PHYSICAL REVIEW D, VOLUME 65, 114003

Hadronic decays of χ_{bJ} into light bottom squarks

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(Received 12 March 2002; published 30 May 2002)

We calculate the rates for inclusive hadronic decay of the three χ_{bJ} states into a pair of light bottom squarks as a function of the masses of the bottom squark and the gluino. We include color-singlet and color-octet configurations. The color-octet contribution is found to be insignificant for the χ_{b0} but can dominate in the χ_{b2} case if current lattice estimates are used for the color-octet matrix element. In comparison with the standard model values, bottom squark decays can increase the predicted hadronic width of the χ_{b0} by as much as 33%, for very small bottom squark masses and gluino masses in the range of 12 GeV, but make a small contribution in the cases of χ_{b1} and χ_{b2}. Data from decays of the χ_{bJ} states could provide significant new bounds on the existence and masses of supersymmetric particles.

DOI: 10.1103/PhysRevD.65.114003 PACS number(s): 12.38.Bx, 13.85.Ni, 13.87.Fh, 14.40.Gx

I. INTRODUCTION

The possible existence of a light bottom squark $\tilde{b}$ with mass similar to or less than that of the bottom quark b is investigated in several theoretical papers [1–7]. This hypothesis is not inconsistent with direct experimental searches and indirect constraints from other observables [8–10]. If the mass $m_{\tilde{b}}$ is less than half of that of the P-wave χ_b resonances, then the direct decay $\chi_b \to \tilde{b}\tilde{b}^*$ could proceed with sufficient rate for observation, particularly in data being accumulated at the Cornell Electron Storage Ring (CESR) facility [11]. In this paper, we compute the expected rates for decays of the three χ_{bJ} states into a pair of bottom squarks as a function of the masses of the bottom squark and the gluino. The mass of the gluino $\tilde{g}$ enters because the gluino is exchanged in the relevant decay subprocesses. The $\tilde{g}$ and the $\tilde{b}$ are the spin-1/2 and spin-0 supersymmetric (SUSY) partners of the gluon (g) and bottom quark. This paper extends previous work of one of us [7] on decays of the Upsilon states, $\Upsilon(nS) \to \tilde{b}\tilde{b}^*, n=1-4$.

In the remainder of this Introduction, we summarize the phenomenological motivation and status of the hypothesis of a light bottom squark along with a light gluino and then present an outline of the remainder of the paper. A recent presentation of the experimental properties of the χ_{bJ} states may be found in Ref. [12].

The hypothesis of a relatively light color-octet gluino $\tilde{g}$ (mass $\simeq 12$ to 16 GeV) that decays with 100% branching fraction into a bottom quark b and a light color-triplet bottom squark $\tilde{b}$ (mass $\simeq 2$ to 5.5 GeV) is proposed in Ref. [1] in order to explain the observed excess cross section for production of bottom quarks at hadron colliders. In this scenario $\tilde{b}$ is the lightest SUSY particle(LSP), and the masses of all other SUSY particles are arbitrarily heavy, i.e., of order the electroweak scale or greater. Improved agreement is obtained with the magnitude and shape of the transverse momentum distribution of bottom-quark production at hadron colliders. The proposal is consistent with measurements of the time-averaged $B^0\overline{B}^0$ mixing probability [1]. The $\tilde{b}$ either picks up a light quark, becoming a spin-1/2 "mesino," perhaps living long enough to escape from a typical collider detector, or it decays promptly via baryon-number R-parity violation into a pair of hadronic jets [10,13].

There are important restrictions on the existence and couplings of bottom squarks from precise measurements of Z^0 decays. A light $\tilde{b}$ would be ruled out unless its coupling to the Z^0 is very small. The squark couplings to the Z^0 depend on the mixing angle $\theta_{\tilde{b}}$. If the light bottom squark ($\tilde{b}_1$) is an appropriate mixture of left-handed and right-handed bottom squarks, its lowest-order (tree-level) coupling to the Z^0 can be arranged to be small [2] if $\sin^2\theta_{\tilde{b}} \sim 1/6$. The exclusion by the CLEO Collaboration [8] of a $\tilde{b}$ with mass 3.5 to 4.5 GeV does not apply since their analysis focuses only on the leptonic decays $\tilde{b} \to c l \tilde{\nu}$ and $\tilde{b} \to c l$. The $\tilde{b}$ need not decay leptonically nor into charm. The search by the DELPHI Collaboration [9] for long-lived squarks in their $\gamma\gamma$ event sample is not sensitive to $m_{\tilde{b}} < 15$ GeV. Bottom squarks make a small contribution to the ratio R of the inclusive cross section for $e^+e^- \to$ hadrons divided by that for $e^+e^- \to \mu^+\mu^-$, requiring an accuracy for detection at the level of 2% or so, much greater than that of current measurements in the 6 to 7% range [14]. In e^+e^- production, resonances in the $\tilde{b}\tilde{b}^*$ system are difficult to extract from backgrounds [15], but $\gamma\gamma$ collisions may be more promising. The angular distribution of hadronic jets produced in e^+e^- annihilation can be examined in order to bound the contribution of scalar-quark production. Spin-1/2 quarks and spin-0 squarks emerge with different distributions, $(1 \pm \cos^2\theta)$, respectively. The angular distribution measured by the CELLO Collaboration [16] is consistent with the production of a single pair of charge-1/3 squarks along with five flavors of quark-antiquark pairs.

The possibility that the gluino may be much less massive than most other supersymmetric particles is intriguing from different points of view [17–20]. An early study by the UA1 Collaboration [21] excludes $\tilde{g}$'s in the mass range $4 < m_{\tilde{g}} < 53$ GeV, but it starts from the assumption that there is a light neutralino $\tilde{\chi}_1^0$ whose mass is less than the mass of the gluino. The conclusion is based on the absence of the expected decay $\tilde{g} \to q + \overline{q} + \not{E}_T$, where $\not{E}_T$ represents the missing energy associated with the $\tilde{\chi}_1^0$. In the scenario discussed here, this decay process does not occur since the bottom squark is the LSP and the $\tilde{\chi}_1^0$ mass is presumed to be large (i.e., >50 GeV). An analysis of 2- and 4-jet events by the ALEPH Collaboration [22] disfavors $\tilde{g}$'s with mass $m_{\tilde{g}} < 6.3$ GeV but not $\tilde{g}$'s in the mass range relevant for the SUSY interpretation of the bottom quark production cross section. A similar analysis is reported by the OPAL Collaboration [23]. A light $\tilde{b}$ is not excluded by the ALEPH analysis. A very precise measurement of the β function in QCD is potentially the most sensitive probe of the existence of a light gluino since the color-octet gluino makes a contribution to β equivalent to that of 3 new flavors of quarks. However, the analysis of data must take into account the effects of SUSY-QCD production processes. The combined ranges of $\tilde{b}$ and $\tilde{g}$ masses proposed in Ref. [1] are compatible with renormalization group equation constraints and the absence of color and charge breaking minima in the scalar potential [3].

