

## Semileptonic $D$ decay into scalar mesons: A QCD sum rule approach

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Semileptonic decays of  $D$  mesons into scalar hadronic states are investigated. Two extreme cases are considered: (a) the meson decays directly into an uncorrelated scalar state of two mesons and (b) the decay proceeds via resonance formation. QCD sum rules including instanton contributions are used to calculate total and differential decay rates under the two assumptions.

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### I. INTRODUCTION

Low lying scalar mesons are an old problem in hadron physics; see the review by Spanier and Törnqvist on scalar mesons in [1] and the literature quoted there. In a recent analysis [2,3] of  $D$ - and  $D_s$ -meson nonleptonic decays distinct signals for strong enhancements in  $S$ -wave  $\pi\pi$  and  $K\pi$  channels have been observed, reviving the interest in these states. In the following we shall refer to the signal in the  $\pi\pi$  channel as  $\sigma$  and in the  $K\pi$  channel as  $\kappa$ . The enhancements can be well described by a Breit-Wigner-type resonance form, with the correct threshold behavior

$$\rho_{X,BW}(s) = \frac{\Gamma_X(s)m_X}{(s-m_X^2)^2 + m_X^2\Gamma_X(s)^2}, \quad (1)$$

where the subscript  $X$  stands for  $\sigma$  or  $\kappa$ . The correct threshold behavior is guaranteed through the  $s$ -dependent width

$$\Gamma_X(s) = \Gamma_{0X} \frac{\lambda^{1/2}(s, m_a^2, m_b^2)}{\lambda^{1/2}(m_X^2, m_a^2, m_b^2)} \frac{m_X^2}{s}. \quad (2)$$

Here  $m_a$  and  $m_b$  are the masses of the mesons in the decay channel and  $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ . In [3] the following parameters were found:

(1) for the  $S$ -wave  $\pi\pi$  channel,  $m_a = m_b = m_\pi$ ,

$$m_\sigma = 0.478 \pm 0.024 \pm 0.017 \text{ GeV};$$

$$\Gamma_{0\sigma} = 0.324 \pm 0.042 \pm 0.021 \text{ GeV}; \quad (3)$$

(2) for the  $S$ -wave  $K\pi$  channel,  $m_a = m_K$ ,  $m_b = m_\pi$ ,

$$m_\kappa = 0.797 \pm 0.019 \pm 0.042 \text{ GeV};$$

$$\Gamma_{0\kappa} = 0.410 \pm 0.043 \pm 0.085 \text{ GeV}. \quad (4)$$

The semileptonic  $D$  decays offer, in principle, much cleaner samples than the nonleptonic decays since there occur no problems connected with the presence of a third strongly interacting particle. In the nonstrange channel important experimental information comes indeed from the analysis of the final state interaction of the pions from leptonic  $K$  decays. For the case of strange mesons, the analysis of  $D$  de-

cays can play a similarly important role. In a recent paper by the FOCUS Collaboration [4] clear evidence was found that in the semileptonic decay  $D^+ \rightarrow K^- \pi^+ \mu^+ \nu$  the  $D$  meson does not decay exclusively into the hadronic vector channel, but that there is interference with a scalar contribution. In this paper we estimate the decay rates of semileptonic  $D$  decays into the scalar  $K\pi$  and  $\pi\pi$  channels. We use the method of QCD sum rules [5] which has been successfully applied to several semileptonic decay processes. For a recent review see [6] and the literature quoted there.

Even if (broad) resonances in the hadronic decay channels exist, it is not clear whether they are due to interactions on the quark level, or if they are rather an effect of interactions in the purely hadronic channel, see for instance [7]. Here a theoretical analysis can be very helpful. The semileptonic decay is supposed to occur on the quark level and therefore the decay rate should depend crucially on the direct coupling of the resonances to the quark currents. We consider two limiting cases:

The observed two-meson final state couples to the corresponding quark-antiquark state only through the  $\sigma$  or  $\kappa$  resonance.

The quark antiquark state couples to uncorrelated final mesons in an  $S$  wave.

Since the signals we investigate in this paper are very broad the finite width has to be taken into account. Otherwise our analysis is based on the same principles and assumptions as those used in the sum-rule analysis of other semileptonic  $D$  decays [8]. We therefore refer to this paper for details and clarifications.

### II. KINEMATICS

We investigate the semileptonic decays

$$D \rightarrow X \ell \bar{\nu}_\ell, \quad (5)$$

where  $X$  might be the  $\sigma, \kappa$  or an uncorrelated  $\pi\pi$  or  $K\pi$  pair in the  $S$  state, as depicted in Fig. 1.

The semileptonic decay of a  $D$  meson with momentum  $p_D$  into a scalar state with total momentum  $p_X$  and invariant mass  $\sqrt{s} = \sqrt{p_X^2}$  is described by the two form factors of the matrix element of the weak current

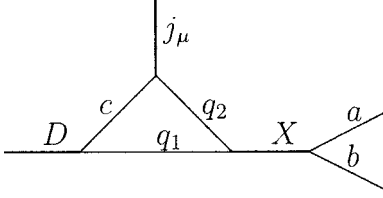


FIG. 1. Schematic picture of the semileptonic vertex of a  $D$ -meson decay.  $c, q_1, q_2$  are quark lines,  $j_\mu$  denotes the weak current.  $X$  might be either a (broad) resonance or represent an uncorrelated two-meson state of mesons  $a$  and  $b$ .

$$\langle X | j_\mu | D \rangle = (p_D + p_X)_\mu f_{+X}(t) + (p_D - p_X)_\mu f_{-X}(t), \quad (6)$$

where  $t = (p_D - p_X)^2$ . In the decay rate the form factor  $f_{-X}$  is multiplied by the difference of the lepton masses and hence is negligible for both  $e$  and  $\mu$  decays.

