

# Black-hole collision with a scalar particle in three-dimensional anti-de Sitter spacetime

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(Received 9 January 2002; published 10 May 2002)

We study the collision between a BTZ black hole and a test particle coupled to a scalar field. We compute the power spectrum, the energy radiated and the plunging waveforms for this process. We show that for late times the signal is dominated by the quasinormal ringing. In terms of the AdS-CFT correspondence the bulk gravity process maps into a thermal state, an expanding bubble and gauge particles decaying into bosons of the associated operator. These latter thermalize on a time scale predicted by the bulk theory.

DOI: 10.1103/PhysRevD.65.104032

PACS number(s): 04.70.-s, 04.30.-w, 04.50.+h, 11.15.-q

## I. INTRODUCTION

Anti-de Sitter (AdS) spacetime has been considered of fundamental meaning within high energy elementary particle physics, especially in supersymmetric theories of gravity such as 11-dimensional supergravity and M theory (or string theory). The dimension  $d$  of AdS spacetime is a parameter which can have values from two to eleven, and where the other spare dimensions either are joined as a compact manifold  $\mathcal{M}$  into the whole spacetime to yield  $\text{AdS}_d \times \mathcal{M}^{11-d}$  or receive a Kaluza-Klein treatment. AdS spacetime appears as the background for black hole solutions and it also plays a further crucial role since it is the near-horizon geometry, separated by a (soft) boundary from an otherwise asymptotically flat spacetime, of some black solutions [1]. In addition, by taking low energy limits at strong coupling and through group theoretic analysis, Maldacena conjectured a correspondence between the bulk of AdS spacetime and a dual conformal field gauge theory (CFT) on the spacetime boundary itself [2]. A concrete method to implement this correspondence is to identify the extremum of the classical string theory action  $I$  for the dilaton field  $\varphi$ , say, at the boundary of AdS, with the generating functional  $W$  of the Green's correlation functions in the CFT for the operator  $\mathcal{O}$  that corresponds to  $\varphi$  [3],  $I_{\varphi_0(x^\mu)} = W[\varphi_0(x^\mu)]$ , where  $\varphi_0$  is the value of  $\varphi$  at the AdS boundary and the  $x^\mu$  label the coordinates of the boundary. The motivation for this proposal can be seen in the reviews [4]. In its strongest form the conjecture requires that the spacetime be asymptotically AdS, the interior could be full of gravitons or containing a black hole. The correspondence realizes the holographic principle; since the bulk is effectively encoded in the boundary, and is also a strong-weak duality, it can be used to study issues of strong gravity using weak CFT or CFT issues at strong coupling using classical gravity in the bulk.

A particularly important AdS dimension is three. In  $\text{AdS}_3$  Einstein gravity is simple; the group of isometries is given by two copies of  $SL(2, R)$ , it has no propagating degrees of freedom, is renormalizable, it allows for the analytical computation of many physical processes extremely difficult or even impossible in higher dimensions, it belongs to the full

string theory compactification scheme [4], the dual  $\text{CFT}_2$  is the low-energy field theory of a D1-D5-brane system which can be thought of as living on a cylinder (the boundary of  $\text{AdS}_3$ ) [5], and it contains the Bañados-Teitelboim-Zanelli (BTZ) black hole. The BTZ black hole is of considerable interest, not only because it can yield exact results, but also because one hopes that the results can qualitatively be carried through to higher dimensions. Several results related to the BTZ black hole itself and to the AdS-CFT correspondence have been obtained [6–9]. The AdS-CFT mapping implies that a black hole in the bulk corresponds to a thermal state in the gauge theory [10]. Perturbing the black hole corresponds to perturbing the thermal state and the decaying of the perturbation is equivalent to the return to the thermal state. Particles initially far from the black hole correspond to a blob (a localized excitation) in the CFT, as the IR-UV duality teaches [11]. The evolution towards the black hole represents a growing size of the blob with the blob turning into a bubble traveling close to the speed of light [7].

In this work we extend some of the previous results and we study in detail the collision between a BTZ black hole and a scalar particle. Generically, a charged particle falling towards a black hole emits radiation of the corresponding field. In higher dimensions it also emits gravitational waves, but since in three dimensions there is no gravitational propagation in the BTZ case there is no emission. Thus, a scalar particle falling into a BTZ black hole emits scalar waves. This collision process is important from the points of view of three-dimensional dynamics and of the AdS-CFT conjecture. Furthermore, one can compare this process with previous works, since there are exact results for the quasinormal mode (QNM) spectrum of scalar perturbations which are known to govern their decay at intermediate and late times [12].

