

Spherically symmetric scalar field collapse in any dimension

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(Received 8 January 2002; published 14 May 2002)

We describe a formalism and numerical approach for studying spherically symmetric scalar field collapse for an arbitrary spacetime dimension d and cosmological constant Λ . The prescription uses a double null formalism, and is based on field redefinitions first used to simplify the field equations in generic two-dimensional dilaton gravity. The formalism is used to construct a code in which d and Λ are input parameters. We reproduce known results in $d=4$ and $d=6$ with $\Lambda=0$, and we present new results for $d=5$ with a zero and negative Λ .

DOI: 10.1103/PhysRevD.65.104036

PACS number(s): 04.70.Dy

I. INTRODUCTION

It is an interesting fact that spherically symmetric gravitational collapse exhibits critical behavior [1]. This is a classical effect in phase space: there are one-parameter families of initial data sets, for a variety of matter fields, such that as the parameter is tuned, a transition from reflection of infalling matter to black hole formation is observed numerically. Two types of behavior are observed close to this transition point. Depending on the matter type, black holes form with zero or nonzero initial mass, and the matter field exhibits discrete or continuous self-similarity. A variety of matter fields have been studied since the initial seminal work by Choptuik. There have also been some extensions beyond spherical symmetry, as well as a semianalytic perturbation theory understanding of the critical exponent. (Recent reviews may be found in Refs. [2,3].) If an exact time-dependent solution were available with the appropriate boundary conditions, this critical behavior would be manifested in the solution of the apparent horizon condition, which is a transcendental equation [4]. However, to date only one time-dependent (and non-self-similar) solution is known, but this does not have the required asymptotically flat boundary conditions [5]. Thus the full partial differential equation (PDE) problem must be studied numerically.

Most numerical studies of spherically symmetric collapse have been in four spacetime dimensions. The only exceptions are the massless minimally coupled scalar field by Garfinkle [7] in six spacetime dimensions, and by Pretorius and Choptuik [8] and Husain and Olivier [9] in three dimensions with a negative cosmological constant. In addition, there exist two papers that study this problem in the much simpler case of the self-similarity ansatz, an ordinary differential equation (ODE) problem, for any spacetime dimension [10,11].

The purpose of this work is to present a formalism and numerical method for studying the gravitational collapse of a spherically symmetric scalar field for any value of spacetime dimension and cosmological constant. The approach reproduces and supplements known results in four and six dimensions with a zero cosmological constant, and gives new results in five spacetime dimensions with a zero and negative cosmological constant.

II. FIELD EQUATIONS

The basic idea for obtaining field equations valid for any dimension is to reduce the d -dimensional Einstein-scalar field equations by imposing spherical symmetry, and then use a field redefinition originally motivated by generic two-dimensional dilaton gravity [12]. This allows the Einstein equations for d -dimensional, spherically symmetric scalar field collapse to be put into a form that can be managed numerically by a single code. In actuality, the formalism and code we describe below are applicable to a more general class of models that includes nonminimal scalar field coupling. This class of theories is quite broad, and contains as a subclass the d -dimensional spherically symmetric case.

Einstein gravity with a cosmological constant in d spacetime dimensions is given by the action

$$S_G^{(d)} = \frac{1}{16\pi G^{(d)}} \int d^d x \sqrt{-g^{(d)}} [R(g^{(d)}) - \Lambda]. \quad (1)$$

The corresponding action for a minimally coupled scalar field is

$$S_M = - \int d^d x \sqrt{-g^{(d)}} (g^{(d)\mu\nu} \partial_\mu \chi \partial_\nu \chi). \quad (2)$$

To impose spherical symmetry, we write the d -dimensional metric $g_{\mu\nu}$ as

$$ds_{(d)}^2 = \bar{g}_{\alpha\beta} dx^\alpha dx^\beta + r^2 (x^\alpha) d\Omega_{(d-2)}, \quad (3)$$

where $d\Omega_{(d-2)}$ is the metric on S^{d-2} and $\alpha, \beta = 1, 2$. This gives the reduced action¹