In Sec. II, we begin with a discussion of factorization. The χ_b is treated as a $b\bar{b}$ bound system and described nonperturbatively. The transition of the (on-shell) $b\bar{b}$ system into the observed final state is calculated in QCD perturbation theory. In Sec. II, we also outline the projection of the on-shell $b\bar{b}$ system into the states of spin, parity, and angular momentum of interest. We present our explicit calculation of $\chi_b \to \tilde{b}\tilde{b}^*$ in Sec. III. We include both color-singlet and color-octet configurations. Results and conclusions are summarized in Sec. IV. When compared with the leading-order expectation in the standard model, the bottom squark contribution can increase the predicted hadronic width of the χ_{b0} by as much as 33% for small bottom squark masses, O(2 to 4 GeV), and gluino masses in the range of 12 GeV. However, bottom squark decays make a negligible contribution in comparison with the standard model value in the χ_{b1} and χ_{b2} cases. The color-octet contribution to bottom squark decays is insignificant for the χ_{b0} but can dominate in the χ_{b2} case if current lattice estimates are used for the color-octet matrix element. Data from decays of the χ_b states could provide significant new bounds on the existence and masses of supersymmetric particles.

II. FACTORIZATION AND CONVENTIONAL DECAYS OF THE χ_{bJ}

To compute the decay of a heavy quarkonium state, we begin with the statement of factorization. The initial χ_b is taken to be a bound state $b\bar{b}$ of bottom quarks. To the extent that the b quarks are heavy enough, the bound state aspects may be described with nonrelativistic quantum chromodynamics (NRQCD) [24]. The decay dynamics are assumed to be described by short-distance perturbative QCD, in turn also justified in part by the large mass of the b quark. The decay rate of the heavy quarkonium state is then expressed in the factored form [25,26]

$$\Gamma(\chi_{bJ}) = \sum_n C_n \langle \chi_{bJ} | \mathcal{O}_n | \chi_{bJ} \rangle. \tag{1}$$

The summation index n stands for the spectroscopic state ${}^{2S+1}L_J^{(1,8)}$ of the $b\bar{b}$ pair. The superscript to the right (1,8) stands for the color quantum number, singlet [27,28] and octet [25], respectively. The short-distance coefficient C_n, describes the inclusive transition of an on-mass-shell $b\bar{b}$ system into the relevant final state, and it is calculable perturbatively in a series in the strong coupling strength $\alpha_s(m_b)$. The nonperturbative matrix element $\langle \chi_{bJ} | \mathcal{O}_n | \chi_{bJ} \rangle$ represents the probability that a χ_{bJ} meson evolves into a free $b\bar{b}$ pair with quantum numbers n. The four quark operator $\mathcal{O}_n$ is defined in Ref. [26].

Although the recent CDF measurement of the polarization of prompt J/ψ's presents a serious challenge for the NRQCD factorization formalism for inclusive charmonium production [29], factorization in the bottomonium case is believed to be safe since m_b is well separated from the long distance scale $m_b v_b^2$. For example, the recent CDF measurement of $\Upsilon(nS)$ polarization agrees with the NRQCD prediction [30] based on matrix elements fitted to run IB data [31]. Furthermore, factorization is safer in the decay process than in production. A recent review can be found in Chap. 9 of Ref. [32].

An alternative, not unrelated physical picture begins with a Fock state expansion of the χ_b into the minimal $|b\bar{b}\rangle$ color-singlet component and higher components such as $|b\bar{b}g\rangle$, with the $b\bar{b}$ pair in a color-octet configuration, $|b\bar{b}q\bar{q}\rangle, |b\bar{b}gg\rangle$, and so forth.

A velocity scaling rule in NRQCD [26] allows one to order the contributions to the decay rate Eq. (1) and to truncate the series on the right-hand side of Eq. (1) in a power series in v_b, the mean velocity of the b in the χ_b rest frame. For χ_{bJ}, there are two potentially equally important contributions to the decay rate. One is the color-singlet spin-triplet P-wave state that scales as $\langle \chi_{bJ} | \mathcal{O}({}^3P_J^{(1)}) | \chi_{bJ} \rangle / m_b^2 \sim m_b^3 v_b^2$. The other is the color-octet spin-triplet S-wave state that is proportional to $\langle \chi_{bJ} | \mathcal{O}({}^3S_1^{(8)}) | \chi_{bJ} \rangle \sim m_b^3 v_b^2$, where we keep the mass dimension while suppressing the v_b dependence of the heavy quark fields and the quarkonium state. The scaling factor v_b^2 of the color-octet matrix element arises from the chromoelectric dipole transition rate from a color-singlet P-wave state into a color-octet spin-triplet state. This color-octet channel was introduced to resolve the infrared problem in the next-to-leading order QCD corrections to light-hadron decays of P-wave quarkonium [25].

In the case of Υ decay to bottom quarks treated in Ref. [7], the leading color-singlet contribution is proportional to $\alpha_s^2 v_b^0$. The leading contribution in the standard model decay of Υ to light hadrons is of order $\alpha_s^3 v_b^0$, with one more power of α_s than in the SUSY case. The SUSY rate is suppressed, however, by the mass of the exchanged gluino in the amplitude and the nonzero bottom squark masses. The leading color-octet contribution in Υ decay is of order $\alpha_s^2 v_b^4$.

In conventional QCD, the χ_{bJ} states are assumed to decay via the transition of the $b\bar{b}$ system into a pair of massless gluons that, in turn, materialize as light hadrons [27]. The short-distance coefficients C_n are known in next-to-leading order in α_s for the color-singlet states $n = {}^3P_J^{(1)}$ [33,34] and for the color-octet state $n={}^3S_1^{(8)}$ [34,35]. We quote the leading-order result in $\alpha_s v_b^2$ [25–27,36]:

$$\Gamma^{\rm SM}(\chi_{b0})=\frac{4\pi\alpha_s^2\mathcal{H}_1}{3m_b^4}\left(1+\frac{n_f m_b^2\mathcal{H}_8}{4\mathcal{H}_1}\right), \tag{2a}$$

$$\Gamma^{\rm SM}(\chi_{b1})=\frac{4\pi\alpha_s^2\mathcal{H}_1}{3m_b^4}\left(0+\frac{n_f m_b^2\mathcal{H}_8}{4\mathcal{H}_1}\right), \tag{2b}$$

$$\Gamma^{\rm SM}(\chi_{b2})=\frac{4\pi\alpha_s^2\mathcal{H}_1}{3m_b^4}\left(\frac{4}{15}+\frac{n_f m_b^2\mathcal{H}_8}{4\mathcal{H}_1}\right). \tag{2c}$$

The superscript SM designates the standard model contribution without SUSY terms. The number of flavors $n_f=4$. The matrix elements in Eq. (2) are the color-singlet term $\mathcal{H}_1=\langle h_b|\mathcal{O}({}^1P_1^{(1)})|h_b\rangle$ and the color-octet term $\mathcal{H}_8=\langle h_b|\mathcal{O}({}^1S_0^{(8)})|h_b\rangle$; h_b is the spin-singlet P-wave bottomonium state. Heavy quark spin symmetry is used in the approximations $\langle\chi_{bJ}|\mathcal{O}({}^3P_J^{(1)})|\chi_{bJ}\rangle=\mathcal{H}_1+O(v_b^2)$, and $\langle\chi_{bJ}|\mathcal{O}({}^3S_1^{(8)})|\chi_{bJ}\rangle=\mathcal{H}_8+O(v_b^2)$. The color-singlet matrix element can be expressed in terms of the derivative of the radial wave function at the origin, $\mathcal{H}_1=(3N_c/2\pi)|R'(0)|^2+O(v_b^2)$. Numerical values of these matrix elements are available from lattice measurements [37,38]:

$$\mathcal{H}_1=2.7\ \ \mathrm{GeV}^5,\quad \mathcal{H}_8/\mathcal{H}_1=2.275\times10^{-3}\ \ \mathrm{GeV}^{-2}. \tag{3}$$

