Constraints on large extra dimensions from neutrino oscillation experiments

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The existence of bulk sterile neutrinos in theories with large extra dimensions can naturally explain small 4-dimensional Dirac masses for the active neutrinos. We study a model with 3 bulk neutrinos and derive various constraints on the size of the extra dimensions from neutrino oscillation experiments. Our analysis includes recent solar and atmospheric data from SNO and Super-Kamiokande, respectively, as well as various reactor and accelerator experiments. In addition, we comment on possible extensions of the model that could accommodate the LSND results, using natural parameters only. Our most conservative bound, obtained from the atmospheric data in the hierarchical mass scheme, constrains the largest extra dimension to have a radius $R < 0.82$ μm. Thus, in the context of the model studied here, future gravitational experiments are unlikely to observe the effects of extra dimensions.

I. INTRODUCTION

The substantial body of theoretical work done on models with large extra dimensions (LED's) has provided us with new insight on some old problems. Even though the original motivations were mostly related to the question of gauge hierarchy [1], various other applications have been found and studied. Among these is an explanation for small but non-vanishing neutrino masses [2]. In the more traditional scenario with a large energy desert between the scales of electroweak symmetry breaking and grand unification, the small Majorana masses can be easily generated through the seesaw mechanism. In the models with LED's, however, this is not possible, since physics at energy scales well above a TeV is no longer described by a renormalizable quantum field theory. There is, however, an alternative mechanism. In these models, all the standard model (SM) particles, including left-handed neutrinos, have to be confined to a 4-dimensional subspace (3-brane) inside the full space-time. On the other hand, if an SM singlet fermion (such as the right-handed neutrino) is present, it can propagate in more than four dimensions. The large volume of the extra dimensions leads to a suppression of the wave function of this fermion on the brane, which in turn allows small Dirac neutrino masses to be generated naturally. This point contributes positively towards a comparison of the models with LED's with those of the traditional energy desert paradigm.

From the four-dimensional point of view, a higher-dimensional SM singlet fermion can be decomposed into a tower of Kaluza-Klein (KK) excitations. These states do not have SM charges, and can therefore be classified as "ster-ile." However, the KK states do not completely decouple from the system: there are mixings between them and the lowest lying, active neutrinos. Thus they can participate in neutrino oscillations, acting effectively as a large number of sterile neutrinos. The implications of this picture have been studied in a number of papers[1] [2–6].

The recent experimental results of the SNO Collaboration [8], in conjunction with the data from Super-Kamiokande [9], have yielded strong constraints on the contribution of any sterile state to the solar neutrino anomaly. Motivated by the new data, we have reexamined the phenomenological constraints on models with extra-dimensional neutrinos. We have considered the bounds coming from reactor and accelerator experiments, as well as measurements of fluxes of solar and atmospheric neutrinos. We have studied a model [4,5] with 3 bulk neutrinos which have Yukawa couplings to the 3 active brane neutrinos. We will refer to this model as the (3, 3) model, where the first integer corresponds to the number of brane neutrinos and the second is the number of bulk neutrinos. Unlike most previous studies, we are considering the situation in which the dominant effects for the solar and atmospheric neutrinos are oscillations amongst the three active states, with oscillations into the sterile KK excitations regarded as a perturbation to be constrained by the data. This framework, which is motivated by the SNO and Super-Kamiokande data, allows us to obtain simple analytic results throughout the analysis. We emphasize that the principal role of the extra dimensions in this case is to provide a natural framework for small Dirac neutrino masses.

In this paper, we will assume that the manifold on which the extra dimensions are compactified is highly asymmetric, with one dimension much larger than the rest. Unlike most collider, astrophysical and cosmological bounds, which are

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[1]There have been a number of other models which combined the ideas of LED with additional ingredients, such as small Majorana masses for the brane neutrinos [7].

largely independent of the compactification manifold's shape and only constrain its volume, the analysis of this paper will yield bounds on the size of the largest dimension. The bounds we obtain are sufficiently tight to rule out (in the context of the model considered here) effectively the possibility that the near-future Cavendish type experiments will observe a signal for large extra dimensions. Although this conclusion clearly depends on the assumed properties of the neutrino sector of the theory, we note that the (3, 3) model seems to be the simplest and most natural way to incorporate small neutrino masses in theories with LED's.

Along with the solar and atmospheric neutrino anomalies, evidence for neutrino oscillations has been reported by the Liquid Scintillation Neutrino Detector (LSND) experiment [10]. The LSND results cannot be accommodated within the minimal (3, 3) model without the introduction of *ad hoc* parameters, such as brane Majorana masses. We propose two extensions of the (3, 3) scheme which can address these results, using only the natural parameters of the model. The first proposal introduces two new sterile states, one on the brane and the other in the bulk. Thus, this model is of the (4, 4) type. The second proposal uses an additional LED of a different radius, in which a sterile bulk state is sequestered.

The paper is organized as follows. In the next section, we describe our setup and introduce the necessary notation. The motivation behind our analysis as well as our approach and choice of parameters are discussed in Sec. III. Section IV discusses constraints on the model from reactor, accelerator, atmospheric, and solar data. We present our results in the context of the hierarchical, inverted, and degenerate mass schemes. In Sec. V, we address the LSND results and outline the aforementioned proposals for accommodating them. Our conclusions are presented in Sec. VI.

II. FORMALISM AND NOTATION

A. General setup

As required in theories with LED's, we will assume that the standard model (SM) fields, including the three families of left-handed neutrinos ν_L^α ($\alpha = e, \mu, \tau$) and the Higgs doublet H, are confined to a four-dimensional brane, while gravitational fields propagate in a space-time with $\delta \geq 2$ additional compactified dimensions of volume V_δ. The fundamental gravitational scale of the higher-dimensional theory M_F is generally taken to be close to the weak scale to solve the hierarchy problem. To reproduce the measured strength of four-dimensional gravity at large distances, we require

$$\bar{M}_{\text{Pl}}^2 = M_F^{\delta+2} V_\delta,\qquad(1)$$

where $\bar{M}_{\text{Pl}} \simeq 2 \times 10^{18}$ GeV is the reduced Planck mass.

We postulate the existence of three families of SM singlet bulk fermions Ψ^α, which can propagate in $4 + \delta$ dimensions. Yukawa couplings of Ψ^α to the left-handed neutrinos on the brane give rise to masses for active neutrino species which are naturally of the right size to accommodate the solar and atmospheric neutrino anomalies. The same couplings lead to mixings between the active species and the higher KK com-

ponents of the bulk fermions, which from the four-dimensional point of view appear as sterile neutrinos.

