Correlations between $\langle p_T \rangle$ and jet multiplicities from the Balitskiĭ-Fadin-Kuraev-Lipatov chain

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(Received 22 November 2001; published 23 May 2002)

Strong correlations between the number of emitted jets and their average transverse momentum are found for the events resulting from the exchange of a single Balitskiĭ-Fadin-Kuraev-Lipatov Pomeron.

DOI: 10.1103/PhysRevD.65.114001 PACS number(s): 13.60.Hb, 12.38.Bx, 13.87.Ce

As is well known, strong correlations are observed experimentally between the average p_T and multiplicities of particles produced in high-energy hadronic collisions [1]. Average p_T grows with multiplicity. This fact can be interpreted as a consequence of multiple hard collisions, which result in a larger number of particles produced and a broadening of the p_T spectrum [2]. An alternative interpretation can be given in terms of color strings which are stretched between the colliding hadrons during the collision. Here larger multiplicities correspond to a larger number of strings, which again leads to a broadening of the p_T spectrum either because of the accumulation of more transverse momentum from the parent partons or because of the interaction between strings [3]. In both cases it is tacitly assumed that with only one hard collision or, alternatively, with only one pair of noninteracting strings there are no correlations between $\langle p_T \rangle$ and multiplicity. Theoretically this assumption can only be tested within the Balitskiĭ-Fadin-Kuraev-Lipatov dynamics, which presents a detailed description of particle (actually jet) production at high energies under certain simplifying assumptions (a fixed small coupling constant). The present calculation is aimed to see whether there exist correlations between $\langle p_T \rangle$ and the number of produced jets in the hard Pomeron described by the BFKL chain of interacting Reggeized gluons.

Note that our definition of a jet is a purely theoretical one. As conventionally assumed in the BFKL model, a jet is a gluon with a large enough transverse momentum $p_T > \mu$. It is believed that it will eventually emerge as an observable jet of hadrons due to parton-hadron duality. The exact manner of its hadronization and experimental observation are beyond the scope of this study.

We shall limit ourselves with the leading order BFKL model both for theoretical and technical reasons. On the theoretical side, the situation with the nonleading terms is far from clear. Their magnitude strongly depends on the renormalization scheme [4]. Their energy behavior does not correspond to the Regge picture [5]. Technically the BFKL equation with the nonleading terms taken into account is untractable for our purpose. Finally it is most improbable that the correlations we study, once present with only the leading terms retained, will vanish when the nonleading terms are taken into account. For the latter reason, we also neglect the nonleading effects due to energy-momentum conservation, which by themselves introduce kinematical correlations between $\langle p_T \rangle$ and the number of jets. In addition, as we find the average p_T result to be very small, of the order 10 GeV/c, their transverse energy can well be neglected, except at the smallest values of $Q2$ and $1/x$.

Our study is closely related to the paper by Kwiecinski, Lewis, and Martin, who calculated the exclusive probabilities to observe a given number of jets from the exchanged hard Pomeron [6]. We shall extensively use both their method of calculation and some of their parametrizations.

The results we present in this paper demonstrate that, in fact, at realistic energies in the BFKL chain there are strong correlations between the number n of produced jets and the average $\langle p_T \rangle_n$ for these events. We find that $\langle p_T \rangle_n$ grows roughly linearly with n with a slope independent of Q^2 in deep inelastic scattering (DIS) and of the same value for purely hadronic collisions. The slope diminishes with rapidity y. So in the limit $y \to \infty$ the correlations are expected to disappear. However, at $y=15$ $(x \sim 10^{-7})$ the value of the slope is still ~ 0.8 GeV/c. These results leave open the question of correlations between $\langle p_T \rangle_n$ and n in the soft Pomeron and in the color string at the currently available energies.

