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Is the truncated SU(N) non-Abelian gauge theory in extra dimensions renormalizable?

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In this article we show that, in the extra dimension model, contrary to the widely accepted conception, the simply truncated ϕ^4 and non-Abelian SU(N) Kaluza-Klein theories are not renormalizable; i.e., the tree level relations of the effective theories cannot sustain the quantum corrections. The breaking down of the tree level relations of the effective theories can be traced back to several factors: the breaking of the higher dimension Lorentz symmetry and higher dimension gauge symmetry, interactions assumed in the underlying Lagrangians, and the dimension reduction and rescaling procedure.

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Renormalization holds a quite special role in the development of the quantum field theory [1]. As we know, quantum corrections of the 4D quantum field theory are generally infinite, and only in a renormalizable theory is it possible through the standard renormalization procedure to remove the ultraviolet divergences in the theory by introducing only few finite counterterms and to make loop contributions (quantum corrections) finite and meaningful.

By considering the degrees of superficial divergence of the irreducible vertices of a specified quantum field theory defined in D dimension, the criterion of renormalizability can be simply formulated [2,3] as

$$\Omega = D - \sum_{i=1}^{n} d_i - \frac{D-1}{2} E_f - \frac{D-2}{2} E_b , \qquad (1)$$

where Ω is the superficial divergence of any a Feynman integral determined by the theory, d_i is the mass dimension of couplings of the theory, and E_f (E_b) is the number of external fermions (bosons). This equation tells us that a theory with couplings of positive or vanishing mass dimension is (super)renormalizable, while a theory with couplings of negative mass dimension is nonrenormalizable. And the nonrenormalizability of a quantum field theory in extra dimensions becomes a straightforward inference due to the fact that any couplings in the theory (except the ϕ^3 in 5D and 6D [4]) will have a negative mass dimension.

In non-Abelian gauge theory, we meet another kind of problem of renormalizability. The theory is unquestionably renormalizable if only judged from the power law given in Eq. (1). But it is not sufficient. In the pure Yang-Mills theory, for instance, there is only one coupling constant in the theory which determines both the trilinear and quartic couplings of vector bosons and the ghost-ghost-vector coupling, as required by the quantum gauge covariance. A subtle problem arises: whether the tree-level gauge structure is preserved after taking into account the quantum corrections, in other words, whether the counterterm determined by, say, the three point Green function, is enough to eliminate the ultraviolet divergences of the four point Green function of the vector boson and the ghost-ghost-vector interaction. As we know, the Becchi-Rouet-Stora-Tyutin (BRST) symmetry [5] and the Slavnov-Taylor identities [6] guarantee the tree level gauge structure of the theory order by order, and the pure Yang-Mills gauge theory is renormalizable [7]. When more particles are added to a non-Abelian gauge theory, if there is no anomaly, we know the theory is still renormalizable, even in the case when the gauge symmetry is spontaneously broken.

The extra dimension theory is a fast developing topic in recent years, and two kinds of extra dimensions can be roughly divided: large extra dimensions where gravity is considered, and small extra dimensions where the standard model is extended to the high dimensions. We are concerned with the latter case here, and there are many papers on both models and phenomenologies of it [8,9]. But, there is an irksome problem about it which is that theoretical predictions are explicitly cutoff dependent even in tree level calculations due to the sum of infinite Kaluza-Klein (KK) excitations. Such a fact can be traced back to the intrinsic nonrenormalizability of the higher dimension quantum theory. Furthermore, the trouble becomes even more serious for the loop processes.

There are papers to regularize the divergent contribution of KK excitations [10], and it seems only the string regularization can provide a solid solution to the problem [11]. Recently, the renormalizable effective theory of the extra dimension is constructed in reference [12], where the mass generation mechanism of the compactification of extra dimension is nonlinearly realized in a technicolor way or in the latticed extra dimension. The (de)constructing way only provides an effective description of the extra dimension theory, but does not prove that an extra dimension theory (or a simply truncated theory) is renormalizable.

To evaluate the contribution of KK excitations, a widely accepted and practical conception indicated in the literature

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is to truncate the infinite KK towers to finite. With the belief that the truncated KK theories are always renormalizable, the tree-level relations among couplings are always used to make theoretical predictions, both in the tree level and one-loop level. However in this paper we will show that the tree level relations of the effective theory might be broken by the quantum corrections. Considering the characteristic power running of extra dimension models, a large deviation from the tree level relations might be caused. Therefore, from either the theoretical respect or the numerical and practical respect, this conception is quite questionable. Below we will detail this problem in two cases: the ϕ^4 theory and the non-Abelian SU(N) gauge theory defined in 5D. In order to contrast and compare, we will also examine the QED and ϕ^3 theory in 5D.

We examine the ϕ^4 theory first. The Lagrangian of the ϕ^4 theory in 5D is defined as

$$L=(\partial_M\phi_{5D})^\dagger(\partial^M\phi_{5D})-m^2(\phi_{5D})^\dagger\phi_{5D} -\frac{\lambda_{5D}}{4}[(\phi_{5D})^\dagger\phi_{5D}]^2, \tag{2}$$

where $M=0,1,2,3,5$. The complex singlet field ϕ_{5D} and quartic coupling λ_{5D} have the mass dimensions 3/2 and -1, respectively. This Lagrangian owns a 5D Lorentz space-time symmetry and global U(1) inner symmetry with the universal phase defined in 5D.

