(a)}_{\mu\nu}(0)=\frac{-N_f g^2}{2a^2}\int_t \frac{1}{\alpha(t,t)^2}\mathrm{tr}[D_0^{-1}(t)\{D(t,t)\gamma_5 H_{(3)1\mu}(t,t,0)-\gamma_5 H_0(t)(\gamma_5 H_{(3)1\mu}(t,t,0))^\dagger\gamma_5 H_0(t)\}$$
$$\times D_0^{-1}(t)\{D(t,t)\gamma_5 H_{(3)1\nu}(t,t,0)-\gamma_5 H_0(t)(\gamma_5 H_{(3)1\nu}(t,t,0))^\dagger\gamma_5 H_0(t)\}] \tag{4.11}$$

and

$$\Pi^{(b)}_{\mu\nu}(0)=\frac{N_f g^2}{2a^2}\int_t \mathrm{tr}\Big[D_0^{-1}(t)\frac{1}{\alpha(t,t)}\{D(t,t)\gamma_5 H_{(3)2\mu\nu}(t,t,0,0)-\gamma_5 H_0(t)(\gamma_5 H_{(3)2\mu\nu}(t,t,0,0))^\dagger\gamma_5 H_0(t)$$
$$-H_0(t)^2\gamma_5 H_{1\mu}(t,t,0)(\gamma_5 H_{1\nu}(t,t,0))^\dagger\gamma_5 H_0(t)-2H_0(t)^2\gamma_5 H_{1\mu}(t,t,0)(\gamma_5 H_0(t))^\dagger\gamma_5 H_{1\nu}(t,t,0)$$
$$-H_0(t)^2\gamma_5 H_0(t)(\gamma_5 H_{1\mu}(t,t,0))^\dagger\gamma_5 H_{1\nu}(t,t,0)+\gamma_5 H_0(t)(\gamma_5 H_{1\mu}(t,t,0))^\dagger\gamma_5 H_0(t)$$
$$\times(\gamma_5 H_{1\nu}(t,t,0))^\dagger\gamma_5 H_0(t)+(\mu\leftrightarrow\nu)\}\Big]. \tag{4.12}$$

The singularity may occur around the region $t\simeq 0$ in each integral. However, the leading order part in t in $\Pi^{(a)}_{\mu\nu}(0)$ and $\Pi^{(b)}_{\mu\nu}(0)$ vanish as

$$\Pi^{(a)}_{\mu\nu}(0)\simeq-\frac{N_f g^2}{2a^2}\int_{t^2<\delta^2}\frac{1}{(t^2)^4}\times\mathrm{tr}\left[\frac{\not t}{t^2}(t^2)^2\gamma_\mu\frac{\not t}{t^2}(t^2)^2\gamma_\nu\right]\simeq 0,$$

$$\Pi^{(b)}_{\mu\nu}(0)\simeq\frac{N_f g^2}{2a^2}\int_{t^2<\delta^2}\mathrm{tr}\left[\frac{\not t}{t^2}\frac{1}{(t^2)^2}(t^2)\not t\gamma_\mu\gamma_\nu\right]\simeq 0,$$

for $\delta\ll 1$. Since both $\Pi^{(a)}_{\mu\nu}(0)$ and $\Pi^{(b)}_{\mu\nu}(0)$ are nonsingular, $\Pi_{\mu\nu}$ does not contain the non-local divergences. From the fact that the $p_\mu\to 0$ limit of $\Pi_{\mu\nu}(p)$ is well defined and finite and that the Ward identity holds, we also conclude that the possible quadratic divergences cancel between diagrams (a) and (b). See also the analysis in Sec. III.

Next we confirm that there are no divergences of the structure such as $a^2p^2\times\infty$, etc. which vanish in the naive continuum limit even if they existed. For this purpose we evaluate $\Pi^{(a)}_{\mu\nu}(p)$ and $\Pi^{(b)}_{\mu\nu}(p)$ for $t^2<\delta^2$ and $ap\sim 0$. We thus examine the behavior of various functions appearing in these amplitudes for $t^2<\delta^2$ and $ap\sim 0$. They are given as follows:

$$D_0^{-1}(t)\simeq(2M_0)^{1/3}\frac{\not t}{t^2},\quad \alpha(t,t)\simeq 3\left(\frac{1}{2M_0}\right)^{4/3}(t^2)^2,$$

$$\gamma_5 H_0(t) \simeq -\left(\frac{1}{2M_0}\right)^{1/3} \not{t}, \quad H_0(t)^2 \simeq \left(\frac{1}{2M_0}\right)^{2/3} t^2,$$

