

Collinear effective theory at subleading order and its application to heavy-light currents

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We consider a collinear effective theory of highly energetic quarks with energy E , interacting with collinear and soft gluons by integrating out collinear degrees of freedom to subleading order. The collinear effective theory offers a systematic expansion in power series of a small parameter $\lambda \sim p_{\perp}/E$, where p_{\perp} is the transverse momentum of a collinear particle. We construct the effective Lagrangian to first order in λ and discuss its features, including additional symmetries such as collinear gauge invariance and reparametrization invariance. Heavy-light currents can be matched from the full theory onto the operators in the collinear effective theory at one loop and to order λ . We obtain heavy-light current operators in the effective theory, calculate their Wilson coefficients at this order, and the renormalization group equations for the Wilson coefficients are solved. As an application, we calculate the form factors for decays of B mesons to light energetic mesons to order λ and at leading-logarithmic order in α_s .

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I. INTRODUCTION

When a B meson decays into light mesons we can explore different kinematic regions, depending on the momenta carried by the light mesons. When a light meson is emitted from a heavy quark with momentum of order Λ_{QCD} , this decay can be successfully described by the heavy quark effective theory (HQET) [1]. The momentum of a heavy quark can be decomposed as $p_b = m_b v + k$, where k is the residual momentum of order Λ_{QCD} . The leading contribution to the decay corresponds to the partonic result, and the corrections can be systematically expanded in power series of $1/m_b$ and α_s . Inclusive decays of heavy mesons with large momentum transfer can be treated in the HQET with the operator product expansion [2]. Exclusive decays with heavy-heavy currents and heavy-light currents can also be treated in the context of the HQET [3,4].

If a light meson from B decays carries a large energy, HQET alone is no longer useful since the large energy of a light meson can be as large as m_b . Then an expansion in $1/m_b$ alone is not appropriate. In this case, however, we can construct a different type of an effective theory by taking the energy E of the energetic light quark to infinity. In this limit, nonperturbative effects can also be systematically obtained. In fact, this effective theory is more complicated than the HQET and the naive power counting in $1/E$ should be modified since the system involves several energy scales.

Another complication arises in decays of a heavy quark with an energetic light quark due to a Sudakov logarithm since there are both collinear and infrared divergences [5]. There has been some discussion of summing Sudakov logarithms using effective field theories [6–9]. Such an approach has an advantage over conventional methods since effective theories are valid beyond perturbation theory, and it is straightforward to go beyond the leading approximation by including higher-dimensional operators. The main advantage of using effective theories in this case is that we can repro-

duce the Sudakov logarithm easily without dividing all the kinematic regions [10], and the calculation is manifest in the calculational procedure. However, we need an effective theory in which logarithms arising at one loop in the effective theory should match logarithms arising at one loop in QCD for any matching scale μ in the minimal subtraction scheme. Only in this case, these logarithms may be summed using the renormalization group equations. The large-energy effective theory suggested by Dugan and Grinstein [11] does not satisfy this criterion since it does not include the effects of collinear gluons properly.

Recently Bauer et al. [12] have proposed a new effective theory called the “collinear-soft effective theory.” If a light quark moves with a large energy, the momentum has three distinct scales. The momentum component in the light cone direction n^{μ} is the largest, of the order of the energy of the quark, E . The transverse momentum is smaller than E , and the momentum component opposite to the light cone direction is the smallest. In order to disentangle the three scales conveniently, a small parameter λ is introduced. The largest component has the momentum of order E . The transverse component is of order $E\lambda$, and the smallest component is of order $E\lambda^2$.

Between E and $E\lambda$, we have collinear modes and soft modes for the light quark. Here we integrate out all the collinear modes above some scale μ , and the result is the effective theory consisting of collinear quarks and soft quarks. The effective theory at this stage is called the collinear-soft effective theory, which we will call the “collinear effective theory” for brevity. Below the scale $E\lambda$ and above $E\lambda^2$, we integrate out all the collinear modes, and there remain only soft modes in the final soft effective theory. This actually corresponds to the large-energy effective theory suggested by Dugan and Grinstein [11], in which there are only soft modes. In Ref. [12] they show that at each stage of the effective theories, the infrared behavior of the full theory is correctly reproduced by including the effects of collinear gluons. Therefore heavy-light currents in the full theory finally can be matched onto operators in the effective theories, their Wilson coefficients are calculable and the renormaliza-

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tion group equation can be solved.

If we consider exclusive B decays via heavy-light currents in the scheme of effective theories, it is sufficient to consider the collinear effective theory between the scale E and $E\lambda$ and integrate out all the degrees of freedom above some scale μ . At this scale, we describe a heavy quark in terms of HQET, and treat an energetic light quark in the collinear effective theory. This limit corresponds to $m_b, E \rightarrow \infty$ with E/m_b fixed. We can calculate the Wilson coefficients of various operators in the effective theory by matching to the full theory and can obtain anomalous dimensions of various operators. In this paper we extend further the idea of the collinear effective theory and derive the effective Lagrangian to subleading order in λ and renormalize the effective theory at one loop. Also we consider the correction to heavy-light currents to order λ and to leading-logarithmic order in α_s .

In Sec. II we briefly review the collinear effective theory, and derive the effective Lagrangian to order λ . We also discuss a collinear gauge invariance in the effective theory. In Sec. III we discuss reparametrization invariance in the collinear effective theory. The reparametrization invariance ensures that the kinetic energy term is not renormalized to all orders in α_s . It is also useful in deriving high-dimensional operators for heavy-light currents in the collinear effective theory and in obtaining the Wilson coefficients and the renormalization behavior of these high-dimensional operators. In Sec. IV we match heavy-light currents between the full QCD and the collinear effective theory, and consider the effects of radiative corrections at one loop. In Sec. V, we compute the anomalous dimensions of various heavy-to-light operators to order λ at one loop, and solve the renormalization group equation for the Wilson coefficients in the collinear effective theory. In Sec. VI we consider form factors of heavy-light currents for the vector and the axial vector currents to order λ . In Sec. VII we present a conclusion and perspectives of the collinear effective theory. In the Appendix we present an explicit calculation to show that the effective Lagrangian at order λ is not renormalized at one loop.

II. COLLINEAR EFFECTIVE THEORY

We construct an effective theory which describes the dynamics of energetic light quarks. A detailed derivation of the effective theory at leading order in λ is described in Refs. [12–15], and we will briefly review the idea. Then we construct the effective theory to order λ . Let us consider a reference frame in which a light quark carries a large energy E . If we neglect the quark mass, the only large parameter in this system is the energy E itself. Since we are interested in decays of heavy mesons to energetic light hadrons, we can conveniently choose a reference frame as the rest frame of a heavy meson, in which the energy of light hadrons is indeed large in the heavy quark limit. In this reference frame, light particles lie close on the light-cone direction n^μ , and we describe their dynamics using the light-cone variables $p = (p^+, p^-, p_\perp)$, where $p^+ = n \cdot p$, and $p^- = \bar{n} \cdot p$. We choose the axis such that $n^\mu = (1, 0, 0, 1)$, $\bar{n}^\mu = (1, 0, 0, -1)$ with $n \cdot \bar{n} = 2$.

For the energetic quark, there are three distinct energy scales, with $p^- \sim 2E$ being large, while p_\perp and p^+ are small. If we take a small parameter as $\lambda \sim p_\perp/p^-$, we can write

$$p^\mu = \bar{n} \cdot p \frac{n^\mu}{2} + (p_\perp)^\mu + n \cdot p \frac{\bar{n}^\mu}{2} \\ = \mathcal{O}(\lambda^0) + \mathcal{O}(\lambda^1) + \mathcal{O}(\lambda^2), \quad (1)$$

since $p^+ p^- \sim p_\perp^2 \sim \lambda^2$. Therefore we have three distinct energy scales $E, E\lambda$ and $E\lambda^2$, making the effective theory more complicated than the HQET. It is similar to the case of non-relativistic QCD (NRQCD) for quarkonium states, in which there are also three distinct scales $m, m\beta$ and $m\beta^2$, where m is the heavy quark mass and β is the typical velocity of a quark inside a quarkonium [16]. The collinear quark can emit either a soft gluon with momentum $k_s = E(\lambda^2, \lambda^2, \lambda^2)$ or a collinear gluon with $k_c = E(\lambda^2, 1, \lambda)$ to the large momentum direction and can still be on its mass shell. Due to the infrared sensitivity with collinear loop momentum, the effective theory is more complicated, and the relevant scales must be treated separately to obtain a consistent power counting method. In the collinear effective theory, the power counting in $1/E$ is troublesome, but the expansion in the small parameter λ offers a consistent power counting and there is no mixing of operators with different powers of λ . This will be discussed in detail in Sec. V.

The Lagrangian in the collinear effective theory can be obtained from the full QCD Lagrangian at tree level by expanding it in powers of λ . The full QCD Lagrangian for massless quarks and gluons is given by

$$\mathcal{L}_{\text{QCD}} = \bar{q} i \not{D} q - \frac{1}{4} G_{\mu\nu}^a G^{\mu\nu a}, \quad (2)$$

where the covariant derivative is $D_\mu = \partial_\mu + ig T^a A_\mu^a$, and $G_{\mu\nu}^a$ is the gluon field strength tensor. We remove large momenta from the Lagrangian, similar to the method employed in the HQET. The quark momentum is split as

$$p = \tilde{p} + k, \quad \tilde{p} \equiv (\bar{n} \cdot p) \frac{n}{2} + p_\perp. \quad (3)$$

The large part of the quark momentum $\bar{n} \cdot p$ and p_\perp , denoted by \tilde{p} , will be removed by defining a new field as

$$q(x) = \sum_{\tilde{p}} e^{-i\tilde{p} \cdot x} q_{n,p}(x). \quad (4)$$

A label p in $q_{n,p}$ refers to only the components $\bar{n} \cdot p$ and p_\perp . The derivative ∂_μ on the field $q_{n,p}$ gives $\mathcal{O}(\lambda^2)$ contributions.