The differential semileptonic decay rate is given by

$$\frac{d^2\Gamma(s, t)}{ds dt} = \frac{G_F^2 |V_{cq_2}|^2}{192\pi^3 m_D^3} \lambda^{3/2}(m_D^2, s, t) f_{+X}^2(t) \frac{\rho_X(s)}{\pi}, \quad (7)$$

where  $G_F$  is the Fermi coupling constant and  $V_{cq_2}$  the Cabibbo-Kobayashi-Maskawa (CKM) transition element from the charmed quark to the quark  $q_2$ . The total width is

$$\Gamma = \int_{(m_a+m_b)^2}^{m_D^2} ds \int_0^{(m_D-\sqrt{s})^2} dt \frac{d^2\Gamma(s, t)}{ds dt}. \quad (8)$$

The spectral distribution in the invariant mass  $\sqrt{s}$  of the hadronic final state is given by

$$\frac{d\Gamma}{d\sqrt{s}} = 2\sqrt{s} \int_0^{(m_D-\sqrt{s})^2} dt \frac{d^2\Gamma(s, t)}{ds dt}. \quad (9)$$

### III. SUM RULES

The  $D$  meson in the initial state is interpolated by the pseudoscalar current

$$j_D(x) = \bar{c}(x) i \gamma_5 q_1(x), \quad (10)$$

where  $c$  is the field of the charmed quark and  $q_1$  that of an up or down quark, summation over spinor and color indices being understood but not indicated explicitly. The final hadronic state  $X$  is interpolated by the scalar current

$$j_X(x) = \bar{q}_1(x) q_2(x), \quad (11)$$

where  $q_2$  represents a light quark field for  $X = \sigma$ , and a strange quark field for  $X = \kappa$ . The semileptonic decay rate is obtained from the time ordered product of the two interpolating fields in Eqs. (10) and (11) and the weak current  $j_\mu^W = \bar{q}_2 \gamma_\mu (1 - \gamma_5) c$ :

$$T_{\mu X}(p_D^2, p_X^2, t) = i^2 \int d^4x d^4y \langle 0 | T [j_D(x) j_\mu^W(0) j_X(y)] | 0 \rangle \times e^{i(p_D \cdot x - p_X \cdot y)}. \quad (12)$$

In order to select the semileptonic decay rates into the lowest lying hadrons we insert intermediate states and obtain the following double dispersion relation in the phenomenological side:

$$T_{\mu X}^{phen}(p_D^2, p_X^2, t) = \frac{1}{\pi^2} \int ds_D ds_X \langle 0 | j_D(0) | D \rangle \langle D | j_\mu^W(0) | X \rangle \times \langle X | j_X(0) | 0 \rangle \times \frac{1}{s_D - p_D^2} \frac{1}{s_X^2 - m_X^2} + \text{contributions of higher resonances}. \quad (13)$$

Introducing

$$\langle 0 | j_D(0) | D \rangle = A_D \rho_D(s_D); \quad p_D^2 = s_D, \quad (14)$$

$$\langle X | j_X(0) | 0 \rangle = A_X \rho_X(s_X); \quad p_X^2 = s_X, \quad (15)$$

and using Eq. (6) we obtain

$$T_{\mu X}^{phen}(p_D^2, p_X^2, t) = \frac{1}{\pi^2} \int ds_D ds_X A_D A_X \frac{\rho_D(s_D)}{s_D - p_D^2} \frac{\rho_X(s_X)}{s_X - p_X^2} \times (f_{+X}(t, s_D, s_X) (p_D + p_X)_\mu + f_{-X}(t, s_D, s_X) (p_D - p_X)_\mu) + \text{contributions of higher resonances}. \quad (16)$$

In the following we concentrate on the relevant form factor  $f_+$  and introduce the factor  $T_+$  which multiplies the vector  $(p_D + p_X)_\mu$

$$T_{+X}^{phen}(p_D^2, p_X^2, t) = \frac{1}{\pi^2} \int ds_D ds_X A_D A_X \frac{\rho_D(s_D)}{s_D - p_D^2} \frac{\rho_X(s_X)}{s_X - p_X^2} \times f_{+X}(t, s_D, s_X) + \text{contributions of higher resonances}. \quad (17)$$

For the  $D$  meson the density  $\rho_D(s_D)$  introduced in Eq. (14) is given by

$$\rho_D(s_D) = \pi \delta(s_D - m_D^2), \quad (18)$$

and we obtain

$$A_D = f_D m_D^2 / m_c, \quad (19)$$

$f_D$  being the  $D$ -meson decay constant in the conventional notation.

For the density  $\rho_X$  we use the two extreme *Ansätze* mentioned above:

The quark-antiquark current  $j_X$  in the  $J = 0^+$  state couples to the  $\sigma$  or  $\kappa$  resonance described by the Breit-Wigner distribution (1)

$$\rho_X(s) = \rho_{X,BW}(s). \quad (20)$$

The quark-antiquark current  $j_X$  couples to an uncorrelated meson pair, the density being described by the density of two-particle phase space

$$\rho_X(s) = \frac{\pi}{16\pi^2} \frac{\lambda(s, m_a^2, m_b^2)^{1/2}}{s}, \quad (21)$$

where  $m_a, m_b$  are the masses of the mesons in the final state.

The three-point function can be evaluated by perturbative QCD if the external momenta are in the deep Euclidean region

$$p_D^2 \ll (m_c + m_1)^2, \quad p_X^2 \ll (m_1 + m_2)^2, \quad t \ll (m_c + m_2)^2. \quad (22)$$

In order to approach the not-so-deep-Euclidean region and to get more information on the nearest physical singularities, nonperturbative power corrections are added to the perturbative contribution

$$T_{+X}^{theor}(p_D^2, p_X^2, t) = \frac{1}{\pi^2} \int_{s_{D,th}} ds_D \int_{s_{X,th}} ds_X \frac{\sigma_{+X}(s_D, s_X, t)}{(s_D - p_D^2)(s_X - p_X^2)} + \sum_{ij} \frac{C_{ij}}{(p_D^2)^i (p_X^2)^j} \langle O_{ij} \rangle. \quad (23)$$

The perturbative contribution is contained in the double spectral function  $\sigma_{+X}$ . The Wilson coefficients  $C_{ij}$  multiplying the power corrections can be evaluated in perturbative QCD. The operators  $O_{ij}$  occur in the operator expansion of the time ordered product Eq. (12); their vacuum expectation values, the condensates  $\langle O_{ij} \rangle$ , are introduced as phenomenological parameters.

