Polarized virtual photon structure function g_2^γ and twist-3 effects in QCD

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We investigate twist-3 effects in the polarized virtual photon structure. The structure functions g_1^γ and g_2^γ of a polarized photon could be experimentally studied in future polarized ep or e^+e^- colliders. The leading contributions to g_1^γ are twist-2 effects, while another structure function g_2^γ, which exists only for the virtual photon target, receives not only twist-2 but also twist-3 contributions. We first show that twist-3 effects actually exist in the box-diagram contributions and we extract the twist-3 part, which can also be reproduced by the pure QED operator product expansion. We then calculate the nontrivial lowest moment $(n=3)$ of the twist-3 contribution to g_2^γ in QCD. For large N_c (the number of colors), the QCD analysis of twist-3 effects in the flavor nonsinglet part of g_2^γ becomes tractable and we can obtain its moments in a compact form for all n.

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I. INTRODUCTION

In recent years, there has been growing interest in the study of the spin structure of the photon. In particular, the first moment of the polarized photon structure function g_1^γ has attracted much attention in the literature [1–5] in connection with the axial anomaly, which is also relevant to the analysis of the nucleon spin structure function $g_1^{\rm nucl}$. The polarized photon structure functions may be extracted from the resolved photon process in the polarized version of the DESY ep collider HERA [6,7]. More directly, they can be measured by polarized e^+e^- collision experiments in future linear colliders (Fig. 1), where $-Q^2$ $(-P^2)$ is the mass squared of the probe (target) photon.

For a real photon $(P^2=0)$ target, there exists only one spin-dependent structure function $g_1^\gamma(x,Q^2)$, which is equivalent to the structure function $W_4^\gamma(x,Q^2)$ $(g_1^\gamma\equiv 2W_4^\gamma)$ discussed some time ago in [8,9]. The leading order (LO) QCD corrections to g_1^γ for a real photon target were first calculated by one of the authors in [10] and later in [11,2], while the next-to-leading order (NLO) QCD analysis was performed in [12,13].

In the case of a virtual photon target $(P^2\neq 0)$, and especially when we deal with the kinematical region $\Lambda^2\ll P^2 \ll Q^2$, where Λ is the QCD scale parameter, we can calculate all the structure functions up to NLO in QCD by the perturbative method, in contrast to the case of the real photon target where in NLO there exist nonperturbative pieces [14,15]. In this context, the spin-independent structure functions $F_2^\gamma(x,Q^2,P^2)$ and $F_L^\gamma(x,Q^2,P^2)$ as well as the parton content of the unpolarized virtual photon have been studied in the above kinematical region in LO [16] and in NLO [17–22]. The target mass effects on both the unpolarized and polarized virtual photon structure functions have been discussed in LO [23]. More recently, the spin-dependent structure function $g_1^\gamma(x,Q^2,P^2)$ of the virtual photon has been investigated up to NLO by the present authors in [24], and also in the second paper of [13].

Generally, for the virtual photon target, there exists another structure function $g_2^\gamma(x,Q^2,P^2)$, which is the analogue of the spin-dependent nucleon structure function $g_2^{\rm nucl}$. In the language of operator product expansion (OPE), it is well known that both twist-2 and twist-3 operators contribute to $g_2^{\rm nucl}$ in the leading order of $1/Q^2$. The same is also true for g_2^γ. In this paper, we shall investigate the twist-3 contribution to g_2^γ in the leading order in QCD, and show that they

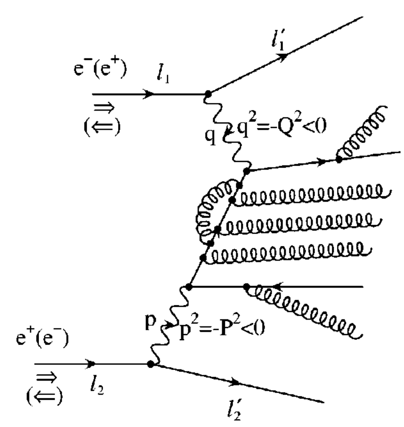

FIG. 1. Deep inelastic scattering on a polarized virtual photon in polarized e^+e^- collision, $e^+e^-\to e^+e^-+$hadrons (quarks and gluons). The arrows indicate the polarizations of the e^+ and e^-. The mass squared of the probe (target) photon is $-Q^2$ $(-P^2)$ $(\Lambda^2 \ll P^2\ll Q^2)$.

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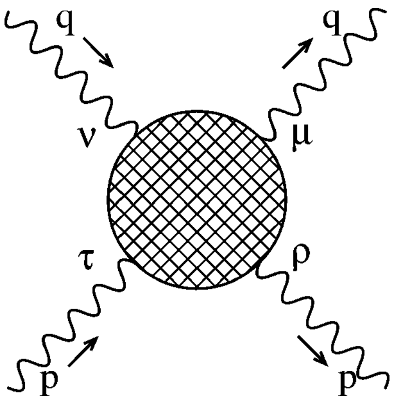

FIG. 2. Forward scattering of a virtual photon with momentum q and another virtual photon with momentum p. The Lorentz indices are denoted by μ, ν, ρ, τ.

are sizable in contrast to the nucleon case, where the experimental data show that the twist-3 contribution appears to be small [25,26].

In the next section we discuss the two structure functions g_1^γ and g_2^γ, which describe the deep inelastic scattering off a polarized virtual photon target. (How the information on g_1^γ and g_2^γ can be extracted from the spin-dependent experiments of $e^+e^-\to e^+e^-+$hadrons is shown in Appendix A.) They are related to the s-channel helicity amplitudes which appear in the forward virtual photon-photon scattering. We examine the box diagrams for the photon-photon scattering to obtain the parton model predictions for g_1^γ and g_2^γ. We then extract a piece which is a deviation from the Wandzura-Wilczek relation [27]. In Sec. III we show in the framework of the pure QED operator product expansion that the extracted piece actually arises from the twist-3 effects. In Sec. IV, we examine the QCD twist-3 effects in g_2^γ in LO. We calculate the nontrivial lowest moment ($n=3$) of the twist-3 contribution to g_2^γ. For large N_c, the QCD analysis of the twist-3 effects in the flavor nonsinglet part of g_2^γ becomes simple and we can obtain its moments in a compact form for all n. The final section is devoted to the conclusion and discussion.

II. $g_2^\gamma(x,Q^2,P^2)$ AND BOX-DIAGRAM CALCULATION

Let us consider the virtual photon-photon forward scattering for $\gamma(q)+\gamma(p)\to\gamma(q)+\gamma(p)$ illustrated in Fig. 2:

$$T_{\mu\nu\rho\tau}(p,q)=i\int d^4x\, d^4y\, d^4z\, e^{iq\cdot x}e^{ip\cdot(y-z)} \times\langle 0|T(J_\mu(x)J_\nu(0)J_\rho(y)J_\tau(z))|0\rangle, \tag{2.1}$$

where J is the electromagnetic current, and q and p are the four-momenta of the probe and target photons, respectively. Its absorptive part is related to the structure tensor $W_{\mu\nu\rho\tau}(p,q)$ for the photon with mass squared $p^2=-P^2$ probed by the photon with $q^2=-Q^2$:

$$W_{\mu\nu\rho\tau}(p,q)=\frac{1}{\pi}\,\mathrm{Im}\,T_{\mu\nu\rho\tau}(p,q). \tag{2.2}$$

The antisymmetric part $W^A_{\mu\nu\rho\tau}$ under the interchange $\mu\leftrightarrow\nu$ and $\rho\leftrightarrow\tau$ can be written in terms of the two spin-dependent structure functions g_1^γ and g_2^γ as [24]

$$W^A_{\mu\nu\rho\tau}=\frac{1}{(p\cdot q)^2}[(I_-)_{\mu\nu\rho\tau}g_1^\gamma-(J_-)_{\mu\nu\rho\tau}g_2^\gamma], \tag{2.3}$$

where the two tensors I_- and J_- are explicitly given by

$$(I_-)_{\mu\nu\rho\tau}\equiv p\cdot q\,\epsilon_{\mu\nu\lambda\sigma}\epsilon_{\rho\tau}{}^{\sigma\beta}q^\lambda p_\beta, \tag{2.4}$$

$$(J_-)_{\mu\nu\rho\tau}\equiv \epsilon_{\mu\nu\lambda\sigma}\epsilon_{\rho\tau\alpha\beta}q^\lambda p^\sigma q^\alpha p^\beta - p\cdot q\,\epsilon_{\mu\nu\lambda\sigma}\epsilon_{\rho\tau}{}^{\sigma\beta}q^\lambda p_\beta. \tag{2.5}$$

In fact, we observe that I_- and J_- are related to those of eight independent kinematic-singularity-free tensors, introduced by Brown and Muzinich [see Eqs. (A3)–(A10) of Ref. [8]] to express the virtual photon-photon forward scattering amplitude, as follows: $I_-=I_2-I_3$ and $J_-=I_7-I_8$.

It may be useful here to see the relations between the structure functions g_i^γ ($i=1,2$) and s-channel helicity amplitudes, which are defined as

$$W(ab|a'b')=\epsilon^*_\mu(a)\epsilon^*_\rho(b)W^{\mu\nu\rho\tau}\epsilon_\nu(a')\epsilon_\tau(b'), \tag{2.6}$$

where $\epsilon_\mu(a)$ represents the photon polarization vector with helicity a, and $a,a'=0,\pm1$, and $b,b'=0,\pm1$. They are related as follows:

$$g_1^\gamma=\frac{1}{2X}\Bigg[\{W(11|11)-W(1-1|1-1)\} - \frac{(p^2q^2)^{1/2}}{p\cdot q}\{W(11|00)+W(01|-10)\}\Bigg],$$

$$g_2^\gamma=\frac{-1}{2X}\Bigg[\{W(11|11)-W(1-1|1-1)\} - \frac{p\cdot q}{(p^2q^2)^{1/2}}\{W(11|00)+W(01|-10)\}\Bigg], \tag{2.7}$$

where $X=(p\cdot q)^2-p^2q^2$.

The photon structure functions g_1^γ and g_2^γ are just the analogues of the nucleon counterparts g_1^{nucl} and g_2^{nucl}, respectively. But it is noted that g_2^γ exists only for the off-shell or virtual photon ($P^2\neq0$) target.

