Structure of screening in QED

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The possibility of constructing charged particles in gauge theories has long been the subject of debate. In the context of QED we have shown how to construct operators which have a particle description. These operators have a gauge invariant decomposition which plays a key role in the infrared dynamics of charges. We have also shown in QCD how antiscreening is generated by one of these factors. In this paper we extend this program by showing how the screening interactions arise through the effects of the other part of the charge.

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INTRODUCTION

The long range nature of the electromagnetic interaction means that the QED coupling cannot be naively switched off. Neglecting this leads to the infra-red problem and the lack of a pole structure in the on-shell Green's functions and *S*-matrix. This has been taken [1] to mean that one cannot describe charged particles in gauge theories. In a series of papers [2–6] we have shown that this conclusion is overly hasty: it is in fact possible to construct gauge invariant operators whose *S*-matrix elements are free of infrared divergences. These fields have been shown to asymptotically recover a particle description of charges and to have a rich structure which is physically reflected in the cancellation of both soft and phase divergences.

Confinement in QCD implies that there may be limitations on our ability to construct gauge invariant color charges [2]. The interquark potential is the most widely used tool for studying color confinement. It is thus essential to understand how the structures of physical charges are reflected in the potential and, ultimately, to identify which structures are responsible for any breakdown of a particle description in QCD. One of the most intriguing aspects of the interquark potential, only investigated in low orders of perturbation theory, is the separation of the potential into screening and anti-screening effects [7–14]. The dominance of anti-screening at short distances yields asymptotic freedom, but these forces are not well understood at large separations or even at higher orders in perturbation theory. We have previously demonstrated [15,16] (in both $2+1$ and $3+1$ dimensions) that the term responsible for the cancellation of soft divergences generates the anti-screening forces between static quarks. We have shown how to construct such dressings at arbitrary orders in perturbation theory. This led us to suggest that the factor responsible for cancelling phase divergences must generate the screening interaction. Here we will show that this is indeed the case.

THE STRUCTURE OF STATIC CHARGES

Many years ago Dirac [17] proposed that a static charged particle should be described by the locally gauge invariant operator

$$\psi_{\mathrm{D}}(x)\equiv\exp\left(-ie\frac{\partial_i A_i}{\nabla^2}(x)\right)\psi(x). \tag{1}$$

His argument for this was that, in addition to the essential requirement of gauge invariance, it has the expected equal-time commutator with the electric field operator

$$[E_i(x),\psi_{\mathrm{D}}(y)]=-\frac{e}{4\pi}\frac{x_i-y_i}{|\boldsymbol{x}-\boldsymbol{y}|^3}\psi_{\mathrm{D}}(y), \tag{2}$$

i.e., it recovers the static Coulombic electric field in $3+1$ dimensions. This argument also works in $2+1$ dimensions.

In [3] we have shown that this electric field requirement is not unique even at lowest order in the coupling. In fact, arguing from a general kinematical point of view (inspired in part by the heavy quark effective theory), we have shown that the correct description of a *static* Abelian charge is given by the dressed field

$$\begin{aligned}h^{-1}(x)\psi(x)&=e^{-ieK(x)}e^{-ie\chi(x)}\psi(x)\\&=\exp\left(-ie\int_{-\infty}^{x_0}\frac{\partial_i E_i}{\nabla^2}(s,\boldsymbol{x})ds\right)\\&\quad\times\exp\left(-ie\frac{\partial_i A_i}{\nabla^2}(x)\right)\psi(x). \end{aligned}\tag{3}$$

The new factor, K, is separately gauge invariant and has a

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vanishing commutator with the electric field in the absence of light charges. We have shown that it (and its generalization for a moving charge) is essential in the cancellation of the phase divergences associated with pair production processes, while the Dirac part, χ, which is essential for gauge invariance, removes the soft divergences.

In order to show that these dressings provide the correct dynamical description of physical charges, we have used them to directly calculate the potential between charges. In the non-Abelian theory we have extended Dirac's proposal to QCD and demonstrated [15,16] that the minimal or soft part of this generalization, i.e., just that structure, χ, required for gauge invariance, produces the anti-screening interaction at order g^4. We will now show that the new factor, K, in Eq. (3) produces the screening effects at the same order of perturbation theory. To this aim we will work in QED.