The phenomenon of radiation emission generated from an infalling particle in asymptotically flat spacetimes has been studied by several authors [13] and most recently in [14], where the results are to be compared to full scale numerical computations for strong gravitational wave emission of astrophysical events [15] which will be observed by the GEO600, Laser Interferometric Gravitational Wave Observatory (LIGO), and VIRGO projects. A scalar infalling particle as a model for calculating radiation reaction in flat spacetimes has been considered in [16]. Many of the techniques have been developed in connection to such spacetimes. Such an analysis has not been carried to nonasymptotically flat spacetimes, which could deepen our understanding of these

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kind of events, and of Einstein's equations. In this respect, for the mentioned reasons, AdS spacetimes are the most promising candidates. As asymptotically flat spacetimes they provide well defined conserved charges and positive energy theorems, which makes them a good testing ground if one wants to go beyond flatness. However, due to different boundary conditions it raises new problems. The first one is that since the natural boundary conditions are boxlike all of the generated radiation will eventually fall into the black hole, thus infinity has no special meaning in this problem, it is as good a place as any other, i.e., one can calculate the radiation passing at any radius  $r$ , for instance near the horizon. Second, in contrast to asymptotically flat spacetimes, here one cannot put a particle at infinity (it needs an infinite amount of energy) and thus the particle has to start from finite  $r$ . This has been posed in [14] but was not fully solved when applied to AdS spacetimes.

## II. FORMULATION OF THE PROBLEM AND BASIC EQUATIONS

We consider a small test particle of mass  $m_0$  and charge  $q_0$ , coupled to a massless scalar field  $\varphi$ , moving along a radial timelike geodesic outside a BTZ black hole of mass  $M$ . The metric outside the BTZ black hole is

$$ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} - r^2 d\theta^2, \quad (1)$$

where  $f(r) = (-M + r^2/l^2)$  and  $l$  is the AdS radius ( $G = 1/8; c = 1$ ). The horizon radius is given by  $r_+ = M^{1/2}l$ . We treat the scalar field as a perturbation, so we shall neglect the back reaction of the field's stress tensor on the metric (this does not introduce large errors [17]). If we represent the particle's worldline by  $x^\mu = x_p^\mu(\tau)$ , with  $\tau$  the proper time along a geodesic, then the interaction action  $\mathcal{I}$  is

$$\begin{aligned} \mathcal{I} = & -\frac{1}{8\pi} \int g^{1/2} \varphi_{;a} \varphi^{;a} d^3y \\ & - m_0 \int (1 + q_0 \varphi) (-g_{ab} \dot{x}^a \dot{x}^b)^{1/2} d\tau, \end{aligned} \quad (2)$$

and thus the scalar field satisfies the inhomogeneous wave equation  $\square \varphi = -4\pi m_0 q_0 \int \delta^3(x^\mu - x_p^\mu(\tau)) (-g)^{-1/2} d\tau$ , where  $g$  is the metric determinant and  $\square$  denotes the covariant wave operator. As the particle moves on a timelike geodesic, we have

$$\dot{t}_p = \frac{\mathcal{E}}{f(r_p)}, \quad \dot{r}_p = -(\mathcal{E}^2 - f(r_p))^{1/2}, \quad (3)$$

where  $\dot{\phantom{x}} \equiv d/d\tau$ , and  $\mathcal{E}$  is a conserved energy parameter. We shall be considering the test particle initially at rest at a distance  $r_0$  [where  $\mathcal{E}^2 = f(r_0)$ ] and at  $\theta_p = 0$ . Expanding the field as

$$\varphi(t, r, \theta) = \frac{1}{r^{1/2}} \phi(t, r) \sum_m e^{im\theta}, \quad (4)$$

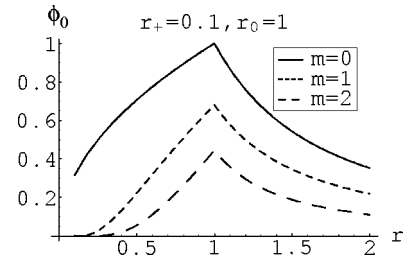


FIG. 1. Initial data  $\phi_0$  for a BTZ black hole with  $r_+ = 0.1$ , and with the particle at  $r_0 = 1$ , for several values of  $m$ , the angular quantum number.