And according to the power law given in Eq. (1), this theory is nonrenormalizable. However, it is helpful to understand the Lagrangian given in Eq. (2) in Wilson's renormalization method [3], which is valid for quantum field theories defined in any dimension of space-time. In this method, the principle of renormalizability is not necessary. The price paid for the sacrifice of this restrictive principle is that one has to include all interactions in the effective Lagrangian permitted by the 5D space-time Lorentz and 5D gauge symmetry, and the number of these operators is infinite. In the ϕ^4 case we consider here, besides the minimal interaction term $(\phi^\dagger\phi)^2$, interactions like $(\phi^\dagger\phi)^3$, $\phi^\dagger\Box^2\phi$, etc. should also be added to the Lagrangian given in Eq. (2). According to the effective theory [13], at low energy region the interaction terms with lower dimensions dominate the behavior of the system. So the Lagrangian given in Eq. (2) can only be understood as being valid below a given ultraviolet cutoff Λ_{UV}^{5D}, where operators with higher dimensions have been greatly suppressed. Therefore the Lagrangian given in Eq. (2) should be only valid for $|P_{5D}|<\Lambda_{UV}^{5D}$, otherwise the unitarity of the S-matrix will be violated if $|P_{5D}|$ is much greater than Λ_{UV}^{5D}. (Here $|P_{5D}|=\sqrt{p_M^2}, M=0,1,2,3,5$, and the metric of space-time is taken as that of a Euculdian one.)

The Lagrangian given in Eq. (2) also has an infrared cutoff Λ_{IR}^{5D} in the compactified extra dimension theories when Λ_{IR}^{5D} approaches the compactification scale $1/R_C$ (R_C is the compactification size). The reason for this infrared cutoff is that near the energy region $1/R_C$ it would be not appropriate any longer to regard the fifth dimension as infinite large and use the 5D Lorentz symmetry and 5D gauge symmetry to restrict operators which might appear in its effective Lagrangian.

For the small extra dimension scenarios, the extra dimensions are always assumed to by compactified and small (say TeV size). In order to match with the low energy regions where the observed world is 4D, the standard dimension reduction method and the matching procedure are used to derive the effective 4D quantum field theory. For example, by assuming that the vacuum manifold has a $M_4\times S^1/Z_2$ structure (the 5D Lorentz space-time symmetry is broken by the vacuum while the U(1) symmetry should also be modified), and by requiring that the Lagrangian is invariant under the orbifold transformation $x_5\to -x_5$, we can assign a boundary condition for the ϕ_{5D}: $\phi_{5D}(x,x_5)=-\phi_{5D}(x,-x_5)$. Then the ϕ_{5D} field can be Fourier expanded as

$$\phi_{5D}(x,x_5)=\phi_{5D}^n\cos\frac{nx_5}{R_c}. \tag{3}$$

Substituting Eq. (3) into the Lagrangian given in Eq. (2) and integrating out the fifth component of the space-time, we get the following reduced effective 4D theory (RE4DT)

$$L^{eff}=L_{kin}+L_{int}, \tag{4}$$

$$L_{kin}=\sum_{n=0}\phi^{n\dagger}\left(-\partial^\mu\partial_\mu-\frac{n^2}{R_c^2}-m^2\right)\phi^n, \tag{5}$$

$$\begin{aligned}L_{int}=-\frac{\lambda}{4}\Bigg\{&(\phi^{0\dagger}\phi^0)^2+\sum_{k,l,m=1}^{\infty}R_1(k,l,m)\\&\times(\phi^{0\dagger}\phi^k\phi^{l\dagger}\phi^{m\dagger}+\text{H.c.})+\sum_{n=1}^{\infty}[4\phi^{0\dagger}\phi^0\phi^{n\dagger}\phi^n\\&+2\,\text{Re}(\phi^{0\dagger}\phi^n\phi^{0\dagger}\phi^n)]\\&+\sum_{k,l,m,n=1}^{\infty}R_2(k,l,m,n)\phi^{k\dagger}\phi^l\phi^{m\dagger}\phi^n\Bigg\},\end{aligned} \tag{6}$$

where $R_i, i=1,\ 2$ are normalization factors and can be understood as the requirement of the momentum conservation of the fifth dimension. Here we omit the subscript 5D for all quantities. To get the RE4DT, the following rescaling relations have been used:

$$\phi_{4D}^0\to\sqrt{2\pi R_c}\phi_{5D}^0,\quad \phi_{4D}^n\to\sqrt{\pi R_c}\phi_{5D}^n,\quad \lambda_{4D}\to\frac{\lambda}{2\pi R_c}. \tag{7}$$

The theory owns a 4D space-time symmetry and the reduced global U(1) symmetry. The RE4DT is invariant under the following transformation:

$$\phi^n\to\exp(i\alpha)\phi^n. \tag{8}$$

It is remarkable that there is an infinite KK towers in the theory, and the zero modes have a different normalization factor than the other KK excitations. Another remarkable fact is that the infinite interactions among KK modes are controlled by only one parameter λ.

Now the effects of high dimension are effectively reflected by the infinite KK towers that appeared in the RE4DT given in Eq. (6). There is no coupling which has a negative mass dimension in the theory, and from the power law, it seems that the theory should be renormalizable and the dimension reduction procedure makes a higher dimension theory to a renormalizable one. But, due to the infinite KK excitations, even if the contribution to a process of each KK excitation is finite, the total result might still be infinite. In this sense, the RE4DT is still nonrenormalizable.

To effectively describe the 5D theory given in Eq. (2), we must match its RE4DT with the underlying 5D theory at a given scale Λ', which should be in the range $\Lambda_{IR}^{5D}<\Lambda<\Lambda_{UV}^{5D}$. Therefore, the infinite KK excitations are truncated by requiring $N'/R_C\approx\Lambda'$ (N'/R_C is the heaviest KK excitation included in the RE4DT $L_{\Lambda'}^{4D}$) and only finite KK excitations are kept in the RE4DT $L_{\Lambda'}^{4D}$. Then finite results could be obtained even for loop processes. It is in this sense the truncated KK theory is renormalizable.

But is that all? Since the couplings among KK modes are controlled by only one parameter λ_{4D}, then it is natural to ask whether it is enough to introduce just only one counterterm to eliminate all ultraviolet divergences in the effective theory? Or in other words, can the tree level structure sustain the quantum corrections? The problem is quite similar to the case for the non-Abelian gauge theory in 4D.