We start by recalling the main points of the formalism employed in [6] to calculate the exclusive probabilities for the production of a given number of jets. To facilitate comparison with [6] we shall borrow their notations. Let the amputated BFKL amplitude be $f(y,k)$, when y is the rapidity and k is the two-dimensional transverse momentum of the virtual (Reggeized) gluon. Function $f(y,k)/k^2$ is interpreted as an unintegrated gluon distribution, related to the conventional gluon distribution by

$$xG(x,Q^2) = \int^{Q^2} \frac{dk^2}{k^2} f\left(\ln \frac{1}{x}, k\right). \tag{1}$$

The BFKL equation for f may be written in the form

$$f(y,k) = f^{(0)}(y,k) + \bar{\alpha}_s \int_0^y dy_1 \int \frac{d^2k_1}{\pi q^2} \times \left(\frac{k^2}{k_1^2} f(y_1,k_1) - f(y,k)\,\theta(k^2-q^2) \right). \tag{2}$$

Here $\bar{\alpha}_s = 3\alpha_s/\pi$, $q = k - k_1$ is the transverse momentum of the emitted (real) gluon, and it was assumed in [6] that the

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driving term $f^{(0)}$ (the impact factor of the target) may also depend on rapidity. To suppress the physically unknown infrared domain and make the equation numerically tractable, the integration over k_1 was constrained in [6] to the interval

$$Q_0^2 < k_1^2 < Q_f^2, \tag{3}$$

with $Q_0 = 1$ GeV/c and $Q_f = 100$ GeV/c. We shall also impose this constraint, implicit in the following equations.

Defining as an observable jet a real gluon with $q^2 \geqslant \mu^2$, one splits the integration over momenta in Eq. (2) into two parts by introducing

$$\theta(q^2 - \mu^2) + \theta(\mu^2 - q^2) = 1 \tag{4}$$

inside the integral. The whole integration kernel is thus split into two parts: a resolved one, K_R, corresponding to emitted gluons with $q^2 > \mu^2$, and an unresolved one, K_{UV}, which combines emission of gluons with $q^2 < \mu^2$ and the subtraction term in Eq. (2). Explicitly, the action in the momentum space of the two kernels is described by

$$(K_R f)(k) = \overline{\alpha}_s k^2 \int \frac{d^2 k_1}{\pi q^2 k_1^2} \theta(q^2 - \mu^2) f(k_1), \tag{5}$$

$$(K_{UV} f)(k) = \overline{\alpha}_s k^2 \int \frac{d^2 k_1}{\pi q^2 k_1^2} \left(\theta(\mu^2 - q^2) f(k_1) - \frac{k_1^2}{k^2} \theta(k^2 - q^2) f(k) \right). \tag{6}$$

Exclusive probabilities to produce n jets are obviously obtained by introducing n operators K_R between the Green's functions of the BFKL equations with kernel K_{UV}. If one presents the full gluon distribution f as a sum of contributions f_n from the production of n jets

$$f(y) = \sum_{n=0} f_n(y), \tag{7}$$

then one gets a recursive relation

$$f_n(y) = \int_0^y dy_1 K(y - y_1) f_{n-1}(y_1), \tag{8}$$

where $K(y)$ is a y-dependent operator in the transverse momentum space

$$K(y) = e^{y K_{UV}} K_R. \tag{9}$$

Equation (8) allows one to successively calculate the relative probabilities to produce $n = 0,1,2,\ldots$ jets starting from the no-jet contribution determined by

$$f_0(y) = e^{y K_{UV}} f^{(0)}(0) + \int_0^y dy_1 e^{(y - y_1) K_{UV}} \frac{d f^{(0)}(y_1)}{d y_1}. \tag{10}$$

In [4] the driving term was chosen to vanish at $y = 0$:

$$f^{(0)}(y,k) = A(1 - e^{-y})^5 e^{-k^2/Q_0^2} \tag{11}$$

(its normalization factor is irrelevant for our purpose).