THE POTENTIAL BETWEEN CHARGES

As usual we identify [2,15,16] the potential with the separation dependent part of the matrix element of the free part of the Hamiltonian in the Fock vacuum

$$\langle 0|h(y')h^{-1}(y)H_0 h(y)h^{-1}(y')|0\rangle. \tag{4}$$

For the purposes of this paper we can neglect higher terms in the expansion of the dressing and simply write

$$h^{-1}(y)=1-ie[K(y)+\chi(y)]. \tag{5}$$

Following our discussion above, we will refer to the K term as the *phase* contribution and χ as the *soft* structure.

The relevant part of the free Hamiltonian in $d+1$ dimensions is

$$H_0=\frac{1}{2}\int \frac{\mathrm{d}^d p}{(2\pi)^d}\tilde{E}_i(\boldsymbol{p},x_0)\tilde{E}_i(-\boldsymbol{p},x_0) \tag{6}$$

$$+\int \frac{\mathrm{d}^d p}{(2\pi)^d}E_p[\tilde{\psi}_-^{\dagger}(-\boldsymbol{p},x_0)\tilde{\psi}_+(\boldsymbol{p},x_0) + \tilde{\psi}_-(\boldsymbol{p},x_0)\tilde{\psi}_+^{\dagger}(-\boldsymbol{p},x_0)], \tag{7}$$

where we have dropped the irrelevant magnetic part of the Hamiltonian and the terms involving the static charges. In the second term here only light fermions are included and our positive and negative frequency decomposition is defined by

$$\tilde{\psi}(\boldsymbol{p},x_0)=\frac{1}{\sqrt{2E_p}}[b^{\alpha}(\boldsymbol{p})u^{\alpha}(\boldsymbol{p})\mathrm{e}^{-iE_p x_0} + d^{\alpha\dagger}(-\boldsymbol{p})v^{\alpha}(-\boldsymbol{p})\mathrm{e}^{iE_p x_0}] = \tilde{\psi}_+(\boldsymbol{p},x_0)+\tilde{\psi}_-(\boldsymbol{p},x_0). \tag{8}$$

We recall that Gauss' law in momentum space reads:

$$p_i\tilde{E}_i(\boldsymbol{p},x_0)=ie\int \frac{\mathrm{d}^d q}{(2\pi)^d}\tilde{\psi}^{\dagger}(\boldsymbol{q},x_0)\tilde{\psi}(\boldsymbol{p}-\boldsymbol{q},x_0), \tag{9}$$

where again, we neglect the heavy, static charges. We stress that this operator identity only holds on gauge invariant states such as those constructed in Eq. (3) and that it is essential to realize that there is an implied normal ordering in the current on the right hand side.

LOWEST ORDER

It is easy to see that at lowest order the momentum space contribution comes from the commutator of the Hamiltonian with the soft terms in the dressings:

$$\tilde{V}(\boldsymbol{q},\boldsymbol{k})=e^2\int \frac{\mathrm{d}^d p}{(2\pi)^d}\langle 0|[\tilde{E}_i(\boldsymbol{p},x_0),\tilde{\chi}(\boldsymbol{q},x_0)] \times[\tilde{E}_i(-\boldsymbol{p},x_0),\tilde{\chi}(\boldsymbol{k},x_0)]|0\rangle = -(2\pi)^d e^2\delta(\boldsymbol{q}+\boldsymbol{k})\frac{1}{\boldsymbol{q}^2}. \tag{10}$$

Performing the $\boldsymbol{k}$ integral recovers the usual result

$$\tilde{V}(\boldsymbol{q})=-e^2\frac{1}{\boldsymbol{q}^2}=-\frac{4\pi\alpha}{\boldsymbol{q}^2}. \tag{11}$$

Note that this gives the correct $(d+1)$-dimensional configuration space Coulombic potential between heavy charges at a separation $\boldsymbol{r}$

$$V(\boldsymbol{r})=-e^2\frac{\Gamma\left(\frac{d}{2}-1\right)}{4\pi^{d/2}}\frac{1}{|\boldsymbol{r}|^{d-2}}. \tag{12}$$

The extension of this soft-soft contribution to the non-Abelian theory gives anti-screening.

In the absence of light charges Eq. (12) is the full result in QED and it is easy to see that there is no contribution from the phase dressing. The presence of n_{f} light fermions, however, modifies the potential, which becomes

$$\tilde{V}(\boldsymbol{q})=-\frac{4\pi\alpha}{\boldsymbol{q}^2}\left\{1+\frac{\alpha}{\pi}\frac{n_{\mathrm{f}}}{3}\ln\left(\frac{\boldsymbol{q}^2}{\mu^2}\right)\right\}. \tag{13}$$

This displays the screening effect of physical matter.

We now want to show that our full dressing generates this screening force in much the same way that the soft part of the dressing yielded the anti-screening interaction. There are now, however, two contributions from the phase part of the dressing and we will analyze them in turn.