In the underlying 5D theory, the answer to this problem is affirmative. To demonstrate the reason, let us consider matching the RE4DT with the underlying 5D theory at another scale Λ'', and for the sake of convenience, we assume that $\Lambda_{IR}^{5D}<\Lambda''<\Lambda'<\Lambda_{UV}^{5D}$. So after invoking the matching procedure at Λ'', we will get the $L_{\Lambda''}^{4D}$ with N'' KK excitations (N'' is determined by $N''/R_C\approx\Lambda''$). There are two differences between the $L_{\Lambda'}^{4D}$ and $L_{\Lambda''}^{4D}$: (1) the numbers of KK excitations are different, the $L_{\Lambda''}^{4D}$ can be obtained by successively integrating out $N'-N''$ KK excitations; (2) the values of couplings $\lambda_{5D}(\Lambda'')$ and $\lambda_{5D}(\Lambda')$ are different, but are related with each other by the renormalization group equation (RGE) of λ_{5D}. However, these two RE4DTs have something in common: the tree-level relations among KK excitations seem to be hold. Since the RGE is valid in the loop level, then it might tantalize one to expect that these tree-level relations would also hold in the RE4DTs in the loop level. However, we will show that is not the case.

To simplify consideration, we truncate the infinite KK excitations and keep only the 0 and 1 modes in the RE4DT. In order to find the consistent solution to the requirement of renormalizability, we rewrite the interaction part of the Lagrangian in a more general form,

$$-L_{int}=\frac{\lambda_{00}}{4}(\phi^{0\dagger}\phi^0)^2+\frac{\lambda_{11}}{4}(\phi^{1\dagger}\phi^1)^2+\frac{\lambda_{01}}{4}[4(\phi^{0\dagger}\phi^0)(\phi^{1\dagger}\phi^1)+2\mathrm{Re}(\phi^{0\dagger}\phi^1\phi^{0\dagger}\phi^1)]. \tag{9}$$

The RE4DT is only a special case of the interaction and gives

$$R\lambda=R\lambda_{00}=R\lambda_{01}=\lambda_{11}, \tag{10}$$

where $R=3/2$. Now we determine the counterterms of the theory. The counterterms, $\delta\lambda_{00}$, $\delta\lambda_{01}$, and $\delta\lambda_{11}$ of λ_{00}, λ_{01}, and λ_{11}, can be directly constructed from the one-loop diagrams. In the dimension regularization and $\overline{MS}$ renormalization scheme, the $\delta\lambda_{00}$, $\delta\lambda_{01}$, and $\delta\lambda_{11}$ are simply determined as

$$\delta\lambda_{00}=\frac{3}{2}\kappa\Delta_\epsilon(\lambda_{00}^2+\lambda_{01}^2), \tag{11}$$

$$\delta\lambda_{01}=\frac{1}{4}\kappa\Delta_\epsilon(\lambda_{01}\lambda_{00}+\lambda_{01}\lambda_{11}+4\lambda_{01}^2), \tag{12}$$

$$\delta\lambda_{11}=\frac{3}{2}\kappa\Delta_\epsilon(\lambda_{11}^2+\lambda_{01}^2), \tag{13}$$

where $\kappa=1/(16\pi^2)$, $\Delta_\epsilon=2/\epsilon-\gamma_E+\log 4\pi$, and $\epsilon=4-D$. With these counterterms, the consistent solution can be easily found. If the RE4DT is renormalizable, we hope that the following relation should hold:

$$\delta\lambda_{00}=\delta\lambda_{01}=\delta\lambda_{11}. \tag{14}$$

Then the consistent solution for this equation requires

$$\lambda_{00}=\lambda_{01}=\lambda_{11}, \tag{15}$$

but the tree level relation given in the Eq. (10) obviously is not satisfying Eq. (15). Therefore it is impossible to just introduce one counterterm $\delta\lambda$ to make the quantum corrections of the theory finite, and the tree-level relation Eq. (10) breaks down. And it is in this sense that the RE4DT is still nonrenormalizable. For the truncated theory with more than one KK excitations, we have the same conclusion.

It is remarkable that from Eqs. (11)–(13) we know the tree level relation $\lambda_{00}=\lambda_{01}$ will also be broken down due to the contribution from λ_{11}, so it is questionable to use the relation at low energy regions when evaluating the contributions of KK excitations to the effective potential of ϕ^0.

Of course, if we forget the dimension reduction and adjust the normalization factor R to be just one, then it is enough to just introduce one counterterm $\delta\lambda$ to make the quantum corrections of the theory finite, at least up to one loop. Obviously, the procedure of normalizing and rescaling in the standard dimension reduction, which makes zero modes different from other KK excitations and produces the normalization factor R_i, is blamed for the nonrenormalizability of the theory. So we conclude here that the nonrenormalizability of the high dimension ϕ^4 theory leaves its trace not only in appearing the infinite KK excitations but in breaking down the tree level relations among couplings with quantum corrections. We also see here that the reduced U(1) symmetry of the theory is not much help for the problem at hand.

Equipped with this experience, it is natural to ask whether the tree level relations of the truncated SU(N) gauge theory can sustain the quantum corrections. Now we consider the case of non-Abelian SU(N) gauge theory. The Lagrangian in 5D is given as

$$\mathcal{L}=-\frac{1}{4}F_{MN}F^{MN}-\frac{1}{2\xi}F^2(A_M)+\bar{c}\frac{\delta F(A_M)}{\delta\alpha}c, \qquad (16)$$

where $F_{MN}=\partial_M A_N-\partial_N A_M+fA_M A_N$, and f is the structure constant of the Lie algebra. $F(A_M)$ is the gauge fixing term and can be assumed [8] to have the form

$$F(A_M)=\partial_M A^M. \qquad (17)$$

The theory has the Lorentz symmetry of 5D space-time and BRST symmetry in 5D. But the theory is nonrenormalizable even if we only judge from the naive power law, since the gauge coupling owns a negative mass dimension. So formally, even though the theory owns a gauge symmetry (BRST symmetry in 5D), it is still nonrenormalizable.

