Exactly solvable matrix models with spontaneous breakdown of SO(*D*) symmetry

Jun Nishimura*

The Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

and Department of Physics, Nagoya University, Furo-cho, Chikusa-ku, Nagoya 464-8602, Japan

(Received 3 February 2002; published 8 May 2002)

In the type IIB matrix model, which was proposed as a nonperturbative formulation of type IIB superstring theory, it is possible that our four-dimensional space-time appears dynamically as a brane in ten dimensions. This in particular requires the spontaneous breakdown of SO(10) symmetry, which was conjectured to be caused by the phase of the fermion integral. We present a concrete example of exactly solvable matrix models in which this happens. The models consist of D bosonic Hermitian matrices coupled to chiral fermions, and the SO(D) symmetry is spontaneously broken precisely due to the phase of the fermion determinant.

DOI: 10.1103/PhysRevD.65.105012 PACS number(s): 11.30.Cp, 11.25.−w, 11.30.Qc

I. INTRODUCTION

One of the biggest puzzles in superstring theory is that the space-time dimensionality which naturally allows a consistent construction of the theory is ten instead of four. A natural resolution of this puzzle is to consider our 4D space-time to appear *dynamically* and the other 6 dimensions to become invisible due to some nonperturbative effects. This may be compared to the situation with QCD in the early 1970s, where quarks were believed to exist according to flavor symmetries and high-energy experiments, while the puzzle was that none of them has ever been observed in reality. The understanding of quark confinement as a nonperturbative phenomenon in non-Abelian gauge theories was important for QCD to be recognized as the correct theory of strong interaction. Likewise, we think it important to try to understand the puzzle of space-time dimensionality in terms of the nonperturbative dynamics of superstring theory.

The issue of the dynamical generation of space-time has been pursued [1–7] in the context of the type IIB matrix model [8]—the Ishibashi-Kawai-Kitazawa-Tsuchiya (IKKT) version of the Matrix Theory [9]. The type IIB matrix model, which was proposed as a nonperturbative definition of type IIB superstring theory in ten dimensions, is a supersymmetric matrix model composed of 10 bosonic matrices and 16 fermionic matrices, and it can be thought of as the zero-volume limit of 10D SU(N) super Yang-Mills theory [16,10–13]. The 10 bosonic matrices represent the dynamical space-time [1] and the model is manifestly invariant under SO(10) transformations, where the bosonic and fermionic matrices transform as a vector and a Majorana-Weyl spinor, respectively. Our four-dimensional space-time may be accounted for if configurations with only four extended directions dominate the integration over the bosonic matrices. This in particular requires the SO(10) symmetry to be spontaneously broken.

Monte Carlo studies of the type IIB matrix model are technically difficult due to the so-called complex-action problem, since the Grassmann integral over fermionic matrices yields a complex quantity in general. If one omits the phase to make standard Monte Carlo methods applicable, the dynamical space-time is observed to be isotropic in ten dimensions at large N [3]. On the other hand, it was found that the phase of the fermion integral enhances the contribution of lower-dimensional configurations considerably [4]. A saddle-point analysis predicts that the dimensionality of the dynamical space-time in the type IIB matrix model should be $3 \leqslant d \leqslant 8$ [4].

In this paper, we present a concrete example of exactly solvable matrix models, in which the spontaneous symmetry breaking (SSB) of SO(D) symmetry occurs precisely due to the phase of the fermion integral. The models consist of D bosonic Hermitian matrices, which are coupled to chiral fermions in an SO(D) invariant manner. The integral over the chiral fermions is complex in general, and its phase favors lower-dimensional configurations just as in the type IIB matrix model. We study the $D=4$ case explicitly and find that the SO(4) symmetry is broken down to SO(3). If we replace the fermion integral by its absolute value, the model is still solvable and exhibits no SSB.