As is evident in Eq. (2b), the decay rate for the χ_{b1} is purely color octet in nature at leading order in α_s because the transition of the $J^{PC}=1^{++}$ color-singlet state into two massless gluons is forbidden. Since $v_b^2\sim0.1$, the color-octet matrix element $\mathcal{H}_8$ is small for bottomonium (for charmonium $v_c^2\sim0.3$ is not negligible). The color-octet contributions in Eq. (2) are estimated to be at the 5% level in the χ_{b0} case and at the 20% level in the χ_{b2} case. The numerical values of the predicted SM hadronic widths in Eq. (2) are approximately 0.8 MeV, 0.04 MeV, and 0.2 MeV for the χ_{b0}, χ_{b1}, and χ_{b2}, respectively.

Derivation of the short-distance term

A practical procedure to extract the short-distance coefficient C_n in Eq. (1) begins from a calculation of the free $b\bar{b}$ amplitude with specified spin, color, and orbital angular momentum. We use P and q to denote the total and the relative momenta of the pair. The heavy quark momentum p_b and its counterpart $p_{\bar{b}}$ are

$$p_b=\frac{P}{2}+q,\quad p_{\bar{b}}=\frac{P}{2}-q. \tag{4}$$

In the rest frame of the $b\bar{b}$ pair, the components of the vectors become $P=(2E_q,\mathbf{0})$, $q=(0,\mathbf{q})$, $p_b=(E_q,\mathbf{q})$, and $p_{\bar{b}}=(E_q,-\mathbf{q})$. In the nonrelativistic limit, the invariant mass of the pair can be expanded as $2E_q=2\sqrt{m_b^2+\mathbf{q}^2}=2m_b+O(\mathbf{q}^2)$.

The amplitude for transition of the free $b\bar{b}$ system into a final state is denoted $\mathcal{M}_{b\bar{b}}$. It includes a spinor factor $u_\alpha^i(p_b,s)\overline{v_\beta^j}(p_{\bar{b}},\bar{s})$ where (s,α,i) and $(\bar{s},\beta,j)$ are the (spin, spinor index, color index) of the b and $\bar{b}$, respectively. The spin-triplet state of the pair can be projected from the outer product of the heavy quark spinors upon multiplying with Clebsch-Gordan coefficients [27,28,39,40]:

$$\mathcal{M}_{b\bar{b}}=\mathcal{M}^{ji}_{\beta\alpha}(p_b,p_{\bar{b}})\sum_{s,\bar{s}}\langle 1\lambda|s,\bar{s}\rangle u_\alpha^i(p_b,s)\overline{v_\beta^j}(p_{\bar{b}},\bar{s}), \tag{5a}$$

$$\sum_{s,\bar{s}}\langle 1\lambda|s,\bar{s}\rangle u_\alpha^i(p_b,s)\overline{v_\beta^j}(p_{\bar{b}},\bar{s}) =\Lambda^{ijk}_{\alpha\beta}\langle 0|\chi^\dagger\sigma^k\psi|b\bar{b}\rangle+\Lambda^{aijk}_{\alpha\beta}\langle 0|\chi^\dagger T^a\sigma^k\psi|b\bar{b}\rangle, \tag{5b}$$

$$\Lambda^{ijk}_{\alpha\beta}=\frac{\delta^{ij}}{N_c}\frac{1}{4m_b}[(\not{p}_b+m_b)\gamma_\mu(\not{p}_{\bar{b}}-m_b)]_{\alpha\beta}L^\mu{}_k, \tag{5c}$$

$$\Lambda^{aijk}_{\alpha\beta}=2T^a_{ij}\frac{1}{4m_b}[(\not{p}_b+m_b)\gamma_\mu(\not{p}_{\bar{b}}-m_b)]_{\alpha\beta}L^\mu{}_k, \tag{5d}$$

where $L^\mu{}_k$ is the boost matrix from the $b\bar{b}$ rest frame to the frame in which the $b\bar{b}$ pair has momentum P. The operators ψ and $\chi^\dagger$ are the annihilation operators for the nonrelativistic b and $\bar{b}$ states, respectively. The projection operator Eq. (5) is valid up to terms of $O(\mathbf{q}^2)$. The projection operator valid to all orders in $\mathbf{q}^n$ is derived in Ref. [40]. The heavy quark state in Eq. (5b) is specified with the nonrelativistic normalization $\langle b(\mathbf{p})|b(\mathbf{q})\rangle=(2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{q})$, whereas the Dirac spinors have relativistic normalization, $\bar{u}u=\bar{v}v=2m_b$.

Substituting the projection operator Eq. (5b) into Eq. (5a) and expanding with respect to $\mathbf{q}$ up to $O(\mathbf{q}^2)$, we can express the amplitude $\mathcal{M}_{b\bar{b}}$ as a linear combination of a color-singlet P-wave term and a color-octet S-wave term:

$$\mathcal{M}_{b\bar{b}}=\mathcal{A}^{ij}\langle 0|\chi^\dagger\mathbf{q}^i\sigma^j\psi|b\bar{b}\rangle+\mathcal{B}^{ai}\langle 0|\chi^\dagger T^a\sigma^i\psi|b\bar{b}\rangle. \tag{6}$$

The P-wave amplitude can be decomposed further into the total angular momentum components, $J=0,1,2$:

$$\begin{aligned}\mathcal{M}_{b\bar{b}}=&\mathcal{A}_0\langle 0|\chi^\dagger\mathcal{K}({}^3P_0^{(1)})\psi|b\bar{b}\rangle+\mathcal{A}_1^i\langle 0|\chi^\dagger\mathcal{K}^i({}^3P_1^{(1)})\psi|b\bar{b}\rangle\\&+\mathcal{A}_2^{ij}\langle 0|\chi^\dagger\mathcal{K}^{ij}({}^3P_2^{(1)})\psi|b\bar{b}\rangle\\&+\mathcal{B}^{ai}\langle 0|\chi^\dagger\mathcal{K}^{ai}({}^3S_1^{(8)})\psi|b\bar{b}\rangle.\end{aligned} \tag{7}$$

The coefficients in Eq. (7) are $\mathcal{A}_0=\mathcal{A}^{ii}/\sqrt{3}$, $\mathcal{A}_1^i=\boldsymbol{\epsilon}^{ijk}\mathcal{A}^{jk}/\sqrt{2}$, and $\mathcal{A}_2^{ij}=(A^{ij}+A^{ji})/2-\delta^{ij}\mathcal{A}^{kk}/3$. The corresponding operators are $\mathcal{K}({}^3P_0)=\mathbf{q}\cdot\boldsymbol{\sigma}/\sqrt{3}$, $\mathcal{K}^i({}^3P_1)=(\mathbf{q}\times\boldsymbol{\sigma})^i/\sqrt{2}$, and $\mathcal{K}^{ij}({}^3P_2)=(\mathbf{q}^i\sigma^j+\mathbf{q}^j\sigma^i)/2-\delta^{ij}\boldsymbol{\sigma}\cdot\mathbf{q}/3$.