Now let us calculate g_1^γ and g_2^γ in the simple parton model by evaluating the box diagrams depicted in Fig. 3. We introduce two projectors $(P_I)^{\mu\nu\rho\tau}$ and $(P_J)^{\mu\nu\rho\tau}$:

$$(P_I)^{\mu\nu\rho\tau}=\frac{1}{4X^2}\Bigg[\left(1+\frac{p^2q^2}{2(p\cdot q)^2}\right)(I_-)^{\mu\nu\rho\tau}+\frac{3}{2}(J_-)^{\mu\nu\rho\tau}\Bigg], \tag{2.8}$$

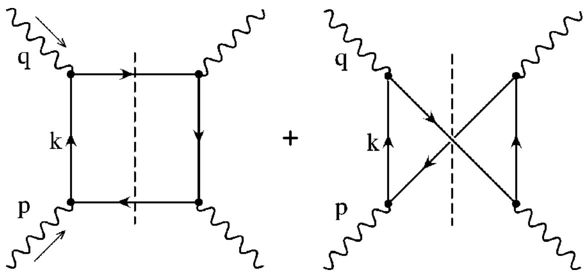

FIG. 3. The box diagrams contributing to g_1^γ and g_2^γ in the pure QED interaction.

$$(P_J)^{\mu\nu\rho\tau}=\frac{1}{4X^2}\left[\frac{3}{2}(I_-)^{\mu\nu\rho\tau}+\left(1+\frac{(p\cdot q)^2}{2p^2q^2}\right)(J_-)^{\mu\nu\rho\tau}\right], \tag{2.9}$$

which satisfy

$$P_I\cdot(I_-)=1,\quad P_I\cdot(J_-)=0,$$
$$P_J\cdot(I_-)=0,\quad P_J\cdot(J_-)=1. \tag{2.10}$$

Then g_1^γ and g_2^γ are given by

$$g_1^\gamma=(p\cdot q)^2(P_I)^{\mu\nu\rho\tau}W^A_{\mu\nu\rho\tau},$$
$$g_2^\gamma=-(p\cdot q)^2(P_J)^{\mu\nu\rho\tau}W^A_{\mu\nu\rho\tau}. \tag{2.11}$$

Applying the projectors P_I and P_J to the box-diagram (massless quark-loop) contributions, and ignoring the power corrections of P^2/Q^2, we obtain

$$g_1^{\gamma(\mathrm{box})}(x,Q^2,P^2)=\frac{3\alpha}{\pi}N_f\langle e^4\rangle\left[(2x-1)\ln\frac{Q^2}{P^2}-2(2x-1)(\ln x+1)\right], \tag{2.12}$$

$$g_2^{\gamma(\mathrm{box})}(x,Q^2,P^2)=\frac{3\alpha}{\pi}N_f\langle e^4\rangle\left[-(2x-1)\ln\frac{Q^2}{P^2}+2(2x-1)\ln x+6x-4\right], \tag{2.13}$$

where $x=Q^2/(2p\cdot q)$, $\alpha=e^2/4\pi$, the QED coupling constant, and $\langle e^4\rangle=\Sigma_{i=1}^{N_f}e_i^4/N_f$, with N_f being the number of active quark flavors. The above results are consistent with those of Ref. [28], where polarized gluon structure functions were considered. It is noted that the first moment of $g_2^{\gamma(\mathrm{box})}$ vanishes, i.e., $g_2^{\gamma(\mathrm{box})}$ satisfies the Burkhardt-Cottingham (BC) sum rule [29]:

$$\int_0^1 dx\, g_2^{\gamma(\mathrm{box})}(x,Q^2,P^2)=0. \tag{2.14}$$

We will see from the OPE analysis in the next section that the BC sum rule for g_2^γ generally holds in the deep-inelastic region $Q^2\gg P^2$. Also note that the sum $g_1^{\gamma(\mathrm{box})}+g_2^{\gamma(\mathrm{box})}$ does not have $\ln Q^2/P^2$ behavior.

In the case of the nucleon, the spin-dependent structure function g_2^{nucl} receives both twist-2 and twist-3 contributions,

$$g_2^{\mathrm{nucl}}(x,Q^2)=g_2^{\mathrm{nucl,tw\text{-}2}}(x,Q^2)+g_2^{\mathrm{nucl,tw\text{-}3}}(x,Q^2), \tag{2.15}$$

and the twist-2 part of g_2^{nucl} is expressed in terms of g_1^{nucl} by the so-called Wandzura-Wilczek (WW) relation [27]:

$$g_2^{\mathrm{nucl,tw\text{-}2}}(x,Q^2)=g_2^{\mathrm{nucl,WW}}(x,Q^2)\equiv-g_1^{\mathrm{nucl}}(x,Q^2)+\int_x^1\frac{dy}{y}g_1^{\mathrm{nucl}}(y,Q^2). \tag{2.16}$$

Thus the difference $\overline{g}_2^{\mathrm{nucl}}=g_2^{\mathrm{nucl}}-g_2^{\mathrm{nucl,WW}}$ contains the twist-3 contribution only. The experimental data so far obtained show that the twist-3 contributions to g_2^{nucl} appear to be negligibly small [25,26].

Now we may ask about the photon structure function g_2^γ: Does g_2^γ also receive twist-3 contributions? If so, are they small as in the nucleon case, or sizable? Does the WW relation also hold for g_2^γ, in other words, is the twist-2 part of g_2^γ expressible in terms of g_1^γ? These issues will be discussed in the next section.

Here let us apply the WW relation to the results of the box-diagram calculation $g_1^{\gamma(\mathrm{box})}$ and $g_2^{\gamma(\mathrm{box})}$ in Eqs. (2.12), (2.13), and define

$$g_2^{\gamma\mathrm{WW,(box)}}(x,Q^2,P^2)\equiv-g_1^{\gamma(\mathrm{box})}(x,Q^2,P^2)+\int_x^1\frac{dy}{y}g_1^{\gamma(\mathrm{box})}(y,Q^2,P^2). \tag{2.17}$$

Then we find that the difference $\overline{g}_2^{\gamma(\mathrm{box})}=g_2^{\gamma(\mathrm{box})}-g_2^{\gamma WW,(\mathrm{box})}$ is given by

$$\overline{g}_2^{\gamma(\mathrm{box})}=\frac{3\alpha}{\pi}N_f\langle e^4\rangle\left[(2x-2-\ln x)\ln\frac{Q^2}{P^2}-2(2x-1)\ln x+2(x-1)+\ln^2 x\right]. \tag{2.18}$$

Its nth moment $\overline{g}_{2,n}^{\gamma(\mathrm{box})}=\int_0^1 dx\, x^{n-1}\overline{g}_2^{\gamma(\mathrm{box})}(x,Q^2,P^2)$ is

$$\overline{g}_{2,n}^{\gamma(\mathrm{box})}=\frac{3\alpha}{\pi}N_f\langle e^4\rangle\frac{n-1}{n}\times\left[-\frac{1}{n(n+1)}\ln\frac{Q^2}{P^2}+\frac{2}{(n+1)^2}-\frac{2}{n^2}\right]. \tag{2.19}$$

In Fig. 4, we have shown the box-diagram contributions to the structure functions $g_1^{\gamma(\mathrm{box})}$, $g_2^{\gamma(\mathrm{box})}$ as well as the $\overline{g}_2^{\gamma(\mathrm{box})}$ given in Eq. (2.18) as functions of x for $Q^2=30\ \mathrm{GeV}^2$ and $P^2=1\ \mathrm{GeV}^2$. We can see that $\overline{g}_2^{\gamma(\mathrm{box})}$ is comparable in magnitude with $g_2^{\gamma(\mathrm{box})}$ for large regions of x. Now it is expected by analogy with the nucleon case that $\overline{g}_2^{\gamma(\mathrm{box})}$ arises from the

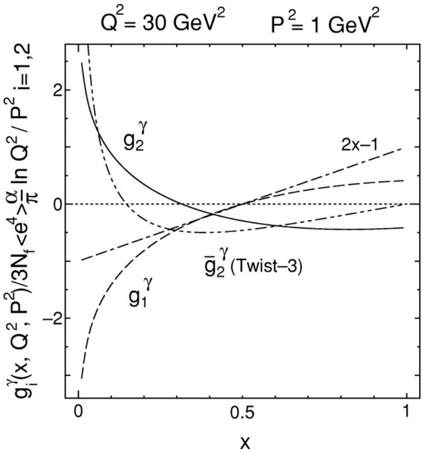

FIG. 4. The box-diagram contributions to $g_1^\gamma(x,Q^2,P^2)$ (dashed line), $g_2^\gamma(x,Q^2,P^2)$ (solid line), and $\overline{g_2^\gamma}(x,Q^2,P^2)$ (dash-double-dotted line) for $Q^2=30\ \mathrm{GeV}^2$ and $P^2=1\ \mathrm{GeV}^2$ for $N_f=3$. The $2x-1$ line shows the leading logarithmic term of g_1^γ.

twist-3 effects. In the next section we will be convinced that $\overline{g_2^{\gamma(\mathrm{box})}}$ is indeed the twist-3 contribution.