II. THE MODEL

The partition function of the model we consider is given by

$$Z = \int dA\, d\psi\, d\bar{\psi}\; e^{-(S_{\rm b}+S_{\psi})}, \tag{1}$$

$$S_{\rm b} = \frac{1}{2} N\, \mathrm{tr}\,(A_\mu^2), \tag{2}$$

$$S_\psi = -\bar{\psi}^f_\alpha (\Gamma_\mu)_{\alpha\beta} A_\mu \psi^f_\beta , \tag{3}$$

where A_μ ($\mu=1,\ldots,D$) are $N\times N$ Hermitian matrices and $\bar{\psi}^f_\alpha$, ψ^f_α are N-dimensional row and column vectors, respectively. [The system has an SU(N) symmetry.] We assume that D is even, but we comment on a generalization to odd D later. The actions (2) and (3) are SO(D) invariant, where the bosonic matrices A_μ transform as a vector, and the fermion fields $\bar{\psi}^f_\alpha$ and ψ^f_α transform as Weyl spinors. The spinor index α runs over $1,\ldots,p$, where $p=2^{D/2-1}$ is the dimension

*Electronic address: nisimura@eken.phys.nagoya-u.ac.jp of the spinor space. The $p\times p$ matrices Γ_μ are the gamma matrices after Weyl projection. The flavor index f runs over $1,\ldots,N_f$. We take the large N limit with $r\equiv N_f/N$ fixed (Veneziano limit) [17]. The fermionic part of the model can be thought of as the zero-volume limit of the system of Weyl fermions in D dimensions interacting with a background gauge field via fundamental coupling.

Integrating out the fermion fields, one obtains

$$Z=\int dA\, e^{-S_{\rm b}}(\det \mathcal{D})^{N_f}, \tag{4}$$

where $\mathcal{D}$ is a $pN\times pN$ matrix given by $\mathcal{D}=\Gamma_\mu A_\mu$. In $D=2$, we find that $\det\mathcal{D}$ transforms under an SO(2) transformation as $\det\mathcal{D}\mapsto e^{i\theta}\det\mathcal{D}$, where θ is the angle of rotation [18,14]. Hence the partition function (4) vanishes in this case [19]. In $D\geqslant 4$, $\det\mathcal{D}$ is SO(D) invariant and so is the model.

The fermion determinant $\det\mathcal{D}$ is complex in general. Under parity transformation, $A_D^P=-A_D$ and $A_i^P=A_i$ (for $i\neq D$), it becomes complex conjugate. From this, it follows that $\det\mathcal{D}$ becomes real if $A_D=0$, or more generally, if $n_\mu A_\mu=0$ for some vector n_μ.

We interpret the D bosonic $N\times N$ Hermitian matrices A_μ as the dynamical space-time as in the type IIB matrix model [1]. The space-time has the Euclidean signature as a result of the Wick rotation, which is always necessary in path-integral formalisms. In the present model, we can obtain the extent of space-time

$$R^2\equiv\left\langle \frac{1}{N}{\rm tr}\,(A_\mu)^2\right\rangle = D+rp \tag{5}$$

using a scaling argument for arbitrary N.

In order to probe the possible SSB of SO(D), we first generalize the bosonic action as

$$S_{\rm b}(\vec{m})=\frac{1}{2}N\sum_\mu m_\mu {\rm tr}\,(A_\mu^2). \tag{6}$$

We calculate the extent in the μth direction

$$\lambda_\mu=\left\langle\frac{1}{N}{\rm tr}\,(A_\mu)^2\right\rangle_{\vec{m}} \qquad \text{(no summation over } \mu)$$

$$=-\frac{2}{N^2}\frac{\partial}{\partial m_\mu}\ln Z(\vec{m}) \tag{7}$$

for arbitrary $\vec{m}$ in the large N limit. Then we take the limit of $m_\nu\to 1$ (for all ν) keeping the order

$$m_1<m_2<\cdots<m_D. \tag{8}$$

If λ_μ do not converge to the same value, it signals the SSB of SO(D) symmetry.