Now we introduce projection operators which project out large components $\xi_{n,p}$ and small components $\xi_{\bar{n},p}$ in the direction n^μ as

$$\xi_{n,p} = \frac{\not{n} \not{\bar{n}}}{4} q_{n,p}, \quad \xi_{\bar{n},p} = \frac{\not{\bar{n}} \not{n}}{4} q_{n,p}. \quad (5)$$

TABLE I. Power counting for the effective theory fields.

	Heavy quark	Collinear quark	Soft gluon	Collinear gluon		
Field	h_v	$\xi_{n,p}$	A_s^μ	$\bar{n} \cdot A_{n,q}$	$n \cdot A_{n,q}$	$A_{n,q}^\perp$
Scaling	λ^3	λ	λ^2	λ^0	λ^2	λ

The fields $\xi_{n,p}$, $\xi_{\bar{n},p}$ satisfy

$$\frac{\not{n}\not{\bar{n}}}{4}\xi_{n,p} = \xi_{n,p}, \quad \frac{\not{\bar{n}}\not{n}}{4}\xi_{\bar{n},p} = \xi_{\bar{n},p}, \quad (6)$$

and

$$\not{n}\xi_{n,p} = 0, \quad \not{\bar{n}}\xi_{\bar{n},p} = 0. \quad (7)$$

We can eliminate the small component $\xi_{\bar{n},p}$ at tree level by using the equation of motion

$$(\bar{n} \cdot p + \bar{n} \cdot iD)\xi_{\bar{n},p} = (\not{p}_\perp + i\not{D}_\perp)\frac{\not{\bar{n}}}{2}\xi_{n,p}, \quad (8)$$

and the Lagrangian can be written in terms of $\xi_{n,p}$. It is convenient to separate the collinear and soft parts in gluon modes as $A^\mu = A_c^\mu + A_s^\mu$ in the covariant derivative D^μ , such that the covariant derivative involves only soft gluons. The typical scale for the collinear gluons is $q^2 \sim \lambda^2$, while the typical scale for the soft gluons is $k^2 \sim \lambda^4$. Since the collinear gluon carries a large momentum $\tilde{q} \equiv (\bar{n} \cdot q, q_\perp)$, derivatives on this field can yield order λ^0 and λ^1 contributions. To make this explicit, we extract the large momentum part containing \tilde{q} by redefining the field $A_c^\mu(x) = \sum_{\tilde{q}} \bar{e}^{-i\tilde{q} \cdot x} A_{n,q}^\mu(x)$. Then the Lagrangian can be written as

$$\begin{aligned} \mathcal{L} = & \bar{\xi}_{n,p'} \left[n \cdot iD - g n \cdot A_{n,q} + (\not{p}_\perp + i\not{D}_\perp - g A_{n,q}^\perp) \right. \\ & \times \frac{1}{\bar{n} \cdot p + \bar{n} \cdot iD - g \bar{n} \cdot A_{n,q}} \\ & \left. \times (\not{p}_\perp + i\not{D}_\perp - g A_{n,q}^\perp) \right] \frac{\not{\bar{n}}}{2} \xi_{n,p}. \quad (9) \end{aligned}$$

Here the summation over the labels \tilde{p} and \tilde{p}' and the phase factors for each collinear field are suppressed. From now on, in order to simplify the notation further, we suppress the label momenta for the collinear fields when there can be no confusion. It should be understood that, when ξ_n and A_n^μ appear, the summation on the label momentum p, q , the large phases, and the conservation of the label momenta are implied. The method to insert all the summations, the phases, and the label momenta are nicely summarized in Ref. [15].

In order to obtain the effective Lagrangian, we expand Eq. (9) in powers of λ . In the power counting of the fields in λ , we follow the procedure of moving all the dependence on λ into the interaction terms to make the kinetic terms of order

λ^0 . This is done by assigning a λ scaling to the fields in the effective theory, as given in Table I [15].

Bauer and Stewart [15] suggested a closed form to include the effects of collinear gluons to all orders. We define an operator $\bar{\mathcal{P}}$ which acts on products of effective theory fields. When acting on collinear fields, $\bar{\mathcal{P}}$ gives the sum of large momentum labels on fields minus the sum of large momentum labels on conjugate fields. Then, for any function f , we have

$$\begin{aligned} f(\bar{\mathcal{P}})(\phi_{q_1}^\dagger \cdots \phi_{q_m}^\dagger \phi_{p_1} \cdots \phi_{p_n}) \\ = f(\bar{n} \cdot p + \cdots + \bar{n} \cdot p_n - \bar{n} \cdot q_1 - \cdots - \bar{n} \cdot q_m) \\ \times (\phi_{q_1}^\dagger \cdots \phi_{q_m}^\dagger \phi_{p_1} \cdots \phi_{p_n}). \quad (10) \end{aligned}$$

The operator $\bar{\mathcal{P}}$ has mass dimension 1, but power counting dimension λ^0 . The conjugate operator $\bar{\mathcal{P}}^\dagger$ acts only to its left and gives the sum of large momenta on conjugate fields minus the sum of large momenta on fields.

Let us consider gauge symmetries of the effective theory. Since there are several gluon modes, there are possible $SU(3)$ color gauge transformations for each mode. We consider gauge symmetries that have support over collinear momenta. The collinear effective theory is invariant under a collinear non-Abelian gauge transformation of the form $U(x) = \exp[i\alpha^a(x)T^a]$. A set of these collinear gauge transformations is a subset of all the gauge transformations, which satisfies $\partial^\mu U \sim E(\lambda^2, 1, \lambda)$. It is useful to decompose this collinear transformation into a sum over the collinear momenta

$$U(x) = \sum_{\tilde{Q}} e^{-i\tilde{Q} \cdot x} \mathcal{U}_{\tilde{Q}}, \quad (11)$$

where $\partial^\mu \mathcal{U}_{\tilde{Q}} \sim \lambda^2$. When we expand the gauge transformation, we obtain simple transformation rules for collinear fermions and gluons. The transformation for collinear fermions and gluons are given by

$$\xi_n \rightarrow \mathcal{U} \xi_n,$$

$$A_n^\mu \rightarrow \mathcal{U} A_n^\mu \mathcal{U}^\dagger - \frac{1}{g} \mathcal{U} \left[\left(\frac{\bar{\mathcal{P}} n^\mu}{2} + \mathcal{P}_\perp^\mu + (in \cdot \partial) \frac{\bar{n}^\mu}{2} \right) \mathcal{U}^\dagger \right]. \quad (12)$$

Here \mathcal{P}_\perp^μ produces a sum of momenta of order λ , and the last term produces a momentum of order λ^2 . And the soft modes transform as $A_s^\mu \rightarrow \mathcal{U} A_s^\mu \mathcal{U}^\dagger$ under a collinear gauge transformation.

Let us define a function W of $\bar{n} \cdot A_n$ such that $W^\dagger \xi_n$ is invariant under the transformation in Eq. (12). The operators W and W^\dagger are defined as

$$W = \left[\exp \left(\frac{1}{\bar{\mathcal{P}}} g \bar{n} \cdot A_n \right) \right], \quad W^\dagger = \left[\exp \left(g \bar{n} \cdot A_n^* \frac{1}{\bar{\mathcal{P}}^\dagger} \right) \right], \quad (13)$$

which satisfy $W^\dagger W = 1$. In the expansion of the exponential, the $1/\bar{\mathcal{P}}$ acts to the right on all gluon fields in the square bracket. Under a collinear gauge transformation, W transforms as [15]

$$W \rightarrow \mathcal{U} W, \quad (14)$$

which makes $W^\dagger \xi_n$ invariant under a collinear gauge transformation. When we expand the exponential in W , we have an infinite series of collinear gluons. But all of them are of order λ^0 and should be included. The operator $\mathcal{P}_\perp - g A_n^\perp$ of order λ transforms as

$$\mathcal{P}_\perp - g A_n^\perp \rightarrow \mathcal{U} (\mathcal{P}_\perp - g A_n^\perp) \mathcal{U}^\dagger \quad (15)$$

under a collinear gauge transformation.

With these transformation properties, we can write the Lagrangian $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1$ in a closed form, including an infinite number of collinear gluons as

$$\begin{aligned} \mathcal{L}_0 &= \bar{\xi}_n \left\{ n \cdot (iD - g A_n) \right. \\ &\quad \left. + (\mathcal{P}_\perp - g A_n^\perp) W \frac{1}{\bar{\mathcal{P}}} W^\dagger (\mathcal{P}_\perp - g A_n^\perp) \right\} \frac{\bar{h}}{2} \xi_n, \\ \mathcal{L}_1 &= \bar{\xi}_n \left\{ iD_\perp W \frac{1}{\bar{\mathcal{P}}} W^\dagger (\mathcal{P}_\perp - g A_n^\perp) \right. \\ &\quad \left. + (\mathcal{P}_\perp - g A_n^\perp) W \frac{1}{\bar{\mathcal{P}}} W^\dagger iD_\perp \right\} \frac{\bar{h}}{2} \xi_n, \end{aligned} \quad (16)$$

where \mathcal{L}_n ($n=0,1$) is the Lagrangian at order λ^n . The expression in Eq. (16) is manifestly invariant under a collinear gauge transformation, and we use the fact that for any function f , $W f(\bar{\mathcal{P}}) W^\dagger = f(\bar{\mathcal{P}} - g \bar{n} \cdot A_n)$.

The Feynman rules for the propagator of a collinear quark and the interaction vertices from \mathcal{L}_0 are shown in Fig. 1. Here γ_\perp^μ is defined as

$$\gamma_\perp^\mu = \gamma^\mu - \frac{\bar{h}}{2} \bar{n}^\mu - \frac{\bar{h}}{2} n^\mu. \quad (17)$$

There are other interaction vertices such as the one with two collinear quark fields and two gluons, and those with triple gluons. We omit them here since they do not contribute to one-loop corrections to order λ in dimensional regularization.