III. OPE ANALYSIS AND PURE QED EFFECTS

Applying the OPE for the product of two electromagnetic currents, we get for the μ-ν antisymmetric part

$$i\int d^4x\, e^{iq\cdot x} T(J_\mu(x)J_\nu(0))^A = -i\epsilon_{\mu\nu\lambda\sigma}q^\lambda \sum_{n=1,3,\ldots}\left(\frac{2}{Q^2}\right)^n q_{\mu_1}\cdots q_{\mu_{n-1}} \times\left\{\sum_i E^n_{(2)i}R^{\sigma\mu_1\cdots\mu_{n-1}}_{(2)i}+\sum_i E^n_{(3)i}R^{\sigma\mu_1\cdots\mu_{n-1}}_{(3)i}\right\}, \tag{3.1}$$

where $R^n_{(2)i}$ and $R^n_{(3)i}$ are the twist-2 and twist-3 operators, respectively, and $E^n_{(2)i}$ and $E^n_{(3)i}$ are corresponding coefficient functions. The twist-2 operators $R^n_{(2)i}$ have totally symmetric Lorentz indices $\sigma\mu_1\cdots\mu_{n-1}$, while the indices of twist-3 operators $R^n_{(3)i}$ are totally symmetric among $\mu_1\cdots\mu_{n-1}$ but antisymmetric under $\sigma\leftrightarrow\mu_i$. Thus the "matrix elements" of operators $R^n_{(2)i}$ and $R^n_{(3)i}$ sandwiched by two photon states with momentum p have the following forms:

$$\langle 0|T(A_\rho(-p)R^{\sigma\mu_1\cdots\mu_{n-1}}_{(2)i}A_\tau(p))|0\rangle_{\mathrm{amp}} = -ia^n_{(2)i}\epsilon_{\rho\tau\alpha}{}^{\{\sigma}p^{\mu_1}\cdots p^{\mu_{n-1}\}}p^\alpha-(\mathrm{traces}), \tag{3.2}$$

$$\langle 0|T(A_\rho(-p)R^{\sigma\mu_1\cdots\mu_{n-1}}_{(3)i}A_\tau(p))|0\rangle_{\mathrm{amp}} = -ia^n_{(3)i}\epsilon_{\rho\tau\alpha}{}^{[\sigma,}p^{\{\mu_1]}\cdots p^{\mu_{n-1}\}}p^\alpha-(\mathrm{traces}), \tag{3.3}$$

where subscript "amp" stands for the amputation of the external photon lines and

$$\epsilon_{\rho\tau\alpha}{}^{\{\sigma}p^{\mu_1}\cdots p^{\mu_{n-1}\}} = \frac{1}{n}\left[\epsilon^\sigma_{\rho\tau\alpha}p^{\mu_1}\cdots p^{\mu_{n-1}} + \sum_{j=1}^{n-1}\epsilon^{\mu_j}_{\rho\tau\alpha}p^{\mu_1}\cdots p^\sigma\cdots p^{\mu_{n-1}}\right], \tag{3.4}$$

$$\epsilon_{\rho\tau\alpha}{}^{[\sigma,}p^{\{\mu_1]}\cdots p^{\mu_{n-1}\}} = \frac{n-1}{n}\epsilon^\sigma_{\rho\tau\alpha}p^{\mu_1}\cdots p^{\mu_{n-1}} - \frac{1}{n}\sum_{j=1}^{n-1}\epsilon^{\mu_j}_{\rho\tau\alpha}p^{\mu_1}\cdots p^\sigma\cdots p^{\mu_{n-1}}. \tag{3.5}$$

Using Eqs. (3.2)–(3.5), we can write down the moment sum rules for g_1^γ and g_2^γ as

$$\int_0^1 dx\, x^{n-1}g_1^\gamma(x,Q^2,P^2)=\sum_i a^n_{(2)i}E^n_{(2)i}(Q^2), \tag{3.6}$$

$$\int_0^1 dx\, x^{n-1}g_2^\gamma(x,Q^2,P^2)=\frac{n-1}{n}\left[-\sum_i a^n_{(2)i}E^n_{(2)i}(Q^2) + \sum_i a^n_{(3)i}E^n_{(3)i}(Q^2)\right], \tag{3.7}$$

From this general OPE analysis we conclude the following.

(i) The BC sum rule [29] holds for g_2^γ:

$$\int_0^1 dx\, g_2^\gamma(x,Q^2,P^2)=0. \tag{3.8}$$

(ii) The twist-2 contribution to g_2^γ is expressed by the WW relation

$$-\frac{n-1}{n}\sum_i a^n_{(2)i}E^n_{(2)i}(Q^2)=\int_0^1 dx\, x^{n-1}g_2^{\gamma\mathrm{WW}}(x,Q^2,P^2), \tag{3.9}$$

with

$$g_2^{\gamma\mathrm{WW}}(x,Q^2,P^2)\equiv -g_1^\gamma(x,Q^2,P^2)+\int_x^1\frac{dy}{y}g_1^\gamma(y,Q^2,P^2). \tag{3.10}$$

(iii) The difference $\overline{g_2^\gamma}=g_2^\gamma-g_2^{\gamma\mathrm{WW}}$ contains only the twist-3 contribution

$$\int_0^1 dx\, x^{n-1}\overline{g_2^\gamma}(x,Q^2,P^2)=\frac{n-1}{n}\left[\sum_i a^n_{(3)i}E^n_{(3)i}(Q^2)\right]. \tag{3.11}$$

Let us now analyze the twist-3 part of g_2^γ in pure QED, i.e., switching off the quark-gluon coupling, in the framework of OPE and the renormalization group (RG) method. In

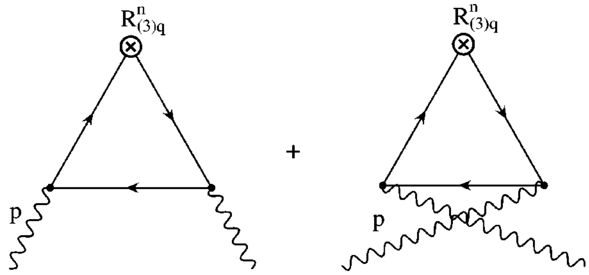

FIG. 5. Triangle diagrams that contribute to the anomalous dimension describing the mixing between the twist-3 quark-bilinear operator $R^n_{(3)q}$ and the photonic operator $R^n_{(3)\gamma}$.

this case the relevant twist-3 operators are the quark and photon operators, which are given, respectively, by

$$R^{\sigma\mu_1\cdots\mu_{n-1}}_{(3)q}=i^{n-1}e_q^2\overline{\psi}\gamma_5\gamma^{[\sigma,}D^{\{\mu_1]}\cdots D^{\mu_{n-1}\}}\psi-(\text{traces}), \tag{3.12}$$

$$R^{\sigma\mu_1\cdots\mu_{n-1}}_{(3)\gamma}=\frac{1}{4}i^{n-1}\epsilon_{\alpha\beta\gamma}{}^{[\sigma,}F^{\alpha\{\mu_1]}\partial^{\mu_2}\cdots\partial^{\mu_{n-1}\}}F^{\beta\gamma}-(\text{traces}), \tag{3.13}$$

where e_q is the quark charge, $D_\mu=\partial_\mu+ieA_\mu$ is the covariant derivative and $\{\}$ means complete symmetrization over the indices, while $[\sigma,\mu_j]$ denotes antisymmetrization on $\sigma\mu_j$. With the above photon operator $R^n_{(3)\gamma}$, we have $a^n_{(3)\gamma}=1$. The coefficient functions corresponding to operators $R^n_{(3)q}$ and $R^n_{(3)\gamma}$

$$\vec{E}^n_{(3)}\left(\frac{Q^2}{\mu^2},\alpha\right)=\begin{pmatrix} E^n_{(3)q}\left(\frac{Q^2}{\mu^2},\alpha\right) \\ E^n_{(3)\gamma}\left(\frac{Q^2}{\mu^2},\alpha\right)\end{pmatrix} \tag{3.14}$$

satisfy the following RG equation to lowest order in α:

$$\mu\frac{\partial}{\partial\mu}\vec{E}^n_{(3)}\left(\frac{Q^2}{\mu^2},\alpha\right)=\gamma_n^{\text{QED}}(\alpha)\vec{E}^n_{(3)}\left(\frac{Q^2}{\mu^2},\alpha\right), \tag{3.15}$$

where $\gamma_n^{\text{QED}}(\alpha)$ is the anomalous-dimension matrix. To lowest order in α, this matrix has the form[1]

$$\gamma_n^{\text{QED}}(\alpha)=\begin{pmatrix} 0 & 0 \\ -\frac{\alpha}{4\pi}K^n_{(3)q} & 0\end{pmatrix}. \tag{3.16}$$

Here $K^n_{(3)q}$ represents the mixing between the photon operator $R^n_{(3)\gamma}$ and the quark operator $R^n_{(3)q}$. Evaluating the triangular diagrams given in Fig. 5 and taking into account the color degrees of freedom in the quark loop, we find

$$K^n_{(3)q}=-24e_q^4\frac{1}{n(n+1)}. \tag{3.17}$$

The solution of Eq. (3.15) is

$$\vec{E}^n_{(3)}\left(\frac{Q^2}{\mu^2},\alpha\right)=\exp\left[-\frac{1}{2}\gamma_n^{\text{QED}}(\alpha)\ln\frac{Q^2}{\mu^2}\right]\vec{E}^n_{(3)}(1,\alpha). \tag{3.18}$$

To lowest order in α, the exponential and the coefficient functions $\vec{E}^n_{(3)}(1,\alpha)$ are written, respectively, as

$$\exp\left[-\frac{1}{2}\gamma_n^{\text{QED}}(\alpha)\ln\frac{Q^2}{\mu^2}\right]=\begin{pmatrix} 1 & 0 \\ \frac{\alpha}{8\pi}K^n_{(3)q}\ln\frac{Q^2}{\mu^2} & 1\end{pmatrix}, \tag{3.19}$$

$$E^n_{(3)q}(1,\alpha)=1+\mathcal{O}(\alpha),$$

$$E_{(3)\gamma}(1,\alpha)=\frac{\alpha}{4\pi}3e_q^4B^n_{(3)\gamma}. \tag{3.20}$$

The "matrix element" $a^n_{(3)q}$ of the quark operator $R^n_{(3)q}$ between the photon states is obtained by evaluating again the triangular diagrams in Fig. 5 and expressed as

$$a^n_{(3)q}=\frac{\alpha}{4\pi}\left(-\frac{1}{2}K^n_{(3)q}\ln\frac{P^2}{\mu^2}+3e_q^4A^n_{(3)q}\right). \tag{3.21}$$

Inserting Eqs. (3.17)–(3.21) into Eq. (3.11) and remembering $a^n_{(3)\gamma}=1$, we obtain for the nth moment of $\overline{g}_2^\gamma$ in pure QED,

$$\overline{g}_2^{\gamma,n}|_{\text{QED}}=\frac{n-1}{n}\frac{\alpha}{4\pi}3e_q^4\left\{-\frac{4}{n(n+1)}\ln\frac{Q^2}{P^2}+A^n_{(3)q}+B^n_{(3)\gamma}\right\}. \tag{3.22}$$

The dependence on the renormalization point μ disappears. And we note that, although $A^n_{(3)q}$ and $B^n_{(3)\gamma}$ are individually renormalization-scheme dependent, the sum $A^n_{(3)q}+B^n_{(3)\gamma}$ is not [30]. The calculation of box diagrams in Fig. 3 gives

$$A^n_{(3)q}+B^n_{(3)\gamma}=8\left\{\frac{1}{(n+1)^2}-\frac{1}{n^2}\right\}. \tag{3.23}$$

Now, adding all the quark contributions of active flavors and replacing $3e_q^4$ in Eq. (3.22) by $3N_f\langle e^4\rangle$, we find that the result is nothing but $\overline{g}_{2,n}^{\gamma(\text{box})}$ given in Eq. (2.19) which is derived from the box-diagram calculation. Thus it is now clear that $\overline{g}_{2,n}^{\gamma(\text{box})}$ is indeed the twist-3 contribution.