III. THE METHOD

We study the model (1) with the anisotropic bosonic action (6) in the large N limit by using a technique known from random matrix theory [15]. Integrating out the bosonic matrices A_μ,

$$Z\sim\frac{1}{\mathcal{N}}\int d\psi d\bar{\psi}\exp\left(-\frac{1}{2N}S_{\rm Fermi}\right), \tag{9}$$

$$S_{\rm Fermi}=(\bar{\psi}^f_\alpha\psi^g_\beta)\Sigma_{\alpha\beta,\gamma\delta}(\bar{\psi}^g_\gamma\psi^f_\delta), \tag{10}$$

$$\Sigma_{\alpha\beta,\gamma\delta}=\sum_\mu\frac{1}{m_\mu}(\Gamma_\mu)_{\alpha\delta}(\Gamma_\mu)_{\gamma\beta}. \tag{11}$$

The normalization factor $\mathcal{N}$ in Eq. (9) is given by

$$\mathcal{N}=\prod_\mu (m_\mu)^{N^2/2}. \tag{12}$$

Here and henceforth, we omit irrelevant $\vec{m}$-independent factors in the partition function.

The four-Fermi action (10) can be written as

$$S_{\rm Fermi}=\Sigma_{\alpha\beta,\gamma\delta}(\Phi^{(+)}_{\alpha\beta,fg}\Phi^{(+)}_{\gamma\delta,fg}-\Phi^{(-)}_{\alpha\beta,fg}\Phi^{(-)}_{\gamma\delta,fg}), \tag{13}$$

$$\Phi^{(+)}_{\alpha\beta,fg}=\frac{1}{2}(\bar{\psi}^f_\alpha\psi^g_\beta+\bar{\psi}^g_\alpha\psi^f_\beta),$$

$$\Phi^{(-)}_{\alpha\beta,fg}=\frac{1}{2}(\bar{\psi}^f_\alpha\psi^g_\beta-\bar{\psi}^g_\alpha\psi^f_\beta). \tag{14}$$

The matrix Σ, where we consider $(\alpha\beta)$ and $(\gamma\delta)$ as single indices, is symmetric, and one can always make it real by choosing the representation of Γ_μ properly. Hence one can diagonalize it as

$$\Sigma_{\alpha\beta,\gamma\delta}=\sum_{\rho\tau}O_{\alpha\beta,\rho\tau}\Lambda_{\rho\tau}O_{\gamma\delta,\rho\tau}, \tag{15}$$

and Eq. (13) can be written as

$$S_{\rm Fermi}=\sum_{\rho\tau}\Lambda_{\rho\tau}\left(\sum_{\alpha\beta}O_{\alpha\beta,\rho\tau}\Phi^{(+)}_{\alpha\beta,fg}\right)^2 -\sum_{\rho\tau}\Lambda_{\rho\tau}\left(\sum_{\alpha\beta}O_{\alpha\beta,\rho\tau}\Phi^{(-)}_{\alpha\beta,fg}\right)^2. \tag{16}$$

Each square in Eq. (16) can be linearized by a Hubbard-Stratonovitch transformation according to

$$\exp(-AQ^2)\sim\int d\sigma\exp\left(-\frac{\sigma^2}{4A}-iQ\sigma\right). \tag{17}$$

Introducing p^2 complex matrices $\hat{\sigma}_{\rho\tau}$ of size N_f, we arrive at

$$Z\sim\frac{1}{\mathcal{N}}\int d\hat{\sigma}d\psi d\bar{\psi}\exp(-NS_G+S_Q), \tag{18}$$

$$S_{\rm G}={\rm Tr}\,(\hat{\sigma}^\dagger_{\rho\tau}\hat{\sigma}_{\rho\tau}); \qquad S_{\rm Q}=\bar{\psi}^f_\alpha\mathcal{M}^{fg}_{\alpha\beta}\psi^g_\beta, \tag{19}$$

where the $p\,N_f \times p\,N_f$ matrix $\mathcal{M}$ is

$$\mathcal{M}^{fg}_{\alpha\beta} = \frac{1}{\sqrt{2}} \sum_{\rho\tau} \sqrt{\Lambda_{\rho\tau}} O_{\alpha\beta,\rho\tau} (\hat{\sigma}_{\rho\tau} + \hat{\sigma}^{\dagger}_{\rho\tau})_{fg}. \tag{20}$$