In order to suppress the condensates of higher dimension and at the same time reduce the influence of higher resonances, the series in Eq. (23) is Borel improved, leading to the mapping

$$f(p^2) \rightarrow \hat{f}(M^2), \quad \frac{1}{(p^2 - m^2)^n} \rightarrow \frac{(-1)^n}{(n-1)!} \frac{e^{-m^2/M^2}}{(M^2)^n}. \quad (24)$$

Furthermore, we make the usual assumption that the contributions of higher resonances are well approximated by the perturbative expression

$$\frac{1}{\pi^2} \int_{s_{0D}}^\infty ds_D \int_{s_{0X}}^\infty ds_X \frac{\sigma_{+X}(s_D, s_X, t)}{(s_D - p_D^2)(s_X - p_X^2)}, \quad (25)$$

with appropriate continuum thresholds  $s_{0D}$  and  $s_{0X}$ . By equating the Borel transforms of the phenomenological expression in Eq. (17) and that of the ‘‘theoretical expression,’’ Eq. (23), we obtain the sum rule

$$f_{+X}(t, m_D^2, \bar{s}_X) f_D \frac{m_D^2}{m_c} e^{-m_D^2/M_D^2} \frac{A_X}{\pi} \times \int_{(m_a+m_b)^2}^{s_{0X}} ds_X \rho_X(s_X) e^{-s_X/M_X^2} = \frac{1}{\pi^2} \int_{s_{D,th}}^{s_{0D}} ds_D \int_{s_{X,th}}^{s_{0X}} ds_X \sigma_{+X}(s_D, s_X, t) \times e^{-s_D^2/M_D^2} e^{-s_X/M_X^2} + \hat{K}_{+X}(M_D^2, M_X^2, t) + \hat{K}_{+I}(M_D^2, M_X^2, t), \quad (26)$$

where  $\bar{s}_X$  is some value below  $s_{0X}$ . In the zero width approximation we have of course  $\bar{s}_X = m_X^2$ .  $\hat{K}_{+X}$  are the Borel transforms of the nonperturbative expressions due to the condensates and  $\hat{K}_I$  is an approximation to the instanton contributions, which might be important in the scalar channel [9].

The decay constant  $f_D$  and the coupling  $A_X$  defined in Eqs. (14), (15), and (19) can also be determined by sum rules obtained from the appropriate two-point functions. Using the same procedure as described above we arrive at

$$\frac{m_D^4}{m_c^2} (f_D^{theor}(M^2))^2 e^{-m_D^2/M^2} = \int_{m_c^2}^{s_{0D}} ds \sigma_D(s) e^{-s/M^2} + \hat{K}_D(M^2), \quad (27)$$

and

$$(A_X^{theor}(M^2))^2 \left( \frac{1}{\pi} \int_{(m_a+m_b)^2}^{s_{0X}} ds \rho_X(s) e^{-s/M^2} \right) = \int_{m_{q_2}^2}^{s_{0X}} ds \sigma_X(s) e^{-s/M^2} + \hat{K}_X(M^2) + \hat{K}_I(M^2). \quad (28)$$

The analysis of the two-point function for the scalar mesons, and the explicit expressions for the functions occurring in Eqs. (26), (27) and (28) are given in Appendixes A and B respectively.

The final sum rule for the form factor is obtained from Eq. (26) by inserting for  $f_D$  and  $A_X$  the expressions  $f_D^{theor}$  and  $A_X^{theor}$  of Eqs. (27) and (28)

$$f_{+X}(t, m_D^2, \bar{s}_X) = e^{m_D^2/M_D^2} \left( \frac{m_D^2}{m_c} f_D(M_D^2) A_X(M_X^2) \frac{1}{\pi} \times \int_{(m_a+m_b)^2}^{s_{0X}} ds_X \rho_X(s_X) e^{-s_X/M_X^2} \right)^{-1} \times \left( \int_{s_{D,th}}^{s_{0D}} ds_D \int_{s_{X,th}}^{s_{0X}} ds_X \sigma_{+X}(s_D, s_X, t) \times e^{-s_D^2/M_D^2} e^{-s_X/M_X^2} + \hat{K}_{+X}(M_D^2, M_X^2, t) + \hat{K}_{+I}(M_D^2, M_X^2, t) \right). \quad (29)$$

The radiative corrections for the scalar and pseudoscalar channels are known to be large [10]. They are expected to be large in the three-point function too. By inserting the sum rule expressions for the two-point functions, Eqs. (27) and (28), in the denominator of the sum rule for the three-point function, Eq. (29), we expect, at least, a partial cancellation of these corrections [11,12].

#### IV. EVALUATION OF THE SUM RULES AND RESULTS

The sum rule Eq. (29) is evaluated in the same way as described in [8], and we only sketch the main steps of this evaluation. In the complete theory, the right-hand side of Eq. (29) should not depend on the Borel variables  $M^2$ . However, in a truncated treatment there will always be some dependence left. Therefore, one has to work in a region where the approximations made are supposedly acceptable and where the result depends only moderately on the Borel variables. To decrease the dependence of the results on the Borel variables  $M^2$ , we take them in the two-point functions at half the value of the corresponding variables in the three-point sum rules, i.e., in Eq. (29) we put

$$M_D'^2 = M_D^2/2 \quad \text{and} \quad M_X'^2 = M_X^2/2. \quad (30)$$

We furthermore choose

$$\frac{M_X^2}{M_D^2} = \frac{m_X^2}{m_D^2 - m_c^2}. \quad (31)$$

We have checked that the results do not depend crucially on this particular choice. If the momentum transfer  $t$  to the lepton pair is larger than a critical value  $t_{cr}$ , non-Landau singularities have to be taken into account [8]. Since anyhow we have to stay away from the physical region, i.e., we must have  $t \ll (m_c + m_1)^2$ , we limit our calculation to the region  $0 < t < t_{cr}$ . In this range the  $t$  dependence can be obtained from the sum rule (29) directly. It can be fitted by a monopole, and extrapolated to the full kinematical region.