where  $m$  is the angular momentum quantum number, the wave equation is given by (after an integration in  $\theta$ )

$$\begin{aligned} \frac{\partial^2 \phi(t, r)}{\partial r_*^2} - \frac{\partial^2 \phi(t, r)}{\partial t^2} - V(r) \phi(t, r) \\ = -\frac{2q_0 m_0 f}{r^{1/2}} \left( \frac{dt}{d\tau} \right)^{-1} \delta(r - r_p), \end{aligned} \quad (5)$$

with  $V(r) = 3r^2/4l^4 - M/2l^2 - M^2/4r^2 + m^2/l^2 - Mm^2/r^2$ , and  $r_* = -M^{-1/2} \operatorname{arccoth}(rM^{-1/2})$ .

## III. THE INITIAL DATA AND BOUNDARY CONDITIONS

In the case we study, and in contrast to asymptotically flat spacetimes where initial data can be pushed to infinity [13], initial data must be provided. Accordingly, we take the Laplace transform  $\Phi(\omega, r)$  of  $\phi(t, r)$  to be

$$\Phi(\omega, r) = \frac{1}{(2\pi)^{1/2}} \int_0^\infty e^{i\omega t} \phi(t, r) dt. \quad (6)$$

Then, Eq. (5) may be written as

$$\frac{\partial^2 \Phi(r)}{\partial r_*^2} + [\omega^2 - V(r)] \Phi(r) = S + \frac{i\omega \phi_0}{(2\pi)^{1/2}}, \quad (7)$$

with  $S = -(f/r^{1/2})(2/\pi)^{1/2}(1/\dot{r}_p)e^{i\omega t}$  being the source function, and  $\phi_0$  the initial value of  $\phi(t, r)$  satisfying  $\partial^2 \phi_0(r, m)/\partial r_*^2 - V(r)\phi_0(r, m) = -(2f/r^{1/2})(dt/d\tau)_{r_0}^{-1} \delta(r - r_p)$ . We have rescaled  $r, r \rightarrow r/l$ , and measure everything in terms of  $l$ , i.e.,  $\phi, r_+$ , and  $\omega$  are to be read,  $(1/l^{1/2}q_0m_0)\phi, r_+/l$  and  $\omega l$ , respectively. One can numerically solve the equation for the initial data  $\phi_0$  by demanding regularity at both the horizon and infinity (for a similar problem see [18]). In Fig. 1, we show the form of  $\phi_0$  for a typical case  $r_+ = 0.1, r_0 = 1$ , and for three different values of  $m, m = 0, 1, 2$ . Other cases like  $r_+ = 1, 10, \dots$  and several values of  $r_0$  can be computed. Large black holes have a direct interpretation in the AdS-CFT conjecture. The results for large or small black holes are nevertheless similar, as we have checked. As a test for the numerical evaluation of  $\phi_0$ , we have checked that as  $r_0 \rightarrow r_+$ , all the multipoles fade away, i.e.,  $\phi_0 \rightarrow 0$ , supporting the no-hair conjecture (that all the multipoles go

to zero). To solve Eq. (7) one has to impose physically sensible boundary conditions, appropriate to AdS spacetimes. In our case the potential diverges at infinity, where  $\phi_0$  vanishes, so we impose reflective boundary conditions [19] there, i.e.,  $\Phi=0$  at infinity. It has been common practice to set  $\Phi \sim F(\omega)e^{-i\omega r_*}$  near the horizon, meaning ingoing waves there. This is an allowed boundary condition as long as  $\phi_0$  vanishes there. However, if  $\phi_0$  does not vanish there, one has to be careful in defining boundary conditions at the horizon. When dealing with AdS spacetimes this detail is crucial to extract the correct information, and it has been overlooked when one deals with asymptotically flat spacetimes as in [14]. Equation (7) together with the source term  $S$  allows us to conclude that near the horizon  $\Phi \sim G(\omega)e^{i\omega r_*} + F(\omega)e^{-i\omega r_*} + i\phi_0/(2\pi)^{1/2}\omega$ . Since we want waves going down the black hole, we shall require

$$\Phi \sim F(\omega)e^{-i\omega r_*} + \frac{i\phi_0}{(2\pi)^{1/2}\omega}, \quad r \rightarrow r_+. \quad (8)$$