$$\gamma_5 H_{1\mu}(t,t+p,-p) \simeq -\left(\frac{1}{2M_0}\right)^{1/3} \gamma_\mu,$$

$$D(t,t+pa) \simeq \left(\frac{1}{2M_0}\right)^{2/3} \{t^2 + (t+pa)^2\},$$

$$\gamma_5 H_{(3)2\mu\nu}(t,t,-pa,pa) \simeq \frac{1}{2M_0} \sum_{l+m+n=1} \{i(i\not{t})^l i\gamma_\mu \times (i(\not{t}+\not{p}a))^m i\gamma_\nu (i\not{t})^n\},$$

where all the higher order terms are nonsingular. Using these expressions, $\Pi^{(a)}_{\mu\nu}(p)$ and $\Pi^{(b)}_{\mu\nu}(p)$ for $t^2<\delta^2$ and $ap\sim 0$ are expressed as

$$\Pi^{(a)}_{\mu\nu}(p) \simeq -\frac{N_f g^2}{2a^2} \int_{t^2<\delta^2} \frac{1}{t^2(t+pa)^2} \times \mathrm{tr}[\not{t}\gamma_\mu(\not{t}+\not{p}a)\gamma_\nu],$$

$$\Pi^{(b)}_{\mu\nu}(p) \simeq \frac{N_f g^2}{2a^2} \int_{t^2<\delta^2} \mathrm{tr}\left[\frac{\not{t}}{t^2}\frac{1}{t^2} f(t,pa)\right],$$

where $f(t,pa)$ is a nonsingular function of t and pa. Since $\Pi^{(b)}_{\mu\nu}(p)$ vanishes in this limit and the expression of $\Pi^{(a)}_{\mu\nu}(p)$ has the same structure as that in continuum theory, we conclude that there are no (unusual) divergences such as $a^2p^2\times\infty$, etc. in the vacuum polarization tensor.

Finally, we investigate the logarithmic divergence. From the above analyses, the logarithmic divergence in $\Pi_{\mu\nu}(p)$ appears from the singular part in the integral for $a\to 0$ and $t\simeq 0$. Since $\Pi^{(b)}_{\mu\nu}(p)$ is nonsingular in this limit, we consider only the amplitude $\Pi^{(a)}_{\mu\nu}(p)$. In Eq. (4.9), we use the Feynman's parameter and shift the integration variables $t_\mu \to t_\mu - ap_\mu x$. We then evaluate the contribution from the integration region $t^2<\delta^2$, $\delta \ll 1$ in the continuum limit. We then have

$$\Pi^{(a)}_{\mu\nu}(p) \simeq -\frac{N_f g^2}{2a^2} \int_{t^2<\delta^2} \frac{d^4t}{(2\pi)^4} \times \int_0^1 dx \frac{x^2(1-x)^2}{\{t^2+p^2a^2x(1-x)\}^6} \frac{\Gamma(6)}{\Gamma(3)^2} \times ((t-pax)^2)^2([t+pa(1-x)]^2)^2 \times \mathrm{tr}[(\not{t}-\not{p}ax)\gamma_\mu(\not{t}+\not{p}a(1-x))\gamma_\nu]. \tag{4.13}$$

After some calculations, the term proportional to $\log a^2p^2$ is extracted as follows:

$$\frac{N_f g^2}{24\pi^2}(p^2\delta_{\mu\nu} - p_\mu p_\nu)\log a^2p^2. \tag{4.14}$$

This expression agrees with the one expected for the continuum theory [23,24] to be consistent with the general analysis in Sec. II.

V. DISCUSSION

We have studied a perturbative aspect of a general class of Dirac operators. To avoid excessive complications, we examined the simplest diagrams of the one-loop fermion correction to the gauge field self-energy tensor. This quantity is related to the one-loop β function and also to the Weyl anomaly. We have confirmed that the perturbative analysis gives the correct result for any $k\geqslant 1$ by using the relation (2.14), in accord with the general analysis of the Weyl anomaly. This correct result is consistent with our previous analyses of the locality of the general operator $H_{(2k+1)}$ and the locality domain of $|F_{\mu\nu}|$ for $H_{(2k+1)}$ [12]: Also, our result does not contradict the general perturbative analysis of lattice theory in [26] if one remembers the locality properties of $H_{(2k+1)}$. We have also confirmed the relation (2.14) for the simplest case $k=1$ by evaluating the self-energy correction in terms of the operator H itself.

When combined with the analysis of chiral anomaly [8], our present analysis shows that all the local anomalies are properly reproduced by our general class of operators D. These analyses give some confidence in the treatment of the fermionic determinant

$$\det H = (\det H_{(2k+1)})^{1/(2k+1)} \tag{5.1}$$

in the possible application to QCD, for example.