IV. QCD EFFECTS

We now switch on the quark-gluon coupling and consider the QCD, effects on $\overline{g}_2^\gamma$, the twist-3 part of g_2^γ. In the case of the nucleon, the analysis of $\overline{g}_2^{\text{nucl}}$, the twist-3 part of the structure function g_2^{nucl}, turns out to be very complicated

[1]We follow the convention used by Bardeen and Buras [14] to write the mixing anomalous dimensions between the photon and other operators.

[31–43]. This is due to the fact that the number of participating twist-3 operators grows with the spin (moment of $\overline{g}_2^{\rm nucl}$) and that these operators mix among themselves through renormalization. Therefore, the Q^2 evolution equation for the moments of $\overline{g}_2^{\rm nucl}$ cannot be written in a simple form, but as a sum of terms, the number of which increases with spin. The same is true for $\overline{g}_2^{\gamma}$.

Writing down the coefficient functions $E^n_{(3)i}(Q^2/\mu^2,g^2,\alpha)$ that correspond to the relevant twist-3 operators $R^n_{(3)i}$ contributing to $\overline{g}_2^{\gamma}$ in a column vector $\vec{E}^n_{(3)}(Q^2/\mu^2,g^2,\alpha)$, the RG equation for $\vec{E}^n_{(3)}$ can be written to lowest order in α as [14]

$$\left(\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}\right)\vec{E}^n_{(3)}\left(\frac{Q^2}{\mu^2},g^2,\alpha\right)=\gamma_n(g^2,\alpha)\vec{E}^n_{(3)}\left(\frac{Q^2}{\mu^2},g^2,\alpha\right), \tag{4.1}$$

where $\beta(g)$ is the QCD β function and γ_n is the anomalous-dimension matrix. The solution is given by

$$\vec{E}^n_{(3)}\left(\frac{Q^2}{\mu^2},g^2,\alpha\right)=\left(T\exp\left[\int_{\bar{g}(Q^2)}^{g}dg'\frac{\gamma_n(g'^2,\alpha)}{\beta(g')}\right]\right)\times\vec{E}^n_{(3)}(1,\bar{g}^2,\alpha). \tag{4.2}$$

The twist-3 photon operator $R^n_{(3)\gamma}$ is again given by Eq. (3.13). In the convention we use now, where the photon coefficient function $E^n_{(3)\gamma}$ is set at the bottom of the column vector $\vec{E}^n_{(3)}$, the matrix γ_n to lowest order in α has the form

$$\gamma_n=\begin{pmatrix}\hat{\gamma}_n(g^2) & 0\\ \vec{K}_n(g^2,\alpha) & 0\end{pmatrix}, \tag{4.3}$$

where $\hat{\gamma}_n$ represents the mixing among hadronic (quark and gluon) operators and the row vector $\vec{K}_n$ describes the mixing between the photon operator $R^n_{(3)\gamma}$ and other hadronic operators. Then the evolution factor is given by [14]

$$T\exp\left[\int_{\bar{g}}^{g}dg'\frac{\gamma_n(g'^2)}{\beta(g')}\right]=\begin{pmatrix}M_n & 0\\ \vec{X}_n & 1\end{pmatrix}, \tag{4.4}$$

with

$$M_n=T\exp\int_{\bar{g}}^{g}dg'\frac{\hat{\gamma}_n(g'^2)}{\beta(g')},$$

$$\vec{X}_n=\int_{\bar{g}}^{g}dg'\frac{\vec{K}_n(g'^2,\alpha)}{\beta(g')}T\exp\left[\int_{\bar{g}}^{g'}dg''\frac{\hat{\gamma}_n(g''^2)}{\beta(g'')}\right]. \tag{4.5}$$

Expanding $\vec{K}_n(g^2,\alpha)$, $\beta(g)$, and $\hat{\gamma}_n(g^2)$ in powers of g,

$$\vec{K}_n(g^2,\alpha)=-\frac{\alpha}{4\pi}\vec{K}_n^{(0)}+\mathcal{O}(\alpha g^2), \tag{4.6}$$

$$\beta(g)=-\frac{g^3}{16\pi^2}\beta_0+\mathcal{O}(g^5),$$

$$\beta_0=\frac{11}{3}N_c-\frac{2}{3}N_f, \tag{4.7}$$

$$\hat{\gamma}_n(g^2)=\frac{g^2}{16\pi^2}\hat{\gamma}_n^{(0)}+\mathcal{O}(g^4), \tag{4.8}$$

we see that the dominant contributions, which behave as $\ln Q^2$, are coming from $\vec{X}_n$, in other words, from the photon coefficient function $E^n_{(3)\gamma}(Q^2/\mu^2,g^2,\alpha)$. Inserting the solution $E^n_{(3)\gamma}$ and $a^n_{(3)\gamma}=1$ into Eq. (3.11), we obtain the following formula for the nth moment of $\overline{g}_2^{\gamma}$ in LO:

$$\int_0^1 dx\,x^{n-1}\overline{g}_2^{\gamma}(x,Q^2,P^2)=\frac{n-1}{n}\frac{2\pi\alpha}{\beta_0}[K_n^{(0)}]_i\left[\int_{\bar{g}^2}^{g^2}\frac{dg'^2}{(g'^2)^2}\times\exp\left(\frac{\hat{\gamma}_n^{(0)}}{2\beta_0}\ln\frac{\bar{g}^2}{g'^2}\right)\right]_{ij}[E^n_{(3)}(1,0)]_j, \tag{4.9}$$

where i and j run over the hadronic (quark and gluon) sector only.

The evaluation of the nth moment of $\overline{g}_2^{\gamma}$ is feasible when n is a small number. But as n gets larger, it becomes a more and more difficult task due to the increase of the number of participating operators and the mixing among these operators. However, we will see that in a certain limit the analysis of the moments becomes tractable. In the following subsections we consider two cases: (1) the nontrivial lowest moment ($n=3$) of $\overline{g}_2^{\gamma}$; and (2) the flavor nonsinglet part of $\overline{g}_2^{\gamma}$ for large N_c. In case (1) the number of participating operators is limited, and we can get all the information on the necessary anomalous dimensions. Thus we obtain the LO QCD prediction for the third moment of $\overline{g}_2^{\gamma}$. In the QCD analysis of photon structure functions, the contributions are divided into two parts, the flavor singlet and nonsinglet parts. In case (2), we show that in the approximation of neglecting terms of order $\mathcal{O}(1/N_c^2)$, we can evade the problem of operator mixing for $\overline{g}_2^{\gamma({\rm NS})}$, the flavor nonsinglet part of $\overline{g}_2^{\gamma}$, and obtain the moments of $\overline{g}_2^{\gamma({\rm NS})}$ in a compact form for all n.

A. The third ($n=3$) moment of $\overline{g}_2^{\gamma}$

Let us start with the analysis of the flavor nonsinglet part. In addition to the photon operator $R^n_{(3)\gamma}$ given by Eq. (3.13), the following four types of twist-3 operator contribute to $\overline{g}_2^{\gamma({\rm NS})}$:

$$R^{\sigma\mu_1\cdots\mu_{n-1}}_{(3)F}=i^{n-1}S'\bar{\psi}\gamma_5\gamma^{\sigma}D^{\mu_1}\cdots D^{\mu_{n-1}}Q^{\rm ch}\psi-(\text{traces}), \tag{4.10}$$

$$R_{(3)l}^{\sigma\mu_1\cdots\mu_{n-1}}=\frac{1}{2n}\{(V_l-V_{n-1-l}+U_l+U_{n-1-l})+(\widetilde{V}_l-\widetilde{V}_{n-1-l}+\widetilde{U}_l+\widetilde{U}_{n-1-l})\}\quad (l=1,...,n-2), \tag{4.11}$$

$$R_{(3)m}^{\sigma\mu_1\cdots\mu_{n-1}}=i^{n-2}mS'\overline{\psi}\gamma_5\gamma^\sigma D^{\mu_1}\cdots D^{\mu_{n-2}}\times\gamma^{\mu_{n-1}}Q^{\rm ch}\psi-(\text{traces}), \tag{4.12}$$

$$R_{(3)E}^{\sigma\mu_1\cdots\mu_{n-1}}=i^{n-2}\frac{n-1}{2n}S'[\overline{\psi}\gamma_5\gamma^\sigma D^{\mu_1}\cdots D^{\mu_{n-2}}\times\gamma^{\mu_{n-1}}(i\not{D}-m)Q^{\rm ch}\psi+\overline{\psi}(i\not{D}-m)\gamma_5\gamma^\sigma D^{\mu_1}\cdots D^{\mu_{n-2}}\times\gamma^{\mu_{n-1}}Q^{\rm ch}\psi]-(\text{traces}), \tag{4.13}$$

with

$$Q^{\rm ch}=Q^2-\langle e^2\rangle\mathbf{1}, \tag{4.14}$$

where Q is the $N_f\times N_f$ quark-charge matrix, $\langle e^2\rangle=\Sigma_{i=1}^{N_f}e_i^2/N_f$, $\mathbf{1}$ is an $N_f\times N_f$ unit matrix with N_f being the number of active flavors, and m represents the quark mass. The symbol S' denotes symmetrization on the indices $\mu_1\mu_2\cdots\mu_{n-1}$ and antisymmetrization on $\sigma\mu_i$. The operators in Eq. (4.11) contain the gluon and photon field strength $G_{\mu\nu}$ and $F_{\mu\nu}$, and their dual tensors $\widetilde{G}_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^{\alpha\beta}$ and $\widetilde{F}_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$. Explicitly, they are given by

$$V_l=+i^n gS'\overline{\psi}\gamma_5 D^{\mu_1}\cdots G^{\sigma\mu_1}\cdots D^{\mu_{n-2}}\times\gamma^{\mu_{n-1}}Q^{\rm ch}\psi-(\text{traces}), \tag{4.15}$$

$$U_l=-i^{n-1}gS'\overline{\psi}D^{\mu_1}\cdots\widetilde{G}^{\sigma\mu_l}\cdots D^{\mu_{n-2}}\times\gamma^{\mu_{n-1}}Q^{\rm ch}\psi-(\text{traces}), \tag{4.16}$$

$$\widetilde{V}_l=-i^n eS'\overline{\psi}\gamma_5 D^{\mu_1}\cdots F^{\sigma\mu_l}\cdots D^{\mu_{n-2}}\times\gamma^{\mu_{n-1}}Q^{\rm ch}\psi-(\text{traces}), \tag{4.17}$$

$$\widetilde{U}_l=+i^{n-1}eS'\overline{\psi}D^{\mu_1}\cdots\widetilde{F}^{\sigma\mu_l}\cdots D^{\mu_{n-2}}\times\gamma^{\mu_{n-1}}Q^{\rm ch}\psi-(\text{traces}), \tag{4.18}$$

where g and e are the QCD and QED coupling constants, respectively. The operator R_E^n in Eq. (4.13) is proportional to the equation of motion (EOM) operator [39,40].