Since we do not take into account radiative corrections we choose the QCD parameters at a fixed renormalization scale of about 1 GeV<sup>2</sup>: the strange and charm mass  $m_s = 0.16$  GeV,  $m_c = 1.3$  GeV, the up and down quark masses are put to zero. We take for the nonstrange quark condensate  $\langle \bar{q}q \rangle = -(0.24)^3$  GeV<sup>3</sup>, for the strange quark condensate  $\langle \bar{s}s \rangle = 0.8 \langle \bar{q}q \rangle$ , and for the mixed quark-gluon condensate  $\langle \bar{q}\sigma Gq \rangle \langle \bar{q}q \rangle = m_0^2 \langle \bar{q}q \rangle$  with  $m_0^2 = 0.8$  GeV<sup>2</sup>.

For the continuum threshold in the  $D$  channel we take from [8]  $s_{0D} = 6$  GeV<sup>2</sup>. The standard value in the  $X$  channel would be  $s_{0X} \approx (m_X + 0.5 \text{ GeV})^2$ , yielding  $s_{0\sigma} \approx 1$  GeV<sup>2</sup> and  $s_{0\kappa} \approx 1.6$  GeV<sup>2</sup>. As an additional condition we use the mass constraint from the two-point function, as described in Appendix A.

We start with the decay  $D \rightarrow \kappa l \bar{\nu}_l$  and first consider the case where the scalar quark current (11) couples directly to the  $\kappa$  signal through a Breit-Wigner distribution (20). In Fig. 2 we show the different contributions to the form factor  $f_{+\kappa}(0)$  in Eq. (29), as a function of the Borel variable  $M_D^2$ ,

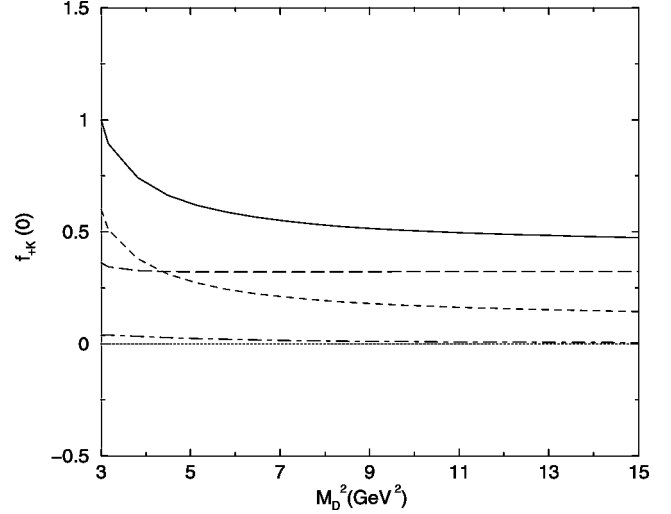


FIG. 2. Dependence of the form factor  $f_{+\kappa}$  at  $t=0$  on the Borel variable  $M_D^2$ . Here the decay is assumed to proceed through resonance formation and the state density is given by Eq. (20). Solid curve: total contribution; long-dashed: perturbative; dashed: quark condensate; dot-dashed: mixed condensate; dotted: instanton contribution.

using the continuum threshold  $s_{0\kappa} = 1.6$  GeV<sup>2</sup>. As the lower limit for the fiducial region in  $M_D^2$  we take that value of  $M_D^2$  where the perturbative contribution is one-half of the total contribution. As an upper limit we take the value  $M_D^2 = 15$  GeV<sup>2</sup>, which is motivated in Appendix A. For such a high value of the Borel parameter the result is very stable but it is largely determined by the choice of the continuum model. The instanton contribution in the fiducial region is completely negligible; the five dimensional mixed condensate is strongly suppressed compared to the three dimensional quark condensate. In the Borel variable of the  $X$  channel the fiducial region corresponds approximately to the range  $1.7 \text{ GeV}^2 \leq M_X^2 \leq 5 \text{ GeV}^2$ .

In the range  $0 \leq t \leq 0.5$  GeV<sup>2</sup> no non-Landau singularities occur for our choices of the continuum thresholds. The momentum dependence of  $f_{+\kappa}$  can, in this  $t$  range, be very well approximated by a monopole expression

$$f_{+\kappa}(t) = \frac{f_{+\kappa}(0)}{1 - \frac{t}{M_p^2}}, \quad (32)$$

and extrapolated to the full physical region.

For  $1.4 \text{ GeV}^2 \leq s_{0\kappa} \leq 1.8 \text{ GeV}^2$  and for values of  $M_D^2$  discussed above, we find, for the form factor at  $t=0$ ,

$$0.48 \leq f_{+\kappa}(0) \leq 0.55, \quad (33)$$

and, for the pole mass,

$$1.9 \text{ GeV} \leq M_p \leq 2.2 \text{ GeV}. \quad (34)$$

The pole mass is considerably smaller than the mass of the charmed pseudovector-meson  $D_{s1}(2536)$  which would fit