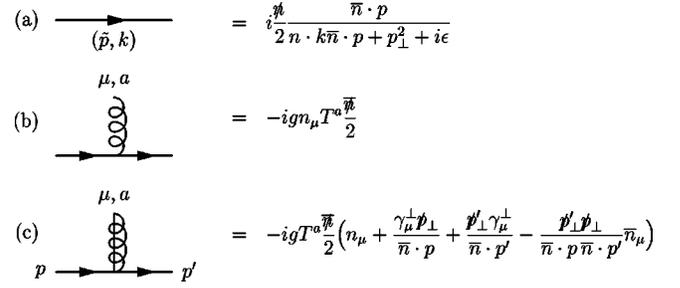


FIG. 1. Feynman rules for \mathcal{L}_0 to order g in the collinear effective theory: (a) collinear quark propagator with label \tilde{p} and residual momentum k , (b) collinear quark interaction with one soft gluon, and (c) collinear quark interaction with one collinear gluon, respectively.

For a heavy quark, we employ HQET for the heavy quark field h_v . The effective Lagrangian for HQET is given by

$$\mathcal{L}_{\text{HQET}} = \bar{h}_v v \cdot i D h_v. \quad (18)$$

The covariant derivative in Eq. (18) contains only soft gluons because the heavy quark field does not couple to collinear gluons. According to the power counting in Table I, the corrections in $1/m_b$ in the HQET Lagrangian are suppressed by λ^2 compared to the leading Lagrangian, and we will not consider them here.

III. REPARAMETRIZATION INVARIANCE

When we decompose a quantity into a large part and a small part, the decomposition is not unique. We can always shift the large part such that a change in the small part compensates this change to make the total quantity unchanged. The physics should be invariant under such a change. The invariance under this shift is called the reparametrization invariance. In HQET, there is a reparameterization invariance [17]. It means that the decomposition of the heavy quark momentum p_b into $m_b v$ and the residual momentum k is not unique. Typically k is of the order of Λ_{QCD} , which is much smaller than m_b . A small change in the four velocity of the order of Λ_{QCD}/m_b can be compensated by a change in the residual momentum. The physics of heavy quarks should be invariant under different decomposition of momenta. A consequence of this reparametrization invariance is that the kinetic energy term in HQET is not renormalized to all orders. Besides, we can obtain higher-dimensional operators for heavy-light currents using the reparametrization invariance. And we can easily obtain the Wilson coefficients and the anomalous dimensions of higher-dimensional operators without any explicit calculation.

A similar reparametrization invariance occurs in the collinear effective theory. The energetic light quark momentum p is given by

$$p^\mu = \frac{\bar{n} \cdot p}{2} n^\mu + p_\perp^\mu + k^\mu. \quad (19)$$

From now on, we will consider a small change of order λ , neglecting terms of order λ^2 , which can be included in a straightforward way. As in HQET, the decomposition of p into n, p_\perp is not unique. A small change in n^μ of order λ can be compensated by a change in p_\perp^μ ,

$$n \rightarrow n + \frac{2\epsilon}{\bar{n} \cdot p}, \quad p_\perp \rightarrow p_\perp - \epsilon, \quad (20)$$

where ϵ is of order λ . And the physics for collinear quarks should be invariant under different decompositions of momenta.

Since n satisfies $n^2=0$, the parameter ϵ must satisfy $n \cdot \epsilon=0$, neglecting terms of order $(\epsilon/\bar{n} \cdot p)^2$. The light quark spinor ξ_n must also change to preserve the constraint $\not{n}\xi_n=0$. Consequently, if ξ_n changes as $\xi_n \rightarrow \xi_n + \delta\xi_n$, $\delta\xi_n$ satisfies

$$\left(\not{n} + \frac{2\not{\epsilon}}{\bar{n} \cdot p} \right) (\xi_n + \delta\xi_n) = 0. \quad (21)$$

To first order in $\epsilon/\bar{n} \cdot p$, one finds

$$\not{n}\delta\xi_n = -\frac{2\not{\epsilon}}{\bar{n} \cdot p} \xi_n. \quad (22)$$

Therefore a suitable choice for the change in ξ_n is

$$\delta\xi_n = -\frac{1}{\bar{n} \cdot p} \frac{\not{n}}{2} \not{\epsilon} \xi_n. \quad (23)$$

The Lagrangian in Eq. (16) must be invariant under the combined changes

$$n \rightarrow n + \frac{2\epsilon}{\bar{n} \cdot p}, \quad \xi_n \rightarrow e^{i\epsilon \cdot x} \left(1 - \frac{1}{\bar{n} \cdot p} \frac{\not{n}\not{\epsilon}}{2} \right) \xi_n, \quad (24)$$

where the prefactor $e^{i\epsilon \cdot x}$ causes a shift $p_\perp \rightarrow p_\perp - \epsilon$. In order to prove the reparametrization invariance, it is convenient to write the Lagrangian \mathcal{L} as

$$\begin{aligned} \mathcal{L} = & \bar{\xi}_n \left\{ n \cdot (iD + \mathcal{P} - gA_n) + (\mathcal{P}_\perp - gA_n^\perp + i\mathcal{D}_\perp) \right. \\ & \left. \times W \frac{1}{\bar{\mathcal{P}}} W^\dagger (\mathcal{P}_\perp - gA_n^\perp + i\mathcal{D}_\perp) \right\} \frac{\not{n}}{2} \xi_n, \end{aligned} \quad (25)$$

where we included $n \cdot \mathcal{P}$ which does not affect the Lagrangian, but the addition makes the Lagrangian manifestly invariant under a collinear gauge transformation.

The change of the Lagrangian is given by

$$\begin{aligned} \delta\mathcal{L} = & \bar{\xi}_n \left[\frac{2\epsilon}{\bar{n} \cdot p} \cdot (\mathcal{P}_\perp - gA_\perp + i\mathcal{D}_\perp) - \not{\epsilon} W \frac{1}{\bar{\mathcal{P}}} W^\dagger (\mathcal{P}_\perp - gA_\perp \right. \\ & \left. + i\mathcal{D}_\perp) \right. \\ & \left. - (\mathcal{P}_\perp - gA_\perp + i\mathcal{D}_\perp) W \frac{1}{\bar{\mathcal{P}}} W^\dagger \not{\epsilon} \right] \frac{\not{n}}{2} \xi_n. \end{aligned} \quad (26)$$

The change $\delta\mathcal{L}$ vanishes, which can be easily seen when we disregard gauge fields. Then the first line in Eq. (26) exactly cancels the second line. Therefore we have proved that the Lagrangian is reparametrization invariant under a shift of order λ . As a result, the kinetic energy terms appearing both in \mathcal{L}_0 and \mathcal{L}_1 are not renormalized. The explicit calculation to show that the kinetic energy term at order λ is not renormalized at one loop is given in Appendix.

We can make a stronger statement by combining the reparametrization invariance and the collinear gauge invariance of the collinear effective theory. In the Lagrangian \mathcal{L}_1 at order λ , the kinetic energy part is given by

$$\bar{\xi}_n \frac{2p_\perp \cdot i\partial_\perp}{\bar{n} \cdot p} \frac{\not{n}}{2} \xi_n, \quad (27)$$

which is not renormalized due to the reparametrization invariance. However, in order to make this part collinear gauge invariant, \mathcal{P}_\perp should be replaced by $\mathcal{P}_\perp - gA_\perp$. There is no constraint from the collinear gauge invariance on whether we should replace the derivative operator with a covariant derivative including a soft gluon. However, if we require the invariance under ultrasoft gauge transformations [15], the derivative operator should be replaced by the covariant derivative. Therefore the extension of the kinetic energy term which is invariant under the collinear and the ultrasoft gauge transformation is given by

$$\begin{aligned} \bar{\xi}_n \left\{ (iD_\perp)_\mu W \frac{1}{\bar{\mathcal{P}}} W^\dagger (\mathcal{P}_\perp^\mu - gA_n^{\perp\mu}) \right. \\ \left. + (\mathcal{P}_\perp^\mu - gA_n^{\perp\mu}) W \frac{1}{\bar{\mathcal{P}}} W^\dagger (iD_\perp)_\mu \right\} \frac{\not{n}}{2} \xi_n. \end{aligned} \quad (28)$$

This is not renormalized to all orders in α_s due to the reparametrization invariance and the gauge invariance. And the remaining part in \mathcal{L}_1 is not renormalized at one loop, hence the whole Lagrangian \mathcal{L}_1 is not renormalized at leading-logarithmic accuracy.

We can fix the form of some corrections at order λ from the operators at λ^0 using the reparametrization invariance. For example, the vector current $\bar{q}\gamma^\mu b$ in the full theory is written as

$$\begin{aligned} \bar{q}\gamma^\mu b \rightarrow & \bar{\xi}_n \left(1 + \frac{\not{n}}{2} \frac{\not{p}_\perp}{\bar{n} \cdot p} \right) \gamma^\mu h_v \\ = & \bar{\xi}_n \gamma^\mu h_v + \bar{\xi}_n \frac{\not{n}}{2} \frac{\not{p}_\perp}{\bar{n} \cdot p} \gamma^\mu h_v, \end{aligned} \quad (29)$$

in the collinear effective theory to order λ . The collinear gauge-invariant form of this operator is given by

$$\bar{\xi}_n \left(1 + \frac{\bar{h}}{2} (\mathcal{P}_\perp - g \mathcal{A}_n^\perp) W \frac{1}{\bar{\mathcal{P}}^\dagger} \right) \Gamma h_v, \quad (30)$$

where the second term is an operator for heavy-light currents at order λ in the effective theory.

IV. MATCHING HEAVY-LIGHT CURRENTS

We consider the matching of heavy-light currents of the form $J = \bar{q} \Gamma b$, where Γ denotes γ^μ or $\gamma^\mu \gamma_5$. Below the scale $\bar{n} \cdot p$, the hadronic current is matched onto currents in the collinear effective theory and the HQET. This introduces a new set of Wilson coefficients. We will match the current operators in the full theory with the current operators in the collinear effective theory and the HQET in a single step neglecting the sum of logarithms of order $\ln(m_b/\bar{n} \cdot p)$, which is quite small since $m_b \sim \bar{n} \cdot p$.