Throughout this paper, we will assume that one of the dimensions is compactified on a circle of radius R which is much larger than the sizes of the other dimensions. In this case, only the KK excitations of the bulk neutrinos corresponding to the largest dimension will be relevant at low energies, and our treatment will be essentially five-dimensional. Apart from its simplicity, this model is interesting for the following reason. LED's could be discovered by Cavendish type experiments which search for deviations of the gravitational force from Newton's law at short distances. At present, these experiments probe distances of order 0.2 mm [11], and sensitivities of order 0.05 mm can be achieved in the near future [12]. If the compactification manifold is symmetric, astrophysical constraints on the radii of the extra dimensions imply that these sensitivities will not be sufficient to detect a signal [13]. However, the astrophysical and other high-energy constraints primarily restrict the *volume* of the extra dimensions, while the Cavendish type experiments are sensitive to the size of the *largest* dimension. Thus, in models with highly asymmetric compactifications, such as the one we are considering, these experiments still have a chance of observing deviations from Newton's law. Unfortunately, the constraints obtained in this paper indicate that even this possibility may be already ruled out, if the neutrino sector of the model has the properties assumed in our study.

As we will see below, the simple setup described here cannot accommodate the positive result reported by the LSND neutrino oscillation experiment. In Sec. V, we will propose two simple extensions of this setup that might be capable of explaining the LSND result.

B. The (3, 3) model

From the 4-dimensional point of view, the 5-dimensional fermion Ψ can be decomposed into two Weyl fermions, ψ_L and ψ_R. The action of the model is given by

$$S = \int d^4x\, dy\, i\bar{\Psi}^\alpha \Gamma_A \partial^A \Psi^\alpha + \int d^4x\, (i\, \bar{\nu}_L^\alpha \gamma_\mu \partial^\mu \nu_L^\alpha$$
$$+ \lambda_{\alpha\beta} H\, \bar{\nu}_L^\alpha \psi_R^\beta(x,0) + \text{H.c.}),\qquad(2)$$

where Γ_A, $A = 0, \ldots, 4$ are the 5-dimensional Dirac matrices. Note that this action conserves lepton number; in particular, we do not introduce Majorana masses for the left-handed neutrinos. The Yukawa couplings $\lambda_{\alpha\beta}$ have dimensions of $(\text{mass})^{-\delta/2}$. Since the only mass scale in the problem at this point is M_F, we introduce dimensionless Yukawa couplings via

$$h_{\alpha\beta} = \lambda_{\alpha\beta} M_F^{\delta/2}.\qquad(3)$$

We will assume that $h_{\alpha\beta}$ are of order one.

Let us decompose the 5-dimensional fermions $\psi_{L,R}$ into a tower of KK states $\psi_{L,R}^{(n)}$, $n = -\infty \ldots \infty$. It turns out that certain linear combinations of the KK states are completely

decoupled from the left-handed neutrinos, and therefore need not be considered. The states that do couple are given by

$$\nu_R^{\alpha(0)} \equiv \psi_R^{\alpha(0)};$$

$$\nu_R^{\alpha(n)} = \frac{1}{\sqrt{2}}(\psi_R^{\alpha(n)} + \psi_R^{\alpha(-n)}), \quad n=1\ldots\infty;$$

$$\nu_L^{\alpha(n)} = \frac{1}{\sqrt{2}}(\psi_L^{\alpha(n)} + \psi_L^{\alpha(-n)}), \quad n=1\ldots\infty. \quad (4)$$

In this notation, the mass terms resulting from Eq. (2) take the form

$$m_{\alpha\beta}^D\left(\bar{\nu}_R^{\alpha(0)}\nu_L^\beta + \sqrt{2}\sum_{n=1}^\infty \bar{\nu}_R^{\alpha(n)}\nu_L^\beta\right) + \sum_{n=1}^\infty \frac{n}{R}\bar{\nu}_R^{\alpha(n)}\nu_L^{\alpha(n)} + \text{H.c.} \quad (5)$$

The Dirac mass matrix is given by

$$m_{\alpha\beta}^D = h_{\alpha\beta}(v M_F/\bar{M}_{\text{Pl}}), \quad (6)$$

where $v \equiv \langle H \rangle = 246$ GeV, and we have used Eq. (3), $(M_F^\delta V_\delta)^{1/2} = \bar{M}_{\text{Pl}}/M_F$ (note that the KK modes have a prefactor proportional to $V_\delta^{-1/2}$), and Eqs. (4). If $M_F/\bar{M}_{\text{Pl}} \ll 1$, as is natural in models with LED's, the common scale of Dirac masses is well below the electroweak symmetry breaking scale v even for order-one Yukawa couplings. For example, with $M_F \sim 100$ TeV, and $0.1 \lesssim h_{\alpha\beta} \lesssim 1$, Dirac masses of the active species will coincide with those required by the solar and atmospheric data. This is the higher-dimensional version of the seesaw mechanism.

Diagonalization of the mass terms in Eq. (5) can be performed in two steps. First, we find 3×3 matrices l and r which diagonalize the Dirac mass matrix, $m_D^d \equiv r^\dagger m^D l = \text{diag}(m_1^D, m_2^D, m_3^D)$. We define

$$\nu_L^\alpha = l^{\alpha i}\nu_L'^i;$$

$$\nu_R^{\alpha(n)} = r^{\alpha i}\nu_R'^{i(n)}, \quad n=0\ldots\infty;$$

$$\nu_L^{\alpha(n)} = r^{\alpha i}\nu_L'^{i(n)}, \quad n=1\ldots\infty. \quad (7)$$

The transformation of the $\nu_L^{\alpha(n)}$ by $r^{\alpha i}$ greatly simplifies the problem by leaving the second term in Eq. (5) diagonal. Then, Eq. (5) takes the form

$$\sum_{i=1}^3 \bar{\nu}_R'^i M_i \nu_L'^i + \text{H.c.} \quad (8)$$

where

$$\nu_L'^i = (\nu_L'^i, \nu_L'^{i(1)}, \nu_L'^{i(2)}, \ldots)^T,$$

$$\nu_R'^i = (\nu_R'^{i(0)}, \nu_R'^{i(1)}, \nu_R'^{i(2)}, \ldots)^T,$$

and M_i is an infinite-dimensional matrix given by

$$M_i = \begin{pmatrix} m_i^D & 0 & 0 & \cdots \\ \sqrt{2}m_i^D & 1/R & 0 & \cdots \\ \sqrt{2}m_i^D & 0 & 2/R & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}. \quad (9)$$

To complete the diagonalization, we need to find infinite-dimensional matrices L and R such that $R_i^\dagger M_i L_i$ is diagonal; then, the mass eigenstates are given by $\tilde{\nu}_L^i = L_i^\dagger \nu_L'^i$, $\tilde{\nu}_R^i = R_i^\dagger \nu_R'^i$. In this paper, we are primarily interested in the decomposition of the flavor-basis brane neutrino states ν_L^α in terms of the mass eigenstates. These are given by

$$\nu_L^\alpha = \sum_{i=1}^3 l^{\alpha i}\sum_{n=0}^\infty L_i^{0n}\tilde{\nu}_L^{i(n)}. \quad (10)$$