At the same time, we recognized that infrared divergences may generally appear in the intermediate stages of perturbative calculations for finite a, which should cancel in the final result. This treatment of infrared divergences in perturbation theory is quite tedious in our generalized operator D. To avoid the infrared complications, the (nonperturbative) Wilsonian effective action, which is supposed to be free of infrared complications, is expected to be essential for the general operator D. As for the perturbative treatment of fully dynamical gauge field such as in the one-loop correction to the fermion self-energy, some auxiliary regulator such as the dimensional regulator may become necessary [22] for a reliable treatment of infrared divergences.

APPENDIX A: FEYNMAN RULES FOR THE GENERAL $H_{(2k+1)}$

We derive the Feynman rules for $H_{(2k+1)}$ theory (not H itself) to calculate the vacuum polarization at the one-loop level on the basis of Eq. (2.14). $H_{(2k+1)}$ has been defined by Eqs. (1.6), (1.7) and (1.8). We expand $H_{(2k+1)}$ up to the second order in the coupling constant g as follows:

$$H_{(2k+1)} = H_{(2k+1)0} + gH_{(2k+1)1} + g^2 H_{(2k+1)2} + \mathcal{O}(g^3). \tag{A1}$$

For this purpose we first need to expand $D_W^{(2k+1)}(x,y)$ in g. $D_W^{(2k+1)}(x,y)$ up to the second order in g is given by

$$D_W^{(2k+1)}(x,y)=a^4\int\frac{d^4p}{(2\pi)^4}\int\frac{d^4q}{(2\pi)^4}e^{ipx-iqy}\times[X_0(p)\delta_P(p-q)+X_1(p,q)+X_2(p,q)+O(g^3)],\tag{A2}$$

where

$$X_0(p)=i\left(\frac{i}{a}\gamma_\mu\sin p_\mu a\right)^{2k+1}+\left(\frac{r}{a}\sum_\mu(1-\cos p_\mu a)\right)^{2k+1}-\left(\frac{m_0}{a}\right)^{2k+1}=-\left(\frac{\sin^2 p_\rho a}{a^2}\right)^k\frac{\gamma_\rho\sin p_\rho a}{a}+M(p),\tag{A3}$$

$$X_1(p,q)\equiv\sum_\mu\int_{-\pi/a}^{\pi/a}\frac{d^4t}{(2\pi)^4}\delta_P(p-t-q)gA_\mu(t)\times X_{1\mu}(p,q,t),\tag{A4}$$

$$X_2(p,q)\equiv\sum_{\mu,\nu}\int_{-\pi/a}^{\pi/a}\frac{d^4k_1}{(2\pi)^4}\int_{-\pi/a}^{\pi/a}\frac{d^4k_2}{(2\pi)^4}\times\delta_P(p-q-k_1-k_2)g^2A_\mu(k_1)A_\nu(k_2)\times X_{2\mu\nu}(p,q,k_1,k_2).\tag{A5}$$

Here we defined

$$X_{1\mu}(p,q,t)=\sum_{l+m=2k}\left\{i\left(\frac{i}{a}\gamma_\rho\sin p_\rho a\right)^l\times\left(i\gamma_\mu\cos\left(q+\frac{t}{2}\right)_\mu a\right)\left(\frac{i}{a}\gamma_\rho\sin q_\rho a\right)^m+\left(\frac{r}{a}\sum_\rho(1-\cos p_\rho a)\right)^l\left(r\sin\left(q+\frac{t}{2}\right)_\mu a\right)\times\left(\frac{r}{a}\sum_\rho(1-\cos q_\rho a)\right)^m\right\}\tag{A6}$$