The squared and spin-averaged amplitude for the ${}^3P_J$ state is

$$\overline{|\mathcal{M}_{b\bar{b}}|^2}=\sum_{J=0}^{2}|\mathcal{A}_J|^2\langle b\bar{b}|\mathcal{O}(^3P_J^{(1)})|b\bar{b}\rangle$$
$$+|\mathcal{B}|^2\langle b\bar{b}|\mathcal{O}(^3S_1^{(8)})|b\bar{b}\rangle. \tag{8}$$

The coefficients $|\mathcal{A}_J|^2$ and $|\mathcal{B}|^2$ are obtained by contracting to rotationally symmetric tensors:

$$|\mathcal{A}_1|^2=\frac{\delta^{ij}}{3}\mathcal{A}_1^i\mathcal{A}_1^{j*}, \tag{9a}$$

$$|\mathcal{A}_2|^2=\frac{1}{5}\left[\frac{1}{2}(\delta^{ik}\delta^{jl}+\delta^{il}\delta^{jk})-\frac{1}{3}\delta^{ij}\delta^{kl}\right]\mathcal{A}_2^{ij}\mathcal{A}_2^{kl*}, \tag{9b}$$

$$|\mathcal{B}|^2=\frac{\delta^{ij}}{3}\mathcal{B}^i\mathcal{B}^{j*}. \tag{9c}$$

The coefficient $\mathcal{B}^{ai}$ of the color-octet term in the amplitude Eq. (7) has a free color index. However, owing to color conservation, the squared term has the form $\mathcal{B}^{ai}\mathcal{B}^{*bj}=\delta^{ab}\mathcal{B}^i\mathcal{B}^{*j}$, and the color index of the color-octet matrix element is contracted with the factor δ^{ab}.

The last step to obtain the short distance coefficients is to introduce the normalization factor $(2M_{\chi_{QJ}})/(2E_q)^2=1/m_Q+O(\mathbf{q}^2)$. This factor appears since we use nonrelativistic normalization in the matrix elements for the $b\bar{b}$ and χ_{bJ} states. The short-distance coefficients are therefore expressed as

$$C(^3P_J^{(1)})=\frac{\Phi}{4m_b^2}|\mathcal{A}_J|^2,\quad C(^3S_1^{(8)})=\frac{\Phi}{4m_b^2}|\mathcal{B}|^2. \tag{10}$$

The nonrelativistic zero-binding energy approximation $m_{\chi_{bJ}}=2m_b$ is used. For a final state composed of two particles with identical mass m_f, the phase space factor $\Phi=(1/8\pi)(1-m_f^2/m_b^2)^{1/2}$.

III. DECAY INTO BOTTOM SQUARKS

In this section we treat the inclusive decay of a χ_b into a pair of bottom squarks $\tilde{b}$ of mass $m_{\tilde{b}}$ carrying four-momenta k_1 and k_2 respectively. The relevant short-distance perturbative subprocess is

$$b+\bar{b}\rightarrow\tilde{b}+\tilde{b}^*. \tag{11}$$

The Feynman diagrams for this subprocess are shown in Fig. 1. In Fig. 1(a), a t-channel gluino $\tilde{g}$ of mass $m_{\tilde{g}}$ is exchanged. The subprocess sketched in Fig. 1(b) in which the $b\bar{b}$ pair annihilates through an intermediate gluon into a $\tilde{b}\tilde{b}^*$ pair is absent in the color-singlet approximation because the initial state is a color singlet. Both Fig. 1(a) and Fig. 1(b) contribute to the color-octet amplitude. Figure 1(b) is independent of the gluino mass, and its contribution survives therefore even in the limit of gluinos with very large mass. Figure 1(a) contributes to both the $^3S_1^{(1)}$ and $^3P_J^{(1)}$ channels whereas the analogous SM process in which gluons are produced and a b quark is exchanged contributes only to $^3P_{0,2}^{(1)}$.

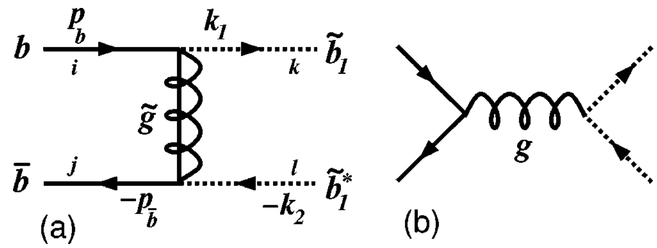

FIG. 1. Feynman diagrams for the process $b\bar{b}\rightarrow\tilde{b}_1\tilde{b}_1^*$.

A. Bottom squark mixing and couplings

The mass eigenstates of the bottom squarks, $\tilde{b}_1$ and $\tilde{b}_2$ are mixtures of left-handed (L) and right-handed (R) bottom squarks, $\tilde{b}_L$ and $\tilde{b}_R$. The mixing is expressed as

$$|\tilde{b}_1\rangle=\sin\theta_{\tilde{b}}|\tilde{b}_L\rangle+\cos\theta_{\tilde{b}}|\tilde{b}_R\rangle, \tag{12}$$

$$|\tilde{b}_2\rangle=\cos\theta_{\tilde{b}}|\tilde{b}_L\rangle-\sin\theta_{\tilde{b}}|\tilde{b}_R\rangle. \tag{13}$$

In our notation, the lighter mass eigenstate is denoted $\tilde{b}_1$. For the case under consideration, the mixing of the bottom squark is determined by the condition that the coupling of the lighter $\tilde{b}$ to the Z boson be small [2], namely $\sin^2\theta_{\tilde{b}}\simeq 1/6$.

We may also express the mass eigenstate $\tilde{b}_1$ in terms of states of definite parity, the $J^P=0^+$ scalar and $J^P=0^-$ pseudo-scalar. Starting from the relationships

$$|0^+\rangle=\frac{1}{\sqrt{2}}(|\tilde{b}_R\rangle+|\tilde{b}_L\rangle), \tag{14}$$

$$|0^-\rangle=\frac{1}{\sqrt{2}}(|\tilde{b}_R\rangle-|\tilde{b}_L\rangle), \tag{15}$$

we obtain

$$|\tilde{b}_1\rangle=\frac{1}{\sqrt{2}}(\cos\theta_{\tilde{b}}+\sin\theta_{\tilde{b}})|0^+\rangle+\frac{1}{\sqrt{2}}(\cos\theta_{\tilde{b}}-\sin\theta_{\tilde{b}})|0^-\rangle. \tag{16}$$

The $\tilde{b}_1$ is a pure $J^P=0^+$ scalar only if $\sin\theta_{\tilde{b}}=\cos\theta_{\tilde{b}}=1/\sqrt{2}$.