The easiest way to find L is to observe that it has to diagonalize the Hermitian matrix $M^\dagger M$, and therefore consists of its eigenvectors. This procedure was performed in Refs. [2,3,5]. The component of L that enters Eq. (10) is given by

$$(L_i^{0n})^2 = \frac{2}{1 + \pi^2\xi_i^2/2 + 2\lambda_i^{(n)2}/\xi_i^2}, \quad (11)$$

where $\xi_i = \sqrt{2}m_i^D R$, and $\lambda_i^{(n)2}$ are the eigenvalues of the matrix $R^2 M^\dagger M$, which are found from the equation

$$\lambda_i - \frac{\pi}{2}\xi_i^2\cot(\pi\lambda_i) = 0. \quad (12)$$

The mass of the mode $\tilde{\nu}_L^{i(n)}$ is simply given by $\lambda_i^{(n)}/R$. Similarly,

$$L_i^{kn} = \frac{k\xi_i}{\lambda_i^{(n)2} - k^2}L_i^{0n}, \quad (13)$$

where $k=1\ldots\infty$ and $n=0\ldots\infty$.

Since our philosophy will be to consider the effects of extra dimensions as small perturbations on top of the oscillations amongst zero modes, we will be mostly interested in the case where $R^{-1} \gg m_i^D$, that is, the limit of small ξ_i. In this limit, we find

$$\lambda_i^{(0)} = \frac{\xi_i}{\sqrt{2}}\left(1 - \frac{\pi^2}{12}\xi_i^2 + \cdots\right), \quad L_i^{00} = 1 - \frac{\pi^2}{12}\xi_i^2 + \cdots$$

$$\lambda_i^{(k)} = k + \frac{1}{2k}\xi_i^2 + \cdots, \quad L_i^{0k} = \frac{\xi_i}{k} + \cdots$$

$$L_i^{k0} = -\frac{\xi_i}{k} + \cdots, \quad L_i^{kk} = 1 - \frac{\xi_i^2}{2k^2} + \cdots, \quad (14)$$

and $L_i^{jk} = O(\xi_i^2)$, where $j \neq k = 1\ldots\infty$. A quantity of crucial interest is the probability of finding a neutrino of flavor β in a beam that was born with flavor α and has traveled a distance L. This probability is given by

$$P_{\alpha\beta}(L) = |A_{\alpha\beta}(L)|^2,$$

$$A_{\alpha\beta}(L) = \sum_{i=1}^{3} l^{\alpha i} l^{\beta i *} A_i(L), \tag{15}$$

where E_ν is the energy of the beam, and

$$A_i(L) = \sum_{n=0}^{\infty} (L_i^{0n})^2 \exp\left(i \frac{\lambda_i^{(n)2} L}{2 E_\nu R^2} \right). \tag{16}$$

Similarly, the probability for ν_α to oscillate into sterile neutrinos is

$$P_{\alpha s}(L) = \sum_{i=1}^{3} \sum_{k=1}^{\infty} |B_{\alpha i(k)}|^2, \tag{17}$$

where

$$B_{\alpha i(k)} = l^{\alpha i} \sum_{n=0}^{\infty} L_i^{0n} L_i^{kn} \exp\left(i \frac{\lambda_i^{(n)2} L}{2 E_\nu R^2} \right). \tag{18}$$

Of course, $\Sigma_{\beta=1}^{3} P_{\alpha\beta}(L) + P_{\alpha s}(L) = 1$.

III. LED STERILE NEUTRINOS AND SNO: MOTIVATION AND CHOICE OF PARAMETERS

The flux of solar neutrinos has been measured by several collaborations using independent experimental techniques [14]. The observed flux is consistently lower than the predictions of the standard solar model (SSM) by a factor of two to three, depending on the spectral sensitivity of the experiment. The most elegant explanation of this deficit is provided by the hypothesis of neutrino oscillations. The neutrinos produced in the Sun are in the flavor (ν_e) eigenstate. If this state is a nontrivial superposition of the mass eigenstates, some of the neutrinos would change flavor on the way to the Earth. Because the terrestrial experiments are predominantly sensitive to electron neutrinos, this effect leads to a suppression of the measured fluxes.

Until recently, the experimental data could be explained either by an oscillation of ν_e into other active species, ν_μ or ν_τ, or by its oscillation into a sterile neutrino sector, denoted by ν_s. However, the best fit was for oscillations into active neutrinos, especially when the Super-Kamiokande day-night and spectrum information was included [9]. The case has recently been greatly strengthened. The SNO Collaboration [8] has unambiguously demonstrated the presence of non-electron active neutrinos in the solar flux. Moreover, the total flux of solar neutrinos implied by SNO and Super-Kamiokande data is in excellent agreement with the SSM calculations [15]. These results mean that only a fraction (if any) of the solar neutrinos get converted into sterile species.

In our framework, there are three active species, namely the left-handed brane neutrinos ν_L^α. These states can mix both among themselves and with the excited KK states of the bulk neutrinos, $\nu_L^{\alpha(n)}$. The bulk neutrinos do not carry any SM charges, and therefore are sterile. Thus the SNO data indicates that the solar neutrino deficit is primarily due to the

oscillations amongst the active brane states, and allows one to put constraints on their mixings with the sterile KK states.

In the model proposed by Dvali and Smirnov [3], the solar neutrino deficit is explained by the oscillations of ν_L^e into the KK excitations of the bulk neutrino of the same flavor. The other active flavors, ν_L^μ and ν_L^τ, play no role. (In the language of Sec. II B, this corresponds to $l^{e1}=1$, $l^{e2}=l^{e3}=0$.) It is immediately obvious that this model is in contradiction with the SNO data: it predicts that no non-electron active neutrinos should be observed in the solar flux. To accommodate the data, it is necessary to introduce non-zero mixing among the active neutrinos.