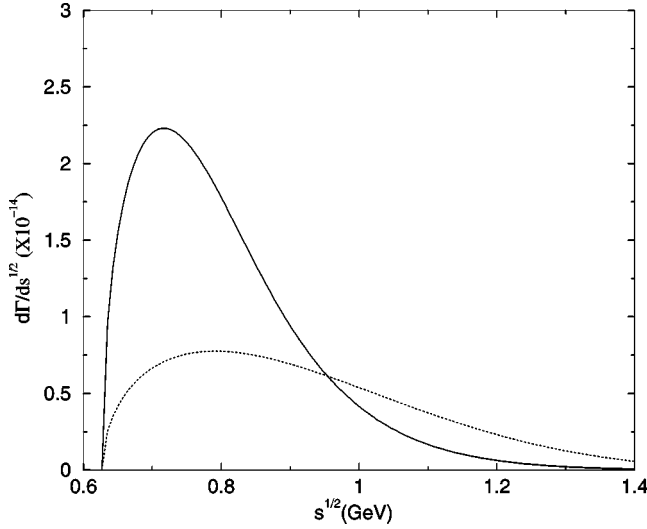


FIG. 3. Invariant mass distribution  $d\Gamma(\sqrt{s})/d\sqrt{s}$  of the hadronic final state in the decay  $D \rightarrow (K\pi)_S \ell \nu$ . Solid: resonance formation; dots: uncorrelated  $K\pi$ .

into the  $t$  channel. In the  $D \rightarrow Kl\nu$  decay the pole mass also came out to be smaller [8] than the mass of the strange vector meson  $D_s^*$  (2114).

In the limits of the Borel variables and the continuum thresholds discussed above we obtain for the total semileptonic decay width

$$\Gamma(D \rightarrow \kappa \ell \nu) = (5.5 \pm 1.0) \times 10^{-15} \text{ GeV}, \quad (35)$$

where we have used  $V_{cs} = 0.97$ . The same calculation done in an—unjustified—zero width approximation would yield a total semileptonic decay width which is about 20% larger.

The spectral distribution  $d\Gamma/d\sqrt{s}$ , Eq. (9), where  $\sqrt{s}$  is the invariant mass of the  $\pi K$  state, is given in Fig. 3, solid line.

Next we investigate the same decay under the assumption that the scalar current in Eq. (11) does not couple to a resonance, but to an uncorrelated  $\pi K$  pair in an  $S$  state; i.e., we use the *Ansatz* (21) for  $\rho_\kappa(s)$  entering the sum rules. In Fig. 4 we show the dependence on the Borel variable  $M_D^2$  of the decay form factor  $f_{+\kappa}(0)$  for  $s_{0\kappa} = 1.6 \text{ GeV}^2$ . Note that now the density (21) describes a two-particle phase space and therefore the dimension of  $f_{+\kappa}$  is different from the previous case.

The masses of the pole fit to the  $t$  dependence are practically the same as for the resonance case; for the form factor at  $t=0$  we obtain

$$5.0 \text{ GeV}^{-1} f_{+\kappa}(0) \leq 7.1 \text{ GeV}^{-1}. \quad (36)$$

The total width comes out for the case of an uncorrelated  $\pi K$  pair in an  $S$  state as

$$\Gamma(D \rightarrow (K\pi)_S \ell \bar{\nu}_\ell) = (3.7 \pm 1.1) \times 10^{-15} \text{ GeV}. \quad (37)$$

The spectral distribution for this case is also shown in Fig. 3, with a dotted line. Although there is no resonance formation the resulting distribution shows a maximum at approximately

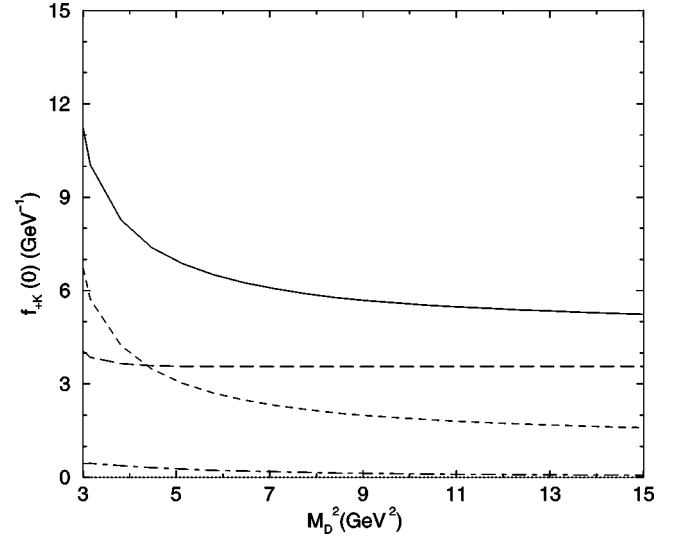


FIG. 4. Dependence of the form factor  $f_{+\kappa}$  at  $t=0$  on the Borel variable  $M_D^2$ . Here the decay is assumed to proceed through an uncorrelated  $\pi K$ -meson pair in an  $S$  state; the density is given by Eq. (21). Solid curve: total contribution; long-dashed: perturbative; dashed: quark condensate; dot-dashed: mixed condensate; dotted: instanton contribution.

the mass of the  $\kappa$ , which is an effect of the decrease of the total phase space near the kinematical limits.

The evaluation of the decay  $D \rightarrow \sigma \ell \nu$  follows exactly the same lines. Here the fiducial range in  $M_D^2$  is chosen according to the same criteria as before and goes approximately from  $M_D^2 = 8 \text{ GeV}^2$  to  $18 \text{ GeV}^2$  corresponding approximately to a range  $1 \text{ GeV}^2 \leq M_\sigma^2 \leq 2.3 \text{ GeV}^2$ . The continuum limit  $s_{0\sigma}$  was chosen between 1 and  $1.6 \text{ GeV}^2$ . The resulting form factors and total decay width are, in the case of a resonance formation with a Breit-Wigner width

$$0.42 \leq f_{+\sigma}(0) \leq 0.57, \quad (38)$$

$$\Gamma(D \rightarrow \sigma \ell \bar{\nu}_\ell) = (8.0 \pm 2.5) \times 10^{-16} \text{ GeV}, \quad (39)$$

and for the case of two uncorrelated  $\pi$  mesons in an  $S$  state

$$5 \text{ GeV}^{-1} \leq f_{+\sigma}(0) \leq 6 \text{ GeV}^{-1} \quad (40)$$

and

$$\Gamma(D \rightarrow (\pi\pi)_S \ell \bar{\nu}_\ell) = (4.5 \pm 1.0) \times 10^{-16} \text{ GeV}. \quad (41)$$

The spectral distributions for both *Ansätze* are shown in Fig. 5.