The fermionic integration yields

$$Z \sim \frac{1}{\mathcal{N}} \int d\hat{\sigma} \exp(-NW[\hat{\sigma}]), \tag{21}$$

where the effective action $W[\hat{\sigma}]$ is given by

$$W[\hat{\sigma}] = S_{\mathrm{G}} - \ln \det \mathcal{M}. \tag{22}$$

Let us first consider the large N limit with finite N_f. Then the evaluation of the partition function amounts to solving the saddle-point equations, which are given by

$$(\hat{\sigma}_{\rho\tau})_{fg} = (\hat{\sigma}^{\dagger}_{\rho\tau})_{fg} = \frac{1}{\sqrt{2}} \sum_{\alpha\beta} (\mathcal{M}^{-1})^{fg}_{\beta\alpha} \sqrt{\Lambda_{\rho\tau}} O_{\alpha\beta,\rho\tau}. \tag{23}$$

Assuming that the flavor SU(N_f) symmetry is not broken, we set $\hat{\sigma}_{\rho\tau} = \sigma_{\rho\tau}\mathbb{1}$, where $\sigma_{\rho\tau} \in \mathbb{C}$. We can further take $\sigma_{\rho\tau}$ to be real, due to Eq. (23). The effective action reduces to

$$W = N_f \{ (\sigma_{\rho\tau})^2 - \ln \det M(\sigma) \}, \tag{24}$$

where the $p \times p$ matrix $M(\sigma)$ is given by

$$M_{\alpha\beta}(\sigma) = \sqrt{2} \sum_{\rho\tau} \sqrt{\Lambda_{\rho\tau}} O_{\alpha\beta,\rho\tau} \sigma_{\rho\tau}. \tag{25}$$

Thus the problem reduces to a system of finite degrees of freedom.

Next we consider the large N limit with $r = N_f/N$ fixed. We assume that r is small, and expand $\hat{\sigma}_{\rho\tau}$ around the (dominant) saddle-point configuration obtained for finite N_f as

$$(\hat{\sigma}_{\rho\tau})_{fg} = \sigma_{\rho\tau}\delta_{fg} + \frac{1}{\sqrt{N}} (\xi_{\rho\tau})_{fg}. \tag{26}$$

The partition function can be evaluated as

$$Z \sim \frac{1}{\mathcal{N}} e^{-NW[\sigma] + C}, \tag{27}$$

where C represents the correction due to the fluctuation $(\xi_{\rho\tau})_{fg}$. The saddle-point contribution $NW[\sigma]$ in Eq. (27) is of order O(rN^2). The correction C grows as N^2 at large N, and thus it may become comparable to $NW[\sigma]$. However, the fact that it can be neglected for finite N_f means that it is suppressed at small r by higher powers than r. Therefore, one can neglect the correction C in the large N limit with $r = N_f/N$ fixed, as far as results up to O(r) are concerned.