We emphasize that $\widetilde{V}_l$ and $\widetilde{U}_l$ (regardless of the quark-charge factor $Q^{\rm ch}$), which are not present in the twist-3 contribution to the nucleon structure function $g_2^{\rm nucl}$, must be included in the analysis of $\overline{g_2^\gamma}$. The reason is that we are here considering not only QCD but also QED, and thus the covariant derivative D_μ should read as

$$D_\mu=\partial_\mu-igA_\mu^aT^a+ieA_\mu, \tag{4.19}$$

where A_μ^a and A_μ are the gluon and photon fields, respectively, and T^a is the color matrix. Then the commutator

$$[D_\mu,D_\nu]=-igG_{\mu\nu}^aT^a+ieF_{\mu\nu} \tag{4.20}$$

leads to the appearance of V_l, U_l, $\widetilde{V}_l$, and $\widetilde{U}_l$ terms. As far as the mixing anomalous dimensions among the hadronic operators, i.e., those given in Eqs. (4.10)–(4.13), are concerned, the terms $\widetilde{V}_l$ and $\widetilde{U}_l$ are irrelevant. But they are indispensable to the correct evaluation of the mixing anomalous dimensions $K_{n,l}$ between the hadronic operator $R_{(3)l}^n$ and photon operator $R_{(3)\gamma}^n$. We need to have $K_{n,l}$ of order $\mathcal{O}(\alpha)$ in the leading logarithm approximation, but $R_{(3)l}^n$ without $\widetilde{V}_l$ and $\widetilde{U}_l$ terms gives $K_{n,l}\sim\mathcal{O}(g^2\alpha)$, since the V_l and U_l terms already have the QCD coupling constant g. In Appendix B we calculate the mixing anomalous dimensions $K_{n,l}^{(0)}$ of order $\mathcal{O}(\alpha)$ for arbitrary n and show that the $\widetilde{U}_l$ term (but not $\widetilde{V}_l$) indeed plays an essential role. Another important consequence of introducing ieA_μ into the covariant derivative D_μ is that with this new term we can show that the photon matrix element of the EOM operator R_E^n [more precisely, $\langle 0|T(A_\rho(-p)R_{(3)E}^{\sigma\mu_1\cdots\mu_{n-1}}A_\tau(p))|0\rangle_{\rm amp}$] actually vanishes at $\mathcal{O}(\alpha)$.

The twist-3 hadronic operators given in Eqs. (4.10)–(4.13) satisfy the following relation [32,37,39,40]:

$$R_{(3)F}^{\sigma\mu_1\cdots\mu_{n-1}}=\frac{n-1}{n}R_{(3)m}^{\mu_1\cdots\mu_{n-1}}+\sum_{l=1}^{n-2}(n-1-l)R_{(3)l}^{\sigma\mu_1\cdots\mu_{n-1}}+R_{(3)}^{\sigma\mu_1\cdots\mu_{n-1}}. \tag{4.21}$$

Hence, including the photon operator, there are, in total, $n+1$ independent operators that contribute to the nth moment of $\overline{g_2^{\gamma({\rm NS})}}$. We have freedom in choosing the hadronic operators as independent bases. But we should keep in mind the following: due to the constraint Eq. (4.21), a different choice of operator bases assigns different values to the coefficient functions at the tree level, which was first pointed out by Kodaira, Yasui, and one of the authors [39]. In the basis of independent operators that includes $R_{(3)F}^n$ but not $R_{(3)m}^n$, the tree level coefficient functions are given by

$$E_{(3)F}^n(\text{tree})=1,\quad E_{(3)l}^n(\text{tree})=0. \tag{4.22}$$

On the other hand, if we eliminate $R_{(3)F}^n$, we have

$$E_{(3)m}^n(\text{tree})=\frac{n-1}{n},\quad E_{(3)l}^n(\text{tree})=n-1-l. \tag{4.23}$$

We always have $E_{(3)E}^n(\text{tree})=0$. So a different choice of the operator bases leads to different forms for the anomalous-dimension matrix and the coefficient functions but the final result for the nth moment of $\overline{g_2^\gamma}$ should be the same (see Appendix C).

Now we take $n=3$ and evaluate the third moment of $\overline{g_2^{\gamma({\rm NS})}}$. From now on we omit the superscripts $n=3$. The relevant hadronic operators are four: $R_{(3)F}$, $R_{(3)1}$, $R_{(3)m}$, and $R_{(3)E}$. Let us take $R_{(3)1}$, $R_{(3)m}$, and $R_{(3)E}$ as independent operators. In these operator bases, the tree level coefficient functions are given by

$$E_{(3)m}(1,0)=\tfrac{2}{3}, \quad E_{(3)1}(1,0)=1. \tag{4.24}$$

The 3×3 anomalous-dimension matrix $\hat{\gamma}^{(0)}$ for hadronic operators has the form

$$\hat{\gamma}^{(0)}=\begin{pmatrix} \hat{\gamma}_{11}^{(0)} & 0 & 0 \\ \hat{\gamma}_{m1}^{(0)} & \hat{\gamma}_{mm}^{(0)} & 0 \\ \hat{\gamma}_{E1}^{(0)} & 0 & \hat{\gamma}_{EE}^{(0)} \end{pmatrix}, \tag{4.25}$$

with [40]

$$\hat{\gamma}_{11}^{(0)}=6C_G-\tfrac{2}{3}C_F, \quad \hat{\gamma}_{mm}^{(0)}=12C_F,$$
$$\hat{\gamma}_{m1}^{(0)}=-\tfrac{4}{9}C_F, \quad \hat{\gamma}_{E1}^{(0)}=-\tfrac{1}{3}C_F. \tag{4.26}$$

Note that we follow the convention of Bardeen and Buras [14] in defining the anomalous-dimension matrix. The matrix $\hat{\gamma}^{(0)}$ is triangular and, therefore, its eigenvalues are $\hat{\gamma}_{11}^{(0)}$, $\hat{\gamma}_{mm}^{(0)}$, and $\hat{\gamma}_{EE}^{(0)}$. In fact we only need the information on the upper left 2×2 submatrix for the analysis, which is decomposed as

$$\tilde{\gamma}^{(0)}|_{(2\times 2)}=\hat{\gamma}_{11}^{(0)}P_1+\hat{\gamma}_{mm}^{(0)}P_2, \tag{4.27}$$

where P_1 and P_2 are projection operators and given by

$$P_1=\begin{pmatrix} 1 & 0 \\ a & 0 \end{pmatrix}, \quad P_2=\begin{pmatrix} 0 & 0 \\ -a & 1 \end{pmatrix}, \tag{4.28}$$

with

$$a=\frac{\hat{\gamma}_{m1}^{(0)}}{\hat{\gamma}_{11}^{(0)}-\hat{\gamma}_{mm}^{(0)}}=\frac{-2C_F}{3(9C_G-19C_F)}. \tag{4.29}$$

The anomalous dimension $K_{n,m}^{(0)}$ is found to be null for all n. So we have $K_m^{(0)}=0$.

Inserting this information into the moment formula for $\overline{g_2^{\gamma}}$ in Eq. (4.9), we find for the third moment of the nonsinglet part $\overline{g_2^{\gamma(\mathrm{NS})}}$

$$\begin{aligned}\overline{g_{2,n=3}^{\gamma(\mathrm{NS})}}&=\int_0^1 dx\, x^2 \overline{g_2^{\gamma(\mathrm{NS})}}(x,Q^2,P^2)\\ &=\frac{2}{3}\frac{\alpha}{4\pi}\frac{2\pi}{\beta_0\alpha_s(Q^2)}K_1^{(0)\mathrm{NS}}\frac{1}{1+\hat{\gamma}_{11}^{(0)\mathrm{NS}}/2\beta_0}\\ &\quad\times\left\{1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{\hat{\gamma}_{11}^{(0)\mathrm{NS}}/2\beta_0+1}\right\},\end{aligned} \tag{4.30}$$

where we have revived the superscript "NS" and $K_1^{(0)\mathrm{NS}}$ is obtained from Eq. (B8) in Appendix B as

$$K_1^{(0)\mathrm{NS}}=-24N_f(\langle e^4\rangle-\langle e^2\rangle^2)\frac{1}{3\times 4}. \tag{4.31}$$

The above result for $\overline{g_{2,n=3}^{\gamma(\mathrm{NS})}}$ is indifferent to the choice of the independent set of operators. In Appendix C we take $R_{(3)F}$, $R_{(3)1}$, $R_{(3)E}$ as independent operators, replacing $R_{(3)m}$ with $R_{(3)F}$, and show that we obtain the same $\overline{g_{2,n=3}^{\gamma(\mathrm{NS})}}$.

Let us move to the third moment of the singlet part $\overline{g_2^{\gamma(\mathrm{S})}}$. For $n=3$, there are five independent hadronic operators contributing to $\overline{g_2^{\gamma(\mathrm{S})}}$, apart from the photon operator $R_{(3)\gamma}^n$. Following the work of Kodaira *et al.* [44], we take $R_{(3)1}^{\mathrm{S}}$, $R_{(3)m}^{\mathrm{S}}$, $R_{(3)E}^{\mathrm{S}}$, $T_{(3)B}$, and $T_{(3)E}$ for an independent set. The first three are the analogues to $R_{(3)1}$, $R_{(3)m}$, $R_{(3)E}$ in the nonsinglet case, obtained by replacing the quark charge factor Q^{ch} with an $N_f\times N_f$ unit matrix **1**. The rest are the Becchi-Rouet-Stora-Tyutin (BRST) exact and the gluon EOM operators, respectively, whose explicit expressions are given in Ref. [44].