The coupling at the three-point vertex in which a b quark enters and a $\tilde{b}_1$ squark emerges [the upper vertex in Fig. 1(a)] is

$$ig_s\sqrt{2}T_{ki}^a[\cos\theta_{\tilde{b}}P_R-\sin\theta_{\tilde{b}}P_L], \tag{17}$$

where i and k are the color indices of the incident b and final $\tilde{b}$, respectively, and a labels the color of the exchanged gluino. Here, $P_L=(1-\gamma_5)/2$ and $P_R=(1+\gamma_5)/2$. At the lower vertex where an antiquark enters and $\tilde{b}_1^*$ emerges, the coupling is

$$ig_s\sqrt{2}T^a_{jl}[\cos\theta_{\tilde{b}}P_L-\sin\theta_{\tilde{b}}P_R], \tag{18}$$

where j and l are the color indices of the incident $\bar{b}$ and final $\tilde{b}^*$, respectively.

B. Subprocess amplitude

The amplitude of the process $b\bar{b}\to\tilde{b}_\lambda\tilde{b}^*_{\bar{\lambda}}$, where $\lambda(\bar{\lambda})=-1$ for $\tilde{b}_L(\tilde{b}^*_L)$ and $\lambda=1$ for $\tilde{b}_R(\tilde{b}^*_R)$, respectively, is

$$\begin{aligned}\mathcal{M}^{ji}_{\beta\alpha}(\lambda,\bar{\lambda})=&\left[\frac{ig_s\bar{\lambda}}{\sqrt{2}}T^a_{jl}(1-\bar{\lambda}\gamma_5)\frac{i}{\not{p}_b-\not{k}_1-m_{\tilde{g}}}\right.\\&\left.\times\frac{ig_s\lambda}{\sqrt{2}}T^a_{ki}(1+\lambda\gamma_5)\right]_{\beta\alpha}\\&+(-ig_sT^a_{ji}\gamma^\mu)_{\beta\alpha}\frac{-ig_{\mu\nu}}{P^2}\\&\times[-ig_sT^a_{kl}(k_1-k_2)^\nu].\end{aligned}\tag{19}$$

Momentum labels are specified in Fig. 1; P is the four-momentum of the χ_b. As remarked earlier, α and β are the spinor indices of the b and $\bar{b}$. The first term in Eq. (19) represents the diagram shown in Fig. 1(a), and the other term is for the diagram of Fig. 1(b). We do not simplify the factors in Eq. (19) so that the Feynman rules can be identified in this expression. The relatively large masses of the exchanged gluino Fig. 1(a) and of the χ_b should justify the use of simple perturbation theory to compute the decay amplitude; g_s is the strong coupling, $\alpha_s=g_s^2/4\pi$.

Using the mixing relation Eq. (13), we can display the explicit mixing angle dependence of the amplitude for the light bottom squark pair final state in the form

$$\begin{aligned}\mathcal{M}^{ji}_{\beta\alpha}=&\sin^2\theta_{\tilde{b}}\mathcal{M}^{ji}_{\beta\alpha}(+,+)+\cos^2\theta_{\tilde{b}}\mathcal{M}^{ji}_{\beta\alpha}(-,-)\\&+\sin\theta_{\tilde{b}}\cos\theta_{\tilde{b}}(\mathcal{M}^{ji}_{\beta\alpha}(+,-)+\mathcal{M}^{ji}_{\beta\alpha}(-,+)).\end{aligned}$$

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Substituting $\mathcal{M}^{ji}_{\beta\alpha}$ into Eq. (5a) and using the projection operator Eq. (5b), we derive an expression in the form of Eq. (6). Useful color factor formulas are

$$[T^a_{ki}T^a_{jl}]\delta_{ij}=\frac{N_c^2-1}{2N_c}\delta_{kl}, \tag{20a}$$

$$[T^a_{ki}T^a_{jl}]T^b_{ij}=-\frac{1}{2N_c}T^b_{kl}. \tag{20b}$$

The partial-wave amplitudes in Eq. (7) are found to be

$$\mathcal{A}_0=\frac{32\pi\alpha_s}{9\sqrt{3}}\delta_{kl}\frac{6\sin\theta_{\tilde{b}}\cos\theta_{\tilde{b}}m_{\tilde{g}}(m_b^2+m_{\tilde{g}}^2-m_{\tilde{b}}^2)-m_b(m_b^2+3m_{\tilde{g}}^2-m_{\tilde{b}}^2)}{(m_b^2+m_{\tilde{g}}^2-m_{\tilde{b}}^2)^2}, \tag{21a}$$

$$\mathcal{A}_1^i=\frac{64\pi\alpha_s}{9\sqrt{2}}\delta_{kl}\frac{k_{1\mu}L^\mu{}_i(\cos^2\theta_{\tilde{b}}-\sin^2\theta_{\tilde{b}})}{(m_b^2+m_{\tilde{g}}^2-m_{\tilde{b}}^2)}, \tag{21b}$$

$$\mathcal{A}_2^{ij}=\frac{64\pi\alpha_s}{9}\delta_{kl}\frac{m_bk_{1\mu}k_{1\nu}L^\mu{}_iL^\nu{}_j}{(m_b^2+m_{\tilde{g}}^2-m_{\tilde{b}}^2)^2}, \tag{21c}$$

$$\mathcal{B}^{ai}=\frac{4\pi\alpha_s}{3}T^a_{kl}\frac{k_{1\mu}L^\mu{}_i}{m_b}\left(\frac{2m_b^2}{m_b^2+m_{\tilde{g}}^2-m_{\tilde{b}}^2}+3\right). \tag{21d}$$

Using the polarization summation relations in Eq. (9), we obtain the short-distance factors of Eq. (10). Substituting these short-distance coefficients into the factorization formula Eq. (1), we derive

$$\Gamma(\eta_b\to\tilde{b}_1\tilde{b}_1^*)=0, \tag{22a}$$

$$\Gamma(\Upsilon\to\tilde{b}_1\tilde{b}_1^*)=\frac{32\pi\alpha_s^2\mathcal{G}_1}{81}\left(1-\frac{m_{\tilde{b}}^2}{m_b^2}\right)^{3/2}\frac{m_b^2}{(m_b^2+m_{\tilde{g}}^2-m_{\tilde{b}}^2)^2}, \tag{22b}$$

$$\Gamma(\chi_{bJ}\to\tilde{b}_1\tilde{b}_1^*)=\frac{4\pi\alpha_s^2\mathcal{H}_1}{3m_b^4}\left(1-\frac{m_{\tilde{b}}^2}{m_b^2}\right)^{1/2}\left(D_J+D_8\frac{m_b^2\mathcal{H}_8}{\mathcal{H}_1}\right), \tag{22c}$$

where $\mathcal{G}_1=\langle\Upsilon|\mathcal{O}(^3S_1^{(1)})|\Upsilon\rangle=(N_c/2\pi)|R(0)|^2$, and $R(0)$ is the Υ wave function at the origin.