and

$$\begin{aligned}X_{2\mu\nu}(p,q,k_1,k_2)=&\sum_{l+m+n=2k-1}\left\{i\left(\frac{i}{a}\gamma_\rho\sin p_\rho a\right)^l\left(i\gamma_\mu\cos\left(p-\frac{k_1}{2}\right)_\mu a\right)\right.\\&\times\left(\frac{i}{a}\gamma_\rho\sin(p-k_1)_\rho a\right)^m\left(i\gamma_\nu\cos\left(q+\frac{k_2}{2}\right)_\nu a\right)\left(\frac{i}{a}\gamma_\rho\sin q_\rho a\right)^n\\&+\left(\frac{r}{a}\sum_\rho(1-\cos p_\rho a)\right)^l\left(r\sin\left(p-\frac{k_1}{2}\right)_\mu a\right)\\&\left.\times\left(\frac{r}{a}\sum_\rho(1-\cos(p-k_1)_\rho a)\right)^m\left(r\sin\left(q+\frac{k_2}{2}\right)_\nu a\right)\left(\frac{r}{a}\sum_\rho(1-\cos q_\rho a)\right)^n\right\}\\&+\frac{1}{2}\sum_{l+m=2k}\left\{i\left(\frac{i}{a}\gamma_\rho\sin p_\rho a\right)^l\left(-ia\gamma_\mu\delta_{\mu\nu}\sin\left(q+\frac{k_1+k_2}{2}\right)_\mu a\right)\left(\frac{i}{a}\gamma_\rho\sin q_\rho a\right)^m\right.\\&\left.+\left(\frac{r}{a}\sum_\rho(1-\cos p_\rho a)\right)^l\left(ar\delta_{\mu\nu}\cos\left(q+\frac{k_1+k_2}{2}\right)_\mu a\right)\left(\frac{r}{a}\sum_\rho(1-\cos q_\rho a)\right)^m\right\},\end{aligned}\tag{A7}$$

where

$$M(p)=\left(\frac{r}{a}\sum_\rho(1-\cos p_\rho a)\right)^{2k+1}-\left(\frac{m_0}{a}\right)^{2k+1},\tag{A8}$$

and l, m, n are the nonnegative integers and δ_P is the periodic lattice delta function. $A_\mu(k)$ is defined by

$$A_\mu(x)=\int_{-\pi/a}^{\pi/a}\frac{d^4k}{(2\pi)^4}A_\mu(k)e^{ik(x-a\hat{\mu}/2)},\tag{A9}$$

and has the properties $A_\mu^\dagger(k)=A_\mu(-k)$ and $A_\mu(k+(2\pi/a)l)=(-1)^{l_\mu}A_\mu(k)$ (l: integer).

Next we want to expand the factor $1/\sqrt{(D_W^{(2k+1)})^\dagger(D_W^{(2k+1)})}$ in $H_{(2k+1)}$ (1.6). But it is very complicated to perform the weak coupling expansion of $1/\sqrt{(D_W^{(2k+1)})^\dagger(D_W^{(2k+1)})}$ directly, if not impossible. Therefore we use the following identity:

$$\frac{1}{\sqrt{X^\dagger X}} = \int_{-\infty}^{\infty} \frac{dt}{\pi} \frac{1}{t^2 + X^\dagger X}. \tag{A10}$$

The weak coupling expansion of the integrand on the right-hand side can be readily performed. After some calculations, we obtain

$$\begin{aligned} H_{(2k+1)} = a^4 \int \frac{d^4 p}{(2\pi)^4} \int \frac{d^4 q}{(2\pi)^4} e^{ipx-iqy} \frac{1}{2} \gamma_5 \Bigg[\left(1 + \frac{X_0(p)}{\omega(p)}\right) \delta_P(p-q) + \left\{\frac{1}{\omega(p)+\omega(q)}\right\} \\ \times \left\{ X_1(p,q) - \frac{X_0(p)}{\omega(p)} X_1^\dagger(p,q) \frac{X_0(q)}{\omega(q)} \right\} + \left\{\frac{1}{\omega(p)+\omega(q)}\right\} \left\{ X_2(p,q) - \frac{X_0(p)}{\omega(p)} X_2^\dagger(p,q) \frac{X_0(q)}{\omega(q)} \right\} \\ + \int_t \left\{\frac{1}{\omega(p)+\omega(q)}\right\} \left\{\frac{1}{\omega(p)+\omega(t)}\right\} \left\{\frac{1}{\omega(t)+\omega(q)}\right\} \\ \times \Bigg\{ -X_0(p) X_1^\dagger(p,t) X_1(t,q) - X_1(p,t) X_0^\dagger(t) X_1(t,q) - X_1(p,t) X_1^\dagger(t,q) X_0(q) \\ + \frac{\omega(p)+\omega(q)+\omega(t)}{\omega(p)\omega(t)\omega(q)} X_0(p) X_1^\dagger(p,t) X_0(t) X_1^\dagger(t,q) X_0(q) \Bigg\} + \mathcal{O}(g^3) \Bigg], \end{aligned} \tag{A11}$$

where

$$\omega(p) = \sqrt{\left(\frac{1}{a^2} \sin^2 p_\rho a\right)^{2k+1} + \left\{ \left(\frac{r}{a} \sum_\rho (1 - \cos p_\rho a)\right)^{2k+1} - \left(\frac{m_0}{a}\right)^{2k+1} \right\}^2}, \tag{A12}$$