The vector-current operator $V^\mu = \bar{q} \gamma^\mu b$ in the full theory can be matched to the effective theory as

$$V^\mu \rightarrow \sum_i C_i(\mu) J_i^\mu + \sum_j B_j O_j^\mu + \sum_k A_k T_k^\mu. \quad (31)$$

The operators J_i are the operators at leading order in λ , and there are three such operators, which are given as

$$J_1^\mu = \bar{\xi}_n W \gamma^\mu h_v, \quad J_2^\mu = \bar{\xi}_n W v^\mu h_v, \quad J_3^\mu = \bar{\xi}_n W n^\mu h_v. \quad (32)$$

Similarly, $\{O_j^\mu\}$ are a complete set of operators at order λ . There are four such operators and a convenient basis for these operators is given by

$$\begin{aligned} O_1^\mu &= \bar{\xi}_n \frac{\bar{h}}{2} (\mathcal{P}_\perp - g \mathcal{A}_\perp) W \frac{1}{\bar{\mathcal{P}}^\dagger} \gamma^\mu h_v, \\ O_2^\mu &= \bar{\xi}_n \frac{\bar{h}}{2} (\mathcal{P}_\perp - g \mathcal{A}_\perp) W \frac{1}{\bar{\mathcal{P}}^\dagger} v^\mu h_v, \\ O_3^\mu &= \bar{\xi}_n \frac{\bar{h}}{2} (\mathcal{P}_\perp - g \mathcal{A}_\perp) W \frac{1}{\bar{\mathcal{P}}^\dagger} n^\mu h_v, \\ O_4^\mu &= \bar{\xi}_n (\mathcal{P}_\perp^\mu - g \mathcal{A}_\perp^\mu) W \frac{1}{\bar{\mathcal{P}}^\dagger} h_v. \end{aligned} \quad (33)$$

The operators in Eqs. (32) and (33) are written in such a way that they are manifestly invariant under a collinear gauge transformation. We also include the nonlocal operators T_k^μ arising from an insertion of the order λ correction to the effective Lagrangian into matrix elements of the leading-order currents, which are defined as

$$T_k^\mu = i \int d^4 y T \{ J_k^\mu(0), \mathcal{L}_1(y) \} \quad (k=1,2,3). \quad (34)$$

Our goal is to calculate the Wilson coefficients $C_i(\mu)$, $B_j(\mu)$ and $A_k(\mu)$ in the leading-logarithmic approximation. The Wilson coefficients are defined by requiring that matrix elements of the vector current in the full theory are the same, to any order in λ , as matrix elements calculated in the effective theory. Before we proceed to explicit calculation, note that there are nontrivial relations between the coefficients $B_j(\mu)$ and $C_j(\mu)$ imposed by the reparametrization invariance. This is because operators of order λ acting on a collinear quark field must always appear in certain combinations with operators of order λ^0 . In our case, there is a unique way in which the operators O_i^μ can be combined with J_i^μ in a reparametrization invariant way, that is,

$$\begin{aligned} \left\langle \bar{\xi}_n \left(1 + \frac{\bar{h}}{2} \frac{\boldsymbol{p}_\perp}{\bar{n} \cdot p} \right) \gamma^\mu h_v \right\rangle + \dots &= \langle J_1^\mu \rangle + \langle O_1^\mu \rangle + \dots, \\ \left\langle \bar{\xi}_n \left(1 + \frac{\bar{h}}{2} \frac{\boldsymbol{p}_\perp}{\bar{n} \cdot p} \right) v^\mu h_v \right\rangle + \dots &= \langle J_2^\mu \rangle + \langle O_2^\mu \rangle + \dots, \end{aligned} \quad (35)$$

$$\begin{aligned} \left\langle \bar{\xi}_n \left(1 + \frac{\bar{h}}{2} \frac{\boldsymbol{p}_\perp}{\bar{n} \cdot p} \right) \left(n^\mu + \frac{2p_\perp^\mu}{\bar{n} \cdot p} \right) h_v \right\rangle + \dots \\ = \langle J_3^\mu \rangle + \langle O_3^\mu \rangle + 2 \langle O_4^\mu \rangle + \dots \end{aligned}$$

This implies that, to all orders in perturbation theory,

$$B_i(\mu) = C_i(\mu) \quad (i=1,2,3), \quad B_4(\mu) = 2C_3(\mu), \quad (36)$$

and the coefficients $C_i(\mu)$ have been calculated at leading-logarithmic order in Ref. [13]. This is our new result and it imposes an important constraint on the theory, which must be obeyed by an explicit calculation.

The operator product expansion of the axial vector current $A^\mu = \bar{q} \gamma^\mu \gamma_5 b$ can be simply obtained from Eq. (31) by replacing $\bar{q} \rightarrow -\bar{q} \gamma_5$ if we perform the calculation using the dimensional regularization with modified minimal subtraction (MS) and the naive dimensional reduction (NDR) scheme with anticommuting γ_5 . We can rewrite the axial current as $A^\mu = -\bar{q} \gamma_5 \gamma^\mu b$. The γ_5 matrix acting on the massless quark q becomes ± 1 depending on the chirality of the quark. Chirality is conserved by the QCD interactions, so the calculation of matching conditions proceeds just as in the vector current case, except that \bar{q} is replaced everywhere by $\bar{q} \gamma_5$. At the end of the calculation, the γ_5 is moved back next to h_v , producing a compensating minus sign for $\gamma^\mu \gamma_5$, but neither for $v^\mu \gamma_5$ nor for $n^\mu \gamma_5$. Thus, for axial vector currents, all the coefficients are the same in magnitude, and only C_1 , B_1 , and A_1 do not change sign, while all the remaining coefficients change sign.

Bauer et al. [12,13] have explicitly showed that the collinear effective theory, indeed, reproduces the infrared behavior of the full theory by including the effects of collinear

gluons. Once we know that the effective theory reproduces the long-distance physics of the full theory, the matching procedure is independent of any long-distance physics, such as infrared singularities, nonperturbative effects and the choice of external states. Thus there is a freedom in choosing the external states and the infrared regularization scheme. We find it most convenient to perform the matching of QCD onto the collinear and the heavy quark effective theory using on-shell external quark states and dimensional regularization for both the ultraviolet and infrared divergences encountered in calculating loop diagrams. This scheme has the great advantage that all loop diagrams in the effective theory vanish, since there is no mass scale other than the renormalization scale μ . It means that matrix elements in the effective theory are given by their tree-level expressions. We assign momentum such that the incoming heavy quark has momentum $p_b = m_b v + k$ (with $2v \cdot k + k^2/m_b = 0$), while the outgoing light energetic quark carries momentum $p = En + p_\perp + k'$ (with $2En \cdot k' + p_\perp^2 = 0$).

The matrix elements of operators can be written as

$$\begin{aligned} \langle J_1^\mu \rangle &= \bar{u}_e(n, s) \gamma^\mu u_h(v, s_b), \\ \langle O_1^\mu \rangle &= \bar{u}_e(n, s) \frac{\not{n}}{2} \frac{\not{p}_\perp}{\not{n} \cdot p} \gamma^\mu u_h(v, s_b), \end{aligned} \quad (37)$$

where $u_e(n, s)$ and $u_h(v, s_b)$ are on-shell spinors for a massless, energetic quark field ξ_n in the collinear effective theory, and a heavy quark field h_v in the HQET, respectively. They satisfy $\not{n} u_e(n, s) = 0$ and $\not{v} u_h(v, s_b) = u_h(v, s_b)$. We compute, in the full theory, the vector current matrix element between on-shell quark states at one-loop order in order to do the matching. The relations of the heavy quark spinors and the light quark spinors between QCD and the effective theory are given by

$$\begin{aligned} u_b(p_b, s_b) &= \left(1 + \frac{\not{k}}{2m_b} \right) u_h(v, s_b) + O(1/m_b^2), \\ u_q(p, s) &= \left(1 - \frac{\not{n}}{2} \frac{\not{p}_\perp}{\not{n} \cdot p} \right) u_e(n, s) + O(\lambda^2). \end{aligned} \quad (38)$$

The correction to the heavy quark field, which involves \not{k} , is suppressed by λ^2 , and it is discarded in our matching at order λ .

We match the coefficients at one loop by employing the dimensional regularization in $D = 4 - 2\epsilon$ dimensions. In the full theory, there is no ultraviolet divergence due to current conservation. The residue at the physical mass pole in the propagator is infrared in nature, and it should be added to the vertex correction. The residue at the physical mass pole for the heavy quark in the $\overline{\text{MS}}$ scheme at order α_s is given by [18]

$$R_b^{(1)} = -\frac{\alpha_s C_F}{4\pi} \left(\frac{2}{\epsilon} + 4 - 6 \ln \frac{m_b}{\mu} \right), \quad (39)$$

and in the HQET, the residue at order α_s is given as

$$R_h^{(1)} = -\frac{\alpha_s C_F}{4\pi} \frac{2}{\epsilon}. \quad (40)$$

The residue for the light quark at order α_s in the collinear effective theory is the same as the residue in the full theory, and it is given as

$$R_q^{(1)} = R_{\xi_n}^{(1)} = \frac{\alpha_s C_F}{4\pi} \frac{1}{\epsilon}. \quad (41)$$

Since the residues for the light quarks are the same, they cancel each other when we match both theories.

The matrix element of the vector current between free quark states with the residues of the external quarks in the full theory can be expressed in terms of the matrix elements in the collinear and the heavy quark effective theory as

$$\begin{aligned} \langle \bar{q} \gamma^\mu b \rangle &= \left\{ 1 - \frac{\alpha_s C_F}{4\pi} \left[\frac{1}{\epsilon^2} + \frac{5}{2\epsilon} - \frac{2}{\epsilon} \ln \frac{xm_b}{\mu} + 2 \ln^2 \frac{xm_b}{\mu} \right. \right. \\ &\quad \left. \left. + \frac{3x-2}{1-x} \ln x + Li_2(1-x) + \frac{\pi^2}{12} + 6 \right] \right\} \\ &\quad \times \langle J_1^\mu + O_1^\mu \rangle + \frac{\alpha_s C_F}{4\pi} \left[\frac{2}{1-x} + \frac{2x}{(1-x)^2} \ln x \right] \\ &\quad \times \langle J_2^\mu + O_2^\mu \rangle + \frac{\alpha_s C_F}{4\pi} \left[-\frac{x}{1-x} + \frac{x(1-2x)}{(1-x)^2} \ln x \right] \\ &\quad \times \langle J_3^\mu + O_3^\mu + 2O_4^\mu \rangle, \end{aligned} \quad (42)$$

where $x = \bar{n} \cdot p / m_b = 2E/m_b$ and $Li_2(x)$ is the dilogarithmic function. Here we have confirmed the consequence of the reparametrization invariance at one loop explicitly. The infrared behavior of the full QCD is reproduced in the collinear effective theory, and the infrared divergences in both theories cancel in matching.