The coefficients are

$$D_0=\frac{8}{27}\frac{m_b^2[6m_{\tilde g}(m_b^2+m_{\tilde g}^2-m_{\tilde b}^2)\sin\theta_{\tilde b}\cos\theta_{\tilde b}-m_b(m_b^2+3m_{\tilde g}^2-m_{\tilde b}^2)]^2}{(m_b^2+m_{\tilde g}^2-m_{\tilde b}^2)^4}, \tag{23a}$$

$$D_1=\frac{16}{27}\left(1-\frac{m_{\tilde b}^2}{m_b^2}\right)\frac{m_b^4(\cos^2\theta_{\tilde b}-\sin^2\theta_{\tilde b})^2}{(m_b^2+m_{\tilde g}^2-m_{\tilde b}^2)^2}, \tag{23b}$$

$$D_2=\frac{64}{135}\left(1-\frac{m_{\tilde b}^2}{m_b^2}\right)^2\frac{m_b^8}{(m_b^2+m_{\tilde g}^2-m_{\tilde b}^2)^4}, \tag{23c}$$

$$D_8=\frac{1}{144}\left(1-\frac{m_{\tilde b}^2}{m_b^2}\right)\left(3+\frac{2m_b^2}{m_b^2+m_{\tilde g}^2-m_{\tilde b}^2}\right)^2. \tag{23d}$$

Equation (22b) was derived previously in Ref. [7]. Equation (22a) indicates that η_b decay is not allowed in lowest order. In the color-singlet case, reduction of the relevant trace provides a lowest-order amplitude proportional to $(\cos^2\theta_{\tilde b}-\sin^2\theta_{\tilde b})p_\eta\cdot(p_\eta-2k_1)$, where p_η and k_1 are the four-vector momenta of the η_b and one of the final $\tilde b$'s. Evaluation in the η_b rest frame shows that this amplitude is zero for a two-body final state.

A symmetric S-wave two-body $\tilde q\tilde q^*$ system can be constructed with one of the pair in a $J^P=0^-$ state and the other a $J^P=0^+$ state. Under charge-conjugation C, a squark $\tilde q$ is transformed into an antisquark $\tilde q^*$ with no overall phase. The two-body $\tilde q\tilde q^*$ system will have $J^{PC}=O^{-+}$, the same quantum numbers as the η_b. Therfore, there appears to be no overall symmetry that would forbid the exclusive lowest-order process $\eta_b\to\tilde b_1\tilde b_1^*$, and Eq. (22a) is not true more generally than in the lowest-order model we use.

As remarked above, the pure color-octet Fig. 1(b) term is independent of the gluino mass, and its contribution survives therefore even in the limit of gluinos with very large mass. Equation (21d) is written in a form that makes this fact transparent; the solitary "3" represents the contribution in the limit that only Fig. 1(b) contributes. If $m_{\tilde b}^2\ll m_{\tilde g}^2$ and $m_b^2\ll m_{\tilde g}^2$, the color-singlet coefficients D_J, Eqs. (23a)–(23c), vanish in proportion to $m_{\tilde g}^{-n}, n=2^{J+1}$, but the color-octet coefficient D_8 remains finite. As $m_{\tilde g}\to\infty$,

$$\Gamma(\chi_{bJ}\to\tilde b_1\tilde b_1^*)\to\frac{4\pi\alpha_s^2\mathcal{H}_8}{3m_b^2}\frac{1}{16}\left(1-\frac{m_{\tilde b}^2}{m_b^2}\right)^{3/2}$$
$$=\Gamma^{\rm SM}(\chi_{b1})\frac{1}{4n_f}\left(1-\frac{m_{\tilde b}^2}{m_b^2}\right)^{3/2}. \tag{24}$$

The momentum of a final state bottom squark in the rest frame of the χ_b is $|k|=\frac{1}{2}\sqrt{m_{\chi_b}^2-4m_{\tilde b}^2}\simeq m_b\sqrt{1-m_{\tilde b}^2/m_b^2}$. We observe that $\Gamma\propto|k|^{2l+1}$, with $l=1$, in the case of Υ decay, Eq. (22b), as expected because the decay of the Υ produces a pair of scalars in a P-wave. In the χ_J case, combining the overall phase space factor $\propto|k|$ in Eq. (22c), with those $\propto|k|^{2J}$ in the expressions for D_J in Eqs. (23a)–(23c), we note that the color-singlet terms have the threshold behaviors expected for decays into S,P, and D wave systems of two spin-0 particles. The overall color-octet coefficient Eqs. (22c) and (23d) is proportional to $|k|^3$.

The dependences on the mixing angle $\theta_{\tilde b}$ in Eqs. (23a)–(23d) can be understood if we recast the results in terms of production of left-handed and right-handed bottom squarks, $\tilde b_L$ and $\tilde b_R$. In the χ_{b0} case, the left-left and right-right combinations are produced with equal rates, leading to a term in Eq. (23a) proportional to $(\cos^2\theta_{\tilde b}+\sin^2\theta_{\tilde b})=1$. In addition, the left-right and right-left combinations are produced with equal rates, leading to the term proportional to $\sin\theta_{\tilde b}\cos\theta_{\tilde b}$. For χ_{b1}, the left-left and right-right combinations are produced with equal but opposite rates, leading to a dependence of $(\cos^2\theta_{\tilde b}-\sin^2\theta_{\tilde b})$. The left-right and right-left combinations do not contribute. In the χ_{b2} case, the the left-left and right-right combinations are produced with equal rates, resulting in a dependence of $(\cos^2\theta_{\tilde b}+\sin^2\theta_{\tilde b})$, and the left-right and right-left combinations do not contribute.

We can obtain a rough estimate of the relative size of the color-octet and color-singlet contributions in Eq. (22c) by beginning with the simplifications $(m_b^2-m_{\tilde b}^2)\ll m_{\tilde g}^2$ and $m_b^2\ll m_{\tilde g}^2$, both reasonably fair approximations for the masses of interest. Doing so, and defining $x=m_b^2/m_{\tilde g}^2$, we find

$$D_8:D_0:D_1:D_2\simeq 1:512x\cos^2\theta_{\tilde b}\sin^2\theta_{\tilde b}/3:256x^2(\cos^2\theta_{\tilde b}-\sin^2\theta_{\tilde b})^2/27:1024x^4/135.$$

Adopting the lattice estimate $m_b^2\mathcal{H}_8/\mathcal{H}_1\simeq 0.05$ [cf. Eq. (3)] along with the typical value $x\simeq 1/10$ for the range of sparticle masses under consideration, we conclude that the color-