Motivated by the SNO data, we will take the following approach. We assume that in the limit in which the sterile KK neutrinos are decoupled ($R \rightarrow 0$ limit) the oscillations amongst the remaining active species provide a good fit to the solar neutrino data. To be specific, we assume that the large mixing angle MSW solution to the solar neutrino problem, which provides the best fit to the data [14], is correct. We fix $\delta m_{sol}^2 = \delta m_{21}^2 \equiv (m_2^D)^2 - (m_1^D)^2 = 3.7 \times 10^{-5}$ eV^2 and take the mixing matrix elements to be $l^{e1} = \cos\beta$, $l^{e2} = \sin\beta$, with $\tan^2\beta = 0.37$. (Varying these parameters over their allowed ranges does not significantly affect our conclusions.) Furthermore, we will assume that the atmospheric neutrino anomaly observed by Super-Kamiokande [17] is also due predominantly to the oscillations between the active species ($\nu_\mu \leftrightarrow \nu_\tau$). This assumption is supported by the analysis of the neutral-current-enhanced events, the zenith angle distribution, and τ-appearance candidates in Super-Kamiokande [18], and by the zenith angle distribution observed by MACRO [19], which strongly disfavor the possibility that the anomaly is due to $\nu_\mu \leftrightarrow \nu_s$ oscillations. A good fit to the Super-Kamiokande data is obtained if we choose the mixing between ν_μ and ν_τ to be maximal, and the mass splitting $|\delta m_{atm}^2| = |\delta m_{32}^2| = 3 \times 10^{-3}$ eV^2. Finally, we (generally) choose l^{e3} to be 0 because of the strong constraint on this matrix element provided by a combination of atmospheric neutrino data and reactor experiments. With these choices, the only remaining free parameters are the radius of the extra dimension R and the absolute scale of the Dirac masses. For the latter, we will consider three possibilities: (1) the "hierarchical" mass scheme, $m_1^D \approx 0$, $m_3^D \gg m_2^D$, implying $m_3^D \approx (\delta m_{atm}^2)^{1/2} \approx 0.055$ eV and $m_2^D \approx (\delta m_{sol}^2)^{1/2} \approx 0.006$ eV; (2) the "inverted" mass scheme, $m_3^D \approx 0$, $m_1^D \approx m_2^D \approx (\delta m_{atm}^2)^{1/2} \approx 0.055$ eV; and (3) the "degenerate" scheme,[2] with $m_1^D \approx m_2^D \approx m_3^D \approx 1$ eV, where the common value has been chosen to be close to the cosmological limit of ~ 4.4 eV on the sum of the light neutrino masses [20]. In the next section, we will present experimental constraints on the radius R in each of these three schemes.

An important advantage of our perturbative approach is that it allows us to obtain simple analytic results throughout

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[2] The original motivation for the degenerate scheme, i.e., mixed hot-cold dark matter models, has now largely disappeared, but it is still a logical possibility. Note also that this scheme requires a fundamental scale M_F larger than 100 TeV.

TABLE I. Upper bounds on R (cm) at 90% C.l. and the corresponding lower bounds on $1/R$ (eV) from various measurements.

Experiment	Experimental bounds		
	Hierarchical (cm, eV)	Inverted (cm, eV)	Degenerate (cm, eV)
CHOOZ	$(9.9 \times 10^{-4}, 0.02)$	$(3.3 \times 10^{-5}, 0.60)$	$(1.8 \times 10^{-6}, 10.9)$
BUGEY	none	$(4.3 \times 10^{-5}, 0.46)$	$(2.4 \times 10^{-6}, 8.3)$
CDHS	none	none	$(5 \times 10^{-6}, 4)$
Atmospheric	$(8.2 \times 10^{-5}, 0.24)$	$(6.2 \times 10^{-5}, 0.32)$	$(4.8 \times 10^{-6}, 4.1)$
Solar	$(1.0 \times 10^{-3}, 0.02)$	$(8.9 \times 10^{-5}, 0.22)$	$(4.9 \times 10^{-6}, 4.1)$

the analysis, making the underlying physical picture quite transparent. A more complete approach would be to fit the data allowing all the parameters of the model [3 Dirac masses, 3 mixing angles, 1 Cabibbo-Kobayashi-Maskawa (CKM) phase and the radius of the extra dimension R] to vary, and then obtain bounds on R by marginalizing over the other variables. The resulting bounds would necessarily be less restrictive than the results of our present approach. However, as we explained above, current experimental data disfavors the possibility that oscillations into sterile states play a dominant role for either solar or atmospheric neutrinos. This implies that the results presented here should provide a good first approximation which could then be refined by a more systematic analysis.

IV. EXPERIMENTAL CONSTRAINTS ON R FROM NEUTRINO OSCILLATION DATA

Having stated our approach, we will use the formalism developed in the previous two sections to derive constraints on the radius of the extra dimension R from a variety of sources, such as reactor and accelerator searches for neutrino oscillations and solar and atmospheric neutrino flux measurements. We will next consider each of these constraints in turn.

A. Reactor and accelerator experiments

Large fluxes of anti-electron neutrinos are produced at nuclear power reactors. If the flux can be either predicted accurately or measured by a nearby detector, measuring the ν_e flux at a certain distance L from the reactor gives the electron neutrino survival probability $P_{ee}(L)$. Several experiments, such as CHOOZ [21] and Bugey [22], have utilized this approach to search for neutrino oscillations. Their results are consistent with no oscillation hypothesis, that is, $P_{ee}(L) \simeq 1$.

The electron neutrino survival probability is determined by Eq. (15),

$$P_{ee}(L) = \left| \sum_{i=1}^{3} |l^{ei}|^2 A_i(L) \right|^2, \quad (19)$$

where A_i is given by Eq. (16). For $l^{e3} \simeq 0$ and the values chosen for the m_i^D, $P_{ee}(L) \approx 1 - P_{es}(L)$. For the case of small ξ that we will be primarily interested in

$$P_{es}(L) \approx \sum_i 2|l^{ei}|^2 \xi_i^2 \sum_{k=1}^{\infty} \left(\frac{1 - \cos \phi_i^k}{k^2} \right) + O(\xi^4), \quad (20)$$

where the oscillation phase is

$$\phi_i^k \equiv \frac{(\lambda_i^{(k)2} - \lambda_i^{(0)2})L}{2E_\nu R^2} \approx \left(\frac{k^2}{R^2} - (m_i^D)^2 \right) \frac{L}{2E_\nu}. \quad (21)$$

It is convenient to evaluate the probabilities numerically, keeping only a finite number of terms in the sums in Eq. (16) or Eq. (20). (The oscillation probability into the k-th KK state is suppressed by k^{-2} for large k.)

For the CHOOZ experiment, the distance from the reactor to the detector is $L \approx 1$ km, and the neutrino energies E_ν range from about $(1-6)$ MeV. The ratio of the measured flux to the no-oscillation Monte Carlo prediction reported by CHOOZ, $\mathcal{R} = 1.01 \pm 0.028(\text{stat}) \pm 0.027(\text{syst})$, implies[3]

$$P_{ee}(L) > 0.942 \quad [P_{es} < 0.058] \quad (22)$$

at 90% C.l. Taking an average value $L/E_\nu \sim 300$ m/MeV, we obtain the following 90% C.l. constraints on the radius of the extra dimension: (1) in the hierarchical scheme, $\xi_2 < 0.43$, corresponding to $1/R > 0.02$ eV; (2) in the inverted scheme, $\xi_1 \approx \xi_2 < 0.13$, $1/R > 0.60$ eV; and (3) in the degenerate scheme, $\xi_1 \approx \xi_2 \approx \xi_3 < 0.13$, $1/R > 10.9$ eV. We present all the corresponding upper bounds on R in Table I. The constraints in the inverted and degenerate schemes are quite a bit stronger than in the, perhaps more natural, hierarchical scheme. There are two reasons for this. First, since $\xi = \sqrt{2}mR$, the same bound on ξ results in stronger bounds on R if the active neutrinos are heavy. Secondly, with the parameters of the CHOOZ experiment, the phases of the oscillations into the first few KK states are small in the hierarchical case, suppressing the oscillations. The higher modes have larger oscillation phases but small mixings.