PHASE-PHASE CONTRIBUTION

The first term we want to calculate comes from the phase-phase analogue of the soft-soft structure in Eq. (10):

$$\tilde{V}_{\mathrm{pp}}(\boldsymbol{q},\boldsymbol{k})=2e^2\int\frac{\mathrm{d}^dp}{(2\pi)^d}E_p\{\mathrm{tr}\langle 0|[\tilde{\psi}_-^\dagger(-\boldsymbol{p},x_0),\tilde{K}(\boldsymbol{q},x_0)][\tilde{\psi}_+(\boldsymbol{p},x_0),\tilde{K}(\boldsymbol{k},x_0)]|0\rangle+\mathrm{tr}\langle 0|[\tilde{\psi}_-(\boldsymbol{p},x_0),\tilde{K}(\boldsymbol{q},x_0)]\times[\tilde{\psi}_+^\dagger(-\boldsymbol{p},x_0),\tilde{K}(\boldsymbol{k},x_0)]|0\rangle\}. \quad (14)$$

After using Gauss' law (9) to rewrite the phase dressing as

$$\tilde{K}(\boldsymbol{p},x_0)=-e\int\frac{\mathrm{d}^dq}{(2\pi)^d}\int_{-\infty}^{x_0}\mathrm{d}s\frac{1}{\boldsymbol{p}^2}\tilde{\psi}^\dagger(\boldsymbol{q},s)\tilde{\psi}(\boldsymbol{p}-\boldsymbol{q},s), \quad (15)$$

we get

$$\tilde{V}_{\mathrm{pp}}(\boldsymbol{q},\boldsymbol{k})=-2(2\pi)^de^4\frac{1}{\boldsymbol{q}^4}\delta(\boldsymbol{q}+\boldsymbol{k})\int\frac{\mathrm{d}^dp}{(2\pi)^d}\frac{E_p}{(E_p+E_{q-p})^2}\mathrm{tr}[\mathcal{P}_-(\boldsymbol{q}-\boldsymbol{p})\mathcal{P}_+(\boldsymbol{p})+\mathcal{P}_+(\boldsymbol{q}-\boldsymbol{p})\mathcal{P}_-(\boldsymbol{p})], \quad (16)$$

where $\mathcal{P}_\pm(\boldsymbol{p})=(\not{p}\pm m)\gamma^0/2E_p$ are the projectors onto positive/negative frequencies.

From the result that

$$\mathrm{tr}(\mathcal{P}_-(\boldsymbol{p})\mathcal{P}_+(\boldsymbol{q}))=\frac{(d+1)n_{\mathrm{f}}}{2E_p2E_q}(E_pE_q+\boldsymbol{p}\cdot\boldsymbol{q}-m^2), \quad (17)$$

we can trivially integrate out $\boldsymbol{k}$ to obtain the phase-phase contribution to the potential at e^4:

$$\tilde{V}_{\mathrm{pp}}(\boldsymbol{q})=-e^4(d+1)n_{\mathrm{f}}\frac{1}{\boldsymbol{q}^4}\int\frac{\mathrm{d}^dp}{(2\pi)^d}\frac{E_pE_{q-p}+\boldsymbol{p}\cdot(\boldsymbol{q}-\boldsymbol{p})-m^2}{E_{q-p}(E_p+E_{q-p})^2}. \quad (18)$$

Expanding around large $\boldsymbol{p}$ here gives the following divergent correction in $d=3-2\epsilon$ dimensions

$$\tilde{V}_{\mathrm{pp}}(\boldsymbol{q})=-\frac{4\pi\alpha_0}{\boldsymbol{q}^2}\frac{\alpha_0}{\pi}\frac{n_{\mathrm{f}}}{3}\left[\frac{1}{\epsilon}-\ln\left(\frac{\boldsymbol{q}^2}{\mu^2}\right)\right]. \quad (19)$$

The sign here, however, corresponds to anti-screening.