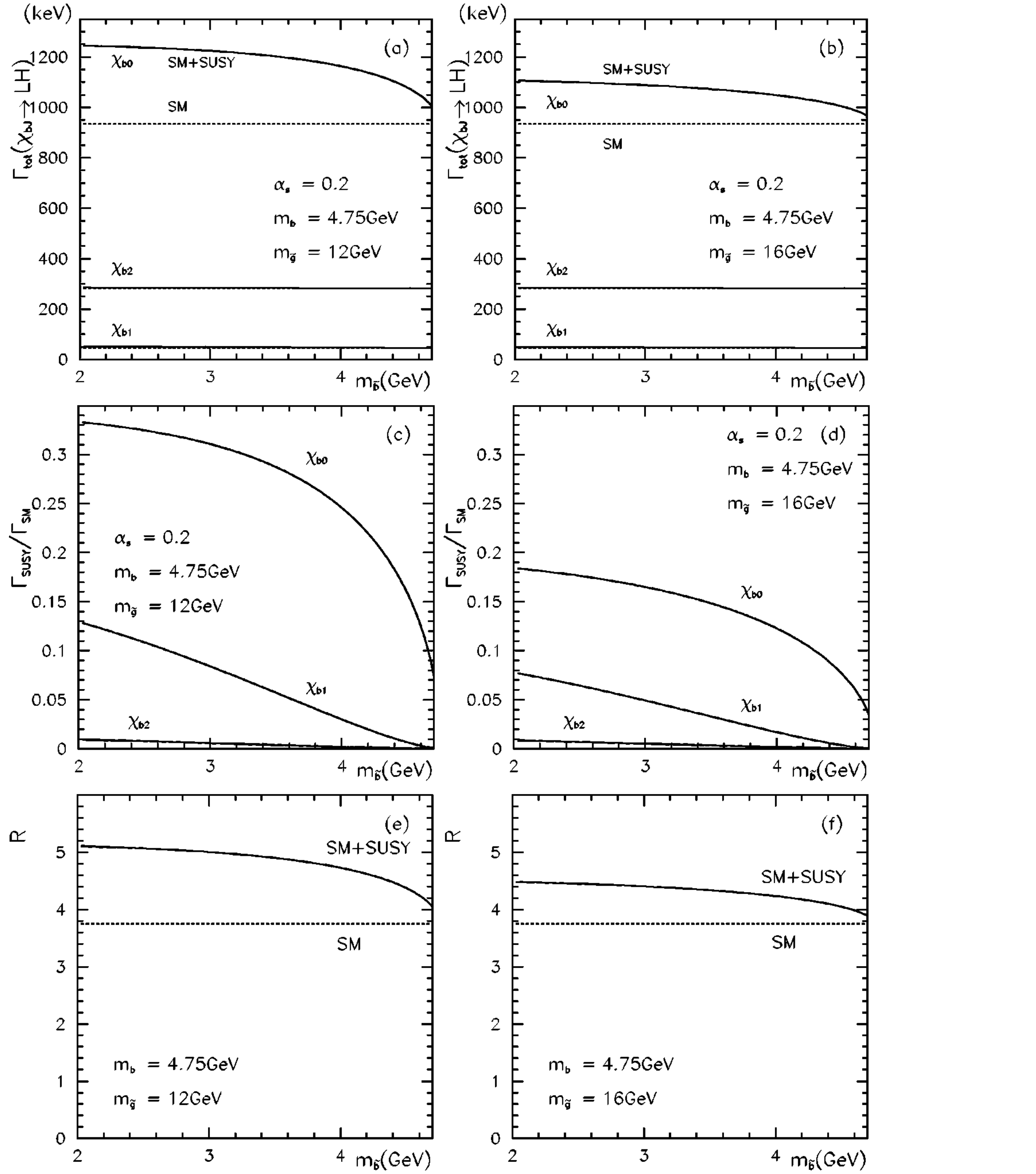

FIG. 2. Hadronic decay rates of the χ_b states as functions of the bottom squark mass are shown in the first row for $\sin\theta_{\tilde b}\cos\theta_{\tilde b}>0$. The left-hand column shows results for $m_{\tilde g}=12$ GeV and the right-hand column for $m_{\tilde g}=16$ GeV. Ratios of the bottom squark decay rates to the standard model predictions are shown in the second row. The third row shows the ratio R defined in Eq. (25).

octet contributions to bottom squark pair production should be roughly a 2% effect for the χ_{b0}, approximately 50% of the answer for χ_{b1}, and dominant in the χ_{b2} case. Nevertheless, as shown in the next section, bottom squark decays are expected to make a small overall contribution to the hadronic widths of the χ_{b1} and χ_{b2}.

IV. PREDICTIONS AND DISCUSSION

The total hadronic decay rates of the χ_{bJ} states are obtained after adding the contributions from conventional two-gluon decay and those for bottom squark decay, Eqs. (2) and (22c). These final decay rates are shown in the first rows of Figs. 2 and 3 and are compared with the standard model expectations. The left-hand sides of the figures show results for the choice $m_{\tilde g}=12$ GeV, and the right-hand sides for $m_{\tilde g}=16$ GeV. Results are displayed as a function of the bottom squark mass in the range 2 GeV$<m_{\tilde b}<m_b$. The choices of $m_{\tilde g}$ and $m_{\tilde b}$ are guided by the results in Ref. [1]. We employ the lattice estimate for the ratio of color-singlet and color-octet components, $\mathcal{H}_8/\mathcal{H}_1$, listed in Eq. (3). For the mixing angle, strong coupling strength, and b quark mass, we use $\sin^2\theta_{\tilde b}=1/6$, $\alpha_s=0.2$, and $m_b=4.75$ GeV. The