Distributions $f_n(y,k)$ themselves are not observable quantities. Physical probabilities are obtained by convoluting f_n with the gluon distribution in the projectile (the projectile impact factor). For the perturbative QCD to be applicable, a reasonable choice is to take the virtual photon as a projectile, as done in [6]. Having in mind that the BFKL picture may only be applied to low values of x, in our calculations we used a simplified expression for the virtual photon impact factor, independent of rapidity, which can be found in [7]. To have some qualitative idea of the situation in purely hadronic collisions, we have also made our calculations for a hadronic projectile with an unperturbative impact factor. For collisions of two identical hadrons it should be identical to the target impact factor which appears as a y-independent driving term $f^{(0)}(k)$ in Eq. (2).

In both cases the exclusive probabilities to observe n jets are given by

$$P_n(y) = \frac{\int (d^2k/k^4) h(k) f_n(y,k)}{\int (d^2k/k^4) h(k) f(y,k)}, \tag{12}$$

where $h(k)$ is the impact factor of the projectile. Both impact factors, $f^{(0)}(k)$ of the target and $h(k)$ of the projectile, should vanish as $k \to 0$. This condition is satisfied by the virtual photon impact factor of [7]. As to the hadronic impact factor $f^{(0)}(k)$, we have chosen it in close similarity with Eq. (11):

$$f^{(0)}(k) = k^2 e^{-k^2/Q_0^2}. \tag{13}$$

As with Eq. (11), the overall normalization is irrelevant.

We are interested in the average values of $\langle q \rangle_n$ in the observed jets, provided their number n is fixed. At this point one has to remember that the momentum k which serves as an argument of $f(y,k)$ refers to the virtual gluon, and not to the emitted one, whose momentum q is hidden inside the kernel K_R. Therefore to find an average of any quantity $\phi(q)$ depending on the emitted real jet momentum, one has to introduce the function $\phi(q)$ into the integral (5), thus changing the kernel K_R to the kernel K_{av} defined by

$$(K_{av} f)(k) = \overline{\alpha}_s k^2 \int \frac{d^2 k_1}{\pi q^2 k_1^2} \theta(q^2 - \mu^2) \phi(q) f(k_1). \tag{14}$$

With n jets, one has to substitute one of the n operators K_R which generate the jets by K_{av}, take a sum of all such substitutions, and divide by n. One has further to integrate over all momenta of the virtual gluon k multiplied by the projectile impact factor, and normalize the result to the total probability to have n jets.

This recipe can be formalized in the following way. Introduce a generalized operator in the virtual gluon momentum space

$$K_1(y) = e^{yK_{UV}}[K_R + K_{av}]. \tag{15}$$

Let the function $F(y,k)$ obey the equation

$$F(y) = f_0(y) + \int_0^y dy_1 K_1(y-y_1) F(y_1). \tag{16}$$

One can split the function F into a sum of contributions F_{nm} corresponding to the action of n operators K_1, out of which $m = 0,1, \dots ,n$, are operators K_{av}:

$$F(y) = \sum_{n=0} \sum_{m=0}^{n} F_{nm}(y). \tag{17}$$

Evidently $F_{n0} = f_n$. We are interested in the contribution $F_{n1} \equiv g_n$ which contains a single operator K_{av}. The average value of interest is determined by

$$\langle \phi(q) \rangle_n = \frac{1}{n} \frac{\int (dk^2/k^4) h(k) g_n(y,k)}{\int (dk^2/k^4) h(k) f_n(y,k)}. \tag{18}$$

In analogy with Eq. (8), one easily sets up a recursion relation for g_n:

$$g_n = \int_0^y dy_1 K(y-y_1) g_{n-1}(y_1) + \int_0^y dy_1 e^{(y-y_1)K_{UV}} K_{av} f_{n-1}(y_1), \tag{19}$$

with the initial condition $g_0(y) = 0$. Together with Eq. (8), this relation allows one to calculate the function g_n for $n = 1,2, \dots$, and then to use Eq. (18) to find the desired averages.