and $X^\dagger \equiv \gamma_5 X \gamma_5$. We write $H_{(2k+1)0}(x,y)$, $H_{(2k+1)1}(x,y)$, and $H_{(2k+1)2}(x,y)$ as follows:

$$H_{(2k+1)0}(x,y) = a^4 \int_p e^{ipx-ipy} H_{(2k+1)0}(p), \tag{A13}$$

$$H_{(2k+1)1}(x,y) = a^4 \sum_\mu \int_{p,q,t} e^{ipx-iqy} \delta_P(p-q-t) A_\mu(t) H_{(2k+1)1\mu}(p,q,t), \tag{A14}$$

$$H_{(2k+1)2}(x,y) = a^4 \sum_{\mu\nu} \int_{p,q,k_1,k_2} e^{ipx-iqy} \delta_P(p-q-k_1-k_2) A_\mu(k_1) A_\nu(k_2) H_{(2k+1)2\mu\nu}(p,q,k_1,k_2), \tag{A15}$$

where $\int_p \equiv \int_{-\pi/a}^{\pi/a} [d^4 p/(2\pi)^4]$. Then from Eq. (A11), $H_{(2k+1)0}(p)$, $H_{(2k+1)1\mu}(p,q,t)$, and $H_{(2k+1)2\mu\nu}(p,q,k_1,k_2)$ can be written as

$$H_{(2k+1)0}(p) = \frac{1}{2} \gamma_5 \left(1 + \frac{X_0(p)}{\omega(p)}\right), \tag{A16}$$

$$\begin{aligned} H_{(2k+1)1\mu}(p,q,t) = \frac{1}{2} \gamma_5 \frac{1}{\omega(p)+\omega(q)} \\ \times \left\{ X_{1\mu}(p,q,t) - \frac{X_0(p)}{\omega(p)} X_{1\mu}^\dagger(p,q,t) \frac{X_0(q)}{\omega(q)} \right\}, \end{aligned} \tag{A17}$$

$$\begin{aligned} H_{(2k+1)2\mu\nu}(p,q,k_1,k_2) = \frac{1}{2} \gamma_5 \frac{1}{\omega(p)+\omega(q)} \Bigg[X_{2\mu\nu}(p,q,k_1,k_2) - \frac{X_0(p)}{\omega(p)} X_{2\mu\nu}^\dagger(p,q,k_1,k_2) \frac{X_0(q)}{\omega(q)} \\ - \frac{1}{(\omega(p)+\omega(p-k_1))(\omega(p-k_1)+\omega(q))} \Big\{ X_{1\mu}(p,p-k_1,k_1) X_{1\nu}^\dagger(q+k_2,q,k_2) X_0(q) \\ + X_{1\mu}(p,p-k_1,k_1) X_0^\dagger(p-k_1) X_{1\nu}(q+k_2,q,k_2) + X_0(p) X_{1\mu}^\dagger(p,p-k_1,k_1) X_{1\nu}(q+k_2,q,k_2) \\ - \frac{\omega(p)+\omega(p-k_1)+\omega(q)}{\omega(p)\omega(p-k_1)\omega(q)} X_0(p) X_{1\mu}^\dagger(p,p-k_1,k_1) X_0(p-k_1) X_{1\nu}^\dagger(q+k_2,q,k_2) X_0(q) \Big\} \Bigg]. \end{aligned} \tag{A18}$$

Note that $q=p-t$ in $H_{(2k+1)1\mu}(p,q,t)$ and $q=p-k_1-k_2$ in $H_{(2k+1)2\mu\nu}(p,q,k_1,k_2)$.

From this weak coupling expansion we can derive the Feynman rules for $H_{(2k+1)}$ theory, which are necessary for the one-loop analyses. To make the structure of the divergences in one-loop amplitudes explicit we derive the Feynman rules for $(1/a^{2k+1})H_{(2k+1)}=(\gamma_5 D)^{2k+1}$. Using Eq. (A16), the fermion propagator $D_0^{-1}(p)$, where $D_0^{-1}(p)$ stands for the inverse of free $(\gamma_5 D)^{2k+1}$, is written as

$$D_0^{-1}(p)=\left(\frac{(s_p^2)^k \not{s}_p}{w(p)+M(p)}+a^{2k+1}\right)\gamma_5, \tag{A19}$$

where $\not{s}_p\equiv\Sigma_\mu\gamma_\mu \sin ap_\mu$ and $s_p^2\equiv\Sigma_\mu \sin^2 ap_\mu$. Using Eq. (A17), we assign the following factor to the fermion-gauge field three-point vertex depicted in Fig. 2