Similar to the argument in the ϕ^4 theory in 5D, the Lagrangian given in Eq. (16) can only be understood as being valid below the ultraviolet cutoff Λ_{UV}^{5D}, otherwise effects of other higher dimension operators will be important or the unitarity condition of the S matrix will be violated.

The vacuum manifold is assumed to have the structure $M_4\times S^1/Z_2$ and the Lorentz symmetry of 5D is spontaneously broken. Considering the fact that the 5D space-time symmetry is broken to 4D space-time symmetry, and the 5D gauge symmetry is broken to 4D gauge symmetry, below we will choose the gauge fixing term

$$F(A_M)=\partial_\mu A^\mu-\xi\partial_5 A^5. \qquad (18)$$

The advantage of choosing this gauge fixing term rather than the one given in Eq. (17) is that physical observables are gauge parameter independent.[1]

By assigning a boundary condition for the vector gauge field

$$A_\mu(x,x_5)=A_\mu(x,-x_5), \qquad (19)$$

and decomposing quantum fields in 5D with $A_\mu(x)=A_\mu^n(x)\cos(nx_5/R_c)$, we get the RE4DT in the following form:

$$L_{4D}^{eff}=L^{00}+L^{ED}, \quad L^{ED}=L_{kin}^{ED}+L_{int}^{ED}, \qquad (20)$$

$$L_{int}^{ED}=L_{K0}^{ED}+L_{KK}^{ED}, \quad L_{K0}^{ED}=L_{K0,tri}^{ED}+L_{K0,qua}^{ED}, \qquad (21)$$

$$L_{kin}^{ED}=A_\mu^0\left[g^{\mu\nu}\partial^\gamma\partial_\gamma-\partial^\mu\partial^\nu\left(1-\frac{1}{\xi}\right)\right]A_\nu^0+\overline{c^0}(-\partial^\mu\partial_\mu)c^0+\sum_{n=1}^{\infty}\frac{1}{2}A_\mu^n\left[g^{\mu\nu}\partial^\gamma\partial_\gamma+g^{\mu\nu}\frac{n^2}{R_c^2}-\partial^\mu\partial^\nu\left(1-\frac{1}{\xi}\right)\right]A_\nu^n$$
$$+\sum_{n=1}^{\infty}\frac{1}{2}A_5^n\left(-\partial^\mu\partial_\mu-\xi\frac{n^2}{R_c^2}\right)A_5^n+\sum_{n=1}^{\infty}\overline{c^n}\left(-\partial^\mu\partial_\mu-\xi\frac{n^2}{R_c^2}\right)c^n, \qquad (22)$$

$$L_{K0,tri}^{ED}=-\frac{1}{2}gf^{abc}\sum_{n=1}^{\infty}(W^{0a\mu\nu}A_\mu^{nb}A_\nu^{nc}+2A_\mu^{0a}A_\nu^{nb}W^{nc\mu\nu})+gf^{abc}\sum_{n=1}^{\infty}A_\mu^{a0}A_5^{nb}\left(\partial^\mu A_5^{nc}+\frac{n}{R_c}A^{nc\mu}\right)$$
$$+gf^{abc}\sum_{n=1}^{\infty}\partial^\mu\overline{c^{na}}A_\mu^{0b}c^{nc}, \qquad (23)$$

where L^{00} represents terms of pure zero modes, $L_{K0,qua}^{ED}$ represents the quartic coupling between the zero and KK modes, and L_{KK}^{ED} represents couplings among KK excitations. Here we omit those interactions among KK excitations. The Lagrangian has a 4D Lorentz space-time symmetry and the reduced BRST symmetry. There is a conservation law of the fifth momentum, which can be viewed as the result from the compactification of the fifth dimension space. Again, it is remarkable that there is an infinite KK tower in the theory, and the zero modes have a different normalization factor than the other KK excitations. And the infinite interactions among KK modes are controlled by only one parameter g, the gauge coupling constant.

The matching procedure will truncate the infinite KK excitations to finite. And the tree level relations among couplings of KK modes are expected to hold if one judges from the underlying theory with the 5D Lorentz space-time symmetry and 5D gauge symmetry.

In order to examine the renormalizability of the truncated theory, as done in the ϕ^4 case, we truncate the infinite KK towers and keep only the 0 and 1 modes in the Lagrangian. And the Lagrangian has the following form:

[1]Reference [14] also used this gauge fixing term.

$$L = L_{kin} + L_{int}, \quad L_{int} = L_{tri} + L_{qua}, \tag{24}$$

$$L_{kin} = A^0_\mu\left[g^{\mu\nu}\partial^\gamma\partial_\gamma - \partial^\mu\partial^\nu\left(1-\frac{1}{\xi}\right)\right]A^0_\nu + \overline{c^0}(-\partial^\mu\partial_\mu)c^0 + \frac{1}{2}A^1_\mu\left[g^{\mu\nu}\partial^\gamma\partial_\gamma + g^{\mu\nu}\frac{1}{R_c^2} - \partial^\mu\partial^\nu\left(1-\frac{1}{\xi}\right)\right]A^1_\nu$$
$$+\frac{1}{2}A^1_5\left(-\partial^\mu\partial_\mu - \xi\frac{1}{R_c^2}\right)A^1_5 + \overline{c^1}\left(-\partial^\mu\partial_\mu - \xi\frac{1}{R_c^2}\right)c^1, \tag{25}$$