The concrete choice of $\phi(q)$ is restricted by the condition of convergence at large q: $\phi(q) < q^2$, as $q \to \infty$. To facilitate our calculation we make a natural choice $\phi(q) = q$:

$$(K_{av} f)(k) = \bar{\alpha}_s k^2 \int \frac{d^2 k_1}{\pi k_1^2 q} \theta(q^2 - \mu^2) f(k_1). \tag{20}$$

With this choice the angular integration in Eq. (20) can be done analytically. With different $\phi(q)$ the magnitude of averages will obviously be different (and growing with α for $\langle q^{2\alpha} \rangle$). However, we do not expect that this will qualitatively change their n-dependence.

We defined our jets by taking $\mu = 2$ GeV/c. As for the cutoffs, we used Eq. (3). To see the influence of the cutoffs we also repeated our calculations with $Q_0 = 0.5$ GeV/c and $Q_f = 1000$ GeV/c. The results slightly change in their absolute values (by no more than 1%–6%) but both the n and y dependences remain the same.

We have calculated the functions f_n and g_n from Eqs. (8) and (19) up to $n = 5$ and $y = 15$. Following [6] we have used the expansion in N Chebyshev polynomials to discretize the kernels in a simple way. The results we present have been obtained with $N = 80$, although, as pointed out in [6], already $N = 20$ gives a reasonable approximation.

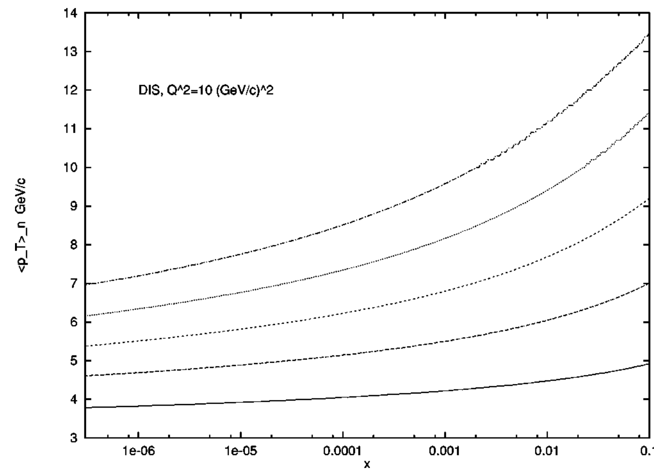

FIG. 1. Average $\langle p_T \rangle_n$ for a fixed number n of jets produced in γ^*-hadron collisions, as a function of x at $Q^2 = 10$ (GeV/c)^2. Curves from bottom to top correspond to $n = 1,2, \dots ,5$.

In Figs. 1–3 we present the averages $\langle q \rangle_n$ for $n = 1$–5 and $x = e^{-y} = 3 \times 10^{-7}$–0.1, for the γ^*-hadron collisions at $Q^2 = 10$, 100, and 1000 (GeV/c)^2. In Fig. 4 we show these averages for the collisions of two identical hadrons with the impact factor (13). As one observes, in all cases $\langle q \rangle_n$ strongly grows with n at all rapidities. The growth is approximately linear

$$\langle p_T \rangle_n \simeq a(y,Q^2) + b(y) n, \tag{21}$$

where both a and b depend on rapidity y, but, at fixed y, b is universal in the sense that it does not depend on Q^2 in DIS and has the same value for pure hadronic collisions. The slope $b(y)$ falls with y: it is equal to 1.25 GeV/c at $y = 7.5$ and 0.8 GeV/c at $y = 15$, so that one may expect that at ultrahigh energies $\langle q \rangle_n$ will become independent of n. The dependence on Q^2 results is rather weak. We attribute it to the fact that in the BFKL model at low x the bulk of the Q^2