$$-\frac{g}{2a^{2k+1}}T^A_{ba}\gamma_5\frac{1}{\omega(p)+\omega(q)}\left\{X_{1\mu}(p,q,t)-\frac{X_0(p)}{\omega(p)}X^\dagger_{1\mu}(p,q,t)\frac{X_0(q)}{\omega(q)}\right\}, \tag{A20}$$

where T^A are $SU(N)$ generators. Using Eq. (A18), we assign the following factor to the fermion-gauge field four-point vertex depicted in Fig. 3

$$\begin{aligned}
&-\frac{g^2}{2a^{2k+1}}(T^AT^B)_{ba}\gamma_5\frac{1}{\omega(p)+\omega(q)}\Bigg[X_{2\mu\nu}(p,q,k_1,k_2)-\frac{X_0(p)}{\omega(p)}X^\dagger_{2\mu\nu}(p,q,k_1,k_2)\frac{X_0(q)}{\omega(q)}\\
&-\frac{1}{(\omega(p)+\omega(p-k_1))(\omega(p-k_1)+\omega(q))}\Big\{X_{1\mu}(p,p-k_1,k_1)X^\dagger_{1\nu}(q+k_2,q,k_2)X_0(q)\\
&+X_{1\mu}(p,p-k_1,k_1)X^\dagger_0(p-k_1)X_{1\nu}(q+k_2,q,k_2)+X_0(p)X^\dagger_{1\mu}(p,p-k_1,k_1)X_{1\nu}(q+k_2,q,k_2)\\
&-\frac{\omega(p)+\omega(p-k_1)+\omega(q)}{\omega(p)\omega(p-k_1)\omega(q)}X_0(p)X^\dagger_{1\mu}(p,p-k_1,k_1)X_0(p-k_1)X^\dagger_{1\nu}(q+k_2,q,k_2)X_0(q)\Big\}\Bigg]+(A,\mu,k_1\leftrightarrow B,\nu,k_2),
\end{aligned} \tag{A21}$$

where we have imposed the Bose symmetry for gauge fields.

In Sec. III of the present paper, we calculate the vacuum polarization tensor at the one-loop level by using these Feynman rules.

APPENDIX B: FEYNMAN RULES FOR THE OPERATOR *H* WITH $k=1$

In this appendix we derive the Feynman rules for the operator H to calculate the vacuum polarization at the one-loop level. For simplicity we consider the case with $k=1$. For the Feynman rules for the operator $H_{(3)}$, we refer to Appendix A. We expand H and $H_{(3)}$ up to the second order in the coupling constant g as follows:

$$H=H_0+gH_1+g^2H_2+\mathcal{O}(g^3), \tag{B1}$$

$$D=\frac{1}{a}\gamma_5H_0+\frac{1}{a}\gamma_5 gH_1+\frac{1}{a}\gamma_5 g^2H_2+\mathcal{O}(g^3),$$

$$\begin{aligned}
H_{(3)}&=H_{(3)0}+gH_{(3)1}+g^2H_{(3)2}+\mathcal{O}(g^3)\\
&=(H_0+gH_1+g^2H_2+\mathcal{O}(g^3))^3 \quad (=H^3)\\
&=H_0^3+g\sum_{0\leqslant m\leqslant 2}H_0^mH_1H_0^{2-m}\\
&\quad+g^2\left(\sum_{0\leqslant m\leqslant 2}H_0^mH_2H_0^{2-m}\right.\\
&\quad\left.+\sum_{0\leqslant l+m\leqslant 1}H_0^lH_1H_0^mH_1H_0^{1-l-m}\right)+\mathcal{O}(g^3).
\end{aligned} \tag{B2}$$

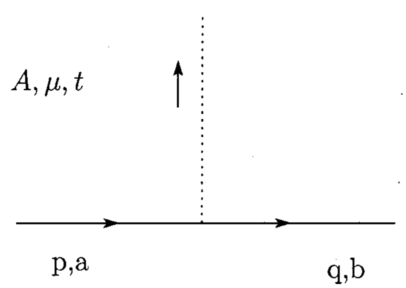

FIG. 2. Fermion-gauge field three-point vertex.

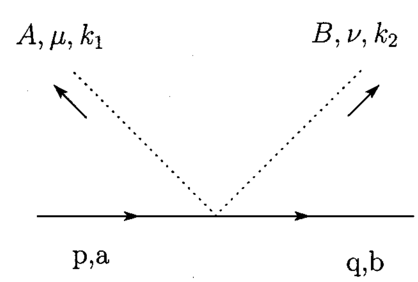

FIG. 3. Fermion-gauge field four-point vertex.