SCREENING

In addition to this phase-phase contribution, there are, though, two more (identical) soft-phase cross-terms. These structures yield

$$\tilde{V}_{\mathrm{sp}}(\boldsymbol{q},\boldsymbol{k})=2e^2\int\frac{\mathrm{d}^dp}{(2\pi)^d}\mathrm{tr}\langle 0|[\tilde{E}_i(\boldsymbol{p},x_0),\tilde{\chi}(\boldsymbol{q},x_0)][\tilde{E}_i(-\boldsymbol{p},x_0),\tilde{K}(\boldsymbol{k},x_0)]|0\rangle. \quad (20)$$

Using Gauss' law (9) this becomes

$$\tilde{V}_{\mathrm{sp}}(\boldsymbol{q},\boldsymbol{k})=2ie^4\frac{1}{\boldsymbol{q}^2}\frac{1}{\boldsymbol{k}^2}\int\frac{\mathrm{d}^dp}{(2\pi)^d}\frac{\mathrm{d}^dp'}{(2\pi)^d}\int_{-\infty}^{x_0}\mathrm{d}s\,\mathrm{tr}\langle 0|[\tilde{\psi}^\dagger(\boldsymbol{p},x_0)\tilde{\psi}(\boldsymbol{q}-\boldsymbol{p},x_0),\tilde{\psi}^\dagger(\boldsymbol{p}',s)\tilde{\psi}(\boldsymbol{k}-\boldsymbol{p}',s)]|0\rangle. \quad (21)$$

After a little algebra, we obtain

$$\tilde{V}_{\mathrm{sp}}(\boldsymbol{q},\boldsymbol{k})=2(2\pi)^de^4\frac{1}{\boldsymbol{q}^4}\delta(\boldsymbol{q}+\boldsymbol{k})\int\frac{\mathrm{d}^dp}{(2\pi)^d}\frac{1}{E_p+E_{q-p}}\mathrm{tr}(\mathcal{P}_-(\boldsymbol{p})\mathcal{P}_+(\boldsymbol{q}-\boldsymbol{p})+\mathcal{P}_-(-\boldsymbol{p})\mathcal{P}_+(\boldsymbol{p}-\boldsymbol{q})). \quad (22)$$

This is then

$$\tilde{V}_{\mathrm{sp}}(\boldsymbol{q},\boldsymbol{k})=e^4(2\pi)^d(d+1)n_{\mathrm{f}}\frac{1}{\boldsymbol{q}^4}\delta(\boldsymbol{q}+\boldsymbol{k})\int\frac{\mathrm{d}^dp}{(2\pi)^d}\frac{E_pE_{q-p}+\boldsymbol{p}\cdot(\boldsymbol{q}-\boldsymbol{p})-m^2}{E_pE_{q-p}(E_p+E_{q-p})}. \quad (23)$$

Note that this is almost identical to the phase-phase term, the only difference being the overall sign and the denominator term in the momentum integral. In $d=3-2\epsilon$ dimensions, we find

$$\tilde{V}_{\rm sp}(\boldsymbol{q}) = +2\frac{4\pi\alpha_0}{\boldsymbol{q}^2}\frac{\alpha_0}{\pi}\frac{n_{\rm f}}{3}\left[\frac{1}{\epsilon}-\ln\left(\frac{\boldsymbol{q}^2}{\mu^2}\right)\right]. \tag{24}$$

Adding this to Eq. (19) we obtain the total (divergent) contribution

$$\tilde{V}(\boldsymbol{q}) = -\frac{4\pi\alpha_0}{\boldsymbol{q}^2}\left\{1-\frac{\alpha_0}{\pi}\frac{n_{\rm f}}{3}\left[\frac{1}{\epsilon}-\ln\left(\frac{\boldsymbol{q}^2}{\mu^2}\right)\right]\right\} \tag{25}$$

up to order α^2. Charge renormalization in QED corresponds to $\alpha_0 = Z_\alpha \alpha$, where $Z_\alpha = 1+(\alpha/\pi)(n_{\rm f}/3)(1/\epsilon)$. We thus see that the divergences cancel as expected and we obtain the usual screening result (13). We have thus shown at next to leading order that the gauge invariant factorization of the charge field is reflected in a gauge invariant decomposition of the detailed structure of the interaction between charges.

CONCLUSION

It has been known for many years that light matter screens charges. What we have shown in this paper is how such matter fields are distributed around physical charges. In a concrete calculation we have seen that the overall screening forces between such charges arise from two distinct, gauge invariant contributions. One has an anti-screening effect, but it is only half the size of the dominant screening term. This separation is not apparent in other methods (such as Wilson loops [18,19] and non-relativistic perturbation theory [7,12]) and it is intriguing to speculate on similar structures in the gluonic screening of QCD.

This result is a further vindication of our approach to the fundamental question of how to describe charged particles in gauge theories. We have seen that, from general principles, the dressing around a charge has a rich structure which is reflected in the infrared properties of the fields and in the forces between charges. This shows a previously unobserved intimate connection between the soft structures (χ) of gauge theories and anti-screening and also between the phase structures (K) and the overall screening effect.

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