$$L_{tri} = gf^{abc}\Bigg\{-\frac{1}{2}(\partial_\mu A^{0a}_\nu - \partial_\nu A^{0a}_\mu)A^{0b\mu}A^{0c\nu} - \frac{1}{2}(\partial_\mu A^{0a}_\nu - \partial_\nu A^{0a}_\mu)A^{1b\mu}A^{1c\nu} - \frac{1}{2}(\partial_\mu A^{1a}_\nu - \partial_\nu A^{1a}_\mu)(A^{0b\mu}A^{1c\nu} + A^{0b\mu}A^{1c\nu})$$
$$+\partial^\mu\overline{c^{0a}}A^{0b}_\mu c^{0c} + \partial^\mu\overline{c^{1a}}A^{0b}_\mu c^{1c} + \partial^\mu\overline{c^{0a}}A^{1b}_\mu c^{1c} + \partial^\mu\overline{c^{1a}}A^{1b}_\mu c^{0c} + \frac{1}{R_c}A^{1a\mu}A^{0b}_\mu A^{1c}_5 + \frac{\xi}{R_c}\overline{c^{1a}}A^{1b}_5 c^{0c} + \partial^\mu A^{1a}_5 A^{0b}_\mu A^{1c}_5\Bigg\}, \tag{26}$$

$$L_{qua} = g^2 f^{abe}f^{cde}\Bigg\{-\frac{1}{4}A^{0a}_\mu A^{0b}_\nu A^{0c\mu}A^{0d\nu} - \frac{1}{2}A^{0a}_\mu A^{0b}_\nu A^{1c\mu}A^{1d\nu} - \frac{1}{2}A^{0a}_\mu A^{1b}_\nu A^{0c\mu}A^{1d\nu} - \frac{1}{2}A^{0a}_\mu A^{1b}_\nu A^{1c\mu}A^{0d\nu}$$
$$+\frac{1}{2}A^{0a}_\mu A^{1b}_5 A^{0c\mu}A^{d1}_5 + \frac{R_1}{2}A^{1a}_\mu A^{1a}_5 A^{1c\mu}A^{d1}_5 - \frac{R_2}{4}A^{1a}_\mu A^{1b}_\nu A^{1c\mu}A^{1d\nu}\Bigg\}, \tag{27}$$

where $R_1 = 1/2$ and $R_2 = 3/2$.

This simplified RE4DT has five particles, where massless zero modes include A^0_μ, and c^0 and the massive first KK excitation includes A^1_μ, c^1, and A_5. There are nine trilinear and five quartic couplings, all are controlled by just one coupling constant g. Generally, in the framework of effective theory, we have only 4D spacetime Lorentz symmetry and 4D SU(N) gauge symmetry of zero mode to restrict permitted operators in the Lagrangian, and each of these couplings might be treated as a free parameter, as we do in the ϕ^4 case. Besides, there might be some extra interactions like $A^1_5A^1_5A^1_5A^1_5$, which is still renormalizable in 4D and is expected to play an important role in the low energy region. However, for the sake of simplicity, we use these tree level relations to calculate and check whether these relations are consistent with the requirement of renormalizability.

In order to simplify the discussion, we omit the renormalization of mass and gauge terms, and only consider the counterterm of the relevant vertices given below

$$\delta L_{int} = \delta Z_{000}A^{0a}_\mu A^{0b}_\nu A^{0c}_\rho + \delta Z_{011}A^{0a}_\mu A^{1b}_\nu A^{1c}_\rho$$
$$+\delta Z_{0000}A^{0a}_\mu A^{0b}_\nu A^{0c}_\rho A^{0d}_\sigma + \delta Z_{0011}A^{0a}_\mu A^{0b}_\nu A^{1c}_\rho A^{1d}_\sigma$$
$$+\delta Z_{1111}A^{1a}_\mu A^{1b}_\nu A^{1c}_\rho A^{1d}_\sigma. \tag{28}$$

If the theory were renormalizable (the tree level relations held), these counterterms should have their structures as given below

$$\delta Z_{000(011)} = c_{000(011)} \quad V_3, \tag{29}$$

$$\delta Z_{(0000),(0011,1111)} = c_{0000,(0011,1111)} \quad V_4, \tag{30}$$

where c_i should be number, and V_3 and V_4 have the following forms

$$V_3 = gf^{abc}[g^{\mu\nu}(p-q)^\rho + g^{\nu\rho}(q-k)^\mu$$
$$+g^{\rho\nu}(k-p)^\nu], \tag{31}$$

$$V_4 = -ig^2[f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho})$$
$$+f^{ace}f^{dbe}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho})$$
$$+f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})]$$
$$= -ig^2[g^{\mu\rho}g^{\nu\sigma}\Sigma^{ac}_{bd} + g^{\mu\sigma}g^{\nu\rho}\Sigma^{ad}_{bc}$$
$$+g^{\mu\nu}g^{\rho\sigma}\Sigma^{ab}_{cd}], \tag{32}$$

where $\Sigma^{ab}_{cd} = f^{ace}f^{bde} + f^{ade}f^{bce}$, and Σ^{ab}_{cd} is unchanged (symmetric) when the indices $a(c)$ and $b(d)$, and (ab) and (cd) interchange with each other.

And if the tree level relations among vertices were preserved after considering the quantum corrections, the following relations should also hold:

$$Z^2_{000} = Z_{A^0}Z_{0000}, \quad Z_{011} = \frac{Z_{A^1}}{Z_{A^0}}Z_{000}, \tag{33}$$

$$Z_{0011} = \frac{Z_{A^1}}{Z_{A^0}}Z_{0000}, \quad Z_{1111} = \frac{Z^2_{A^1}}{Z^2_{A^0}}R_2 Z_{0000}, \tag{34}$$

where Z_{A^0} and Z_{A^1} are the renormalization constants of wave functions and Z_{000}, Z_{011}, Z_{0000}, Z_{0011}, and Z_{1111} are the renormalization constants of the corresponding vertices.

However, if those counterterms do not have the expected structures or the above expected relations do not hold, we can necessarily conclude that the theory is not consistent with the requirement of renormalizability, i.e., the theory is nonrenormalizable.