¹In most cases, one should perform dimensional reduction at the level of the field equations to guarantee that one obtains the correct solution space. It is well known [12] that in the present case the reduced field equations correspond to the field equations obtained from the reduced action.

$$\begin{aligned}
S_{\text{TOT}} &:= S_G + S_M \\
&= \mathcal{V}^{(n)} \int d^2x \sqrt{-\bar{g}} \left[\frac{1}{2G} \left(\frac{n}{8(n-1)} \bar{\phi}^2 [R(\bar{g}) - \Lambda] \right. \right. \\
&\quad \left. \left. + \frac{1}{2} |\partial \bar{\phi}|^2 + \frac{n^2}{8l^2} \bar{\phi}^{(2n-4)/n} - l^n \bar{\phi}^2 |\partial \chi|^2 \right] \right], \quad (4)
\end{aligned}$$

where $n = d - 2$, $\mathcal{V}^{(n)}$ is the volume of the unit n sphere, and

$$l^n = G^{(d)}, \quad (5)$$

$$\frac{1}{2G} = \frac{8(n-1)}{16\pi n}, \quad (6)$$

$$\bar{\phi} = \left(\frac{r}{l} \right)^{n/2}. \quad (7)$$

The key simplification in our formalism is achieved by the following conformal reparametrization of the metric, which eliminates the kinetic term for $\bar{\phi}$ from the action [12]. Let

$$g_{\alpha\beta} = \Omega^2(\bar{\phi}) \bar{g}_{\alpha\beta}, \quad (8)$$

where

$$\Omega^2(\bar{\phi}) = C \exp \left(\frac{1}{2} \int \frac{d\bar{\phi}}{(d\bar{D}/d\bar{\phi})} \right), \quad (9)$$

$$\bar{D}(\bar{\phi}) = \frac{n}{8(n-1)} \bar{\phi}^2, \quad (10)$$

and C is an arbitrary constant. Now define a dimensionless “dilaton” field

$$\phi = \bar{D}(\bar{\phi}) = \frac{n}{8(n-1)} \left(\frac{r}{l} \right)^n. \quad (11)$$

Note that ϕ is proportional to the area of the n sphere at radius r . With these redefinitions, the reduced action takes the simpler form

$$\begin{aligned}
S_{\text{TOT}} &= \frac{1}{2G} \int d^2x \sqrt{-g} [\phi R(g) + V^{(n)}(\phi)] \\
&\quad - \int d^2x \sqrt{-g} H^{(n)}(\phi) |\partial \chi|^2, \quad (12)
\end{aligned}$$

where

$$H^{(n)}(\phi) \equiv \frac{8(n-1)}{n} \phi, \quad (13)$$

$$\begin{aligned}
V^{(n)}(\phi) &\equiv \frac{1}{n} \left(\frac{8(n-1)}{n} \right)^{1/n} \phi^{1/n} \left[-l^2 \Lambda + \frac{n^2}{8} \right. \\
&\quad \left. \times \left(\frac{8(n-1)}{n} \right)^{(n-2)/n} \phi^{-2/n} \right], \quad (14)
\end{aligned}$$

and the overall factor of $\mathcal{V}^{(n)}$ has been dropped.

For arbitrary functions $V(\phi)$ and $H(\phi)$, the action (12) is that of generic dilaton gravity theory coupled to a scalar field in two spacetime dimensions. This theory has been studied in great detail [12]. The vacuum equations ($\chi=0$) can be solved exactly. By choosing an adapted coordinate system in which the dilaton ϕ plays the role of the spatial coordinate (i.e., $x = l\phi$), the vacuum solution for the metric is

$$ds^2 = -[j(\phi) - 2GM] dt^2 + [j(\phi) - 2GM]^{-1} dx^2, \quad (15)$$

where M plays the role of mass, and

$$j(\phi) \equiv \int_0^\phi d\tilde{\phi} V(\tilde{\phi}). \quad (16)$$

Note that we have dropped the superscript n denoting space-time dimension, since the above solution applies to the generic case. For the specific case of d -dimensional spherically symmetric gravity,

$$\begin{aligned}
j^{(n)}(\phi) &= \frac{1}{n} \left(\frac{8(n-1)}{n} \right)^{1/n} \left[-l^2 \Lambda \left(\frac{n}{n+1} \right) \phi^{(n+1)/n} \right. \\
&\quad \left. + \frac{n^3}{8(n-1)} \left(\frac{8(n-1)}{n} \right)^{(n-2)/n} \phi^{(n-1)/n} \right]. \quad (17)
\end{aligned}$$