IV. EXACT RESULTS IN 4D

Let us solve the saddle-point equations explicitly in the simplest case $D = 4$. We choose Γ_i ($i = 1,2,3$) to be Pauli matrices and $\Gamma_4 = i\mathbb{1}$. The matrix M is a 2×2 matrix

$$M(\sigma) = \begin{pmatrix} a + ib & ic + d \\ ic - d & a - ib \end{pmatrix}, \tag{28}$$

$$a = \sqrt{\rho_4}\sigma_{11}; \qquad b = \sqrt{\rho_3}\sigma_{22}$$
$$c = \sqrt{\rho_1}\sigma_{12}; \qquad d = \sqrt{\rho_2}\sigma_{21}, \tag{29}$$

where we have introduced the notation

$$\rho_\mu = \sum_\nu (-1)^{\delta_{\mu\nu}} (m_\nu)^{-1}. \tag{30}$$

The saddle-point equations are

$$\sigma_{11} = \Delta^{-1}\rho_4\sigma_{11}; \qquad \sigma_{12} = \Delta^{-1}\rho_1\sigma_{12}$$
$$\sigma_{21} = \Delta^{-1}\rho_2\sigma_{21}; \qquad \sigma_{22} = \Delta^{-1}\rho_3\sigma_{22}, \tag{31}$$

where $\Delta = a^2 + b^2 + c^2 + d^2$. Equation (31) implies that Δ should take one of the four possible values ρ_1, ρ_2, ρ_3 and ρ_4. In each case, the effective action is evaluated as $W = N_f(1 - \ln \Delta)$.

When the parameters $\vec{m}$ obey the order (8), the dominant saddle point is given by $\Delta = \rho_4$. Thus the partition function can be obtained as

$$Z \sim \frac{1}{\mathcal{N}} e^{NN_f \ln \rho_4}. \tag{32}$$

Using Eq. (7) we get

$$\lambda_\mu = (m_\mu)^{-1} \pm 2r \frac{1}{\rho_4} (m_\mu)^{-2}, \tag{33}$$

where the $\pm$ symbol should be $+$ for $\mu = 1,2,3$ and $-$ for $\mu = 4$. In the limit of $m_\nu \to 1$ (for all ν), one obtains

$$\lambda_1 = \lambda_2 = \lambda_3 = 1 + r; \qquad \lambda_4 = 1 - r, \tag{34}$$

which means that the SO(4) is spontaneously broken down to SO(3). We note that $R^2 = \sum_\mu \lambda_\mu = 4 + 2r$ agrees with the finite N result (5). The SSB is associated with the formation of a condensate $\langle \overline{\psi^f_\alpha} \psi^f_\alpha \rangle$, which is invariant under SO(3), but not under full SO(4). We remind the reader that Eq. (34) gives the result up to O(r) as we discussed in the previous section.

V. THE PHASE OF THE DETERMINANT

In order to clarify the role played by the phase of the determinant $\det \mathcal{D}$, let us consider the model

$$Z' = \int dA e^{-S_{\mathrm{b}}} |\det \mathcal{D}|^{N_f}. \tag{35}$$

This model can be obtained by replacing half of the N_f Weyl fermions ψ in Eq. (1) by Weyl fermions χ with opposite chirality. Namely, Eq. (35) can be rewritten as

$$Z' = \int dA d\psi d\bar{\psi} d\chi d\bar{\chi} \; e^{-(S_b + S_\psi + S_\chi)}, \tag{36}$$

$$S_\chi = -\bar{\chi}^f_\alpha (\Gamma^\dagger_\mu)_{\alpha\beta} A_\mu \chi^f_\beta . \tag{37}$$

We use a representation of gamma matrices in which Γ_i [$i=1,\ldots,(D-1)$] are Hermitian and $\Gamma_D = i\mathbb{1}$. Note that the flavor index f now runs over $f=1,\ldots,N_f/2$.

We can solve the above model with the anisotropic bosonic action (6) in the large N limit using the same method as before. The four-Fermi action reads

$$S'_{\rm Fermi} = (\bar{\psi}^f_\alpha \psi^g_\beta)\Sigma_{\alpha\beta,\gamma\delta}(\bar{\psi}^g_\gamma \psi^f_\delta) + (\bar{\chi}^f_\alpha \chi^g_\beta)\Sigma_{\alpha\beta,\gamma\delta}(\bar{\chi}^g_\gamma \chi^f_\delta) + (\bar{\psi}^f_\alpha \chi^g_\beta)\tilde{\Sigma}_{\alpha\beta,\gamma\delta}(\bar{\chi}^g_\gamma \psi^f_\delta) + (\bar{\chi}^f_\alpha \psi^g_\beta)\tilde{\Sigma}_{\alpha\beta,\gamma\delta}(\bar{\psi}^g_\gamma \chi^f_\delta), \tag{38}$$