## V. SUMMARY AND CONCLUSIONS

The semileptonic decays of  $D$  mesons into scalar hadrons are very similar to those into pseudoscalars. The same role played by the vector part in the pseudoscalar case is played by the weak pseudovector for the decay into scalars. Therefore, the theoretical expressions are very similar, only terms proportional to the mass of the strange quark change sign. This leads to a small reduction of the form factors compared to the decay into a pseudoscalar state. The main difference of

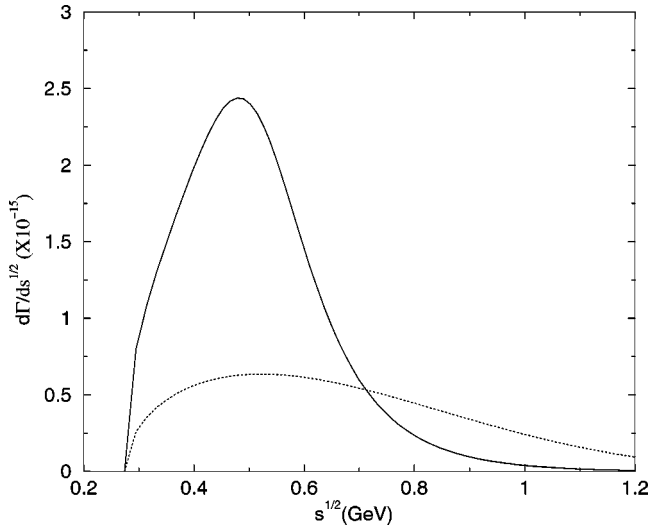


FIG. 5. The invariant mass distribution of the hadronic final state of the rate in the decay  $D \rightarrow (\pi\pi)_S \ell \nu$ . Solid: resonance formation; dots: uncorrelated  $K\pi$ .

the semileptonic decay rates into  $K$  and  $\kappa$  is due to the different phase space. The decay into  $\sigma$  mesons is suppressed by the small value of the weak ( $c, d$ ) matrix element  $V_{cd} \approx 0.225$ . The spectral distributions of the invariant masses of the  $K\pi$  and  $\pi\pi$  systems are given in Figs. 3 and 5. Their maxima are well below the masses of the  $\kappa$  and the  $\sigma$ , and also the forms are quite different from Breit-Wigner distributions. The increase of the spectral distributions is steeper and the fall-off substantially faster than for the corresponding Breit-Wigner forms. This is an effect of the total phase space in the semileptonic decay.

If the scalar current does not couple directly to the resonance but only to an uncorrelated meson pair, the decay rate is reduced by a factor two compared with the decay into resonances, but nevertheless an enhancement near 0.8 and 0.5 GeV is visible due to the total final state phase space (see Figs. 3 and 5). With good statistics a discrimination between the two extreme cases should be possible. This would add valuable information on the nature of the intriguing low lying scalar resonances. In reality things might be complicated by a coupling of the interpolating current 11 on the quark level to a resonance as well as to an uncorrelated meson pair. In this case one has to construct a new density  $\rho_X$  and use it instead of the densities in Eq. (20) or (21), in order to calculate the form factors  $f_X(t)$ . The procedure that follows is the same.

#### ACKNOWLEDGMENTS

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#### APPENDIX A: EVALUATION OF THE TWO-POINT SUM RULES

In Fig. 6 we display the different contributions to  $A_\kappa$ , as

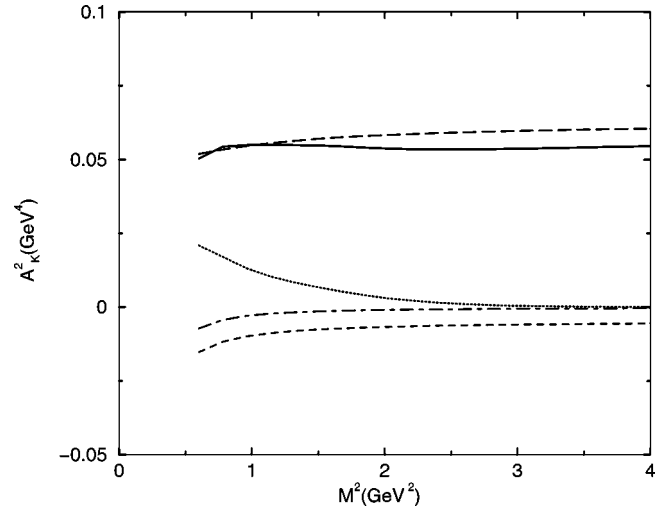


FIG. 6. Borel mass dependence of  $A_\kappa^2$ . Solid curve: total contribution; long-dashed: perturbative; dashed: quark condensate; dot-dashed: mixed condensate; dotted: instanton contribution.

a function of the Borel variable  $M_\kappa'^2$  using the *Ansatz* in Eq. (20) for  $\rho_\kappa(s)$  and  $s_{0\kappa} = 1.4 \text{ GeV}^2$ . As expected the instanton contribution is particularly important for small values of the Borel variable. In spite of the weak dependence of the sum rule results on  $M_\kappa'^2$ , it is well known that radiative corrections could be large in this case. Therefore, we do not attach great significance to the high stability of our results.