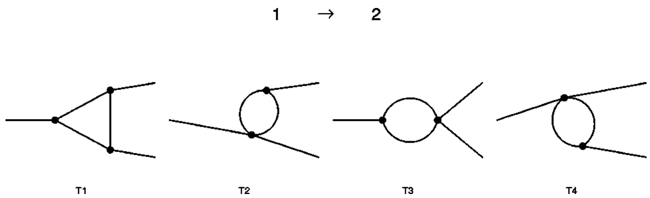

FIG. 1. The topologies of $1\rightarrow 2$ processes.

Before starting to extract those counterterms of vertices, we write down [here we use the Feynman and 't Hooft gauge and work in the dimension regularization and modified minimal subtraction ($\overline{\rm MS}$) renormalization scheme] the wave function renormalization of A^0, A^1, and A_5:

$$Z_{A^0}=1+\left(N_{VB}\times\frac{10}{3}-\frac{N_S}{3}\right)C_{div}\,, \tag{35}$$

$$Z_{A^1}=1+\frac{19}{3}\,C_{div}\,, \tag{36}$$

$$Z_{A_5}=1+4\,C_{div}\,, \tag{37}$$

where $C_{div}=g^2\kappa\Delta_\epsilon C_2(G)$. The N_{VB} is to count the number of adjoint representations of vector bosons and their ghosts, N_S is to count the number of adjoint representation of the scalar, and in our case $N_{VB}=2$, $N_S=1$. $C_2(G)$ is the Casimir operator of the adjoint representation of gauge group G. It is remarkable that the above result gives $Z_{A^1}=Z_{A^0}$.

Now we start to construct the relevant counterterms up to the one-loop level through the corresponding five processes, $A^0\rightarrow A^0A^0$, $A^0\rightarrow A^1A^1$, $A^0A^0\rightarrow A^0A^0$, $A^0A^0\rightarrow A^1A^1$, and $A^1A^1\rightarrow A^1A^1$, respectively. The relevant topologies of Feynman diagrams are given in Figs. 1 and 2, respectively.

The counterterms of the relevant trilinear couplings are given below:

$$\delta Z_{000}=\left(N_{VB}\times\frac{4}{3}-\frac{N_S}{3}\right)C_{dv}\,V_3\,, \tag{38}$$

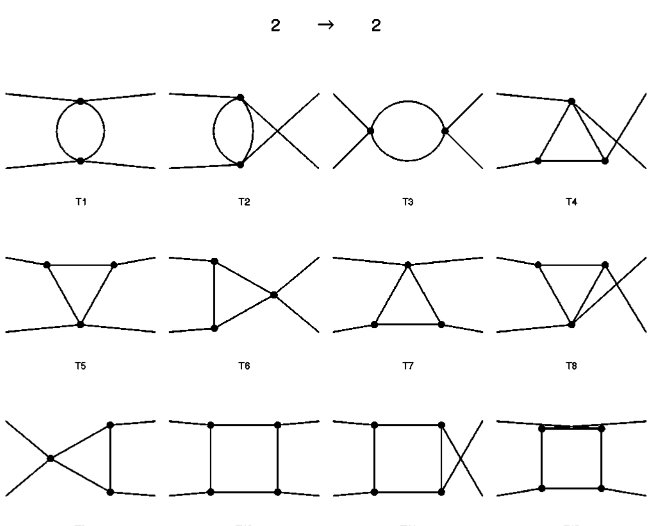

FIG. 2. The topologies of $2\rightarrow 2$ processes.

$$\delta Z_{011}=2\times\frac{4}{3}\,C_{div}\,V_3+\frac{9}{2}\,C_{div}f^{abc}(R_2-1)\times[g^{\mu\nu}p^\rho-g^{\mu\rho}p^\nu]. \tag{39}$$

p is the incoming momentum of A^0_μ. Then the renormalization constant of the trilinear coupling of zero modes can be given as

$$Z_{000}=1+\left(N_{VB}\times\frac{4}{3}-\frac{N_S}{3}\right)C_{div}\,. \tag{40}$$

The counterterms of the relevant quartic couplings are given as follows:

$$\delta Z_{0000}=-\left(N_{VB}\times\frac{2}{3}+\frac{N_S}{3}\right)C_{div}\,V_4\,, \tag{41}$$

$$\delta Z_{0011}=-\frac{4}{3}\,C_{div}\,V_4+(R_2-1)T_{01}\,, \tag{42}$$

$$\delta Z_{1111}=-\frac{4}{3}\,C_{div}\,V_4+R_1^2S_{11}+(R_2-1)T_{11}+(R_2^2-1)U_{11}\,, \tag{43}$$

where T_{01}, S_{11}, T_{11} and U_{11} are given as

$$T_{01}=\frac{\kappa g^2\Delta_\epsilon}{4}\left\{g^{\mu\nu}g^{\rho\sigma}\left[-\frac{5}{2}\Sigma^{ab}_{cd}C_2(G)-5S^{ab}_{cd}\right]+g^{\mu\rho}g^{\sigma\nu}[4f^{abe}f^{cde}+2f^{eag}f^{gch}f^{hdi}f^{ibe}]+g^{\mu\sigma}g^{\rho\nu}[-4f^{abe}f^{cde}+2f^{eag}f^{gbh}f^{hci}f^{ide}]\right\}, \tag{44}$$

$$S_{11}=\kappa g^2\Delta_\epsilon\left\{g^{\mu\nu}g^{\rho\sigma}\left[\frac{1}{2}\Sigma^{ab}_{cd}C_2(G)+S^{ab}_{cd}\right]+g^{\mu\rho}g^{\nu\sigma}\left[\frac{1}{2}\Sigma^{ac}_{bd}C_2(G)+S^{ac}_{bd}\right]+g^{\mu\sigma}g^{\nu\rho}\left[\frac{1}{2}\Sigma^{ad}_{bc}C_2(G)+S^{ad}_{bc}\right]\right\}, \tag{45}$$