#### IV. GREEN'S FUNCTION SOLUTION

To proceed we must find a solution to Eq. (7) through a Green's function analysis. A standard treatment [14] invokes contour integration to calculate the integrals near the horizon. There is no need for this here; by demanding regularized integrals the correct boundary conditions appear in a natural way (see [20] for a regularization of the Teukolsky equation). Let  $\Phi^\infty$  and  $\Phi^H$  be two independent solutions of the homogeneous form of Eq. (7), satisfying  $\Phi^H \sim e^{-i\omega r_*}, r \rightarrow r_+$ ;  $\Phi^H \sim A(\omega)r^{1/2} + B(\omega)r^{-3/2}, r \rightarrow \infty$ ;  $\Phi^\infty \sim C(\omega)e^{i\omega r_*} + D(\omega)e^{-i\omega r_*}, r \rightarrow r_+$ ;  $\Phi^\infty \sim 1/r^{3/2}, r \rightarrow \infty$ . Define  $h^H$  through  $dh^H/dr_* = -\Phi^H$  and  $h^\infty$  through  $dh^\infty/dr_* = -\Phi^\infty$ . We can then show that  $\Phi$  given by

$$\begin{aligned} \Phi = & \frac{1}{W} \left[ \Phi^\infty \int_{-\infty}^r \Phi^H S dr_* + \Phi^H \int_r^\infty \Phi^\infty S dr_* \right] \\ & + \frac{i\omega}{(2\pi)^{1/2}W} \left[ \Phi^\infty \int_{-\infty}^r h^H \frac{d\phi_0}{dr_*} dr_* + \Phi^H \int_r^\infty h^\infty \frac{d\phi_0}{dr_*} dr_* \right. \\ & \left. + (h^\infty \phi_0 \Phi^H - h^H \phi_0 \Phi^\infty)(r) \right], \quad (9) \end{aligned}$$

is a solution to Eq. (7) and satisfies the boundary conditions. The Wronskian  $W = 2i\omega C(\omega)$  is a constant. Near infinity, we get from Eq. (9) that

$$\begin{aligned} \Phi(r \rightarrow \infty) = & \frac{1}{W} \left[ \Phi_{(r \rightarrow \infty)}^\infty \int_{-\infty}^\infty \Phi^H S dr_* \right] \\ & + \frac{i\omega}{(2\pi)^{1/2}W} \left[ \Phi_{(r \rightarrow \infty)}^\infty \int_{-\infty}^\infty h^H \frac{d\phi_0}{dr_*} dr_* \right. \\ & \left. + (h^\infty \phi_0 \Phi^H - h^H \phi_0 \Phi^\infty)(r \rightarrow \infty) \right]. \quad (10) \end{aligned}$$

Now, in our case, this is just zero, as it should be, because both  $\Phi^\infty, \phi_0 \rightarrow 0$ , as  $r \rightarrow \infty$ . However, if one is working with

asymptotically flat space, as in [14], where  $\Phi^\infty \rightarrow e^{i\omega r_*}$  at infinity, we get (recalling that  $\phi_0 \rightarrow 0$ )

$$\begin{aligned} \Phi(r \rightarrow \infty) = & \frac{1}{W} \left[ \Phi_{(r \rightarrow \infty)}^\infty \int_{-\infty}^\infty \Phi^H S dr_* \right] \\ & + \frac{i\omega}{(2\pi)^{1/2}W} \Phi_{(r \rightarrow \infty)}^\infty \int_{-\infty}^\infty h^H \frac{d\phi_0}{dr_*} dr_*, \quad (11) \end{aligned}$$

and where each integral is well defined. In particular, integrating by parts the second integral can be put in the form

$$\begin{aligned} \int_{-\infty}^\infty h^H \frac{d\phi_0}{dr_*} dr_* = & [h^H \phi_0]_{-\infty}^\infty + \int_{-\infty}^\infty \Phi^H \phi_0 dr_* \\ = & \frac{i\phi_0 e^{-i\omega r_*}}{\omega}(r \rightarrow -\infty) + \int_{-\infty}^\infty \Phi^H \phi_0 dr_*. \quad (12) \end{aligned}$$