It can easily be verified by making the appropriate substitutions and conformal reparametrization (8) that the physical line element $ds_{(d)}^2$ corresponding to Eq. (15) is precisely that of a d -dimensional de Sitter or anti-de Sitter black hole with mass M .² It is important to note that the metric (15) is singular at $\phi=0$ even when $M=0$. Up to numerical constants, j goes to zero as

$$j(\phi) \rightarrow \phi^{1-1/n} \quad (18)$$

near $\phi=0$. This is not a physical singularity since the physical metric \bar{g} is indeed the Minkowski metric when M and Λ are zero. Nonetheless, the vanishing of $j(\phi)$ will affect the choice of boundary conditions in our numerical method.

We now examine the field equations that derive from Eq. (12) in double null coordinates, for which the metric may be parametrized as

$$ds^2 = -2l g(u, v) \phi'(u, v) du dv, \quad (19)$$

where the prime denotes partial differentiation with respect to the null coordinate v . (Recall that this is just the $u-v$ part of the physical metric.) The corresponding field equations are

$$\phi' = -\frac{l}{2} V^{(n)}(\phi) g \phi', \quad (20)$$

²In order to get the overall scale factor right, one must choose the constant C in Eq. (9) appropriately.

$$\frac{g' \phi'}{g H^{(n)}(\phi)} = 2G(\chi')^2, \quad (21)$$

$$[H^{(n)}(\phi)\chi']' + [H^{(n)}(\phi)\dot{\chi}]' = 0. \quad (22)$$

In the above, the dot refers to differentiation with respect to u , which is treated like the “time” coordinate for the purposes of the following numerical integration. Remarkably, for arbitrary n , Eqs. (20)–(22) are virtually identical in form to those studied in [9] in the context of $(2+1)$ -dimensional AdS gravity.³ However, the boundary conditions are special in the case of $n=1$ ($d=3$), so we will not consider this case further. Except where explicitly stated, we henceforth restrict consideration to $n \geq 2$ (which means spacetime dimension $d \geq 4$).

The evolution equations may be put in a form more useful for numerical solution by defining the variable

$$h = \chi + \frac{2\phi\chi'}{\phi'}. \quad (23)$$

This effectively replaces the scalar field χ by h . The evolution equations are

$$\dot{\phi} = -\tilde{g}/2, \quad (24)$$

$$\dot{h} = \frac{1}{2\phi}(h-\chi)(g\phi V - \tilde{g}), \quad (25)$$

where

$$\tilde{g} = \int_u^v (g\phi' V) dv' \quad (26)$$

and χ is now to be considered a functional of h and g given by

$$\chi = \frac{1}{2\sqrt{\phi}} \int_u^v dv' \left[\frac{h\phi'}{\sqrt{\phi}} \right] + \frac{K_3(u)}{\sqrt{\phi}}. \quad (27)$$

The integration constant $K_3(u)$ must be zero because the definition of h requires $h=\chi$ at $\phi=0$.⁴ The function g is a functional of h and ϕ , obtained by integrating the constraint (21):

$$g = K_1(u) \exp \left[4\pi \int_u^v dv' \frac{\phi'}{\phi} (h-\chi)^2 \right], \quad (28)$$

where $K_1(u)$ is again an integration constant (i.e., independent of the “spatial” coordinate v). We consider the case of a spherically symmetric, collapsing shell of matter, with no black hole in the interior, initially. Thus, our boundary con-

dition should be such as to guarantee that the metric at $r=0$ (which translates to $\phi=0$) goes over to the vacuum solution (15). By transforming the vacuum ($M=0$) metric (15) to double null coordinates (19), we obtain a metric of the form