The Wilson coefficients C_i for J_i^μ at the renormalization scale μ are given by

$$\begin{aligned} C_1(\mu) &= 1 - \frac{\alpha_s C_F}{4\pi} \left[2 \ln^2 \left(\frac{xm_b}{\mu} \right) - 5 \ln \frac{m_b}{\mu} \right. \\ &\quad \left. + \frac{3x-2}{1-x} \ln x + 2Li_2(1-x) + \frac{\pi^2}{12} + 6 \right], \\ C_2(\mu) &= \frac{\alpha_s C_F}{4\pi} \left[\frac{2}{1-x} + \frac{2x}{(1-x)^2} \ln x \right], \\ C_3(\mu) &= \frac{\alpha_s C_F}{4\pi} \left[-\frac{x}{1-x} + \frac{x(1-2x)}{(1-x)^2} \ln x \right], \end{aligned} \quad (43)$$

and the coefficients B_j are given as

$$B_i(\mu) = C_i(\mu) \quad (i=1,2,3), \quad B_4(\mu) = 2C_3(\mu). \quad (44)$$

This relation is expected from the reparametrization invariance, and the operators O_i^μ have the same anomalous dimension as those of the leading operators J_i^μ . The explicit calculation that the operators O_1 to O_4 have the same ultraviolet behavior as their corresponding leading operators is shown in Sec. V.

The coefficients A_i are given by the product of those for J_i^μ and \mathcal{L}_1 , and they are given by

$$A_i(\mu) = C_i(\mu). \quad (45)$$

The fact that the Wilson coefficients A_i are the same as C_i is because the effective Lagrangian \mathcal{L}_1 at order λ is not renormalized at leading-logarithmic order.

V. RENORMALIZATION GROUP IMPROVEMENT

The perturbative expansion of the Wilson coefficients contains large logarithms of the type $[\alpha_s \ln(2E/\mu)]^n$, which should be summed to all orders. We employ the renormalization group to improve one-loop results. The reason why we choose λ as the small parameter is because various operators with different orders of λ do not mix in this power counting. If we choose to expand in powers of $1/E$, when we renormalize operators, a factor E in the numerator could be induced from loop calculations. This is expected since the propagator of a collinear quark explicitly involves E in the $1/E$ expansion. Therefore higher-dimensional operators in $1/E$ can mix with those operators with one less power of m_b or E , and a power counting in $1/E$ is inappropriate. However, if we expand the effective Lagrangian in powers of λ , such mixing never occurs, and we can do the power counting in λ consistently.

In general, the coefficients of the operators with the same power of λ mix into themselves and satisfy a renormalization group equation of the form

$$\mu \frac{d}{d\mu} C(\mu) = \gamma(\mu) C(\mu). \quad (46)$$

Since Eq. (46) is homogeneous, we can reproduce the exponentiation of Sudakov logarithm.

The renormalization of the operators J_i^μ at order λ^0 was performed in Ref. [13]. The counterterm for the operators J_i^μ in the effective theory using the Feynman gauge is given by

$$Z_i = 1 + \frac{\alpha_s C_F}{4\pi} \left[\frac{1}{\epsilon^2} - \frac{2}{\epsilon} \ln \frac{2E}{\mu} + \frac{5}{2\epsilon} \right]. \quad (47)$$

This counterterm is the same for all J_i^μ , and is independent of the Dirac structure of the operators since the propagators and the vertices in the collinear effective theory do not alter the Dirac structure of the operators. Furthermore, there is no operator mixing. The anomalous dimensions are given by

$$\gamma_i = Z_i^{-1} \left(\mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial g} \right) Z_i, \quad (48)$$

$$= -gT^a \bar{n} \left[\frac{\gamma_1^\mu}{\bar{n} \cdot (p+q)} + \frac{\not{p}_1 \bar{n}^\nu}{\bar{n} \cdot (p+q) \bar{n} \cdot q} \right] \Gamma_i^\mu$$

FIG. 2. Feynman rules for the operator O_i^μ ($i=1,2,3$) containing a collinear gluon at order λ . Here $\Gamma_i^\mu = \gamma^\mu$, v^μ and n^μ for $i=1,2,3$, respectively. The momentum of the gluon is outgoing.

where

$$\mu \frac{\partial}{\partial \mu} Z_i = \frac{\alpha_s(\mu) C_F}{2\pi\epsilon}, \quad (49)$$

$$\beta \frac{\partial}{\partial g} Z_i = -\frac{\alpha_s C_F}{2\pi} \left(\frac{1}{\epsilon} - 2 \ln \frac{2E}{\mu} + \frac{5}{2} \right).$$

Here we have used $\beta = -g\epsilon + O(g^3)$. This gives the anomalous dimension

$$\gamma_i = -\frac{\alpha_s(\mu) C_F}{2\pi} \left(\frac{5}{2} - 2 \ln \frac{2E}{\mu} \right). \quad (50)$$

The divergence in Eq. (49) is cancelled, and solving the renormalization group equation Eq. (46), we obtain

$$C_i(\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(2E)} \right)^{(C_F/2\beta_0)(5-8\pi/\beta_0\alpha_s)} \left(\frac{2E}{\mu} \right)^{2C_F/\beta_0} C_i(2E), \quad (51)$$

where $\beta_0 = 11 - 2n_f/3$, and $C_i(2E)$ are the Wilson coefficients at $\mu = \bar{n} \cdot p = 2E$, as given in Eq. (43).

At order λ , we need to renormalize the operators O_i^μ . Let us first consider the renormalization of O_1^μ to O_3^μ . The Feynman rules for the vertex from these operators with a collinear gluon are given in Fig. 2. The Feynman diagrams to renormalize the operators O_i^μ ($i=1,2,3$) at order α_s are shown in Fig. 3. Since the loop calculation does not alter the Dirac structure, we can treat the renormalization of these operators in the same way for all the three operators. The Feynman diagrams in Fig. 3 give the amplitude

$$M_i^{(1)\mu} = -\frac{\alpha_s C_F}{4\pi} O_i^\mu \left[\frac{1}{\epsilon^2} + \left(2 - 2 \ln \frac{\bar{n} \cdot p}{\mu} \right) \frac{1}{\epsilon} \right]. \quad (52)$$

Note that there is no mixing for the operators O_i^μ . If we add the residues from the propagators of a heavy quark and a collinear quark, we have the counterterm

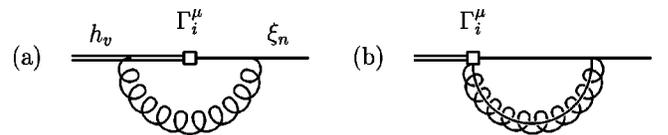


FIG. 3. Feynman diagrams for the renormalization of O_i^μ ($i=1,2,3$) at one loop.

$$Z_i^{(1)} = 1 + \frac{\alpha_s C_F}{4\pi} \left[\frac{1}{\epsilon^2} - \frac{2}{\epsilon} \ln \frac{2E}{\mu} + \frac{5}{2\epsilon} \right] \quad (i=1,2,3), \quad (53)$$

which is identical to the counterterm for the leading operators J_i^μ . We can do the same calculation for the operator O_4^μ and it turns out that the operator O_4^μ has the same dependence on ϵ as J_3^μ . And the counterterm is also given by Eq. (53). Therefore the operators O_i^μ ($i=1, \dots, 4$) have the same anomalous dimensions as the leading operators. This is the explicit proof of the reparametrization invariance at one loop and order λ .

For the time-ordered products T_k^μ , the anomalous dimensions are the same as those of J_k^μ because the Lagrangian \mathcal{L}_1 in defining T_k^μ is not renormalized at one loop. Here we see that the reparametrization invariance and the gauge invariance influence the structure of the theory. Furthermore, since there is no mixing, the perturbative corrections to heavy-light currents take a simple form to order λ .

VI. APPLICATION TO FORM FACTORS

As an application of the collinear effective theory, we can consider the form factors for B mesons into light mesons. We consider the kinematic region in which the energy E of the light quark is large,

$$E = \frac{m_b^2 - q^2}{2m_b} \sim \frac{m_b}{2}, \quad q = p_b - p_q, \quad (54)$$

which equivalently means that the momentum transfer squared through the weak current is small $q^2 \ll m_b^2$. In this case, the off-shellness of the light quark is $p_q^2 = 2Ek_+$, where $k_+ \sim \Lambda_{\text{QCD}}$, thus $\lambda \sim \sqrt{\Lambda_{\text{QCD}}/m_b}$. Therefore our formulation to order λ gives the correction to the form factors at order $\sqrt{\Lambda_{\text{QCD}}/m_b}$. For simplicity, we will consider the form factors for the vector and the axial vector currents.

The form factors for \bar{B} decays into light pseudoscalar and vector mesons from the vector current $V^\mu = \bar{q}\gamma^\mu b$, and the axial vector current $A^\mu = \bar{q}\gamma^\mu\gamma_5 b$ are defined as

$$\begin{aligned} \langle P(p') | V^\mu | \bar{B}(p) \rangle &= f_+(q^2) \left[p^\mu + p'^\mu - \frac{M^2 - m_P^2}{q^2} q^\mu \right] \\ &\quad + f_0(q^2) \frac{M^2 - m_P^2}{q^2} q^\mu, \end{aligned}$$

$$\langle V(p', \epsilon^*) | V^\mu | \bar{B}(p) \rangle = \frac{2V(q^2)}{M + m_V} i \epsilon^{\mu\nu\alpha\beta} \epsilon_\nu^* p'_\alpha p_\beta, \quad (55)$$

$$\begin{aligned} &\langle V(p', \epsilon^*) | A^\mu | \bar{B}(p) \rangle \\ &= 2m_V A_0(q^2) \frac{\epsilon^* \cdot q}{q^2} q^\mu + (M + m_V) \\ &\quad \times A_1(q^2) \left[\epsilon^{*\mu} - \frac{\epsilon^* \cdot q}{q^2} q^\mu \right] - A_2(q^2) \frac{\epsilon^* \cdot q}{M + m_V} \\ &\quad \times \left[p^\mu + p'^\mu - \frac{M^2 - m_V^2}{q^2} q^\mu \right], \end{aligned}$$

where $q = p - p'$, m_P (m_V) is the mass of the pseudoscalar (vector) meson, ϵ_μ^* is the polarization vector of the vector meson, and M is the mass of a \bar{B} meson. We use the sign convention $\epsilon^{0123} = -1$.