In the zero width approximation one can obtain a sum rule for the mass by performing the logarithmic derivative of the right- and left-hand sides of Eq. (27) with respect to  $M^{-2}$ , assuming that  $f_D$  is independent of  $M^{-2}$ . In Fig. 7 we display the logarithmic derivative of the left- and right-hand sides of the sum rule (28), again assuming that  $A_\kappa$  is independent of the Borel variable.

For  $s_{0\kappa} = 1.4 \text{ GeV}^2$  there is a reasonable overlap be-

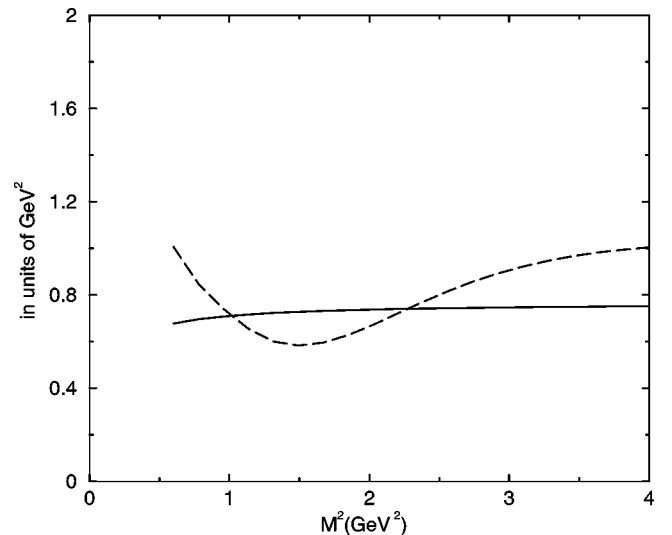


FIG. 7. Borel mass dependence of the RHS (dashed line) and the LHS (solid line) of the sum rule generated by the logarithmic derivative of Eq. (28) with respect to  $M^{-2}$  for the case of the  $\kappa$ .

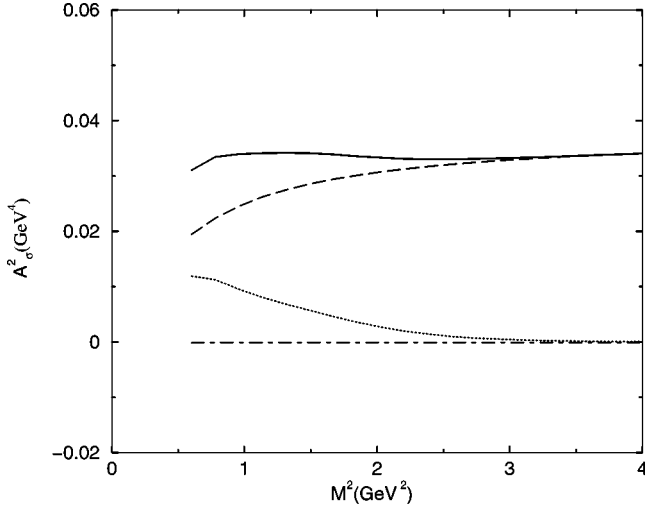


FIG. 8. Borel mass dependence of  $A_\sigma^2$ . Solid curve: total contribution; long-dashed: perturbative; dashed: quark condensate; dot-dashed: mixed condensate; dotted: instanton contribution.

tween the two sides of the sum rule in the Borel window  $0.9 \leq M^2 \leq 2 \text{ GeV}^2$ , and this defines the range for the Borel variable  $M_X'^2$  in Eq. (29). Since all Borel variables are related by Eqs. (30) and (31), we obtain from the above range approximately  $5 \leq M_D^2 \leq 15 \text{ GeV}^2$ .

In the case of  $\sigma$ , similar results are obtained and in Fig. 8 we show [for  $s_{0\sigma} = 1.2 \text{ GeV}^2$  and the ansatz in Eq. (20) for  $\rho_\sigma(s)$ ] that the instanton contribution (dotted line) is even more important in this case, giving a very stable result as a function of the Borel mass. The quark condensate and mixed condensate contributions (dashed line) are zero since they are now proportional to the light quark mass, which we take equal to zero.

In Fig. 9 we show the left-hand side (LHS) and the RHS of the sum rule generated by the logarithmic derivative of

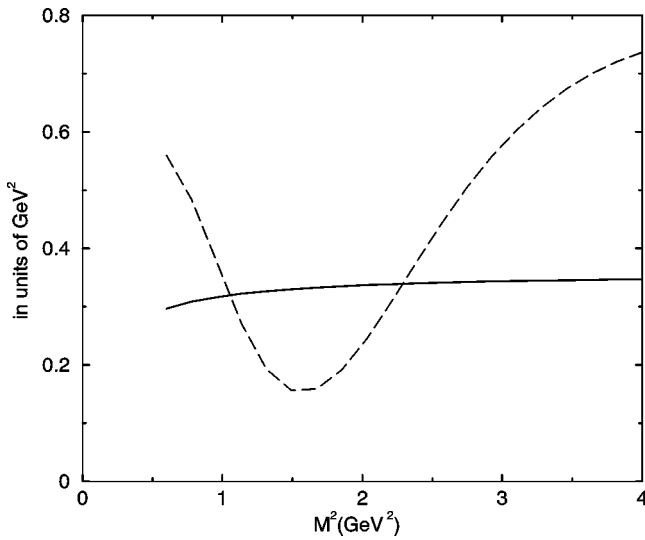


FIG. 9. Borel mass dependence of the RHS (dashed line) and the LHS (solid line) of the sum rule generated by the logarithmic derivative of Eq. (28) with respect to  $M^{-2}$  for the case of the  $\sigma$ .

Eq. (28) with respect to  $M^{-2}$ , as a function of the Borel mass, using the *Ansatz* in Eq. (20) for  $\rho_\sigma(s)$  and  $s_{0\sigma} = 1.2 \text{ GeV}^2$ . We see that there is again a good overlap between the two sides of the sum rule in the Borel window  $1.0 \leq M^2 \leq 2.5 \text{ GeV}^2$ . Since the mass of the particle is related with the square root of the LHS of this sum rule, from this figure we see that our result is compatible with  $m_\sigma = 0.5 \text{ GeV}$  found in [2].