$$ds^2 = -j(\phi) du dv, \quad (29)$$

where $u = t - \phi^*$ and $v = t + \phi^*$, with the generalized “tor-
toise coordinate” ϕ^* defined by

$$\phi^* = l \int_0^\phi \frac{d\phi}{j(\phi)}. \quad (30)$$

With these definitions, $\phi=0$ corresponds to the surface $v=u$. Moreover, it follows that for the vacuum solution

$$\phi' \equiv \frac{\partial \phi}{\partial v} = \frac{1}{2} \frac{j(\phi)}{l}. \quad (31)$$

Comparing metric (29) to our general form (19), we see that $g=1$ for the vacuum solution. Since we would like the numerical solution to approach the vacuum at $\phi=0$, the above analysis determines the required boundary conditions. In particular, the integration constant $K_1(u)=1$, and

$$\dot{\phi} \equiv \frac{\partial \phi}{\partial u} \rightarrow -\frac{1}{2} \frac{j(\phi)}{l}, \quad (32)$$

which vanishes at $\phi=0$ in agreement with the expression (26).

III. NUMERICAL METHOD

The numerical scheme uses a v (“space”) discretization to obtain a set of coupled ODEs:

$$h(u, v) \rightarrow h_i(u), \quad \phi(u, v) \rightarrow \phi_i(u), \quad (33)$$

where $i=0, \dots, N$ specifies the v grid. Initial data for these two functions are prescribed on a constant v slice, from which the functions $g(u, v)$, $\tilde{g}(u, v)$ are constructed. Evolution in the “time” variable u is performed using the fourth-order Runge-Kutta method. The general scheme is similar to that used in [13], together with some refinements used in [6]. This procedure was also used for the three-dimensional collapse calculations in [9].

The initial scalar field configuration $\chi(\phi, u=0)$ is most conveniently specified as a function of ϕ rather than r . (Recall that $\phi \propto r^n$.) This together with the initial arrangement of the radial points $\phi(v, u=0)$ fixes all other functions. We used the initial specification $\phi(0, v)=v$.

We consider two types of initial scalar field configurations: the Gaussian and “tanh” functions

$$\chi_G(u=0, \phi) = a \phi \exp \left[-\left(\frac{\phi - \phi_0}{\sigma} \right)^2 \right] \quad (34)$$

and

$$\chi_T(u=0, \phi) = a \tanh(\phi). \quad (35)$$

³For $n=1$, they are identical as expected.

⁴We will see below that although ϕ' goes to zero at $\phi=0$, it does so slowly enough to guarantee that the second term in Eq. (23) vanishes at $\phi=0$.

These choices permit us to test “universality,” which is defined as the independence of the details of the collapse, such as the critical exponent, on the choice of initial data shapes and parameters. Although universality may be tested for a large variety of shapes and parameters, the emphasis here is on different spacetime dimensions, so we have restricted attention to varying the amplitude parameter for these two shapes.

The initial values of the other functions are determined in terms of the above by computing the integrals for g_n and \tilde{g}_n using Simpson’s rule. In all cases, we used values of $\phi_0 = 1$ and $\sigma = 0.3$ for the Gaussian initial data.

The boundary conditions at fixed u are

$$\phi_k = 0, \quad \tilde{g}_k = 0, \quad g_k = 1, \quad (36)$$

where k is the index corresponding to the position of the origin $\phi = 0$. (In the algorithm used, all grid points $0 \leq i \leq k-1$ correspond to ingoing rays that have reached the origin and are dropped from the grid; see below.) These conditions are equivalent to $r(u, u) = 0$, $g|_{r=0} = g(u, u) = 1$, and guarantee regularity of the metric at $r = 0$. Notice that for our initial data, ϕ_k and hence h_k are initially zero, and therefore they remain zero at the origin because of Eq. (25).

As evolution proceeds via the Runge-Kutta procedure, the entries in the ϕ_i array sequentially reach 0, at which point they are dropped from the grid. Thus the radial grid loses points with evolution. This is similar to the procedure used in [13] and [6].