where $\tilde{\Sigma}$ can be obtained from Σ by replacing m_D by $-m_D$. Similarly to Eq. (15), it can be diagonalized as

$$\tilde{\Sigma}_{\alpha\beta,\gamma\delta} = \sum_{\rho\tau} \tilde{O}_{\alpha\beta,\rho\tau}\tilde{\Lambda}_{\rho\tau}\tilde{O}_{\gamma\delta,\rho\tau}. \tag{39}$$

In order to linearize Eq. (38), we have to introduce four sets of $\hat{\sigma}_{\rho\tau}$ matrices, which we denote as $\hat{\sigma}^\psi_{\rho\tau}$, $\hat{\sigma}^\chi_{\rho\tau}$, $\hat{\sigma}^S_{\rho\tau}$ and $\hat{\sigma}^A_{\rho\tau}$. As before, we set $\hat{\sigma}^\psi_{\rho\tau} = \sigma^\psi_{\rho\tau}\mathbb{1}$, where $\sigma^\psi_{\rho\tau} \in \mathbb{R}$, etc. Introducing a new complex variable $\tilde{\sigma}_{\rho\tau} = (1/\sqrt{2})(\sigma^S_{\rho\tau} + i\sigma^A_{\rho\tau})$, the effective action becomes

$$W' = \frac{N_f}{2}(S'_{\rm G} - \ln\det M'), \tag{40}$$

$$S'_{\rm G} = (\sigma^\psi_{\rho\tau})^2 + (\sigma^\chi_{\rho\tau})^2 + 2|\tilde{\sigma}_{\rho\tau}|^2 \tag{41}$$

$$M' = \begin{pmatrix} M(\sigma^\psi) & \tilde{M}(\tilde{\sigma}) \\ \tilde{M}(\tilde{\sigma}^*) & M(\sigma^\chi) \end{pmatrix}. \tag{42}$$

The $p\times p$ matrices $M(\sigma^\psi)$ and $M(\sigma^\chi)$ are the same as Eq. (25) except that $\sigma_{\rho\tau}$ is replaced by $\sigma^\psi_{\rho\tau}$ and $\sigma^\chi_{\rho\tau}$, respectively. The new $p\times p$ matrix $\tilde{M}(\tilde{\sigma})$ is given by

$$\tilde{M}_{\alpha\beta}(\tilde{\sigma}) = \sqrt{2}\sum_{\rho\tau}\sqrt{\tilde{\Lambda}_{\rho\tau}}\tilde{O}_{\alpha\beta,\rho\tau}\tilde{\sigma}_{\rho\tau}. \tag{43}$$

The set of solutions to the saddle-point equations is richer than before. There are solutions with $\tilde{\sigma}_{\rho\tau} = \tilde{\sigma}^*_{\rho\tau} = 0$. In this case, the problem reduces to the previous one. However, there is another class of solutions in which $\sigma^\psi_{\rho\tau} = \sigma^\chi_{\rho\tau} = 0$.

Let us consider the $D=4$ case. The matrix $\tilde{M}$ is a 2×2 matrix

$$\tilde{M}(\tilde{\sigma}) = \begin{pmatrix} \tilde{a} + i\tilde{b} & i\tilde{c} + \tilde{d} \\ i\tilde{c} - \tilde{d} & \tilde{a} - i\tilde{b} \end{pmatrix}, \tag{44}$$

$$\tilde{a} = \sqrt{\rho}\,\tilde{\sigma}_{11}; \qquad \tilde{b} = \sqrt{\rho_{34}}\,\tilde{\sigma}_{22}$$

$$\tilde{c} = \sqrt{\rho_{14}}\,\tilde{\sigma}_{12}; \qquad \tilde{d} = \sqrt{\rho_{24}}\,\tilde{\sigma}_{21}, \tag{45}$$

where we have introduced the notations

$$\rho = \sum_\nu (m_\nu)^{-1}; \quad \rho_{\mu\lambda} = \sum_\nu (-1)^{\delta_{\mu\nu} + \delta_{\lambda\nu}}(m_\nu)^{-1}. \tag{46}$$