## APPENDIX B: PERTURBATIVE AND NONPERTURBATIVE CONTRIBUTIONS TO THE TWO- AND THREE-POINT FUNCTIONS

In all this work we take into account the mass of the strange quark at most squared and neglect the mass of the light quarks. For the scalar meson  $X$ , we consider the particular case of  $\kappa$ , since the  $\sigma$  can be easily obtained from it by neglecting the strange quark mass. The perturbative contributions for the two-point functions defined in Sec. III are

$$\sigma_D(s) = \frac{3}{8\pi^2} \frac{(s - m_c^2)^2}{s} \quad (\text{B1})$$

and

$$\sigma_\kappa(s) = \frac{3}{8\pi^2} (s - 2m_s^2). \quad (\text{B2})$$

The nonperturbative contributions including the quark and mixed condensates are

$$\hat{K}_D(M^2) = -m_c \langle \bar{q}q \rangle e^{-m_c^2/M^2} \left[ 1 + \frac{m_0^2}{2M^2} \left( 1 - \frac{m_c^2}{2M^2} \right) \right] \quad (\text{B3})$$

and

$$\hat{K}_\kappa(M^2) = m_s e^{-m_s^2/M^2} \left( \langle \bar{q}q \rangle + \frac{\langle \bar{s}s \rangle}{2} + \frac{m_0^2}{2M^2} \langle \bar{q}q \rangle \right), \quad (\text{B4})$$

where we have defined  $\langle \bar{q}g_s \sigma \cdot Gq \rangle = m_0^2 \langle \bar{q}q \rangle$ . The instanton induced contribution is given by

$$\hat{K}_I(M^2) = \frac{\bar{n}}{2\bar{m}_s\bar{m}_q} M^2 z^2 \int_{z^2/4}^{\infty} dx \frac{x^2}{(x - z^2/4)^2} e^{-x^2/(x - z^2/4)}, \quad (\text{B5})$$

where  $z = M\bar{\rho}$ , and we have introduced the average instanton size,  $\bar{\rho}$ , and the instanton number density,  $\bar{n}$ , given by [9]

$$\bar{n} \simeq \frac{1}{2} \text{ fm}^{-4}, \quad \bar{\rho} \simeq \frac{1}{3} \text{ fm}. \quad (\text{B6})$$

In Eq. (B5)  $\bar{m}_s$  and  $\bar{m}_q$  are the effective quark masses which we take to be  $\bar{m}_s = 400 \text{ MeV}$  and  $\bar{m}_q = 300 \text{ MeV}$ .

The perturbative double spectral function is obtained by using the Cutkosky rules and, in the case of the meson  $\kappa$ , is given by

$$\rho_{+\kappa}(s_D, s_X, t) = \frac{-3}{8\pi^2\lambda^{3/2}} \left[ -2s_D s_X t + m_c^2 s_X (s_D - s_X + t) + m_s^2 s_D (s_X - s_D + t) + m_c m_s \times (m_c^2 (s_D - s_X - t) + m_s^2 (s_X - s_D - t) - (s_D - s_X)^2 + t(s_D + s_X)) \right] \times \Theta(s_D - m_c^2) \Theta(s_X - s_{Xmin}) \Theta(s_{Xmax} - s_X), \quad (\text{B7})$$

where

$$s_{Xmin} = \frac{m_c^2 (-m_c^2 + m_s^2 + s_D + t) + m_s^2 s_D - s_D t + (m_c^2 - s_D) \sqrt{\lambda(m_c^2, m_s^2, t)}}{2m_c^2} \quad (\text{B8})$$

and

$$s_{Xmax} = \frac{m_c^2 (-m_c^2 + m_s^2 + s_D + t) + m_s^2 s_D - s_D t - (m_c^2 - s_D) \sqrt{\lambda(m_c^2, m_s^2, t)}}{2m_c^2}. \quad (\text{B9})$$

The nonperturbative contributions, including the quark and mixed condensates, which survive the double Borel transformation in  $p_D^2$  and  $p_X^2$  are

$$\hat{K}_{+\kappa}(M_D^2, M_X^2, t) = \langle \bar{q}q \rangle e^{-m_c^2/M_D^2} \left[ \frac{m_c - m_s}{2} - m_0^2 \left( m_c^2 \frac{m_c - m_s}{8M_D^4} - \frac{2m_c - m_s}{6M_D^2} + \frac{(4m_c^2 + m_c m_s - 2t)(m_c - m_s)}{24M_D^2 M_X^2} - \frac{m_c - 2m_s}{6M_D^2} + \frac{(m_c m_s - 2t)(m_c - m_s)}{24M_D^2 M_X^2} \right) \right], \quad (\text{B10})$$

and the instanton induced contribution is given by

$$\hat{K}_{+I}(M_D^2, M_X^2, t) = \frac{8\bar{\rho}^4 t}{\pi^2} \frac{\bar{n}}{\bar{m}_s \bar{m}_q} \int_0^\infty dr ds du d\gamma \frac{(\gamma M_D^2 - 1) M_X^4}{(1 + 4\gamma s)^2 (\gamma s M_D^2 - 4M_X^2)^2 \gamma s^2} \times \sqrt{\frac{ru}{(s-r)(A_0-u)}} e^{-\bar{\rho}^2(r+u)} e^{-\gamma m_c^2} e^{-\alpha_0 \bar{\rho}^2} e^{-tA_0/sM_D^2}, \quad (\text{B11})$$

where

$$A_0 = \frac{s(\gamma M_D^2 - 1)}{1 + 4s\gamma} \quad (\text{B12})$$

and

$$\alpha_0 = \frac{(s + (1 + 4s\gamma)t)M_X^2}{s + (1 + 4s\gamma)t - 4M_X^2}. \quad (\text{B13})$$

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