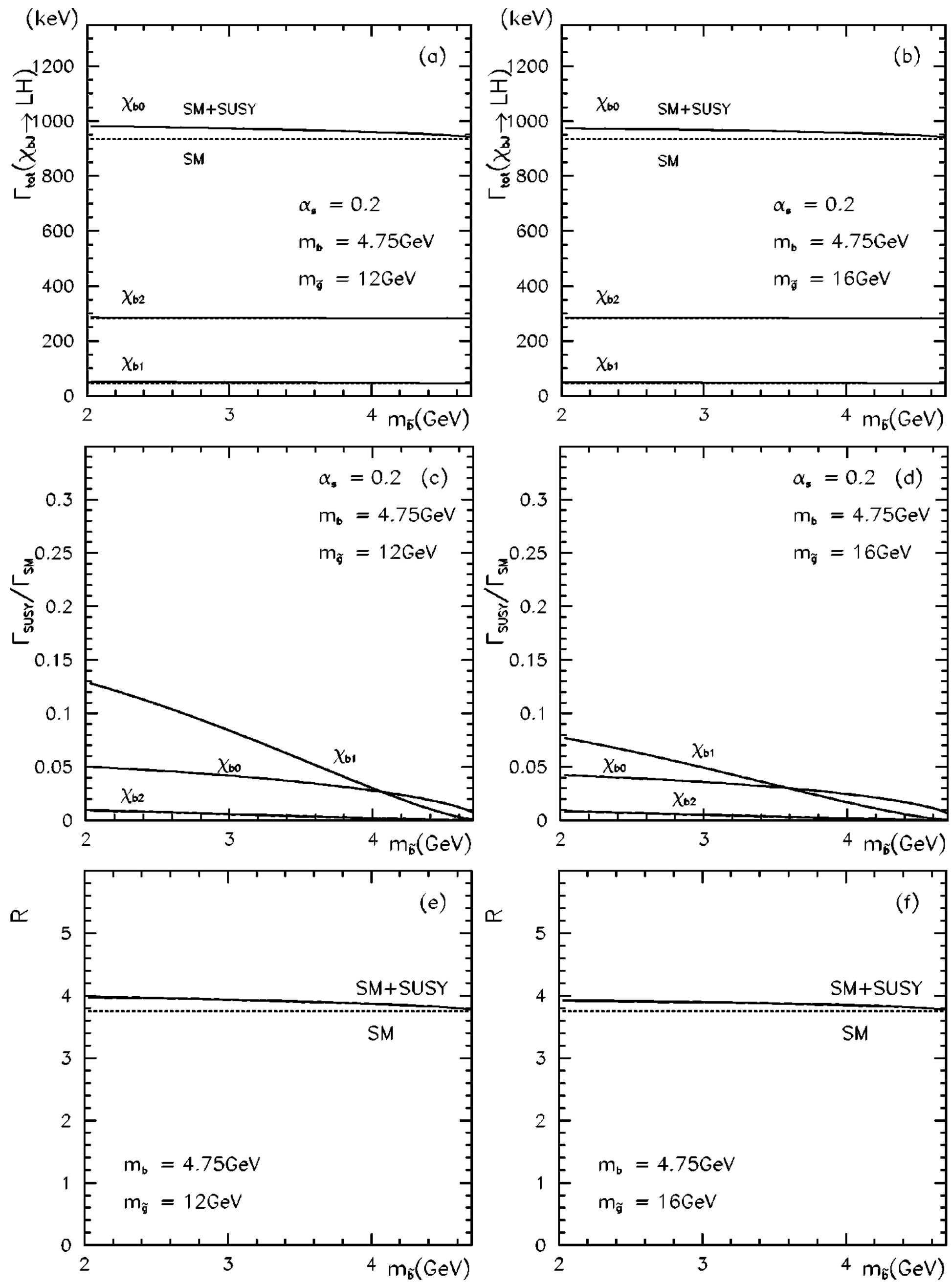

FIG. 3. As in Fig. 2 but with $\sin\theta_{\tilde b}\cos\theta_{\tilde b}<0$.

sign of the product $\sin\theta_{\tilde b}\cos\theta_{\tilde b}$ is not constrained by experiment. We consider both possibilities; results for $\sin\theta_{\tilde b}\cos\theta_{\tilde b}>0$ are shown in Fig. 2, and those for $\sin\theta_{\tilde b}\cos\theta_{\tilde b}<0$ in Fig. 3. The substantial differences in the χ_{b0} case between Fig. 2 and Fig. 3 arise from the cancellation in the numerator of Eq. (23a) when $\sin\theta_{\tilde b}\cos\theta_{\tilde b}<0$. Owing to the phase space and angular momentum threshold suppression factors $|k|^{2J+1}$, the SUSY contributions vanish as $m_{\tilde b}\to m_b(|k|\to 0)$ for all χ_{bJ} (as well as for the Υ).

The ratios of the decay rates through the bottom squark final state to the standard model prediction are shown in the second row of Figs. 2 and 3. The dependence on the overall factor $4\pi\alpha_s^2\mathcal{H}_1/3m_b^4$ cancels in this ratio, removing all dependence on the strong coupling strength $\alpha_s(m_b)$ (at the order in perturbation theory at which we work [35]) and much of the dependence on the b quark mass m_b and on the matrix element $\mathcal{H}_1$. The contributions of bottom squarks decays to the hadronic widths are small in comparison to the standard model values except for the χ_{b0}. As seen in Fig. 2, the SUSY contribution can be as large as 33% of the standard model value for small $m_{\tilde b}, m_{\tilde g}=12$ GeV, and positive $\sin\theta_{\tilde b}\cos\theta_{\tilde b}$. The fraction decreases as $m_{\tilde b}$ and $m_{\tilde g}$ increase, and is reduced if $\sin\theta_{\tilde b}\cos\theta_{\tilde b}<0$. By contrast, for small $m_{\tilde b}$, the SUSY contributions are at most 15% and 1% of the standard model values in the χ_{b1} and χ_{b2} cases. Inspection of Eqs. (2c) and (22c) provides an easy explanation for the small role of the

SUSY contribution in the χ_{b2} case. We observe that $\Gamma^{\rm SM} \sim (4/15)\Gamma_o$ whereas

$$\Gamma^{\rm SUSY} \sim D_8 m_b^2 \Gamma_o \mathcal{H}_8 / \mathcal{H}_1 \sim (1/20) D_8 \Gamma_o .$$

Since $D_8 \sim 1/16$, we obtain $\Gamma^{\rm SUSY}/\Gamma^{\rm SM} \sim 1\%$.

We comment on results obtained if the color-octet configuration is absent or, equivalently, the matrix element $\mathcal{H}_8$ is much smaller than estimated in Eq. (3). The conclusions regarding χ_{b0} are essentially unchanged since the color-singlet contribution is dominant for both the SM and bottom squark decays. For χ_{b1}, the SM color-singlet term is absent at leading order; the SUSY decay mode would therefore dominate at this order. In the χ_{b2} case, the color-singlet rate for decay into bottom squarks is negligible compared to the SM color-singlet rate for gg decay. Overall, in the absence of color-octet contributions the predictions for the ratio $\Gamma_{\rm SUSY}/\Gamma_{\rm SM}$ would be qualitatively similar in the χ_{b0} and χ_{b2} cases to those in Figs. 2 and 3, but very different for the χ_{b1}.

A ratio R [25] may be formed in which the color-octet contributions cancel:

$$R \equiv \frac{\Gamma(\chi_{b0}) - \Gamma(\chi_{b1})}{\Gamma(\chi_{b2}) - \Gamma(\chi_{b1})}. \tag{25}$$

This ratio depends only on the short-distance coefficients. Results are shown in the third row of Figs. 2 and 3. The ratio is enhanced significantly by the SUSY contribution for small $m_{\tilde{b}}, m_{\tilde{g}} = 12$ GeV, and positive $\sin\theta_{\tilde{b}} \cos\theta_{\tilde{b}}$.

Direct observation of χ_b decay into bottom squarks requires an understanding of the ways that bottom squarks may manifest themselves [1,10]. If the $\tilde{b}$ lives long enough, it will pick up a light quark and turn into a $J = 1/2$ B mesino, $\tilde{B}$, the superpartner of the B meson. The mass of the mesino could fall roughly in the range 3 to 6 GeV for the interval of $\tilde{b}$ masses favored by the analysis of the bottom quark cross section. Charged B-mesino signatures in χ_{bJ} decay include single back-to-back equal momentum tracks in the center-of-mass; measurably lower momentum than lepton pairs; an angular distribution consistent with the decay of a state of spin J into two fermions; and ionization, time-of-flight, and Cherenkov signatures typical of a particle whose mass is heavier than that of a proton. On the other hand, possible baryon-number-violating R-parity-violating decays of the bottom squark lead to $u+s$, $c+d$, and $c+s$ final states. These final states of four light quarks may be indistinguishable from conventional hadronic final states mediated by the two-gluon intermediate state. Since the bottom squark decay mode is predicted to be substantial for χ_{b0}'s, decays of the χ_{b0} primarily via the $\tilde{B}$ possibility would offer an excellent opportunity to observe bottom squarks or to place significant limits on their possible masses.

ACKNOWLEDGMENTS

We acknowledge valuable conversations with Geoff Bodwin, Zack Sullivan, and Tim Tait. Work in the High Energy Physics Division at Argonne National Laboratory is supported by the U. S. Department of Energy, Division of High Energy Physics, under Contract No. W-31-109-ENG-38.

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