We can calculate these form factors systematically in powers of λ in the collinear effective theory. The matrix elements in the full theory are matched to the matrix elements in the collinear effective theory using Eq. (31). However, here we do not include interactions where a collinear gluon is exchanged with the spectator quarks inside a B meson. In Ref. [19] it was argued that these spectator effects could be of the same order in λ and $1/m_b$ as the soft contributions, but they are suppressed by a power of $\alpha_s(\sqrt{m_b}\Lambda_{\text{QCD}})$. They are therefore just as important as the one-loop corrections to the matching coefficients such as $C_i(\mu)$. Here we apply the collinear effective theory to the soft contributions only. It means that the effective theory applies to light mesons produced in an asymmetric configuration, in which a single quark from the b decay carries almost all the momentum.

If we consider this process as light-cone dominated, this is not a typical configuration. A typical configuration is for both a quark and antiquark that have nearly equal momentum. And spectator interactions can play an important role in this configuration. In heavy-to-heavy transitions such as $B \rightarrow D$ in the heavy quark limit, the interactions of a heavy quark with the soft degrees of freedom around the heavy quark do not change even when there is a transition. On the contrary, in heavy-to-light transitions, the soft degrees of freedom around the heavy quark experience an abrupt change. If an energetic quark and the soft degrees of freedom move somehow elastically with almost the same velocity, we can safely consider the interaction of an energetic quark with the soft degrees of freedom in terms of the collinear effective theory. This corresponds to the soft contribution to form factors. If only an energetic quark is pushed to the light-cone direction, the soft degrees of freedom around the heavy quark should arrange themselves to follow the energetic quark to form light mesons. In this process, hard gluons should be exchanged between the energetic quark and the previous soft degrees of freedom in the heavy quark. This corresponds to the hard spectator interaction. This hard spectator interaction should be considered separately, and we leave the hard spectator contributions for future study.

A convenient way to evaluate hadronic matrix elements in the effective theory is to associate the spin wave function

$$\mathcal{M}(v) = \sqrt{M} \frac{1 + \not{v}}{2} \times \begin{pmatrix} -\gamma_5 \\ \not{\epsilon} \end{pmatrix} \begin{array}{l} \text{pseudoscalar meson } P, \\ \text{vector meson } V, \end{array} \quad (56)$$

with the eigenstates of the effective Lagrangian, where M is the mass of the meson. The form factors in the effective theory can be written as

$$\langle L(n) | \bar{\xi}_n \Gamma h_v | \bar{B}(v) \rangle = \text{tr} [A_L(E) \bar{\mathcal{M}}_L \Gamma \mathcal{M}_B], \quad (L=P, V) \quad (57)$$

where Γ denotes a Dirac structure, and

$$\bar{\mathcal{M}}_L = \begin{pmatrix} -\gamma_5 \\ \not{\epsilon}^* \end{pmatrix} \frac{\bar{h} \not{h}}{4}, \quad \text{where } L=P, V, \quad (58)$$

$$\mathcal{M}_B = \frac{1 + \not{v}}{2} (-\gamma_5)$$

are the spin wave functions associated with a light meson and a \bar{B} meson, respectively. The normalization factor \sqrt{M} appearing in \mathcal{M} is absorbed in $A_L(E)$. The function $A_L(E)$ contains the long-distance dynamics, and it is independent of the Dirac structure Γ in the current. The most general form for $A_L(E)$ is given by

$$\Xi_L(E) = \xi_{1L}(E) + \xi_{2L}(E) \not{v} + \xi_{3L}(E) \not{h} + \xi_{4L}(E) \not{h} \not{v}, \quad (59)$$

but due to the properties of the projection operators in $\bar{\mathcal{M}}_L$ and \mathcal{M}_B , not all of them are independent. For $L=P$, there is one independent term, and for $L=V$, there are two independent terms.

Charles et al. [20] have shown that there are only three independent matrix elements in heavy-to-light transitions by employing the HQET and the large-energy effective theory to obtain the leading result in $1/E$. However, this is not sufficient to describe heavy-to-light decays because interactions with collinear gluons should be included. Though the argument is different, there are also three independent matrix elements in the collinear effective theory.

At order λ , we have the form factors of the form

$$\langle L(n) | \bar{\xi}_n p_\perp^\mu \Gamma h_v | \bar{B}(v) \rangle = \text{tr} [A_L^\mu(E) \bar{\mathcal{M}}_L \Gamma \mathcal{M}_B], \quad (60)$$

where $A_L^\mu(E)$ contains the long-distance dynamics and they are independent of the Dirac structure Γ of the current. Since the operator is proportional to p_\perp^μ , the only allowed vector component for A_L^μ is γ_\perp^μ . Therefore the most general form for A_L^μ is given by

$$A_L^\mu(E) = \gamma_\perp^\mu [a_{1L}(E) + a_{2L}(E) \not{v} + a_{3L}(E) \not{h} + a_{4L}(E) \not{h} \not{v}]. \quad (61)$$

As in the case of $\Xi_L(E)$, all the terms are not independent due to the projection operators in $\bar{\mathcal{M}}_L$ and \mathcal{M}_B . For $L=P$ there is only one independent term, and for $L=V$ there are two independent terms. Similarly, the matrix elements of the time-ordered products T_i can be written as

$$\langle L(n) | i \int d^4y T \{ J_i^\mu(0), \mathcal{L}_1(y) \} | \bar{B}(v) \rangle = \text{tr} [B_L^\mu(E) \bar{\mathcal{M}}_L \Gamma \mathcal{M}_B], \quad (62)$$

and the most general form for $B_L^\mu(E)$ is written as

$$B_L^\mu(E) = \gamma_\perp^\mu [b_{1L}(E) + b_{2L}(E) \not{v} + b_{3L}(E) \not{h} + b_{4L}(E) \not{h} \not{v}], \quad (63)$$

because \mathcal{L}_1 is of order λ and it typically depends on p_\perp^μ . Here also we have one independent term for $L=P$, and two independent terms for $L=V$.

In summary, we can write the parameters describing the long-distance physics as

$$\Xi_P(E) = 2E \xi_P, \quad \Xi_V(E) = E \not{h} \left(\xi_\perp - \frac{\not{v}}{2} \xi_\parallel \right),$$

$$A_P^\mu(E) = \frac{a_P}{2} \gamma_\perp^\mu, \quad A_V^\mu(E) = \gamma_\perp^\mu \frac{\not{h}}{2} \left(a_{V1} + \frac{\not{v}}{2} a_{V2} \right), \quad (64)$$

$$B_P^\mu(E) = b_P \gamma_\perp^\mu, \quad B_V^\mu(E) = \gamma_\perp^\mu \frac{\not{h}}{2} \left(b_{V1} - \frac{\not{v}}{2} b_{V2} \right).$$

Note that the convention for the longitudinal form factor ξ_\parallel is the same as that of Ref. [19], and is related to the corresponding form factor ζ_\parallel defined in Ref. [20] by $\xi_\parallel(E) = (m_V/M) \zeta_\parallel(E)$. The matrix elements of all the operators can be expressed in terms of these nonperturbative parameters. At order λ^0 , the matrix elements for pseudoscalar bosons are given by

$$\langle P | \bar{\xi}_n \gamma^\mu h_v | \bar{B} \rangle = 2E \xi_P n^\mu, \quad \langle P | \bar{\xi}_n \gamma^\mu \gamma_5 h_v | \bar{B} \rangle = 0,$$

$$\langle P | \bar{\xi}_n v^\mu h_v | \bar{B} \rangle = 2E \xi_P v^\mu, \quad \langle P | \bar{\xi}_n v^\mu \gamma_5 h_v | \bar{B} \rangle = 0, \quad (65)$$

$$\langle P | \bar{\xi}_n n^\mu h_v | \bar{B} \rangle = 2E \xi_P n^\mu, \quad \langle P | \bar{\xi}_n n^\mu \gamma_5 h_v | \bar{B} \rangle = 0.$$

For vector mesons, the matrix elements are written as

$$\langle V | \bar{\xi}_n \gamma^\mu h_v | \bar{B} \rangle = 2E \xi_\perp i \epsilon^{\mu\nu\alpha\beta} \epsilon_\nu^* n_\alpha v_\beta,$$

$$\langle V | \bar{\xi}_n v^\mu h_v | \bar{B} \rangle = \langle V | \bar{\xi}_n n^\mu h_v | \bar{B} \rangle = 0,$$

$$\langle V | \bar{\xi}_n \gamma^\mu \gamma_5 h_v | \bar{B} \rangle = 2E \xi_\perp (\epsilon^{*\mu} - (\epsilon^* \cdot v) n^\mu) + 2E \xi_\parallel (\epsilon^* \cdot v) n^\mu, \quad (66)$$

$$\langle V | \bar{\xi}_n v^\mu \gamma_5 h_v | \bar{B} \rangle = -2E \xi_\parallel (\epsilon^* \cdot v) v^\mu,$$

$$\langle V | \bar{\xi}_n n^\mu \gamma_5 h_v | \bar{B} \rangle = -2E \xi_\parallel (\epsilon^* \cdot v) n^\mu.$$