$$T_{11}=-\frac{\kappa g^2\Delta_\epsilon}{4}\{g^{\mu\nu}g^{\rho\sigma}[23\Sigma^{ab}_{cd}C_2(G)+30S^{ab}_{cd}]+g^{\mu\rho}g^{\nu\sigma}[23\Sigma^{ac}_{bd}C_2(G)+30S^{ac}_{bd}]+g^{\mu\sigma}g^{\nu\rho}[23\Sigma^{ad}_{bc}C_2(G)+30S^{ad}_{bc}]\}, \tag{46}$$

$$U_{11}=\kappa g^2\left\{g^{\mu\nu}g^{\rho\sigma}\left[\frac{7}{2}\Sigma^{ab}_{cd}C_2(G)+3S^{ab}_{cd}\right]+g^{\mu\rho}g^{\nu\sigma}\left[\frac{7}{2}\Sigma^{ac}_{bd}C_2(G)+3S^{ac}_{bd}\right]+g^{\mu\sigma}g^{\nu\rho}\left[\frac{7}{2}\Sigma^{ad}_{bc}C_2(G)+3S^{ad}_{bc}\right]\right\}, \tag{47}$$

where $S^{ab}_{cd}=f^{eaf}f^{fcg}f^{gbh}f^{hde}+f^{eaf}f^{fdg}f^{gbh}f^{hce}$. S_{11} is the contribution of scalar A^1_5 in the two-point one loop, T_{11} is from the three-point one loop with one $A^1A^1A^1A^1$ vertex, and U_{11} is from the diagrams with two $A^1A^1A^1A^1$ vertices. And the convention of indices is given as $A^{ia}_\mu \to A^{jb}_\nu A^{jc}_\rho$ and $A^{ia}_\mu A^{ib}_\nu \to A^{jc}_\rho A^{jd}_\sigma$. Substituting R_i into the sum of $R_1^2 S_{11}+(R_2-1)T_{11}+(R_2^2-1)U_{11}$ we get

$$\kappa g^2\Delta_\epsilon\left\{g^{\mu\nu}g^{\rho\sigma}\left[\frac{13}{8}\Sigma^{ab}_{cd}C_2(G)+\frac{1}{4}S^{ab}_{cd}\right]+g^{\mu\rho}g^{\nu\sigma}\left[\frac{13}{8}\Sigma^{ac}_{bd}C_2(G)+\frac{1}{4}S^{ac}_{bd}\right]+g^{\mu\sigma}g^{\nu\rho}\left[\frac{13}{8}\Sigma^{ad}_{bc}C_2(G)+\frac{1}{4}S^{ad}_{bc}\right]\right\}. \quad (48)$$

So neither δZ_{1111} nor δZ_{0011} nor δZ_{011} has the expected structure.

The quartic coupling of the zero modes can be formulated as

$$Z_{0000}=1-\left(N_{VB}\times\frac{2}{3}+\frac{N_S}{3}\right)C_{div}. \quad (49)$$

The renormalizability of the zero modes part can be easily checked, since the relation $Z^2_{000}=Z_{A^0}Z_{0000}$ indeed hold. The nonrenormalizability of the KK excitations is obvious from the results given above. The difference of Z_{000} and Z_{011} can be explained by two facts: the first one is that there is no interaction term of the form $\partial^\mu A_5 A^1_\mu A_5$, since this term is forbidden by the requirement of the conservation of the fifth momentum and is eliminated in the procedure of integrating out the fifth space. There is indeed one diagram in which A_5 contributes superficially divergently, but it is finite. So the scalar contributes to the $A^0\to A^1A^1$ convergently. The second one is related with the normalization factor of the quartic interaction $A^1A^1A^1A^1$, which provides the terms related with the normalization factors R_i. The differences between δZ_{0000} and $\delta Z_{0011(1111)}$, can also be explained by these two facts.

So, we see here that more than one counterterm is necessarily needed in order to eliminate all ultraviolet divergences for the processes we consider. In other words, the tree level relations among couplings given by simply truncating the infinite KK tower are not consistent with the requirement of a renormalizable theory. And it is in this sense that the simply truncated theory is nonrenormalizable. As explained above, in the non-Abelian SU(N) gauge theory case, it is the R_i and the forbidden trilinear coupling $\partial^\mu A_5 A^1_\mu A_5$ that conspire to make the truncated theory nonrenormalizable. Therefore, in order to eliminate all divergences in the theory, the more generic effective Lagrangian with one KK excitation which respects the 4D Lorentz space-time symmetry, the 4D zero mode gauge symmetry and the fifth momentum conservation law should have the following form:

$$\begin{aligned} L=&-\frac{1}{2}\mathrm{Tr}[F_{\mu\nu}F^{\mu\nu}]-\frac{1}{2}\mathrm{Tr}[\overline{F}_{\mu\nu}\overline{F}^{\mu\nu}]-M_C^2\mathrm{Tr}[\overline{A}_\mu\overline{A}^\mu]\\ &-\lambda_{21}\mathrm{Tr}[\overline{A}_\mu D^\mu D^\nu\overline{A}_\nu]-\lambda_{31}\mathrm{Tr}[F_{\mu\nu}\overline{A}^\mu\overline{A}^\mu]\\ &-\lambda_{41}\mathrm{Tr}[\overline{A}^\mu\overline{A}^\nu]\mathrm{Tr}[\overline{A}_\mu\overline{A}_\nu]\\ &-\lambda_{42}\mathrm{Tr}[\overline{A}^\mu\overline{A}_\mu]\mathrm{Tr}[\overline{A}^\nu\overline{A}_\nu]-\mathrm{Tr}[D^\mu A^1_5 D_\mu A^1_5]\\ &-M_C^2\mathrm{Tr}[A^1_5A^1_5]-\lambda_{33}\mathrm{Tr}[\overline{A}^\mu\overline{A}_\mu A^1_5]\\ &-\lambda_{43}\mathrm{Tr}[\overline{A}^\mu\overline{A}_\mu]\mathrm{Tr}[A^1_5A^1_5]\\ &-\lambda_{44}\mathrm{Tr}[\overline{A}^\mu A^1_5]\mathrm{Tr}[\overline{A}_\mu A^1_5]\\ &-\lambda_{45}\mathrm{Tr}[A^1_5A^1_5A^1_5A^1_5]+\cdots, \end{aligned} \quad (50)$$