For the first class of solutions, the effective action at each saddle point is given by $W' = N_f(1 - \ln\rho_\nu)$, where $\nu = 1,2,3,4$. For the second class of solutions, the saddle-point equations become

$$\tilde{\sigma}^*_{11} = \tilde{\Delta}^{-1}\rho\tilde{\sigma}_{11}; \qquad \tilde{\sigma}^*_{12} = \tilde{\Delta}^{-1}\rho_{14}\tilde{\sigma}_{12}$$

$$\tilde{\sigma}^*_{21} = \tilde{\Delta}^{-1}\rho_{24}\tilde{\sigma}_{21}; \qquad \tilde{\sigma}^*_{22} = \tilde{\Delta}^{-1}\rho_{34}\tilde{\sigma}_{22}, \tag{47}$$

and their complex conjugates, where $\tilde{\Delta} = \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{d}^2$. Due to Eq. (47), $|\tilde{\Delta}|$ should take one of the four values ρ, ρ_{14}, ρ_{24} and ρ_{34}. In each case, the effective action is evaluated as $W' = N_f(1 - \ln|\tilde{\Delta}|)$.

Thus for arbitrary $\vec{m}$, we find that the dominant saddle point is given by the second class of the solutions with $|\tilde{\Delta}| = \rho$ and the partition function is obtained as

$$Z' \sim \frac{1}{\mathcal{N}} e^{NN_f \ln\rho}. \tag{48}$$

Using Eq. (7) we get

$$\lambda_\mu = (m_\mu)^{-1} + 2r\frac{1}{\rho}(m_\mu)^{-2} \to 1 + \frac{1}{2}r, \tag{49}$$

in the limit of $m_\nu \to 1$ (for all ν), which means that SO(4) is preserved. A nonvanishing condensate $\langle \bar{\psi}^f_\alpha \chi^f_\alpha + \bar{\chi}^f_\alpha \psi^f_\alpha \rangle$ breaks chiral symmetry, but not SO(4).

VI. SUMMARY AND DISCUSSION

One can generalize the model to odd D by considering Dirac fermions instead of Weyl fermions. In fact, such a model can be obtained from the even D model considered here by taking the $m_D \to \infty$ limit. The result for the 3D case can thus be read off from Eq. (33) as $\lambda_\mu = 1 + \frac{2}{3}r$ for all μ, which preserves the SO(3) symmetry. We note that in the odd D models the fermion determinant for each flavor is real, but it is not necessarily positive. However, for even N_f one obtains a real positive weight, and for odd N_f the sign of the weight is independent of N_f. This explains the absence of SSB in the 3D model. Based on a similar argument, we speculate that the SO(D) symmetry of *even D* models is spontaneously broken down to SO($D-1$) in general.

Our model provides a concrete example in which the spontaneous breakdown of SO(D) symmetry is caused by the phase of the fermion determinant. While this demonstrates the conjectured mechanism for the SSB in the type IIB matrix model, the actual dimensionality of the dynamical space-time should be determined by the dynamics, which must be very different from the present model. In particular, it is known that supersymmetry makes the dynamical space-time behave like a branched-polymer system [1], which is expected to be much easier to collapse due to the external force caused by the effects we have studied here.