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[3]These results and similar ones throughout the paper are obtained by the conservative method of assuming a Gaussian distribution for $P_{ee}(L)$ in the physical region $0 \leqslant P_{ee}(L) \leqslant 1$, using the central value and uncertainty from $\mathcal{R}$ but renormalizing so that the total probability for finding any value of $P_{ee}(L)$ in the physical region is unity.

These results were obtained numerically using Eq. (16), but it is instructive to discuss the approximate formula (20), which yields almost identical results, even in the hierarchical mass scheme. In the hierarchical case,

$$P_{es}(L) \approx 2 \sin^2\beta \, \xi_2^2 \sum_{k=1}^{\infty} \left(\frac{1-\cos\phi_2^k}{k^2} \right). \qquad (23)$$

The phases ϕ_2^k of the first few KK states are not large, so the $\cos\phi_2^k$ terms must be kept. We have verified that integrating over the neutrino energy E_ν, which smooths the oscillations, gives results almost identical to using the average L/E_ν. In the inverted and degenerate cases, the $\phi_{1,2}^k$ are large and the $\cos\phi_{1,2}^k$ average to zero. Since $\xi_1 \approx \xi_2$ in these cases, one finds $P_{es}(L) \approx \xi_2^2 \pi^2/3$.

Our assumption $l^{e3}=0$ yields the most conservative limit. Relaxing this assumption would result in more stringent limits because $1-P_{ee}(L)$ would receive new positive contributions $\sim |l^{e3}|^2(2+(\pi^2/3)\xi_3^2)$, where the two terms are respectively due to oscillations into active neutrinos and into the third KK tower.

The analysis of the Bugey experiment is very similar. The results are presented in Table I. Because of the small distance between the reactor and the detector in this experiment, it is not possible to put significant bounds on R in the hierarchic scheme: oscillation phases are substantial only for very high KK modes, but the mixings of those modes with the active neutrino are severely suppressed. In the other two mass schemes, the bounds are similar to CHOOZ, although somewhat weaker due to a lower precision of the flux measurement.

Neutrino oscillations can also be searched for at accelerators, where beams of muon neutrinos can be produced. The CDHS experiment [23] at CERN has searched for the disappearance of muon neutrinos from a beam with average energy $E_\nu \approx 3$ GeV by measuring the ratios of the numbers of events in two detectors at 130 and 885 m from the source. The negative results of this experiment can, in principle, yield bounds on R. However, for the L/E_ν range of the experiment we obtain significant bounds only for the degenerate scheme. Averaging over the neutrino energies in the CDHS range and comparing with the measured ratio, we obtain the constraint that $1/R$ must be greater than around 6 eV, with a small allowed window around 4 eV. Because of the imprecise nature of the degenerate solution, we just quote $1/R>4$ eV.

More recently, the CHORUS [24] and NOMAD [25] Collaborations have searched for an *appearance* of tau neutrinos in a beam originally composed of ν_μ. The non-observation of tau neutrino events has led to strong bounds on the probability of $\nu_\mu \rightarrow \nu_\tau$ oscillations, of order $P<10^{-4}$. Unfortunately, this constraint does not put significant bounds on R. With the parameters of these experiments, the phases of the oscillations into the first 10–100 modes are small in hierarchical and inverted schemes. For higher modes, the oscillation amplitudes are of order ξ^2/n^2, since the KK mode has then to oscillate back into ν_τ to be detected. Thus, the corresponding probability is severely suppressed: $P_{\mu\tau} \sim \xi^4/n^4$.

The resulting constraints are of the order $\xi \sim 1$ and are not competitive with bounds from neutrino disappearance experiments. For the degenerate scheme the oscillation phases could be substantial even for low-lying modes, but because of unitarity cancellations the resulting oscillation probabilities are severely suppressed, and no useful bound can be obtained. (The cancellations appearing in this case are analogous to those discussed in Sec. V.)

Another accelerator experiment, LSND, has reported evidence for neutrino oscillations. We postpone the discussion of this result until Sec. V.

B. Atmospheric neutrinos

The Super-Kamiokande experiment [17] has measured the fluxes of electron and muon neutrinos produced in the atmosphere by cosmic rays. While the flux of electron neutrinos is well described by a Monte Carlo (MC) simulation based on the no-oscillation hypothesis, the muon neutrino flux deviates significantly from this prediction. In particular, the flux of up-going muon neutrinos is suppressed in comparison with the down-going flux. The data, including the corresponding zenith angle distribution, is consistent with the hypothesis that on their way through the Earth the up-going muon neutrinos undergo oscillations that convert them into ν_τ's. (An alternative possibility of $\nu_\mu \rightarrow \nu_s$ oscillations is strongly disfavored [18].) Fitting the data within the two-flavor scheme yields the mass difference $|\delta m_{32}^2| \approx 3 \times 10^{-3}$ eV^2, and the mixing angle $\theta_{23} \approx \pi/4$ (maximal mixing).

For the case we are considering, the oscillation of $\nu_{\mu,e}$ into sterile KK states will involve large oscillation phases and therefore lead to a suppression of the overall ν_e and/or ν_μ fluxes that is independent of the zenith angle. The Super-Kamiokande Collaboration [17] have included the possibility of such L/E_ν-independent flux changes in their analysis to take into account the theory uncertainties in the absolute and relative flux calculations. However, their results can also be used to constrain L/E_ν-independent oscillations into sterile states, provided the theoretical uncertainties in the fluxes are properly included.

The best-fit values of the Super-Kamiokande data [17], assuming a $\nu_\mu \leftrightarrow \nu_\tau$ analysis, suggest that there is a slight overall excess in the ratio of the measured to the central value of the simulated number of events. The data also yield a deficit for the ratio of the μ-like to e-like events, assuming the same two-flavor analysis. In the analysis of the data [17], the overall excess and the deficit mentioned above are parametrized by $\alpha=0.034\pm0.25$ and $\beta_s=-0.059\pm0.08$, respectively, where the observed sub-GeV ν_e and ν_μ rates relative to the number expected for no oscillations into sterile states are

$$r_{\nu_e} = (1+\alpha)\left(1 - \frac{\beta_s}{2}\right)$$

$$r_{\nu_\mu} = (1+\alpha)\left(1 + \frac{\beta_s}{2}\right). \qquad (24)$$

The subscript s refers to the fact that this value is obtained from the sub-GeV events, which yield the most stringent constraints. We have assumed that the errors are dominated by the theoretical flux uncertainties. Quantitatively, the experimental results imply

$$-\alpha = \frac{1}{2}[P(\nu_\mu \to \nu_s) + P(\nu_e \to \nu_s)] \qquad (25)$$

and

$$\beta_s = P(\nu_e \to \nu_s) - P(\nu_\mu \to \nu_s). \qquad (26)$$

In what follows, we use the one-sided 90% C.l. upper bounds on $-\alpha$ and $\pm\beta_s$ to derive our constraints.