At each u step, a check is made to see if an apparent horizon has formed by observing the function

$$ah \equiv g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi = -\frac{\dot{\phi}}{lg}, \quad (37)$$

whose vanishing signals the formation of an apparent horizon.⁵ For each run of the code with fixed amplitude a , this function is scanned from larger to smaller radial values after each Runge-Kutta iteration, and evolution is terminated if the value of this function reaches 10^{-3} . The corresponding radial coordinate value is recorded as R_{ah} . In the subcritical case, it is expected that all the radial grid points reach zero without detection of an apparent horizon. This is the signal of pulse reflection.

The results (a, R_{ah}) are collated as in [1], by seeking a relationship of the form

$$R_{ah} \propto (a - a_*)^\gamma, \quad (38)$$

⁵It is worth noting that there exist special foliations of black hole spacetimes which have no apparent horizon [14]. Therefore, there is in principle the possibility that a numerical scheme that encounters such a slicing may fail to detect black hole formation. This manifestly does not happen with the double null coordinates used in our study, as demonstrated by the form of the static solution described above and our results below.

where a_* is the critical amplitude which separates the black hole and reflection solutions. R_{ah} is the basic dimensional scale if a black hole forms, and is linearly related to black hole mass in four spacetime dimensions.

To improve numerical accuracy near $\phi = 0$, we follow a procedure similar to [6], where all functions on a constant u surface are expanded in power series in ϕ at $\phi = 0$, and the first three values of the constraint integrals are derived using the respective power series. We write

$$h = h_0 + h_1 \phi \quad (39)$$

and calculate the parameters h_0, h_1 using the linear least-squares fit for the first 15 points in $h_i(u)$.⁶ From this, the expressions for χ , g , and \tilde{g} follow. The remaining $N-3$ values of these functions are computed from their integral using Simpson’s rule for equally spaced points. This linear fit near $\phi = 0$ is necessary because it elegantly handles the problematic $1/\phi$ factor in the h evolution equation, which would persist even if a finer mesh were used. The v derivatives of functions (needed for computing g and \tilde{g}) are calculated using $f'_i = (f_{i+1} - f_{i-1})/2\Delta v$ with end-point values determined by linear extrapolation: $f'_1 = 2f'_2 - f'_3$ and $f'_N = 2f'_{N-1} - f'_{N-2}$.

Further comments on the procedure are the following. The number of v grid points decreases as ingoing null geodesics cross $r = 0$, and so a reflected pulse cannot be followed back out toward infinity.⁷ Also, again due to the loss of grid points, and hence resolution, we are not able to observe the detailed behavior of the scalar field very near criticality. An additional numerical adjustment concerns the enforcement of boundary conditions at the origin: there is the gravitational tendency for the matter to pile up at the origin as the collapse proceeds. However, the formalism has the competing implicit condition $h(u, u) = 0$ (i.e., at $\phi = 0$). This can lead to a shift of this boundary condition under evolution. It is rectified by adjusting the scalar field function at each time step by adding a constant shift at all points. This shift is of order 10^{-5} or less at each time step, and therefore there is a minor loss of accuracy, but a corresponding gain in stability.

The code was tested for grid sizes ranging from 2000 to 6000 points, and with the u and v step sizes ranging from 10^{-2} to 10^{-4} , for the two types of initial data used, as well as the vacuum case of a vanishing scalar field. These tests established that the code converges. Further tests of the code include the reproduction of the known results in four and six dimensions, which also demonstrates the accuracy of the results we obtain. All the results presented below were for a grid size of 6000 points, with u and v step sizes of 10^{-3} .

Finally, we point out that this procedure allows a more accurate analysis of the supercritical case than the subcritical

⁶There is nothing fundamental about this number, since the behavior of the scalar field turns out to be very linear over the first several points; the results for h_0 and h_1 were virtually insensitive if the number of points used varied by a few on either side of 15.

⁷A modification of our procedure along the lines suggested in [17] may allow the tracking of the reflected pulse to future null infinity.