Now we want to know the expressions of H_0, H_1, and H_2. We have obtained H_0 in our previous paper [12,9]. In momentum space H_0 is written as

$$H_0(p)=\gamma_5\left(\frac{1}{2}\right)^{2/3}\left(\frac{1}{\omega(p)}\right)^{2/3}\left\{(\omega(p)+M(p))^{2/3}-(\omega(p)-M(p))^{1/3}\frac{\not{s}_p}{a}\right\},$$

$$=\gamma_5\left(\frac{1}{2\,\omega(p)}\right)^{2/3}\left(\frac{1}{\omega(p)+M(p)}\right)^{1/3}\times\left\{-\left(\frac{s_p^2}{a^2}\right)\frac{\not{s}_p}{a}+\omega(p)+M(p)\right\},\qquad\text{(B3)}$$

where $\not{s}_p\equiv\Sigma_\mu\gamma_\mu\sin ap_\mu$ and $s_p^2\equiv\Sigma_\mu\sin^2 ap_\mu$. One can easily check that $H_{(3)0}(p)=(H_0(p))^3$.

Next we derive the expression of H_1. We write H_1 as follows:

$$H_1=a^4\sum_\mu\int_{p,q,t}e^{ipx-iqy}\delta_P(p-q-t)A_\mu(t)\times H_{1\mu}(p,q,t),\qquad\text{(B4)}$$

where $\int_{p,q,t}\equiv\int[d^4p/(2\pi)^4]\int[d^4q/(2\pi)^4]\int[d^4t/(2\pi)^4]$. Using Eq. (B2) at first order in g, we obtain

$$H_{(3)1\mu}(p,q,t)=\sum_{0\leqslant m\leqslant 2}H_0(p)^mH_{1\mu}(p,q,t)H_0(q)^{2-m}$$
$$=D(p,q)H_{1\mu}(p,q,t)+H_0(p)H_{1\mu}(p,q,t)H_0(q),\qquad\text{(B5)}$$

where

$$D(p,q)=H_0(p)^2+H_0(q)^2,\qquad\text{(B6)}$$

$$H_0(p)^2=\left\{\frac{1}{2}\left(1+\frac{M(p)}{\omega(p)}\right)\right\}^{1/3}.\qquad\text{(B7)}$$

Now we consider an ansatz as

$$H_{1\mu}(p,q,t)=\frac{1}{\alpha}(D(p,q)H_{(3)1\mu}(p,q,t)-H_0(p)H_{(3)1\mu}(p,q,t)H_0(q)).\qquad\text{(B8)}$$

Substituting this ansatz for $H_{1\mu}$ into Eq. (B5), we easily obtain the expression of α as

$$\alpha(p,q)=D^2(p,q)-H_0^2(p)H_0^2(q)$$
$$=\sum_{0\leqslant m\leqslant 2}H_0(p)^{2m}H_0(q)^{2(2-m)}.\qquad\text{(B9)}$$

Thus we have obtained the expression of H_1.

Further we derive the expression of H_2, performing the similar procedure as deriving H_1. We first write H_2 as follows:

$$H_2=a^4\sum_{\mu\nu}\int_{p,q,k_1,k_2}e^{ipx-iqy}\delta_P(p-q-k_1-k_2)\times A_\mu(k_1)A_\nu(k_2)H_{2\mu\nu}(p,q,k_1,k_2).\qquad\text{(B10)}$$

Using Eq. (B2) at the second order in g, we obtain

$$H_{(3)2\mu\nu}(p,q,k_1,k_2)=\sum_{0\leqslant m\leqslant 2}H_0(p)^mH_{2\mu\nu}(p,q,k_1,k_2)H_0(q)^{2-m}+\sum_{0\leqslant l+m\leqslant 1}H_0(p)^lH_{1\mu}(p,p-k_1,k_1)H_0(p-k_1)^mH_{1\nu}(p-k_1,q,k_2)H_0(q)^{1-l-m}$$
$$=D(p,q)H_{2\mu\nu}(p,q,k_1,k_2)+H_0(p)H_{2\mu\nu}(p,q,k_1,k_2)H_0(q)+H_{1\mu}(p,p-k_1,k_1)H_{1\nu}(p-k_1,q,k_2)H_0(q)+H_{1\mu}(p,p-k_1,k_1)H_0(p-k_1)H_{1\nu}(p-k_1,q,k_2)+H_0(p)H_{1\mu}(p,p-k_1,k_1)H_{1\nu}(p-k_1,q,k_2).\qquad\text{(B11)}$$