In the hierarchical mass scheme, the active neutrinos predominantly mix with the $\tilde{\nu}_L^{3(k)}$, since $\xi_3 \gg \xi_2$, $\xi_1 \approx 0$. Since $l^{e3} = 0$, this mixing can only occur for the muon neutrinos, decreasing their flux compared to the best-fit value; the electron neutrino flux remains unchanged. Equation (26) implies that at 90% C.l.,

$$P(\nu_\mu \to \nu_s) - P(\nu_e \to \nu_s) < 0.17. \qquad (27)$$

Thus, ignoring the ν_e oscillations, and in the limit $\phi_3^k \gg 1$, we obtain $P(\nu_\mu \to \nu_s) = \xi_3^2 \pi^2/6 < 0.17$, which yields $1/R > 0.24$ eV.

In the inverted mass scheme with $\xi_1 \approx \xi_2 = \xi$ and $\xi_3 \approx 0$, two KK towers, $\tilde{\nu}_L^{1(k)}$ and $\tilde{\nu}_L^{2(k)}$, contribute to the oscillations. Both electron and muon neutrinos can mix with these states, so both fluxes are reduced. It turns out that the suppression is more significant for the electron neutrino, so we derive our constraint from the upper bound on β_s. Again, assuming the phase energy averaging, we find $P(\nu_e \to \nu_s) = 2P(\nu_\mu \to \nu_s) = \xi^2 \pi^2/3$. Thus, $\xi^2 \pi^2/6 < 0.1$, which yields $1/R > 0.32$ eV.

Finally, in the degenerate mass scheme, with $\xi_1 \approx \xi_2 \approx \xi_3 = \xi$, all three KK towers can mix with the active neutrinos with an approximately equal strength. In this case, the oscillations into the KK states suppress the electron and muon neutrino fluxes by the *same amount*. In this case, we use Eq. (25) to find $(1/2)[P(\nu_\mu \to \nu_s) + P(\nu_e \to \nu_s)] < 0.39$, at 90% C.l. We have $P(\nu_\mu \to \nu_s) = P(\nu_e \to \nu_s) = \xi^2 \pi^2/3$, where phases are energy averaged. This yields $\xi^2 \pi^2/3 < 0.39$ and hence $1/R > 4.1$ eV. As we can see, the bounds from the atmospheric data are quite stringent. These results are summarized in Table I.

C. Solar neutrinos

As we already discussed in Sec. III, the results of the SNO Collaboration [8] make it unlikely that oscillations into sterile KK neutrinos play a major role in explaining the solar neutrino deficit. In particular, combining the SNO [8] and Super-Kamiokande [9] results, and using the new SSM estimate of the flux of ^{8}B neutrinos [15] that incorporates a recent more accurate measurement of the ^{7}Be$(p,\gamma)^8$B reaction [16], one obtains

$$\sum_{\alpha=\mu,\tau} P(\nu_e \to \nu_\alpha) = 0.62 \pm 0.21,$$

$$P(\nu_e \to \nu_s) = 0.083 \pm 0.21,$$

$$P(\nu_e \to \nu_e) = 0.30 \pm 0.05 \qquad (28)$$

for the probabilities for an initial ^{8}B neutrino in the SNO–Super-Kamiokande energy range to oscillate into ν_μ, ν_τ, or a sterile state, or to remain a ν_e, respectively. This leads to

$$P(\nu_e \to \nu_s) < 0.40 \qquad (29)$$

at 90% C.l. (The corresponding limit would be 0.41 using the older cross sections.)

To compute the fluxes of solar neutrinos, we need to take into account matter effects in the Sun. In vacuum, the flavor-basis active neutrino states can be decomposed into the mass eigenstates according to

$$\nu_\alpha = \sum_{i=1}^{3} \sum_{n=0}^{\infty} U_{\alpha i}^{(n)} \tilde{\nu}^{i(n)}. \qquad (30)$$

[This is just Eq. (10) of Sec. II B; we have dropped the subscript L to avoid cluttering and defined $U_{\alpha i}^{(n)} = l^{\alpha i} L_i^{0n}$.] In matter, an additional, flavor-dependent effective mass term is generated for active neutrino species. The decomposition (30) is modified by this effect

$$\nu_\alpha = \sum_{i=1}^{3} \sum_{n=0}^{\infty} U_{\alpha i}^{m(n)} \tilde{\nu}_m^{i(n)}, \qquad (31)$$

where $\tilde{\nu}_m^{i(n)}$ and $U_{\alpha i}^{m(n)}$ are respectively the eigenstates and mixing matrix elements in matter. These of course depend on the matter density, and therefore change as the neutrino travels through the Sun. We assume that neutrino propagation is adiabatic, that is, the fraction of each of the mass eigenstates in the beam remains constant. This is an excellent approximation for the large mixing angle (LMA) solar neutrino parameters that we are using [14,26]. Since all the phases associated with solar neutrino oscillations are large, we can neglect the interference effects between the matter eigenstates. Then, the oscillation probability is given by a simple formula [26]

$$P(\nu_e \to \nu_f) = \sum_{i,n} P(\nu_e \to \tilde{\nu}_m^{i(n)}) \cdot P(\tilde{\nu}^{i(n)} \to \nu_f)$$

$$= \sum_{i,n} |U_{ei}^{m(n)}|^2 |U_{fi}^{(n)}|^2. \qquad (32)$$

In this equation and below, $\tilde{\nu}_m^{i(n)}$ and $U_{\alpha i}^{m(n)}$ are evaluated at the neutrino production point in the core of the Sun, and we have assumed that the neutrino flux is measured in the vacuum (as is the case for solar neutrinos.)

The values of the mixing angles $U_{ei}^{m(n)}$ in Eq. (32) can be computed exactly if the neutrino energy and electron and neutron densities in the region of the Sun where the neutri-

nos are created are known. This in turn depends on the number and properties of the Mikheyev-Smirnov-Wolfenstein (MSW) resonances [26] passed by the neutrino on its way to the vacuum.