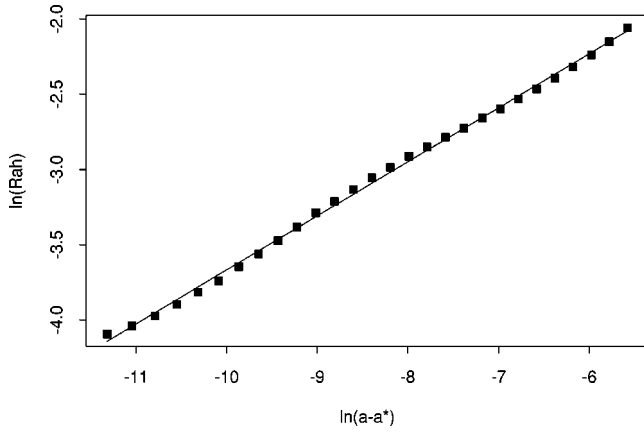


FIG. 1. Logarithmic plot of apparent horizon radius R_{ah} versus initial scalar field amplitude $(a - a_*)$ in four spacetime dimensions for Gaussian initial data. $\gamma = 0.36$.

one, again because of the loss of grid points as the evolution proceeds. This is less of an issue in the supercritical case because the number of points lost depends on the initial pulse amplitude, and our termination condition is such that typically one-quarter of the original number of points are still present at termination, for the closest approach to criticality. We could of course monitor the subcritical case up to this point as well in an attempt to observe the self-similarity of the scalar field, but the very nature of this behavior requires a way to replace lost grid points, as observed in [6]. For this reason, we focus on supercritical evolutions.

IV. RESULTS AND CONCLUSIONS

The code was first tested to recalculate known results in four and six dimensions. It was then run for the five-dimensional case with zero, positive, and negative cosmological constant. All the calculations were performed for amplitudes above the threshold for black hole formation, and for initial data specified in both Gaussian and tanh forms, parametrized by amplitude a . The figures below show the scaling law Eq. (38). The squares represent the points

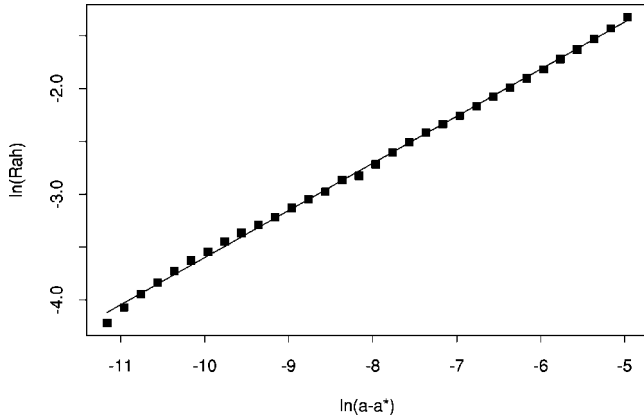


FIG. 2. Logarithmic plot of apparent horizon radius R_{ah} versus initial scalar field amplitude $(a - a_*)$ in six spacetime dimensions for Gaussian initial data. $\gamma = 0.44$.

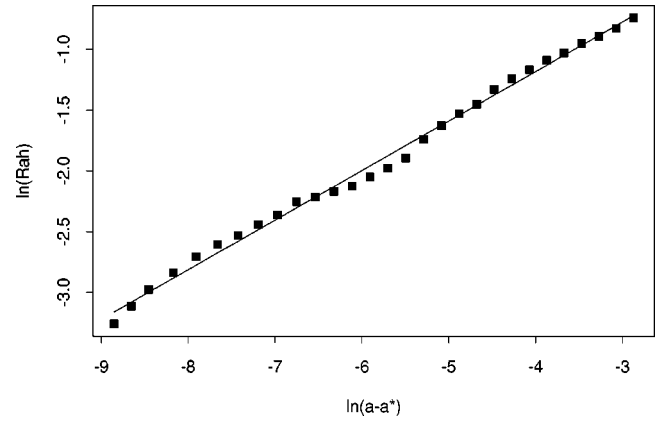


FIG. 3. Logarithmic plot of the apparent horizon radius R_{ah} versus initial scalar field amplitude $(a - a_*)$ in five spacetime dimensions with zero cosmological constant for tanh initial data. $\gamma = 0.41$.