Here, the final sum converges, but not each term in it. Expression (12) is just expression (3.15) in [14], although it was obtained imposing incorrect boundary conditions and not well defined regularization schemes. Due to the fact that the initial data vanishes at infinity, the results in [14] are left unchanged. In this work, we are interested in computing the wave function  $\Phi(r, \omega)$  near the horizon ( $r \rightarrow r_+$ ). In this limit we have

$$\begin{aligned} \Phi(r \sim r_+) = & \frac{1}{W} \left[ \Phi^H \int_{r_+}^\infty \Phi^\infty S dr_* \right] \\ & + \frac{i\omega}{(2\pi)^{1/2}W} \Phi^H \left[ \int_{r_+}^\infty \Phi^\infty \phi_0 dr_* \right. \\ & \left. - (h^\infty \phi_0)(r_+) \right] + \frac{i\phi_0(r_+)}{(2\pi)^{1/2}\omega}, \quad (13) \end{aligned}$$

where an integration by parts has been used. Fortunately, one can obtain an exact expression for  $\Phi^\infty$  in terms of hypergeometric functions [12]. The results for  $\Phi^\infty$  and  $W$  are

$$\Phi^\infty = \frac{1}{r^{3/2}(1-M/r^2)^{i\omega/2M^{1/2}}} F\left(a, b, 2, \frac{M}{r^2}\right), \quad (14)$$

$$\begin{aligned} W = 2i\omega \frac{2^{i\omega/M^{1/2}} \Gamma(2) \Gamma\left(-\frac{i\omega}{M^{1/2}}\right)}{M^{3/4} \Gamma\left(1 + i\frac{m-\omega}{2\sqrt{M}}\right) \Gamma\left(1 - i\frac{m+\omega}{2\sqrt{M}}\right)}. \quad (15) \end{aligned}$$

Here,  $a = 1 + i[(m-\omega)/2\sqrt{M}]$  and  $b = 1 - i[(m+\omega)/2\sqrt{M}]$ . So, to find  $\Phi$  we only have to numerically integrate (13). We have also determined  $\Phi^\infty$  numerically by imposing the boundary conditions above. The agreement between the nu-

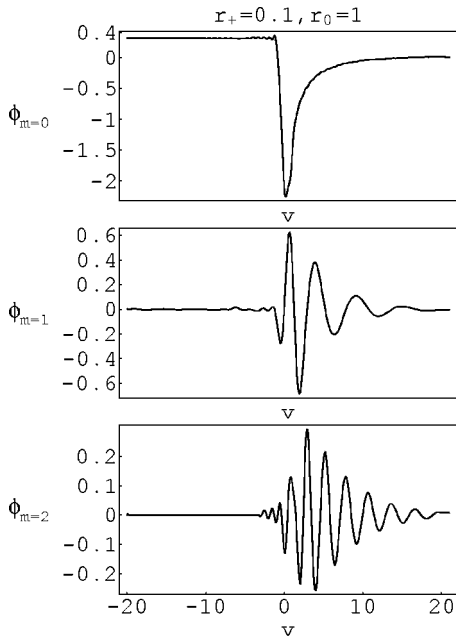


FIG. 2. Waveforms  $\phi(v)$  for a  $r_+ = 0.1$ ,  $r_0 = 1$  BTZ black hole, for the three lowest values of  $m$ .

merically computed  $\Phi^\infty$  and Eq. (14) was excellent. To find  $\phi(t, r)$  one must apply the inverse Laplace transformation to  $\Phi(\omega, r)$ . Integrating the wave equation in this spacetime is simpler than in asymptotically flat space, in the sense that, due to the boundary conditions at infinity the solution is more stable, and less effort is needed to achieve the same accuracy. Using a similar method to that in [14], we estimate the error in our results to be limited from above by 0.5%.