The tree level coefficient functions corresponding to these operators are

$$E_{(3)1}^{\mathrm{S}}(1,0)=1, \quad E_{(3)m}^{\mathrm{S}}(1,0)=\tfrac{2}{3}, \tag{4.32}$$

and the others are zero. The mixing anomalous dimensions among these operators have been calculated and form a 5×5 matrix. The physically relevant part is the following 2×2 submatrix:

$$\hat{\gamma}^{(0)\mathrm{S}}=\begin{pmatrix} \hat{\gamma}_{11}^{(0)\mathrm{S}} & \hat{\gamma}_{1m}^{(0)\mathrm{S}} \\ \hat{\gamma}_{m1}^{(0)\mathrm{S}} & \hat{\gamma}_{mm}^{(0)\mathrm{S}} \end{pmatrix}, \tag{4.33}$$

with [32,33,44]

$$\hat{\gamma}_{11}^{(0)\mathrm{S}}=6C_G-\tfrac{2}{3}C_F+\tfrac{4}{3}N_f, \quad \hat{\gamma}_{1m}^{(0)\mathrm{S}}=0,$$
$$\hat{\gamma}_{m1}^{(0)\mathrm{S}}=-\tfrac{4}{9}C_F, \quad \hat{\gamma}_{mm}^{(0)\mathrm{S}}=12C_F. \tag{4.34}$$

The matrix $\hat{\gamma}^{(0)\mathrm{S}}$ is triangular and, therefore, the same procedures as the nonsinglet case can be applied here. We obtain for the third moment of the singlet part $\overline{g_2^{\gamma(\mathrm{S})}}$

$$\begin{aligned}\overline{g_{2,n=3}^{\gamma(\mathrm{S})}}&=\int_0^1 dx\, x^2 \overline{g_2^{\gamma(\mathrm{S})}}(x,Q^2,P^2)\\ &=\frac{2}{3}\frac{\alpha}{4\pi}\frac{2\pi}{\beta_0\alpha_s(Q^2)}K_1^{(0)\mathrm{S}}\frac{\langle e^2\rangle}{1+\hat{\gamma}_{11}^{(0)\mathrm{S}}/2\beta_0}\\ &\quad\times\left\{1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{\hat{\gamma}_{11}^{(0)\mathrm{S}}/2\beta_0+1}\right\},\end{aligned} \tag{4.35}$$

with

$$K_1^{(0)\mathrm{S}}=-24N_f\langle e^2\rangle\frac{1}{3\times 4}. \tag{4.36}$$

In Fig. 6, we have plotted the Q^2 evolution of the flavor singlet and nonsinglet components as well as the total of the third moment of $\overline{g_2^{\gamma}}(x,Q^2,P^2)$ in units of α/π for $P^2=1\ \mathrm{GeV}^2$ with $N_f=3$. The flavor singlet and nonsinglet

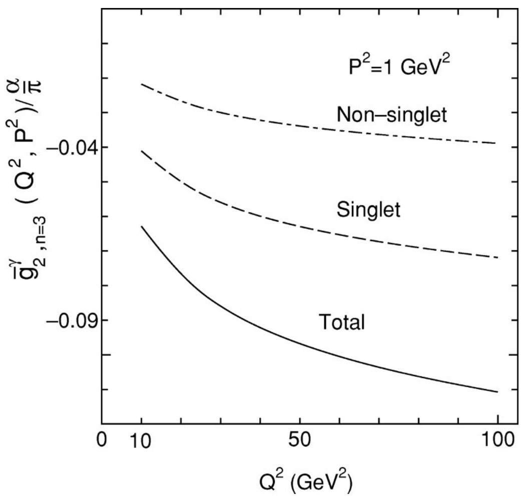

FIG. 6. The third $(n=3)$ moment of $\overline{g}_2^{\gamma}(x,Q^2,P^2)$ in units of α/π as a function of Q^2 for $P^2=1\ \mathrm{GeV}^2$ with $N_f=3$. The dash-dotted (dashed) line corresponds to the flavor nonsinglet (singlet) component. The solid line represents the sum of the two components, the total $\overline{g}_{2,n=3}^{\gamma}/(\alpha/\pi)$.

components show somewhat different types of Q^2 dependence, and the singlet component gives a larger contribution due to the charge factor.

B. Flavor nonsinglet part of $\overline{g}_2^{\gamma}$ for large N_c

For the case of the flavor nonsinglet nucleon structure function $g_2^{\mathrm{nucl(NS)}}$, it has been observed by Ali, Braun, and Hiller (ABH) [45] that in the large N_c limit the twist-3 part $\overline{g}_2^{\mathrm{nucl(NS)}}$ obeys a simple Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation [46]. In their formalism of working directly with the nonlocal operator contributing to the twist-3 part of $\overline{g}_2^{\mathrm{nucl(NS)}}$, they showed that local operators involving gluons effectively decouple from the evolution equation for large N_c, which is the number of colors. Later, the ABH result for $\overline{g}_2^{\mathrm{nucl(NS)}}$ was reproduced by one of the authors [47] in the framework of the standard OPE and RG method. At large N_c, the operators involving gluon field strength $G_{\mu\nu}$ decouple from the evolution equation of $\overline{g}_2^{\mathrm{nucl(NS)}}$, and the whole contribution in LO is represented by one type of operator. The same is true for the flavor nonsinglet part of $\overline{g}_2^{\gamma}$.

Let us take $R_{(3)F}^n$, $R_{(3)l}^n$, $R_{(3)E}^n$ as independent hadronic operators, eliminating $R_{(3)m}^n$. The advantage of this choice of operator basis is that from Eq. (4.22) the hadronic coefficient functions take simple forms at the tree level [39]:

$$E_{(3)F}^n(1,0)=1,\quad E_{(3)l}^n(1,0)=0 \quad \text{for}\ \ l=1,\ldots,n-2. \tag{4.37}$$

The mixing anomalous dimensions for these operators are very complicated. However, it was found in Ref. [47] that, in the approximation of neglecting terms of order $\mathcal{O}(1/N_c^2)$ and thus putting $2C_F=C_G$, the $(F,\ F)$ and $(l,\ F)$ elements are reduced to the simple expressions

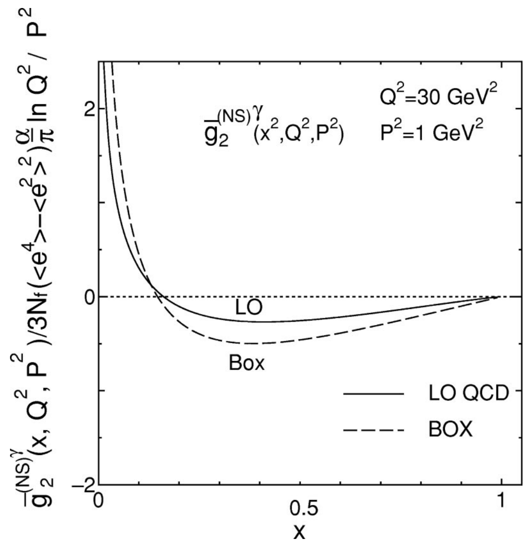

FIG. 7. The box-diagram (dashed line) and the QCD LO (solid line) contributions for large N_c to the flavor nonsinglet photon structure function $\overline{g}_2^{\gamma(\mathrm{NS})}(x,Q^2,P^2)$ for $Q^2=30\ \mathrm{GeV}^2$ and $P^2=1\ \mathrm{GeV}^2$ for $N_f=3$.

$$\hat{\gamma}_{n,FF}^{(0)}=8C_F\left(S_n-\frac{1}{4}-\frac{1}{2n}\right)\quad \text{with}\ \ S_n=\sum_{j=1}^{n}\frac{1}{j}, \tag{4.38}$$

$$\hat{\gamma}_{n,lF}^{(0)}=0\quad \text{for}\ \ l=1,\ldots,n-2. \tag{4.39}$$

Note that the corrections are of $\mathcal{O}(1/N_c^2)$, about 10% for QCD $(N_c=3)$.

Inserting the above results (4.37)–(4.39) into Eq. (4.9), we find that, for large N_c, the nth moment of $\overline{g}_2^{\gamma(\mathrm{NS})}$ is given by

$$\int_0^1 dx\, x^{n-1}\overline{g}_2^{\gamma(\mathrm{NS})}(x,Q^2,P^2) = \frac{n-1}{n}\frac{\alpha}{4\pi}\frac{2\pi}{\beta_0\alpha_s(Q^2)}K_{n,F}^{(0)}\frac{1}{1+\hat{\gamma}_{n,FF}^{(0)}/2\beta_0} \times\left\{1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{\hat{\gamma}_{n,FF}^{(0)}/2\beta_0+1}\right\}, \tag{4.40}$$

with

$$K_{n,F}^{(0)}=-24N_f(\langle e^4\rangle-\langle e^2\rangle^2)\frac{1}{n(n+1)}. \tag{4.41}$$

We now perform the Mellin transform of Eq. (4.40) to get $\overline{g}_2^{\gamma(\mathrm{NS})}(x,Q^2,P^2)$ as a function of x. The result is plotted in Fig. 7. Compared with the pure QED box-graph contribution, we find that the LO QCD effects are sizable and tend to suppress the structure function $\overline{g}_2^{\gamma(\mathrm{NS})}$ in both the large x and small x regions, so that the vanishing $n=1$ moment of $\overline{g}_2^{\gamma(\mathrm{NS})}$, i.e., the BC sum rule, is preserved.

As for $\overline{g}_2^{\gamma(\mathrm{S})}$, the flavor singlet part of $\overline{g}_2^{\gamma}$, it is expected that a similar simplification may occur for large N_c and its moments may be written in a compact form for all n as in the case of $\overline{g_2^{\gamma(\mathrm{NS})}}$. At the moment we do not know how to solve the mixing problem in the flavor singlet sector to get an analytically simple formula for the moments of $\overline{g_2^{\gamma(\mathrm{S})}}$ for large N_c. This is an interesting subject which should be pursued.

V. CONCLUSION

In the OPE of two electromagnetic currents, we expect the presence of the twist-3 operators in addition to the usual twist-2 operators. From a study of the lepton-nucleon polarized deep inelastic scattering, we have learned that the twist-3 contribution does not show up as a sizable effect, since the nucleon matrix elements of the twist-3 operators are found to be small in experiments.