For the energies of the ^{8}B neutrinos observed by Super-Kamiokande and SNO, an MSW resonance can only occur for mass-squared splittings less than about 10^{-4} eV^2. Hence, in our case, resonant conversion is only possible between the two zero-mode mass eigenstates, $\tilde{\nu}^{1(0)}$ and $\tilde{\nu}^{2(0)}$. The higher mass states, which are mainly the sterile KK excitations, do not undergo resonant conversion and are little affected by matter effects for the masses required by the reactor and atmospheric data. Thus, there is only one resonance to consider. We will assume that all solar neutrinos observed by SNO and Super-Kamiokande cross this resonance, and that for all of them the propagation through the resonance is adiabatic, which are excellent approximations for the large mixing angle (LMA) parameter region. The electron neutrino mixing angles in the core of the Sun are then

$$|U_{e1}^{m(0)}|^2=0, \quad |U_{e2}^{m(0)}|^2=|U_{e1}^{(0)}|^2+|U_{e2}^{(0)}|^2, \quad (33)$$

with all the other matrix elements $U_{ei}^{m(n)}$ being equal to their vacuum values.

Formulas (32) and (33), together with Eq. (17) and the small-ξ expansions of the vacuum mixing angles (14), allow us to estimate the sterile component of the solar neutrino flux. For example, in the hierarchical mass scheme,

$$P(\nu_e\to\nu_s)=1-\sum_{\alpha=e,\mu,\tau}P(\nu_e\to\nu_\alpha)=\frac{\pi^2}{6}(1+\sin^2\beta)\xi_2^2. \quad (34)$$

The experimental bound $P(\nu_e\to\nu_s)<0.40$ at 90% C.l. then implies $1/R>0.02$ eV. The analysis in the other two mass schemes is analogous except that $P(\nu_e\to\nu_s)=\pi^2\xi_2^2/3$, implying $1/R>0.22$ eV (inverted scheme) and $1/R>4.1$ eV (degenerate scheme). These results are shown in Table I.

V. THE LSND EXPERIMENT

The LSND experiment has searched for $\bar{\nu}_\mu\to\bar{\nu}_e$ oscillations and reports an excess of $\bar{\nu}_e$ events, corresponding to an oscillation probability $P(\bar{\nu}_\mu\to\bar{\nu}_e)=(0.264\pm0.067\pm0.045)\%$ [10]. Similar results are obtained for $\nu_\mu\to\nu_e$. The data suggests that the oscillations occur for 0.2 eV$^2\lesssim\delta m^2\lesssim10$ eV^2, with the lower δm^2 values favored by the nonobservation of an oscillation signal in the KARMEN experiment [27]. Thus, it seems that mass scales of ~1 eV must be present in any model that attempts to describe the LSND results. In particular, one must introduce a fourth neutrino to account for LSND as well as the solar and atmospheric results, and this must be a sterile ν_s due to the constraint on the number of light active neutrinos from the Z width [28]. Many authors have analyzed such four-neutrino schemes [29]. The best fits are obtained in the so-called 2+2 schemes, in which there are two closely spaced pairs or states, with the pairs separated by ~1 eV and a small mix-

ing between the pairs to account for the LSND results. However, the simplest versions, in which the pairs consist of either (a) (ν_e,ν_s) and (ν_μ,ν_τ), or (b) (ν_e,ν_τ) and (ν_μ,ν_s), are now excluded by the solar and atmospheric data, so that the partners of the ν_e and ν_μ would have to be mixtures of ν_s and ν_τ. The alternative $3+1$ (or $3+p$, with $p>1$) scheme involves three closely-spaced active neutrinos (similar to the hierarchical, inverted or degenerate schemes), separated from one or more mainly sterile states by about 1 eV. $\nu_\mu\to\nu_e$ oscillations are therefore a sub-leading effect involving the mixing of both ν_μ and ν_e with the sterile state. Such schemes are strongly disfavored because it is difficult to obtain a large enough effect for LSND while still respecting the reactor and accelerator ν_e and ν_μ disappearance constraints, but are still barely possible.

Although the (3, 3) model contains KK states of mass ~1 eV in the hierarchical scheme for large mode numbers $k\sim100$, transitions among the active states are suppressed by the smallness of $|l^{e3}|$ (so far set to zero) and δm^2 for the active states, as well as by the $1/k$ factors in the mixing. Even allowing $|l^{e3}|\approx0.2$, calculations based on our formalism still fall an order of magnitude short of the central value for $P(\bar{\nu}_\mu\to\bar{\nu}_e)$ quoted above. The inverted and the degenerate schemes result in roughly the same or even smaller probabilities. In generic $3+1$ or $3+p$ schemes the rate is simply proportional to the fourth power of mixing angles θ, i.e., $P(\bar{\nu}_\mu\to\bar{\nu}_e)\propto\theta_{e4}^2\theta_{\mu4}^2$. However, in our model there are cancellations between the two KK towers so that $P(\bar{\nu}_\mu\to\bar{\nu}_e)\propto|\xi_2^2-\xi_1^2|^2=4|\delta m_{sol}^2R^2|^2$ (or $|l^{e3}|^2|\delta m_{atm}^2R^2|^2$ for $l^{e3}\neq0$). Thus, there are no enhancements from the larger Dirac masses in these schemes, and also R cannot be larger than in the hierarchical case because of other constraints.

Hence, it seems that an extension of the (3, 3) model is needed in order to address the LSND results. Several authors [7] have considered the introduction of Majorana masses of unknown origin for the brane states. In the following, we propose two other extensions that seem to provide the necessary ingredients to accommodate the LSND data. We give only simple estimates of the effects. To determine whether these extensions can indeed provide a consistent explanation of all neutrino oscillation data requires a more careful study, which is outside the scope of the present work.

A. The (4, 4) model

Consider two extra sterile neutrinos, denoted by N^s and ν_L^s, residing in the bulk and on the brane, respectively; ν_L^s is left-handed. A mass term that couples these two states is consistent with all the symmetries of the theory. It has the form

$$\mathcal{L}_{mass}=\lambda M_F\bar{\nu}_L^s\nu_R^s(x,0)+\text{H.c.}, \quad (35)$$

where λ is a coupling analogous to $\lambda_{\alpha\beta}$ in Eq. (2), and ν_R^s is the right-handed component of N^s. Note that M_F is the only natural choice for the mass scale. Introducing a dimensionless Yukawa coupling $h=\lambda M_F^{\delta/2}$ and performing the KK decomposition of ν_R^s, we obtain mass terms of the form

$$m_s \left(\bar{\nu}_R^{s(0)} \nu_L^s + \sqrt{2} \sum_{n=1}^{\infty} \bar{\nu}_R^{s(n)} \nu_L^s \right) + \text{H.c.,} \qquad (36)$$

where the mass scale is given by

$$m_s = h \frac{M_F^2}{\bar{M}_{\text{Pl}}}. \qquad (37)$$

For $M_F \sim 100$ TeV and $h \sim 1$, this mass scale is in the 1 to 10 eV range. Thus, the scale required to account for the LSND results has been obtained without any fine-tuning, using only the natural parameters of the theory. Note that the symmetries of the theory also allow the brane Yukawa couplings between ν_R^s and the active left-handed neutrinos ν_L^α, which will lead to mixings between the active species and the fourth sterile KK tower. It seems plausible that the solar, atmospheric and LSND results could be accommodated simultaneously in this framework; however, a more careful analysis is clearly called for to verify this.