## V. NUMERICAL RESULTS FOR THE WAVEFORMS AND SPECTRA

To better understand the numerical results, we first point out that the QNM frequencies for this geometry, calculated by Cardoso and Lemos [12] (see also [21] for a precise relation between these QNM frequencies and the poles of the correlation functions on the CFT side), are

$$\omega_{\text{QNM}} = \pm m - 2iM^{1/2}(n+1). \quad (16)$$

In Fig. 2 we show the waveforms for the  $r_+ = 0.1$ ,  $r_0 = 1$  black hole, as a function of the advanced null-coordinate  $v = t + r_*$ . This illustrates in a beautiful way that QNMs govern the late time behavior of the waveform. For example, for  $m=0$ ,  $\omega_{\text{QNM}} = -0.2i(n+1)$ , one expects to find a purely decaying perturbation. This is evident from Fig. 2. For  $m=1$ ,  $\omega_{\text{QNM}} = 1 - 0.2i(n+1)$ , so the signal should ring (at late times) with frequency one. This is also clearly seen from Fig. 2. For  $m=2$  we have the same kind of behavior. For large negative  $v$  and fixed  $t$  one has large negative  $r_*$ , so one is near the horizon. Thus  $\phi(v \rightarrow -\infty)$  in Fig. 2 should give the same values as  $\phi_0$  at  $r_+$  in Fig. 1, which is the case. The energy spectra peak at higher  $\omega$  when compared to the fundamental  $\omega_{\text{QNM}}$  as is evident from Fig. 3, which means that higher modes are excited. The total radiated energy as a

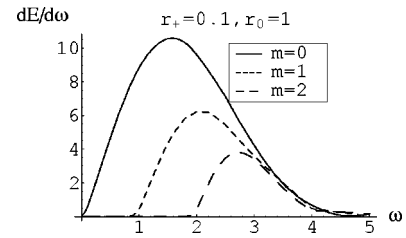


FIG. 3. Typical energy spectra, shown here for  $r_+ = 0.1$  and  $r_0 = 1$ .

function of  $m$  goes to zero slower than  $1/m$  implying that the total radiated energy diverges. However, this divergence can be normalized by taking a minimum size  $L$  for the particle with a cut off given by  $m_{\text{max}} \sim (\pi/2)(r_+/L)$  [13]. We have calculated for  $r_+ = 0.1$  the total energy for the cases  $m = 0, 1, 2$ , yielding  $E_{m=0} \approx 26$ ,  $E_{m=1} \approx 12$ , and  $E_{m=2} \approx 6$ . An estimation of the total energy for a particle with  $m_{\text{max}} \approx 1000$  yields  $E_{\text{total}} \approx 80$  (the energy is measured in units of  $q_0^2 m_0^2$ ). We have also computed the radiated energy for several values of  $r_0$  and verified that it is not a monotonic function of  $r_0$ . For small values of  $r_0$  the energy radiated is a linear function of  $(r_0 - r_+)$ , for intermediate  $r_0$  it has several peaks, and it grows monotonically for large  $r_0$ . The zero frequency limit (ZFL) depends only on the initial data and one can prove that it is given by  $(dE/d\omega)_{\omega \rightarrow 0} = \phi_0^2$ . This is to be contrasted to the ZFL for outgoing gravitational radiation in asymptotically flat spacetimes [22] where it depends only on the initial velocity of the test particle.

## VI. CONCLUSIONS

In conclusion, we have obtained for the first time the plunging waveforms, the power spectrum, and the energy radiated for the collision of a scalar particle with the BTZ black hole. We have shown these quantities for small black holes. For large black holes the results are qualitatively the same, with the one difference that the ringing is much shorter (these results together with results for higher dimensional black holes will be reported elsewhere). For the AdS-CFT correspondence we have added to previous works the precise evolution of an infalling probe and its radiation. This has implications in the strongly coupled CFT: to the black hole corresponds a thermal bath, to the infalling probe corresponds an expanding bubble, and to the scalar field waves correspond particles decaying into bosons of the associate operator of the gauge theory. Both the bubble and the particles in the CFT thermalize with the characteristic time scale calculated through the gravity in the bulk  $1/\text{Im}[\omega_{\text{QNM}}]$ , oscillate according to Fig. 2, which for late times yields the oscillation frequency  $\text{Re}[\omega_{\text{QNM}}]$ , and radiate according to Fig. 3. This is hard to calculate by direct means in the strongly coupled regime of the gauge theory.

## ACKNOWLEDGMENTS

We thank Amaro Rica da Silva for discussions and Observatório Nacional–Rio de Janeiro for hospitality. This work was partially funded by FCT-Portugal through project PESO/PRO/2000/4014 and through PRAXIS XXI program.



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