In this paper, we investigated the twist-3 effects in g_2^γ for a virtual photon target, in the pure QED interaction as well as in LO QCD. We found that the twist-3 contribution is appreciable for the photon case in contrast to the nucleon case. In this sense, the virtual photon structure function g_2^γ provides us with a good testing ground for studying the twist-3 effects. We expect that the future polarized versions of the *ep* or e^+e^- colliders may bring us important information on the polarized photon structure. More thorough QCD analysis including the flavor-singlet part is now under way.

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APPENDIX A: TWO-PHOTON PROCESS $e^+e^-\to e^+e^-$+HADRONS

The information on the polarized structure functions g_1^γ and g_2^γ can be extracted from experiments on the two-photon annhilation process with polarized e^+e^- beams as shown in Fig. 1:

$$e^{\pm}(l_1)e^{\mp}(l_2)\to e^{\pm}(l_1')e^{\mp}(l_2')\gamma(q)\gamma(p)$$
$$\to e^{\pm}(l_1')e^{\mp}(l_2')+\text{hadrons}, \tag{A1}$$

with the virtual photon momenta $q=l_1-l_1'$ and $p=l_2-l_2'$. The cross section for this process is written as [48]

$$d\sigma^P=\frac{(4\pi\alpha)^2}{p^2q^2}\rho_{1(\mathrm{pol})}^{\mu\nu}\rho_{2(\mathrm{pol})}^{\rho\tau}M^*_{\mu\rho}M_{\nu\tau}$$
$$\times\frac{(2\pi)^4\delta(p+q-P_X)d\Gamma}{4[(l_1\cdot l_2)^2-m^4]^{1/2}}$$
$$\times\frac{d^3l_1'd^3l_2'}{2E_1'2E_2'(2\pi)^6}, \tag{A2}$$

where m is the electron mass, $E_{1,2}'$ are scattered electron (positron) energies, $P_X=\Sigma_i p_i$, and $d\Gamma=\Pi_i d^3p_i/2p_{i0}(2\pi)^3$ are the total momentum and the phase-space volume, respectively, of the hadron system produced, and $M_{\nu\tau}$ is the transition amplitude for $\gamma\gamma\to$hadrons. For polarized e^+e^- beams, the photon density matrices $\rho_{1(\mathrm{pol})}^{\mu\nu}$ and $\rho_{2(\mathrm{pol})}^{\rho\tau}$ are given by

$$\rho_{1(\mathrm{pol})}^{\mu\nu}=\frac{1}{-q^2}\mathrm{Tr}\left[\frac{\gamma_5\not{s}_1}{2}(\not{l}_1+m)\gamma^\mu(\not{l}_1'+m)\gamma^\nu\right]$$
$$=2im\epsilon^{\mu\nu\alpha\beta}s_{1\alpha}q_\beta/(-q^2), \tag{A3}$$

$$\rho_{2(\mathrm{pol})}^{\rho\tau}=2im\epsilon^{\rho\tau\alpha\beta}s_{2\alpha}p_\beta/(-p^2), \tag{A4}$$

where $s_{1,2}$ are the initial e^+ (e^-) polarization vectors. When the incident beams are longitudinally polarized and at high energies, the polarization vectors are expressed as

$$s_i^\mu=h_i\frac{l_i^\mu}{m}\quad(i=1,2), \tag{A5}$$

with $h_i=\pm1$ representing the helicity states of the beams.

The absorptive part of the $\gamma\gamma$ forward scattering amplitude $W_{\mu\nu\rho\tau}$ in Eq. (2.2) is related to the following integrated quantity over the phase-space volume of the produced hadron system:[2]

$$(4\pi\alpha)W_{\mu\nu\rho\tau}(p,q)=\frac{1}{2\pi}\int M^*_{\mu\rho}M_{\nu\tau}(2\pi)^4$$
$$\times\delta(p+q-P_X)d\Gamma. \tag{A6}$$

Applying $\rho_{1(\mathrm{pol})}^{\mu\nu}$ and $\rho_{2(\mathrm{pol})}^{\rho\tau}$ to $W_{\mu\nu\rho\tau}$ [actually to $W^A_{\mu\nu\rho\tau}$ in Eq. (2.3)], we find for the longitudinally polarized beams

$$\rho_{1(\mathrm{pol})}^{\mu\nu}\rho_{2(\mathrm{pol})}^{\rho\tau}W_{\mu\nu\rho\tau}=4h_1h_2\left[\left\{\frac{4l_1\cdot l_2}{p\cdot q}+1-\frac{2}{y}-\frac{2}{r}\right\}g_1^\gamma\right.$$
$$\left.+4\left\{\frac{l_1\cdot l_2}{p\cdot q}-\frac{1}{yr}\right\}g_2^\gamma\right], \tag{A7}$$

where we have introduced the variables

$$y\equiv\frac{p\cdot q}{l_1\cdot p},\quad r\equiv\frac{p\cdot q}{l_2\cdot q}. \tag{A8}$$

Hence the difference between the cross sections for the two-photon annihilation process with e^+e^- beams polarized parallel and antiparallel to each other is given by

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[2]Our definition of $W_{\mu\nu\rho\tau}$ and, therefore, g_1^γ and g_2^γ, is such that they are proportional to $e^2(=4\pi\alpha)$, and not to e^4 in conformity to the nucleon case.

$$d\sigma^{\uparrow\uparrow}-d\sigma^{\uparrow\downarrow}=\frac{d^3l_1'd^3l_2'}{E_1'E_2'}\frac{\alpha^3}{\pi^2(l_1\cdot l_2)p^2q^2}\times\left[\left\{\frac{4l_1\cdot l_2}{p\cdot q}+1-\frac{2}{y}-\frac{2}{r}\right\}g_1^\gamma+4\left\{\frac{l_1\cdot l_2}{p\cdot q}-\frac{1}{yr}\right\}g_2^\gamma\right]. \quad (A9)$$

In particular for colliding beams, the laboratory is considered to be the c.m. reference frame. We have

$$l_1=(E,0,0,E), \quad l_2=(E,0,0,-E),$$

$$l_1'=(E_1',E_1'\sin\theta_1\cos\phi_1,E_1'\sin\theta_1\sin\phi_1,E_1'\cos\theta_1), \quad (A10)$$

$$l_2'=(E_2',E_2'\sin\theta_2\cos\phi_2,E_2'\sin\theta_2\sin\phi_2,-E_2'\cos\theta_2),$$

where θ_1, ϕ_1, and $\pi-\theta_2$, ϕ_2 are the polar and azimuthal angles for the final leptons l_1' and l_2', respectively. Then we obtain

$$d\sigma^{\uparrow\uparrow}-d\sigma^{\uparrow\downarrow}=\frac{E_1'E_2'dE_1'dE_2'd\cos\theta_1 d\cos\theta_2 d\phi}{\pi E^2}\times\frac{\alpha^3}{p^2q^2(p\cdot q)}\Big[\{(E+E_1')(E+E_2')+(E+E_1'\cos\theta_1)(E+E_2'\cos\theta_2)-E_1'E_2'\sin\theta_1\sin\theta_2\cos\phi\}g_1^\gamma+\frac{4}{p\cdot q}E^2E_1'E_2'\{(1-\cos\theta_1)(1-\cos\theta_2)-2\sin\theta_1\sin\theta_2\cos\phi\}g_2^\gamma\Big], \quad (A11)$$

where $\phi=\phi_1-\phi_2$, and

$$q^2=-2EE_1'(1-\cos\theta_1),$$

$$p^2=-2EE_2'(1-\cos\theta_2), \quad (A12)$$

$$p\cdot q=(E-E_1')(E-E_2')+(E-E_1'\cos\theta_1)(E-E_2'\cos\theta_2)-E_1'E_2'\sin\theta_1\sin\theta_2\cos\phi.$$

APPENDIX B: CALCULATION OF $K_{n,l}^{(0)}$

In this appendix we present details of the calculation of $K_{n,l}^{(0)}$, the mixing anomalous dimension between the hadronic and photon operators $R_{(3)l}^n$ and $R_{(3)\gamma}^n$, for arbitrary n. The expressions for the operators $R_{(3)l}^n$ and $R_{(3)\gamma}^n$ are given in Eqs. (4.11) and (3.13), respectively (we put aside the charge factor $Q^{\rm ch}$). As a standard procedure, we introduce a lightlike vector Δ_μ ($\Delta^2=0$) to symmetrize the Lorentz indices and to eliminate the trace terms, and we define

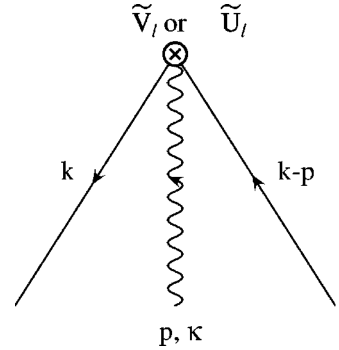

FIG. 8. The tree-level three-point vertices of $\widetilde{V}_l$ and $\widetilde{U}_l$.

$$R_{(3)l}^n\cdot\Delta\equiv R_{(3)l}^{\sigma\mu_1\cdots\mu_{n-1}}\Delta_{\mu_1}\cdots\Delta_{\mu_{n-1}},$$

$$R_{(3)\gamma}^n\cdot\Delta\equiv R_{(3)\gamma}^{\sigma\mu_1\cdots\mu_{n-1}}\Delta_{\mu_1}\cdots\Delta_{\mu_{n-1}}. \quad (B1)$$

Our first task is to evaluate the amputated two-point function with $R_{(3)\gamma}^n\cdot\Delta$ embedded between two photon fields at $\mathcal{O}(1)$. We find

$$\langle\gamma|R_{(3)\gamma}^n\cdot\Delta|\gamma\rangle\equiv\langle 0|T(A_\rho(-p)R_{(3)\gamma}^n\cdot\Delta A_\tau(p))|0\rangle_{\rm amp}=i\frac{n-1}{n}\{\epsilon_{\rho\tau}^{\sigma\beta}p_\beta(p\cdot\Delta)^{n-1}-\epsilon_{\rho\tau}^{\alpha\beta}\Delta_\alpha p_\beta p^\sigma(p\cdot\Delta)^{n-2}\}. \quad (B2)$$