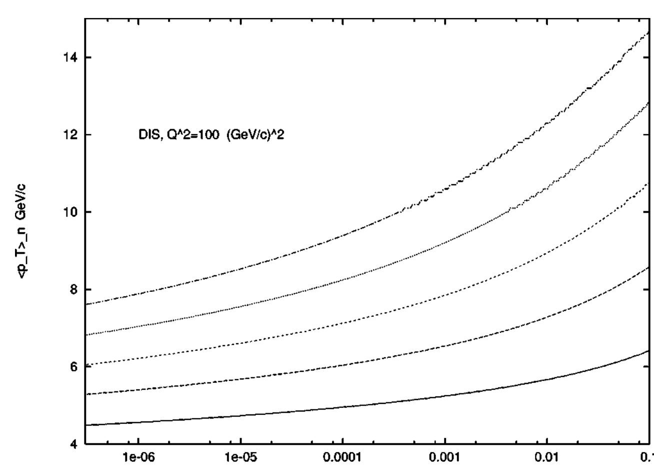

FIG. 2. Average $\langle p_T \rangle_n$ for a fixed number n of jets produced in γ^*-hadron collisions, as a function of x at $Q^2 = 100$ (GeV/c)^2. Curves from bottom to top correspond to $n = 1,2, \dots ,5$.

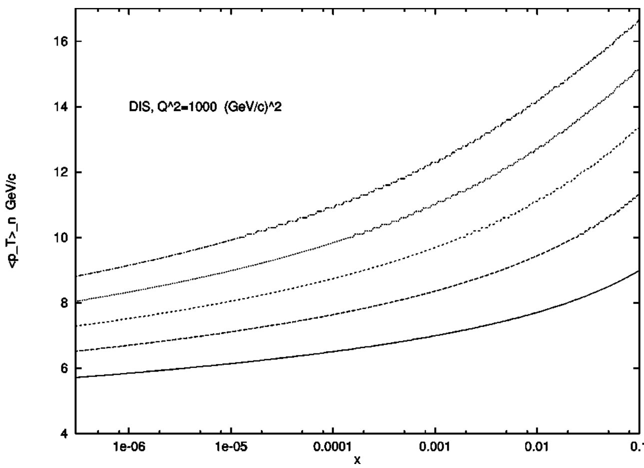

FIG. 3. Average $\langle p_T \rangle_n$ for a fixed number n of jets produced in γ^*-hadron collisions, as a function of x at $Q^2=1000\ (\mathrm{GeV}/c)^2$. Curves from bottom to top correspond to $n=1,2,\ldots,5$.

dependence is separated in the form of the overall factor (linear in Q at $x\to 0$), which cancels between the numerator and denominator in Eq. (18).

As an interesting by-product of our study we find that the averages $\langle q \rangle_n$ go down with rapidity for all $n \geqslant 2$. This is quite unexpected, since, as is well-known, in the BFKL approach an overall average $\langle q \rangle$ rapidly grows with y [[8] and Eq. (25) below]. It seems that this growth is totally explained by the growth of the average number of jets $\langle n \rangle$.

Passing to discussion, we first point out that it is an open question as to which kinematical conditions and to what degree the BFKL Pomeron may describe realistic hadronic processes. Emissions of high-p_T jets in γ^*-hadron collisions seem to be a suitable place to see the BFKL signatures. Our results show that in such emissions strong positive correlations are predicted between $\langle p_T \rangle$ and the number of jets, already for a single Pomeron exchange. This indicates that in fact such correlations do not require multiple rescatterings nor Pomeronic interactions, but that they are already present in the basic mechanism of jet production. Obviously, this conclusion cannot be directly applied to particle production in the soft region, and so the empirically invoked absence of such correlations for particle production from the color string does not really contradict our results. It can only be tested in its own framework by confronting color string predictions with the experimental data.