(a, R_{ah}) and the lines are the least-squares fit to these points.

The four-dimensional results for the Gaussian initial data are illustrated in Fig. 1. The least-squares fit gives a slope of $\gamma = 0.36$, in good agreement ($\sim 4\%$) with earlier studies [1–3]. The figure also shows the oscillation about the fit line, again in accord with earlier work.

The six-dimensional results for Gaussian data appear in Fig. 2. Our result for the critical exponent is $\gamma = 0.44$. For comparison, the result in Ref. [7] is $\gamma = 0.424$.

The results for the four- and six-dimensional calculations for the tanh initial data are $\gamma = 0.35$ and $\gamma = 0.41$, respectively. This provides a further check of our code, and further evidence of the insensitivity of the critical exponent to the shape of the initial data (“universality”) in both four and six dimensions. Note that in six dimensions this is the first evidence for universality, since Ref. [7] contains results only for a specific Gaussian form of initial data, different from the one used here. Note also that the agreement of our apparent horizon radius scaling results with those of the earlier works cited shows that the apparent horizon appears to be a fairly good approximation to the event horizon of the long-time

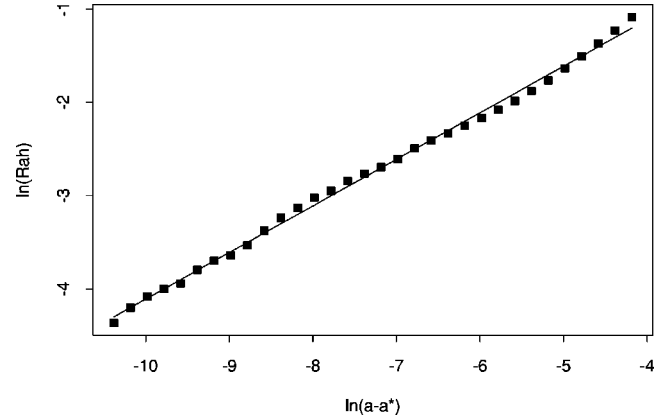


FIG. 4. Logarithmic plot of apparent horizon radius R_{ah} versus initial scalar field amplitude $(a - a_*)$ in five spacetime dimensions with $\Lambda = -1$ for Gaussian initial data. $\gamma = 0.49$.

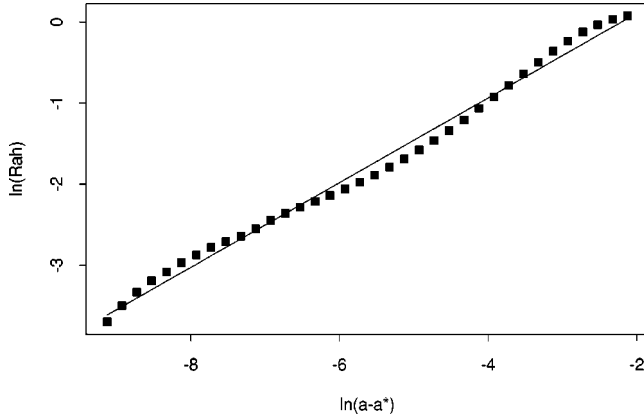


FIG. 5. Logarithmic plot of apparent horizon radius R_{ah} versus initial scalar field amplitude $(a - a_*)$ in five spacetime dimensions with $\Lambda = 0$ for Gaussian initial data. $\gamma = 0.52$.

static limit (insofar as these earlier studies actually find this limit).

These tests of our formalism and code establish the consistency of our results with the earlier works mentioned above, and set the stage for new calculations for arbitrary values of the cosmological constant. Although our code allows calculations for any dimension, we focus on the five-dimensional case mainly because results already exist for three, four, and six dimensions.

With $\Lambda = 0$, for the tanh initial data, we find a critical exponent of $\gamma = 0.41$ (Fig. 3). This value falls between the four- and six-dimensional cases as conjectured in [7].