Next we calculate the one-loop diagram for the two-point function with $R_{(3)l}^n\cdot\Delta$ sandwiched by two photon fields,

$$\langle\gamma|R_{(3)l}^n\cdot\Delta|\gamma\rangle\equiv\langle 0|T(A_\rho(-p)R_{(3)l}^n\cdot\Delta A_\tau(p))|0\rangle_{\rm amp}, \quad (B3)$$

which should be $\mathcal{O}(\alpha)$. The operator $R_{(3)l}^n$ is made up of four terms, V_l, U_l, $\widetilde{V}_l$, and $\widetilde{U}_l$, whose expressions are given in Eqs. (4.15)–(4.18). Since the V_l and U_l terms already have the QCD coupling constant g, their contributions are $\mathcal{O}(g^2\alpha)$. So we work with the $\widetilde{V}_l$ and $\widetilde{U}_l$ terms. The three-point "basic" vertices of $\widetilde{V}_l\cdot\Delta$ and $\widetilde{U}_l\cdot\Delta$ depicted in Fig. 8 are given, respectively, by

$$\widetilde{\mathcal{V}}_{l,k}=e\gamma_5(k\cdot\Delta)^{l-1}[k^\sigma\Delta_\kappa-(k\cdot\Delta)g^\sigma_\kappa]\times[(k-p)\cdot\Delta]^{n-2-l}\not{\Delta}, \quad (B4)$$

$$\widetilde{\mathcal{U}}_{l,\kappa}=ie(k\cdot\Delta)^{l-1}\epsilon_\kappa^{\sigma\alpha\beta}\Delta_\alpha k_\beta\times[(k-p)\cdot\Delta]^{n-2-l}\not{\Delta}. \quad (B5)$$

The $\mathcal{O}(\alpha)$ contributions come from the two diagrams shown in Fig. 9. Inspecting the form of $\widetilde{\mathcal{V}}_{l,\kappa}$ we easily see that the loop diagrams for $\widetilde{V}_l\cdot\Delta$ fail to produce a term that is proportional to $\langle\gamma|R_{(3)\gamma}^n\cdot\Delta|\gamma\rangle$. In fact, after the loop integral we find that both the $\widetilde{V}_l$ and $\widetilde{V}_{n-1-l}$ terms give a null result.

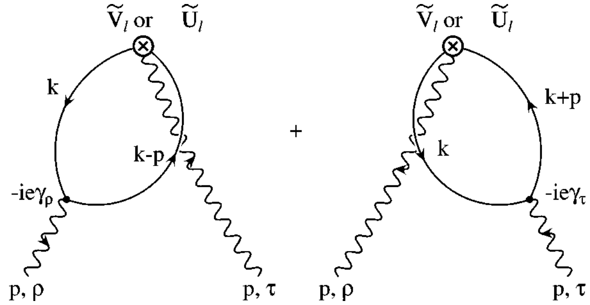

FIG. 9. One-loop diagrams at $\mathcal{O}(\alpha)$ contributing to the Green's functions of $\widetilde{V}_l$ and $\widetilde{U}_l$ with two photons as external lines.

On the other hand, the one-loop diagrams for $\widetilde{U}_l$ and $\widetilde{U}_{n-1-l}$ give terms proportional to $\langle\gamma|R^n_{(3)\gamma}\cdot\Delta|\gamma\rangle$. The logarithmically divergent part of the $\widetilde{U}_l$ contribution, for example, has the following form:

$$\frac{1}{2n}\langle\gamma|\widetilde{U}_l\cdot\Delta|\gamma\rangle=\frac{\alpha}{4\pi}\frac{12}{n-1}(-1)^{n-l}B(l+2,n-l)\times\langle\gamma|R^n_{(3)\gamma}\cdot\Delta|\gamma\rangle\ln\Lambda^2, \tag{B6}$$

where the Beta function $B(l+2,n-l)$ has appeared from the Feynman-parameter integral

$$\int_0^1 dx\, x^{l+1}(x-1)^{n-1-l}=(-1)^{n-1-l}B(l+2,n-l). \tag{B7}$$

Hence, adding together the $\widetilde{U}_{n-1-l}$ contribution, we find the mixing anomalous dimension between the $R^n_{(3)l}$ and $R^n_{(3)\gamma}$ operators (apart from the quark-charge factor) as

$$K^{(0)}_{n,l}=-\frac{24}{n-1}[(-1)^{n-l}B(l+2,n-l)+(-1)^{l+1}B(n+1-l,l+1)]. \tag{B8}$$

In particular, for $n=3$ and so $l=1$, we have $K^{(0)}_{n=3,1}=-24[1/(3\times 4)]$ except for the quark-charge factor.

Now it is interesting to note the following identity:

$$\sum_{l=1}^{n-2}(n-1-l)\times\frac{1}{n-1}[(-1)^{n-l}B(l+2,n-l)+(-1)^{l+1}B(n+1-l,l+1)]=\frac{1}{n}-\frac{1}{n+1}\quad(\text{for odd integer } n), \tag{B9}$$

which is a direct consequence of the relation (4.21) satisfied by the twist-3 operators. We know that $K^{(0)}_{n,m}=0$ from the null result for the mixing anomalous dimension between the mass operators $R^n_{(3)m}$ and $R^n_{(3)\gamma}$. Also we know that $K^{(0)}_{n,E}=0$ since the photon matrix element of the EOM operator $R^n_{(3)E}$ vanishes. Thus we have the relation

$$K^{(0)}_{n,F}=\sum_{l=1}^{n-2}(n-1-l)K^{(0)}_{n,l}. \tag{B10}$$

The identity (B9) ensures that the relation indeed holds true. For $n=3$ we have $K^{(0)}_{n=3,F}=K^{(0)}_{n=3,1}$.

APPENDIX C: REANALYSIS OF $\overline{g}^{\gamma(\mathrm{NS})}_{2,n=3}$

In this appendix we reanalyze $\overline{g}^{\gamma(NS)}_{2,n=3}$, the third moment of $\overline{g}^{\gamma(\mathrm{NS})}_2$. (The superscripts $n=3$ and "NS" are omitted.) We choose $R_{(3)F}$, $R_{(3)1}$, $R_{(3)E}$ as independent operators, replacing $R_{(3)m}$ with $R_{(3)F}$. In these operator bases, the tree level coefficient functions are given by

$$E_{(3)F}(1,0)=1,\quad E_{(3)1}(1,0)=0. \tag{C1}$$

The relevant 2×2 anomalous-dimension matrix $\widetilde{\gamma}^{(0)}$ has the form

$$\widetilde{\gamma}^{(0)}=\begin{pmatrix}\widetilde{\gamma}^{(0)}_{FF} & \widetilde{\gamma}^{(0)}_{F1}\\ \widetilde{\gamma}^{(0)}_{1F} & \widetilde{\gamma}^{(0)}_{11}\end{pmatrix}, \tag{C2}$$

with

$$\widetilde{\gamma}^{(0)}_{FF}=\tfrac{34}{3}C_F,\quad \widetilde{\gamma}^{(0)}_{F1}=-\tfrac{2}{3}C_F,\quad \widetilde{\gamma}^{(0)}_{1F}=6C_G-12C_F,\quad \widetilde{\gamma}^{(0)}_{11}=6C_G, \tag{C3}$$

which are obtained from Eqs. (33)–(38) of Ref. [47]. (Note that we follow the convention of Bardeen and Buras in defining the anomalous-dimension matrix which is the *transposed* one as given in Ref. [47].) Then the eigenvalues of $\widetilde{\gamma}^{(0)}$ and the corresponding projection operators, such that $\widetilde{\gamma}^{(0)}=\lambda_1P_1+\lambda_2P_2$, are found to be

$$\lambda_1=12C_F,\quad \lambda_2=6C_G-\frac{2}{3}C_F, \tag{C4}$$

$$P_1=\frac{1}{b+1}\begin{pmatrix}1 & -b\\ -1 & b\end{pmatrix},\quad P_2=\frac{1}{b+1}\begin{pmatrix}b & b\\ 1 & 1\end{pmatrix}, \tag{C5}$$

with $b=C_F/[9(2C_F-C_G)]$. Comparing the results in Eq. (4.26), we note that $\lambda_1=\hat{\gamma}^{(0)\mathrm{NS}}_{mm}$ and $\lambda_2=\hat{\gamma}^{(0)\mathrm{NS}}_{11}$. This is a consequence of the fact that a different choice of the operator bases leads to different forms for the anomalous-dimension matrix but its eigenvalues remain the same. Now inserting these $E_{(3)i}(1,0)$, λ_i, and P_i for $i=F,1$ into the moment formula for $\overline{g}^{\gamma}_2$ in Eq. (4.9), we find that

$$\overline{g}_{2,n=3}^{\gamma(\mathrm{NS})} \propto \frac{K_F^{(0)}}{\overline{g}^2(b+1)} \left[\frac{1}{1+\lambda_1/2\beta_0} \left\{ 1 - \left(\frac{\overline{g}^2}{g^2} \right)^{\lambda_1/2\beta_0+1} \right\} + \frac{b}{1+\lambda_2/2\beta_0} \left\{ 1 - \left(\frac{\overline{g}^2}{g^2} \right)^{\lambda_2/2\beta_0+1} \right\} \right] + \frac{K_1^{(0)}}{\overline{g}^2(b+1)} \left[\frac{-1}{1+\lambda_1/2\beta_0} \left\{ 1 - \left(\frac{\overline{g}^2}{g^2} \right)^{\lambda_1/2\beta_0+1} \right\} + \frac{1}{1+\lambda_2/2\beta_0} \left\{ 1 - \left(\frac{\overline{g}^2}{g^2} \right)^{\lambda_2/2\beta_0+1} \right\} \right]. \tag{C6}$$

From Appendix B, we observe that $K_F^{(0)} = K_1^{(0)}$ for $n=3$. Thus $\overline{g}_{2,n=3}^{\gamma(NS)}$ is reduced to

$$\overline{g}_{2,n=3}^{\gamma(\mathrm{NS})} \propto \frac{1}{\overline{g}^2} \frac{K_1^{(0)}}{1+\lambda_2/2\beta_0} \left\{ 1 - \left(\frac{\overline{g}^2}{g^2} \right)^{\lambda_2/2\beta_0+1} \right\}, \tag{C7}$$

which coincides with the expression for $\overline{g}_{2,n=3}^{\gamma(NS)}$ given in Eq. (4.30) since $\lambda_2 = \hat{\gamma}_{11}^{(0)\mathrm{NS}}$.

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