where $\overline{F}^{\mu\nu}=D^\mu\overline{A}^\nu-D^\nu\overline{A}^\mu$, $D^\mu=\partial^\mu-ig[A^\mu,.]$, $\overline{A}=\Sigma_a A^{1a}T^a$, T^a are the generators of the gauge group, the Tr means to sum over the generators of the gauge group, and the omitted terms are related with gauge fixing and ghost terms. The effective Lagrangian is invariant under the following transformation:

$$\begin{aligned} &A\to A'=UAU^{-1}-\frac{i}{g}(\partial U)U^{-1},\\ &\overline{A}\to\overline{A}'=U\overline{A}U^{-1},\\ &A^1_5\to A^{1'}_5=U\overline{A}^1_5U^{-1}. \end{aligned} \quad (51)$$

After matching this generic effective Lagrangian with the truncated RE4DT at the matching scale Λ, the ultraviolet boundary condition of couplings λ_i in Eq. (50) is fixed. Below the matching scale Λ, these couplings will develop in terms of their RGEs, respectively.

Comparing the extra dimension model with the renormalizable SU(5) unification model in 4D, there is a similarity between these two theories: the breaking of the tree level relations. In the SU(5) unification model, the SM is the effective theory of SU(5) GUT theory for the energy scale below the GUT scale Λ_{GUT}. At the Λ_{GUT}, there are tree-level relations among the couplings of gauge groups $SU(3)\times SU(2)\times U(1)$. Below the Λ_{GUT}, due to the decoupling of Higgs multiplets and the SU(5) gauge symmetry breaking, the gauge couplings develop, respectively, and the tree level relations of them are broken by the quantum corrections.

There is a difference between these two theories: there are extra operators in the extra dimension model generated by quantum corrections. Compared with the renormalizable SU(5) where all renormalizable terms of the subgroup $SU(3)\times SU(2)\times U(1)$ have been contained in the Lagrangian of SU(5) theory, the extra dimension SU(N) theory is unlucky in this respect, since the extra interaction terms, like $\mathrm{Tr}[A^1_5A^1_5A^1_5A^1_5]$, although not permitted by the 5D Lorentz and 5D SU(N) gauge symmetry, have to be introduced in order to remove divergences from the theory.

In order to pinpoint the reasons for the breaking down of tree level relations and the appearance of extra operators, we consider the dimension reduction and rescaling procedure of the renormalizable ϕ^4 theory defined in 4D. Assuming that the z direction is compactified, by using the dimension reduction and matching procedure, we will get its RE3DT defined at a scale Λ_{UV}^{3D}. Since the RE3DT is a super-renormalizable theory, vertex corrections are finite and there is no need to introduce any counterterm for the couplings of the KK modes. However, after considering the quantum corrections, the finite loop contributions still break the tree-level relations among couplings of KK modes, the direct reason is still the different normalization factor between zero mode and KK excitations.

In the simply truncated effective ϕ^4 and SU(N) effective theories, either in 4D or in 3D, the tree level relations among couplings cannot hold in the quantum corrections, although they are supposed to hold in their underlying theories. The fundamental reason for the breaking down of tree level relations seems to be related to the higher dimension Lorentz symmetry and higher dimension gauge symmetry breaking, and the dimension reduction and rescaling procedure itself.

We examined the truncated QED theory, where only the vector boson is assumed to propagate in the bulk. The Lagrangian of the theory in 5D has the form

$$L=-\frac{1}{4}F^{MN}F_{MN}+\bar{\psi}(i\gamma^{\mu}D_{\mu}-m)\psi\delta(x_5) \qquad (52)$$

where $F_{MN}=\partial_M A_N-\partial_N A_M$, $M=0,1,2,3,5$, $\mu=0,1,2,3$, ψ is defined in the 3-brane and $D_\mu\psi=\partial_\mu\psi-igA_\mu\psi$. This theory is nonrenormalizable in 5D due to the fact that the gauge coupling constant g has a negative mass dimension. We find that up to one-loop level, the tree level coupling structure is unchanged by the quantum corrections. The reason seems to be simple: the bilinear interaction vertices and normalization factors of the theories do not undermine the tree level relations among couplings in these two cases, not as in the ϕ^4 and non-Abelian SU(N) gauge theories where the normalization factors of quartic couplings or forbidden terms break down the tree level relations. We also examined the real scalar ϕ^3 theory in 5D, and this theory is superrenormalizable according to the power law. The Lagrangian is given by

$$L=\frac{1}{2}(\partial_M\phi_{5D})(\partial^M\phi_{5D})-\frac{1}{2}m^2(\phi_{5D})^2-\frac{\lambda_{5D}}{3!}(\phi_{5D})^3. \qquad (53)$$

And again we find that up to the one-loop level, the coupling structure of its truncated theory is unchanged by the quantum corrections.

In summary, up to the one loop level, by truncating KK excitations to only one, we examined the renormalization of the truncated KK theories of ϕ^4 theory, the non-Abelian gauge SU(N) theory, the QED theory, and the ϕ^3 theory defined in 5D, and found that the normalization factors of four KK excitations or the forbidden missing terms or both undermine the tree-level structure of the simply truncated theories in quantum corrections. We conclude that the breaking of the higher dimension Lorentz symmetry and higher dimension gauge symmetry, interactions assumed in the underlying Lagrangians, and the dimension reduction and rescaling procedure play their roles in the breaking down of the tree level relations.

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