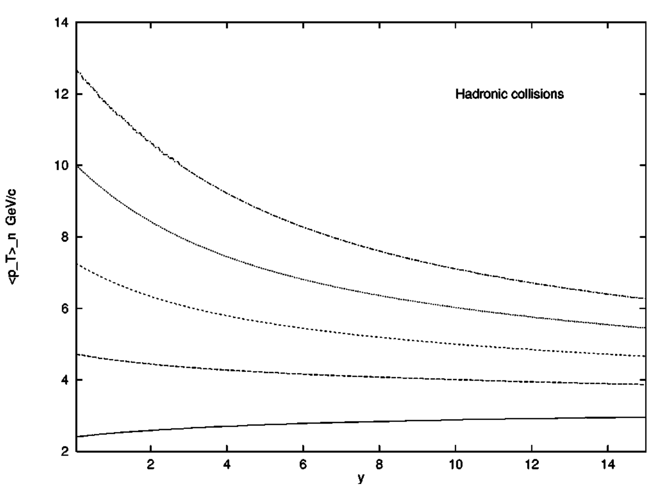

FIG. 4. Average $\langle p_T \rangle_n$ for a fixed number n of jets produced in hadronic collisions, as a function of y. Curves from bottom to top correspond to $n=1,2,\ldots,5$.

Our result (21) has been obtained for a relatively small number of jets and at energies corresponding to $y \leqslant 15$. If extrapolated to all n and energies it would lead to a relation between the overall averages

$$\langle p_T \rangle \propto \langle n \rangle. \tag{22}$$

However, it is well-known that this relation does not hold in the BFKL model at asymptotic energies, when $\langle p_T \rangle$ grows much faster than $\langle n \rangle$. This more or less known fact can be demonstrated by a simple calculation. Indeed the inclusive cross-section $I(y,y_1,q)$ to produce a jet at rapidity y_1 and with a transverse momentum q, in a collison with an overall rapidity $y \gg 1$, is given by [9]

$$I(y,y_1,q)=\frac{\overline{\alpha}_s}{4\pi}\sigma(y)\frac{1}{q^2}[1-\Phi(z)], \tag{23}$$

where

$$z=b\ln q, \quad b=\sqrt{\frac{y}{ay_1(y-y_1)}}, \quad a=14\overline{\alpha}_s\zeta(3). \tag{24}$$

Here Φ is the error function and $\sigma(y)$ is the total cross section. Since $b\sim 1/\sqrt{y}\ll 1$, the term with Φ is actually important only at large $\ln q$ when it cuts the distribution in q at $\ln q \sim \sqrt{y}$. For this reason the scale of q is unimportant, so that one may safely fix it by setting $\mu=1$. The average value of any positive power β of the transverse momentum is easily found to be

$$\langle p_T^\beta \rangle = \frac{\int_0^y dy_1 \int d^2q q^\beta I(y,y_1,q)}{\int_0^y dy_1 \int d^2q I(y,y_1,q)} = \frac{1}{\lambda^2} e^{\lambda^2}\Phi(\lambda),$$

$$\lambda=(1/2)\beta\sqrt{ay}. \tag{25}$$

Evidently $\langle p_T^\beta \rangle$ grows exponentially with y for any $\beta>0$.

Thus our results cannot be valid for all n and energies and refer precisely to the values of n and energies for which the calculations were done. It is interesting to note that relations similar to Eq. (22), with $\langle p_T \rangle$ substituted by $\langle p_T^2 \rangle$, were earlier obtained from gluon saturation [10], and in the percolation approach [11].

As mentioned, an unexpected result obtained in our calculation is that $\langle p_T \rangle_n$ at fixed $n \geqslant 2$ fall with energy. As seen in Figs. 1–4 this fall is not dramatic at energies at which we can expect the BFKL Pomeron to be relevant ($y>10$). Still, it is quite appreciable, especially in view of the naive belief that the average transverse momentum should grow with energy. At present we do not see any plausible explanation of this phenomenon. Certainly it deserves further investigation including higher y and/or n. We hope that it can be tested experimentally as a possible signature of the BFKL Pomeron.

It is a pleasure to thank C. Pajares for interesting discussions and for pointing out to us some relevant references. We also thank J. Castro and G. Parente for their useful comments. This work was supported by the Secretaría de Estado de Educación y Universidades of Spain and by the RFFI grant 01-02-17137 (Russia).

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