Now we define

$$H'_{(3)2\mu\nu}(p,q,k_1,k_2)\equiv H_{(3)2\mu\nu}-\{H_{1\mu}H_{1\nu}H_0+H_{1\mu}H_0H_{1\nu}+H_0H_{1\mu}H_{1\nu}\}.\qquad\text{(B12)}$$

Using $H'_{(3)2\mu\nu}$, Eq. (B11) is written as

$$H'_{(3)2\mu\nu}=DH_{2\mu\nu}+H_0(p)H_{2\mu\nu}H_0(q).\qquad\text{(B13)}$$

The structure of this equation is the same as that of Eq. (B5). Therefore $H_{2\mu\nu}$ is written as

$$H_{2\mu\nu}=\frac{1}{\alpha(p,q)}(D(p,q)H'_{(3)2\mu\nu}-H_0(p)H'_{(3)2\mu\nu}H_0(q)). \tag{B14}$$

Finally we obtain the expression of $H_{2\mu\nu}$ in momentum space as follows:

$$\begin{aligned}H_{2\mu\nu}(p,q,k_1,k_2)=\frac{1}{\alpha(p,q)}\{&D(p,q)H_{(3)2\mu\nu}(p,q,k_1,k_2)-H_0(p)H_{(3)2\mu\nu}(p,q,k_1,k_2)H_0(q)\\&-H_0(q)^2H_{1\mu}(p,p-k_1,k_1)H_{1\nu}(p-k_1,q,k_2)H_0(q)-D(p,q)H_{1\mu}(p,p-k_1,k_1)\\&\times H_0(p-k_1)H_{1\nu}(p-k_1,q,k_2)-H_0(p)^2H_0(p)H_{1\mu}(p,p-k_1,k_1)H_{1\nu}(p-k_1,q,k_2)\\&+H_0(p)H_{1\mu}(p,p-k_1,k_1)H_0(p-k_1)H_{1\nu}(p-k_1,q,k_2)H_0(q)\}.\end{aligned} \tag{B15}$$

From this weak coupling expansion we can derive the Feynman rules for D in the case of $k=1$, which are necessary for the one-loop analysis. Using Eq. (B3), the fermion propagator $D_0^{-1}(p)$ is written as

$$D_0^{-1}(p)=\frac{aH_0(p)^2\left(\frac{s_p^2}{a^2}\right)\frac{\not{s}_p}{a}}{\omega(p)+M(p)}+aH_0(p)^2. \tag{B16}$$

Using Eq. (B8) and Eq. (B9), we assign the following expression to the fermion-gauge field three-point vertex depicted in Fig. 2:

$$-\frac{g}{a}T^A_{ba}\frac{1}{\alpha(p,q)}\{D(p,q)\gamma_5H_{(3)1\mu}(p,q,t)-\gamma_5H_0(p)(\gamma_5H_{(3)1\mu}(p,q,t))^\dagger\gamma_5H_0(q)\}, \tag{B17}$$

where T^A are $SU(N)$ generators. Using Eq. (B15), we assign the following expression to the fermion-gauge four-point vertex depicted in Fig. 3,

$$\begin{aligned}&-\frac{g^2}{a}(T^AT^B)_{ba}\frac{1}{\alpha(p,q)}\{D(p,q)\gamma_5H_{(3)2\mu\nu}(p,q,k_1,k_2)-\gamma_5H_0(p)(\gamma_5H_{(3)2\mu\nu}(p,q,k_1,k_2))^\dagger\gamma_5H_0(q)\\&-H_0(q)^2\gamma_5H_{1\mu}(p,p-k_1,k_1)(\gamma_5H_{1\nu}(p-k_1,q,k_2))^\dagger\gamma_5H_0(q)\\&-D(p,q)\gamma_5H_{1\mu}(p,p-k_1,k_1)(\gamma_5H_0(p-k_1))^\dagger\gamma_5H_{1\nu}(p-k_1,q,k_2)\\&-H_0(p)^2\gamma_5H_0(p)(\gamma_5H_{1\mu}(p,p-k_1,k_1))^\dagger\gamma_5H_{1\nu}(p-k_1,q,k_2)\\&+\gamma_5H_0(p)(\gamma_5H_{1\mu}(p,p-k_1,k_1))^\dagger\gamma_5H_0(p-k_1)\\&\times(\gamma_5H_{1\nu}(p-k_1,q,k_2))^\dagger\gamma_5H_0(q)\}+(A,\mu,k_1\leftrightarrow B,\nu,k_2).\end{aligned} \tag{B18}$$

We perform a one-loop calculation in Sec. IV on the basis of these Feynman rules.

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