With $\Lambda = -1$, and Gaussian initial data, we find a critical exponent of $\gamma = 0.49$ (Fig. 4). All of the graphs show an oscillation about the least-squares fit line. This is a known feature for zero cosmological constant, and is concomitant with discrete self-similarity of the critical solution. Our results for the negative cosmological constant also show this feature, which indicates that the critical solution for this case also has discrete self-similarity.

We find that in five dimensions the critical exponent appears not to be universal, at least in the supercritical approach to computing it. The Gaussian initial data yielded $\gamma = 0.52$ for $\Lambda = 0$, in comparison to $\gamma = 0.41$ for the tanh initial data (Fig. 5).

The reason for this is not clear to us and it would be worthwhile to calculate the exponents using the subcritical approach suggested in Ref. [15], where the Ricci scalar at the origin is calculated near criticality from below. It is also worth noting that a similar lack of universality is manifested in the three-dimensional AdS case using the supercritical apparent horizon method of computing γ [16].

We also carried out a preliminary investigation of the positive cosmological constant case in five dimensions. This is an interesting case because of the presence of a cosmological horizon in addition to the potential apparent horizon. Figure 6 shows graphs of the scalar field and apparent hori-

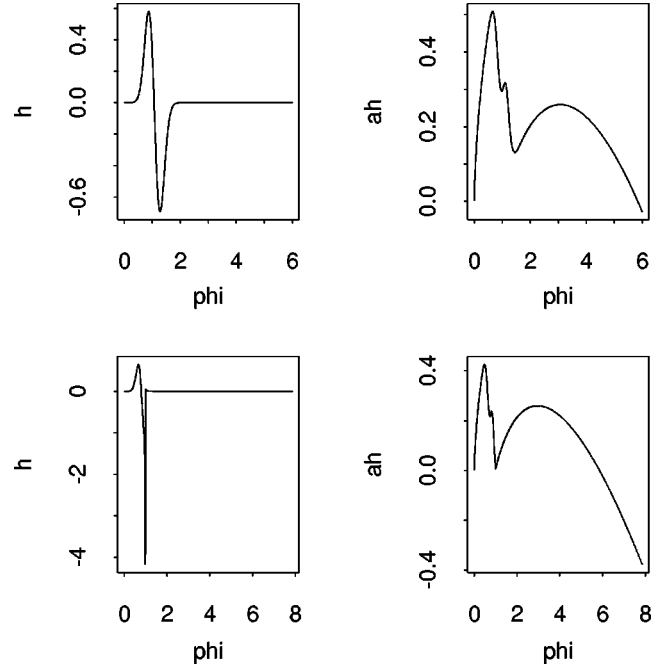


FIG. 6. Plots of the scalar field h and the ah function Eq. (37) prior to (top row) and nearer apparent horizon formation in five spacetime dimensions with positive Λ . Note the expansion of the ϕ grid from 6 to 8 in the bottom two graphs.

zon functions h and ah in the left and right columns, respectively, as functions of ϕ , prior to and at the onset of apparent horizon formation in the two successive rows. Note the location of the cosmological horizon in the right-hand column near $\phi = 5.8$. We find that the function $\phi(u, v)$ evolved such that instead of the radial grid contracting as for the zero and negative Λ cases, it expanded as the scalar field moved towards the origin. This feature is visible in Fig. 6: the range of ϕ in the lower graphs has expanded to 8 from 6. In fact, the closer the onset of apparent horizon formation is, the larger the range of the ϕ variable (and hence the radial grid). This prevented us from extracting accurate apparent horizon radii since the interesting features became confined to an ever-shrinking part of the grid. We hope to study this in detail in future work.

In summary, we have described a formalism and code for studying spherically symmetric gravitational collapse of a scalar field for any d and Λ , presented new results in five dimensions, and given evidence for universality in six dimensions. In future work, we will present results of a systematic analysis of the critical exponent as a function of both spacetime dimension and cosmological constant.

ACKNOWLEDGMENTS

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada. G.K. would like to thank J. Gegenberg for helpful conversations.

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