Using the above relations, we can determine the heavy-to-light form factors at leading order in λ and α_s ,

$$f_+(q^2) = \frac{f_0(q^2)}{X} = \xi_P(E), \quad \frac{2\hat{m}_V}{X} A_0(q^2) = \xi_{\parallel}(E),$$

$$\frac{1+\hat{m}_V}{X} A_1(q^2) = \frac{V(q^2)}{1+\hat{m}_V} = \xi_{\perp}(E), \quad (67)$$

$$\frac{A_2(q^2)}{1+\hat{m}_V} = \xi_{\perp}(E) - \xi_{\parallel},$$

where $X=2E/M$, $\hat{m}_V=m_V/M$. From the results in Sec. IV we can include the perturbative corrections, which change the relation between form factors. We find that, at leading order in λ and at leading logarithmic order in α_s ,

$$f_+ = \xi_P(E) \left[C_1 + \frac{X}{2} C_2 + C_3 \right],$$

$$\frac{f_0}{X} = \xi_P(E) \left[C_1 + \left(1 - \frac{X}{2} \right) C_2 + C_3 \right],$$

$$\frac{V}{1+\hat{m}_V} = C_1 \xi_{\perp}(E),$$

$$\frac{2\hat{m}_V}{X} A_0 = \xi_{\parallel}(E) \left[C_1 + \left(1 - \frac{X}{2} \right) C_2 + C_3 \right], \quad (68)$$

$$\frac{1+\hat{m}_V}{X} A_1 = C_1 \xi_{\perp}(E),$$

$$\frac{A_2}{1+\hat{m}_V} = C_1 \xi_{\perp}(E) - \left(C_1 + \frac{X}{2} C_2 + C_3 \right) \xi_{\parallel}(E).$$

These results are the same as those derived by Bauer et al. [13], though our basis is different from theirs. In Ref. [19] Beneke and Feldmann have calculated the soft contribution to the form factors using the large-energy effective theory. As we have stressed, the matching to the full theory is impossible in this case. However, they judiciously absorbed the infrared divergences into the nonperturbative parameters such as ξ_P , ξ_{\perp} or ξ_{\parallel} by observing the Dirac structure of the matrix elements. In the process, the nonperturbative parameters are defined at each order in α_s . Since we can match the collinear effective theory to the full theory, we can check their calculations. We find that their perturbative corrections in Eqs. (30), (32) and (33) in Ref. [19] are correct when we compare them with the exact results in the collinear effective theory. Now we include the nonperturbative corrections at order λ , along with the perturbative correction.

At order λ , the matrix elements of O_i^{μ} for pseudoscalar mesons are given as

$$\left\langle P \left| \bar{\xi}_n \frac{\vec{h}}{2} \not{p}_{\perp} \gamma^{\mu} h_v \right| \bar{B} \right\rangle = a_P (2v^{\mu} - n^{\mu}),$$

$$\left\langle P \left| \bar{\xi}_n \frac{\vec{h}}{2} \not{p}_{\perp} \gamma^{\mu} \gamma_5 h_v \right| \bar{B} \right\rangle = 0,$$

$$\left\langle P \left| \bar{\xi}_n \frac{\vec{h}}{2} \not{p}_{\perp} v^{\mu} h_v \right| \bar{B} \right\rangle = a_P v^{\mu},$$

$$\left\langle P \left| \bar{\xi}_n \frac{\vec{h}}{2} \not{p}_{\perp} v^{\mu} \gamma_5 h_v \right| \bar{B} \right\rangle = 0, \quad (69)$$

$$\left\langle P \left| \bar{\xi}_n \frac{\vec{h}}{2} \not{p}_{\perp} n^{\mu} h_v \right| \bar{B} \right\rangle = a_P n^{\mu},$$

$$\left\langle P \left| \bar{\xi}_n \frac{\vec{h}}{2} \not{p}_{\perp} n^{\mu} \gamma_5 h_v \right| \bar{B} \right\rangle = 0,$$

$$\langle P | \bar{\xi}_n \not{p}_{\perp}^{\mu} h_v | \bar{B} \rangle = \langle P | \bar{\xi}_n \not{p}_{\perp}^{\mu} \gamma_5 h_v | \bar{B} \rangle = 0,$$

and for vector mesons we have

$$\left\langle V \left| \bar{\xi}_n \frac{\vec{h}}{2} \not{p}_{\perp} \gamma^{\mu} \gamma_5 h_v \right| \bar{B} \right\rangle = a_{V2} \epsilon^{*} \cdot v (2v^{\mu} - n^{\mu}),$$

$$\left\langle V \left| \bar{\xi}_n \frac{\vec{h}}{2} \not{p}_{\perp} \gamma^{\mu} h_v \right| \bar{B} \right\rangle = 0,$$

$$\left\langle V \left| \bar{\xi}_n \frac{\vec{h}}{2} \not{p}_{\perp} v^{\mu} \gamma_5 h_v \right| \bar{B} \right\rangle = -a_{V2} \epsilon^{*} \cdot v v^{\mu},$$

$$\left\langle V \left| \bar{\xi}_n \frac{\vec{h}}{2} \not{p}_{\perp} v^{\mu} h_v \right| \bar{B} \right\rangle = 0, \quad (70)$$

$$\left\langle V \left| \bar{\xi}_n \frac{\vec{h}}{2} \not{p}_{\perp} n^{\mu} \gamma_5 h_v \right| \bar{B} \right\rangle = -a_{V2} \epsilon^{*} \cdot v n^{\mu},$$

$$\left\langle V \left| \bar{\xi}_n \frac{\vec{h}}{2} \not{p}_{\perp} n^{\mu} h_v \right| \bar{B} \right\rangle = 0,$$

$$\langle V | \bar{\xi}_n \not{p}_{\perp}^{\mu} \gamma_5 h_v | \bar{B} \rangle = -a_{V1} [\epsilon^{*\mu} - (\epsilon^{*} \cdot v) n^{\mu}],$$

$$\langle V | \bar{\xi}_n \not{p}_{\perp}^{\mu} h_v | \bar{B} \rangle = a_{V1} i \epsilon^{\mu\nu\alpha\beta} \epsilon_{\nu}^{*} n_{\alpha} v_{\beta}.$$

Finally, for the time-ordered products, we have

$$\langle P | i \int d^4 y T \{ \bar{\xi}_n \gamma^{\mu} h_v(0) \mathcal{L}_1(y) \} | \bar{B} \rangle = b_P n^{\mu},$$

$$\langle P | i \int d^4 y T \{ \bar{\xi}_n v^{\mu} h_v(0) \mathcal{L}_1(y) \} | \bar{B} \rangle = b_P v^{\mu}, \quad (71)$$

$$\langle P|i \int d^4y T\{\bar{\xi}_n n^\mu h_v(0) \mathcal{L}_1(y)\}|\bar{B}\rangle = b_P n^\mu,$$

and the time-ordered products involving the heavy-light currents with γ_5 vanish. For the matrix elements of the time-ordered products for vector mesons, we find

$$\begin{aligned} \langle V|i \int d^4y T\{\bar{\xi}_n \gamma^\mu h_v(0) \mathcal{L}_1(y)\}|\bar{B}\rangle &= b_{V1} i \epsilon^{\mu\nu\alpha\beta} \epsilon_v^* n_\alpha v_\beta, \\ \langle V|i \int d^4y T\{\bar{\xi}_n \gamma^\mu \gamma_5 h_v(0) \mathcal{L}_1(y)\}|\bar{B}\rangle &= b_{V1} [\epsilon^{*\mu} \\ &\quad - (\epsilon^* \cdot v) n^\mu] + b_{V2} \epsilon^* \cdot v n^\mu, \end{aligned}$$

$$\langle V|i \int d^4y T\{\bar{\xi}_n v^\mu h_v(0) \mathcal{L}_1(y)\}|\bar{B}\rangle = 0,$$

$$\langle V|i \int d^4y T\{\bar{\xi}_n n^\mu h_v(0) \mathcal{L}_1(y)\}|\bar{B}\rangle = 0, \quad (72)$$

$$\langle V|i \int d^4y T\{\bar{\xi}_n v^\mu \gamma_5 h_v(0) \mathcal{L}_1(y)\}|\bar{B}\rangle = -b_{V2} \epsilon^* \cdot v v^\mu,$$

$$\langle V|i \int d^4y T\{\bar{\xi}_n n^\mu \gamma_5 h_v(0) \mathcal{L}_1(y)\}|\bar{B}\rangle = -b_{V2} \epsilon^* \cdot v n^\mu.$$

Combining all these form factors, we obtain in the collinear effective theory

$$\begin{aligned} \langle P|V^\mu|\bar{B}\rangle &= 2En^\mu \left[(C_1 + C_3) \xi_P + \frac{1}{2E} [a_P(-B_1 + B_3) + b_P(A_1 + A_3)] \right] + 2Ev^\mu \left[C_2 \xi_P + \frac{1}{2E} [a_P(2B_1 + B_2) + b_P A_2] \right], \\ \langle V|V^\mu|\bar{B}\rangle &= 2Ei \epsilon^{\mu\nu\alpha\beta} \epsilon_v^* n_\alpha v_\beta \left[C_1 \xi_\perp + \frac{1}{2E} (B_4 a_{V1} + A_1 b_{V1}) \right], \\ \langle V|A^\mu|\bar{B}\rangle &= 2E \epsilon^{*\mu} \left[C_1 \xi_\perp + \frac{1}{2E} (B_4 a_{V1} + A_1 b_{V1}) \right] - 2E(\epsilon^* \cdot v) n^\mu \left[C_1 \xi_\perp - (C_1 + C_3) \xi_\parallel + \frac{1}{2E} [(B_1 - B_3) a_{V2} + B_4 a_{V1} \right. \\ &\quad \left. + A_1 b_{V1} - (A_1 + A_3) b_{V2}] \right] + 2E(\epsilon^* \cdot v) v^\mu \left[C_2 \xi_\parallel + \frac{1}{2E} [(2B_1 + B_2) a_{V2} + A_2 b_{V2}] \right]. \end{aligned} \quad (73)$$