B. Two large dimensions

As discussed above, the small rate expected for LSND in the canonical (3,3) model is due to a cancellation between the contributions of two KK towers, so that even in the inverted and degenerate schemes the rate is proportional to $|\xi_2^2 - \xi_1^2|^2$ or $|l^{e3}|^2 |\xi_3^2 - \xi_2^2|^2$ rather than $|\xi_2|^4$ or $|l^{e3}|^2 |\xi_3|^4$. This cancellation can be traced to the fact that all of the bulk neutrinos propagate in the same large dimension. The cancellation could be broken in the case in which there are two or more extra dimensions of unequal radii. In the following, we consider a simple phenomenological model in which the three bulk neutrinos each propagate in only one extra dimension, of radii R_1, R_2, and R_3, respectively, where two or more of the R_i may be different. We make no attempt to construct a full model or derive all of the constraints, for our goal is only to illustrate a possibility.

One can then show that in the limit when the $1/R_i$ are all large compared to the Dirac masses m_i^D, the amplitude $A_{\mu e}(L)$ in Eq. (15) becomes

$$A_{\mu e}(L) = \left[-\frac{\pi^2}{6} \Xi \Xi^\dagger + \Xi P \Xi^\dagger \right]_{\mu e} + \cdots, \qquad (38)$$

where $\Xi \equiv \sqrt{2} m^D \mathbf{R}$, m^D is the analog of the Dirac mass matrix in Eq. (6), $\mathbf{R}$ is the 3×3 matrix diag(R_1, R_2, R_3), and the matrix P is given by

$$P \equiv \sum_{k=1}^{\infty} \frac{e^{i\phi^k}}{k^2}, \qquad (39)$$

with $\phi^k \equiv [(k^2 L)/(2 E_\nu)] \mathbf{R}^{-2}$. Writing $m^D = l^\dagger m_D^d r$, where $m_D^d = \text{diag}(m_1^D, m_2^D, m_3^D)$ is diagonal, we see that $A_{\mu e}(L)$ does not vanish for $m_1^D \approx m_2^D$ provided that the left and right unitary matrices l and r are not equal. As a simple two-family example with l and r given by rotations with angles β_L and β_R, respectively, one finds

$$A_{\mu e}(L) = 2(m_2^D)^2 (-s_L c_R + c_L s_R)(c_L c_R + s_L s_R)$$

$$\times \left[\left(P_{11} - \frac{\pi^2}{6} \right) R_1^2 - \left(P_{22} - \frac{\pi^2}{6} \right) R_2^2 \right], \qquad (40)$$

where $s_{L,R} \equiv \sin \beta_{L,R}$, and similarly for $c_{L,R}$. This is easily generalized to the three-family case that can accommodate atmospheric neutrino results. Thus, it is clear that the $\nu_e \leftrightarrow \nu_\mu$ oscillation rates can be enhanced compared to the (3, 3) model. However, a full investigation of this proposal is beyond the scope of this paper.

VI. CONCLUSIONS

In this paper, we have studied neutrino oscillations in the context of theories with LED's. In particular, we focused on a 5-dimensional model with three active brane and three sterile bulk neutrinos, coupled to the 4-dimensional Higgs field. This setup yields small Dirac masses for the active species, in addition to a tower of sterile KK neutrinos. An attractive feature of this (3, 3) model is that it does not introduce any *ad hoc* parameters, such as small 4-dimensional Majorana masses. With an extra dimension of radius R, the lightest KK neutrinos have masses $\sim 1/R$. Our approach was based on the assumption that a three-flavor analysis of the data provides the best fit for the observed oscillations, and we have treated the effect of the LED as a perturbation. Hence, in our treatment, the Dirac masses are taken to be much smaller than $1/R$.

The recent solar neutrino data from the SNO Collaboration, in conjunction with the results of the Super-Kamiokande experiment for atmospheric neutrinos, significantly constrain the size of the effect of sterile states on the solar neutrino deficit. In particular, models based solely on active-sterile oscillations, as in Ref. [3], are now ruled out. This fact motivated us to determine what bounds the new data place on the contributions of KK sterile states to neutrino oscillations. Since these sterile states originate in the extra dimensions, we obtained constraints on the size R of the largest of the extra dimensions. In addition to the new solar data analysis, we also performed analyses of reactor, accelerator, and atmospheric data. The (3, 3) model studied here cannot accommodate the LSND data. However, we proposed two natural extensions of this model to address the LSND results. One is a (4, 4) model with two extra sterile neutrinos, one on the brane and another in the bulk, which can yield a $3+1$, or, less naturally, a $2+2$ four-neutrino scheme. These are phenomenologically similar to four neutrino schemes obtained in other theoretical frameworks, yielding equally good or poor descriptions of the data. However, there are no Majorana masses, so there is a conserved lepton number and neutrinoless double beta decay is forbidden. The other extension uses two LED's of unequal radii, in each of which a bulk neutrino is sequestered. We provided estimates of the effects in these two cases and concluded that the LSND results seem to be accommodated in our proposals, although a more careful analysis is warranted.

In this paper, we have concentrated on constraints from laboratory, solar and atmospheric neutrino oscillation experi-

ments. There are also complementary constraints from cosmology and astrophysics. In particular, the degenerate scheme may be excluded by consideration of big bang nucleosynthesis, cosmic microwave background radiation, and diffuse extra-galactic background radiation [30], but the hierarchical and inverted schemes are consistent. There are also potentially very strong limits on all of the schemes coming from the constraints on the energy carried away by the sterile KK modes from deep inside a supernova core. Qualitative estimates [31] concentrating on a single family suggest that the constraints may be considerably stronger than those in Table I. A more detailed study would be very useful. Bounds on the model considered here can also be derived from studying the effects of KK neutrinos in electroweak processes [6].

Our main results are contained in Table I. These are to be interpreted as constraints on the size of the largest of the extra dimensions, regardless of their total number. If, as seems plausible, the pattern of masses in the neutrino sector is like those of the other fermionic sectors, and hence hierarchical, the strongest bound comes from the atmospheric neutrino data and requires R to be less than about 0.8 μm. The bounds in the inverted and degenerate mass schemes are

even stronger. On the other hand, the currently discussed Cavendish type experiments have sensitivities only of order 50 μm, and an improvement of two orders of magnitude would be necessary to reach the region allowed in our model. Thus, to the degree that the (3, 3) model is a natural context for small Dirac masses in theories with LED's, our analysis suggests that neutrino oscillations seem to rule out the possibility of observing the gravitational effects of the extra dimensions in the foreseeable future.

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