As we mentioned earlier, standard Monte Carlo simulation of the type IIB matrix model is difficult precisely due to the existence of the phase. However, we have recently proposed a new method to circumvent this problem [7]. Analytical approaches using approximations such as the one in Ref. [13] may be useful as well. We hope that our model will serve also as a toy model for testing new methods to determine the dimensionality of the dynamical space-time in the type IIB matrix model.

ACKNOWLEDGMENTS

The author would like to thank S.M. Nishigaki and J.J. Verbaarschot for correspondence on exact results in random matrix theory. He is also grateful to K.N. Anagnostopoulos, H. Kawai and G. Vernizzi for discussions, and to F. Sugino for pointing out an error in the first version of this paper.

[1] H. Aoki, S. Iso, H. Kawai, Y. Kitazawa, and T. Tada, Prog. Theor. Phys. **99**, 713 (1998).
[2] T. Hotta, J. Nishimura, and A. Tsuchiya, Nucl. Phys. **B545**, 543 (1999).
[3] J. Ambjǿrn, K.N. Anagnostopoulos, W. Bietenholz, T. Hotta, and J. Nishimura, J. High Energy Phys. **07**, 011 (2000).
[4] J. Nishimura and G. Vernizzi, J. High Energy Phys. **04**, 015 (2000); Phys. Rev. Lett. **85**, 4664 (2000).
[5] P. Bialas, Z. Burda, B. Petersson, and J. Tabaczek, Nucl. Phys. **B592**, 391 (2001); Z. Burda, B. Petersson, and J. Tabaczek, *ibid.* **B602**, 399 (2001).
[6] J. Ambjǿrn, K.N. Anagnostopoulos, W. Bietenholz, F. Hofheinz, and J. Nishimura, Phys. Rev. D **65**, 086001 (2002).
[7] K.N. Anagnostopoulos and J. Nishimura, hep-th/0108041.
[8] N. Ishibashi, H. Kawai, Y. Kitazawa, and A. Tsuchiya, Nucl. Phys. **B498**, 467 (1997).
[9] T. Banks, W. Fischler, S.H. Shenker, and L. Susskind, Phys. Rev. D **55**, 5112 (1997).
[10] W. Krauth, H. Nicolai, and M. Staudacher, Phys. Lett. B **431**, 31 (1998).
[11] J. Ambjǿrn, K.N. Anagnostopoulos, W. Bietenholz, T. Hotta, and J. Nishimura, J. High Energy Phys. **07**, 013 (2000).
[12] P. Austing and J.F. Wheater, J. High Energy Phys. **02**, 028 (2001); **04**, 019 (2001).
[13] S. Oda and F. Sugino, J. High Energy Phys. **03**, 026 (2001); F. Sugino, *ibid.* **07**, 014 (2001).
[14] F.R. Klinkhamer and J. Nishimura, Phys. Rev. D **63**, 097701 (2001).
[15] K.B. Efetov, Adv. Phys. **32**, 53 (1983); J.J. Verbaarschot, H.A. Weidenmuller, and M.R. Zirnbauer, Phys. Rep. **129**, 367 (1985).
[16] See also [10–13] for works on other dynamical aspects of the type IIB matrix model and simplified models.
[17] When $N_f = N$, the fermion fields can be written in terms of square matrices $(\Psi_\alpha)_{if}$ and $(\overline{\Psi}_\alpha)_{fi}$, so that the fermion action reads $S_\psi = -\mathrm{tr}\,(\overline{\Psi}_\alpha (\Gamma_\mu)_{\alpha\beta} A_\mu \Psi_\beta)$.
[18] This breaking of SO(2) symmetry, which actually comes from the fermion measure, has been observed recently in exact results of 2D U(1) chiral gauge theories, where *CPT* invariance is also broken due to an anomaly [14].
[19] If we use the anisotropic bosonic action (6), the partition function of the $D=2$ model can be calculated as $Z \sim \mathcal{N}^{-1}\{(m_1)^{-1} - (m_2)^{-1}\}^{rN^2/2}$ in the large N limit, where $\mathcal{N}$ is given by Eq. (12).