From these relations we can obtain the form factors to order λ and to leading-logarithmic order in α_s as

$$\begin{aligned} f_+ &= \left[C_1 + \frac{X}{2} C_2 + C_3 \right] \left[\xi_P + \frac{1}{2E} (a_P + b_P) \right] \\ &\quad - (2-X) C_1 \frac{a_P}{2E}, \end{aligned}$$

$$\begin{aligned} \frac{f_0}{X} &= \left[C_1 + \left(1 - \frac{X}{2}\right) C_2 + C_3 \right] \left[\xi_P + \frac{1}{2E} (a_P + b_P) \right] \\ &\quad - X C_1 \frac{a_P}{2E}, \end{aligned}$$

$$\begin{aligned} \frac{2\hat{m}_V}{X} A_0 &= \left[C_1 + \left(1 - \frac{X}{2}\right) C_2 + C_3 \right] \left[\xi_\parallel + \frac{1}{2E} (a_{V2} + b_{V2}) \right] \\ &\quad - X C_1 \frac{a_{V2}}{2E}, \end{aligned} \quad (74)$$

$$\frac{1 + \hat{m}_V}{X} A_1 = \frac{V}{1 + \hat{m}_V} = C_1 \left(\xi_\perp + \frac{b_{V1}}{2E} \right) + C_3 \frac{a_{V1}}{E},$$

$$\begin{aligned} \frac{A_2}{1 + \hat{m}_V} &= C_1 \left(\xi_\perp + \frac{b_{V1}}{2E} \right) + C_3 \frac{a_{V1}}{E} \\ &\quad - \left[C_1 + \frac{X}{2} C_2 + C_3 \right] \left[\xi_\parallel + \frac{1}{2E} (a_{V2} + b_{V2}) \right] \\ &\quad + (2-X) C_1 \frac{a_{V2}}{2E}. \end{aligned}$$

Here we keep \hat{m}_V explicitly even though $\hat{m}_V \sim \Lambda_{\text{QCD}}/m_b \sim \lambda^2$ in our power counting. It is because meson masses are inserted in the definition of form factors in Eq. (55) without regard to the power counting in the collinear effective theory. However, we neglect the terms proportional to the mass squared of the light meson compared to M^2 . And we use the relations among the Wilson coefficients to express the result in terms of C_i only.

At leading order in λ , there are three unknown nonperturbative parameters $\xi_P(E)$, $\xi_\perp(E)$, and $\xi_\parallel(E)$. These are dimensionless functions. While the Isgur-Wise function in HQET is normalized to one at maximal momentum transfer due to the heavy quark symmetry, there is no constraint in the normalization of these unknown parameters [19]. At order λ , there are six additional nonperturbative parameters: $a_P(E)$, $a_{V1}(E)$, $a_{V2}(E)$, $b_P(E)$, $b_{V1}(E)$, and $b_{V2}(E)$. In our convention, all these parameters have mass dimension, for example,

$$\begin{aligned}
\text{(a)} \quad & \text{---} \bullet \text{---} = \frac{i \bar{n} \cdot p}{\bar{n} \cdot p} \\
\text{(b)} \quad & \text{---} \bullet \text{---} = -g T^a \bar{n} \cdot p^\mu / \bar{n} \cdot p \\
\text{(c)} \quad & \text{---} \bullet \text{---} = g T^a \bar{n} \left[\frac{\bar{n}^\mu}{\bar{n} \cdot p \bar{n} \cdot p'} (2 p_\perp \cdot k_\perp + (p'_\perp - p_\perp) k_\perp) - \frac{\gamma_\perp^\mu k_\perp}{\bar{n} \cdot p} - \frac{k_\perp \gamma_\perp^\mu}{\bar{n} \cdot p'} \right]
\end{aligned}$$

$$\frac{a_P}{E} \sim \lambda \sim \sqrt{\frac{\Lambda_{\text{QCD}}}{m_b}}, \quad (75)$$

where the last relation comes from the kinematics. The remaining five unknown parameters are of the same order in λ . Therefore Eq. (74) is our result for the form factors to order $\sqrt{\Lambda_{\text{QCD}}/m_b}$.

There are interesting relations among the form factors in the effective theory. At zeroth order in λ and α_s , those relations are given by

$$f_+ = \frac{f_0}{X} (= \xi_P), \quad \frac{V}{1 + \hat{m}_V} = \frac{1 + \hat{m}_V}{X} A_1 (= \xi_\perp), \quad (76)$$

$$\frac{2 \hat{m}_V}{X} A_0 = \frac{1 + \hat{m}_V}{X} A_1 - (1 - \hat{m}_V) A_2 (= \xi_\parallel).$$

These relations are modified at order λ and at leading-logarithmic order in α_s as

$$\begin{aligned}
f_+ - \frac{f_0}{X} &= -(1-X) \left[C_2 \left(\xi_P + \frac{1}{2E} (a_P + b_P) \right) + \frac{1}{E} C_1 a_P \right], \\
\frac{V}{1 + \hat{m}_V} &= \frac{1 + \hat{m}_V}{X} A_1, \\
\frac{2 \hat{m}_V}{X} A_0 &= \frac{1 + \hat{m}_V}{X} A_1 - (1 - \hat{m}_V) A_2 \\
&\quad + (1-X) C_2 \left[\xi_\parallel + \frac{1}{2E} (a_{V2} + b_{V2}) \right] \\
&\quad + (1-X) C_1 \frac{a_{V2}}{E}.
\end{aligned} \quad (77)$$

Note that the second relation in Eq. (76) still holds to order λ and at leading-logarithmic order in α_s . And the tree-level results hold only in the limit $X \rightarrow 1$.

VII. CONCLUSION

We have shown that heavy meson decays in which light mesons are emitted with large energy can be consistently described by the collinear effective theory combined with the HQET. And we can obtain a systematic expansion of the effective Lagrangian in powers of λ . Heavy-light currents

FIG. 4. Feynman rules for the effective Lagrangian \mathcal{L}_1 to order g : (a) collinear quark without an external gluon, (b) collinear quark interaction with a soft gluon, and (c) collinear quark interaction with a collinear gluon, and k^μ denotes residual momentum of order λ^2 .

can also be expanded consistently in powers of λ , and the Wilson coefficients of various operators in the effective theory can be computed by matching the effective theory to the full theory. It is crucial to note that the collinear effective theory reproduces the infrared behavior of the full theory by including the effects of collinear gluons.

There is a reparametrization invariance in the collinear effective theory, in which a slight change of the light-cone direction n^μ can be compensated by a change of p_\perp to make the physics invariant under this transformation. If we also require that the theory be invariant under collinear gauge transformations, we can prove that the effective Lagrangian \mathcal{L}_1 at order λ is not renormalized. This reparametrization invariance is also useful in deriving the operators of order λ from the operators of order λ^0 . The Wilson coefficients and the anomalous dimensions can be obtained from the operators which are related by the reparametrization invariance. The reparametrization invariance and the collinear gauge invariance put a serious constraint in the structure of heavy-light currents in the collinear effective theory.

The development of the collinear effective theory casts a renewed view on heavy quark decays in which light quarks are emitted with large energy. Bauer et al. [21] have considered nonleptonic decays using the collinear effective theory, and found that the decay $B \rightarrow D \pi$ is factorized in the heavy quark limit to all orders in α_s . It will be interesting to look into nonleptonic decays of B mesons in the context of the collinear effective theory including higher-order corrections in λ .

What we have not considered here is hard spectator effects, in which spectator quarks interact with the energetic quark through hard gluons. As Beneke and Feldmann [19] pointed out, this contribution can be as important as the soft contribution to the form factors. If we can analyze the hard spectator contribution also in the scheme of the collinear effective theory, we will have a better understanding of form factors in this kinematic region. This is the next subject to be developed.

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APPENDIX: RENORMALIZATION OF \mathcal{L}_1 AT ORDER α_s

In this appendix we show explicitly that the effective Lagrangian \mathcal{L}_1 at order λ in Eq. (16) is not renormalized at one

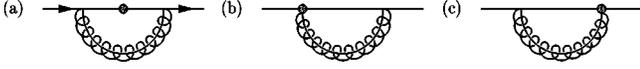


FIG. 5. Feynman diagrams for the renormalization of \mathcal{L}_1 at one loop.

loop. The Feynman rules for the Lagrangian \mathcal{L}_1 to order g is shown in Fig. 4. The derivative is of order λ^2 , and it is replaced by the residual momentum k in momentum space. We will concentrate on the first term in \mathcal{L}_1 , which is of the form

$$O_1 = \bar{\xi}_n \frac{\not{p}_\perp i \not{p}_\perp + i \not{p}_\perp \not{p}_\perp}{\not{n} \cdot \not{p}} \frac{\not{h}}{2} \xi_n, \quad (\text{A1})$$

which is shown in Fig. 4(a). Other terms in \mathcal{L}_1 contribute to the renormalization of O_1 at order λ along with the radiative corrections of O_1 . In order to show that \mathcal{L}_1 is not renormalized, we have to consider all the radiative corrections for the operators shown in Fig. 4. However, we will concentrate on the renormalization of O_1 , since other terms have the same renormalization behavior as O_1 at leading-logarithmic order.

The Feynman diagrams to renormalize O_1 are shown in Fig. 5. And the corresponding diagrams with a soft gluon exchange vanish due to the vertex structure. All the diagrams in Fig. 5 are zero using dimensional regularization for on-shell external states, and the coefficient of O_1 is given by the

tree-level value. In order to see the renormalization group behavior, we have to extract the ultraviolet divergent part by putting the external quark off the mass shell by $p^2 = p_\perp^2$. We will show only the ultraviolet divergent parts here. Calculating the Feynman diagram in Figs. 5(a), 5(b), and 5(c), we obtain

$$M_a = \frac{\alpha_s C_F}{4\pi} \frac{1}{\epsilon} O_1, \quad M_b = -\frac{\alpha_s C_F}{4\pi} \frac{3}{\epsilon} O_1, \quad M_c = \frac{\alpha_s C_F}{4\pi} \frac{3}{\epsilon} O_1, \quad (\text{A2})$$

respectively. Therefore the sum of all the diagrams is given by

$$M = M_a + M_b + M_c = \frac{\alpha_s C_F}{4\pi} \frac{1}{\epsilon} O_1. \quad (\text{A3})$$

When we add the wave function renormalization to this amplitude, the ultraviolet divergences cancel, and the anomalous dimension of O_1 is zero. Therefore we have shown that the operator O_1 is not renormalized at order α_s explicitly. In fact, we have to consider one-loop corrections to the remaining operators in \mathcal{L}_1 . But no other operators are renormalized though we do not show them here. As a result, the Wilson coefficients A_k of the time-ordered products in Eq. (45) come from the Wilson coefficients of the operators J_i^μ alone and not from \mathcal{L}_1 .

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