Scalar and spinor particles in the spacetime of a domain wall in string theory

V. B. Bezerra,[1,*] L. P. Colatto,[2,†] M. E. X. Guimarães,[3,‡] and R. M. Teixeira Filho[4,§]
[1]*Departamento de Física, Universidade Federal da Paraíba, João Pessoa, Brazil*
[2]*Instituto de Física, Universidade de Brasília, Brasília, Brazil*
[3]*Departamento de Matemática, Universidade de Brasília, Brasília, Brazil*
[4]*Instituto de Física, Universidade Federal da Bahia, Salvador, Brazil*

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We consider scalar and spinor particles in the spacetime of a domain wall in the context of low energy effective string theories, such as the generalized scalar-tensor gravity theories. This class of theories allows for an arbitrary coupling of the wall and the (gravitational) scalar field. First, we derive the metric of a wall in the weak-field approximation and we show that it depends on the wall's surface energy density and on two post-Newtonian parameters. Then, we solve the Klein-Gordon and the Dirac equations in this spacetime. We obtain the spectrum of energy eigenvalues and the current density in the scalar and spinor cases, respectively. We show that these quantities, except in the case of the energy spectrum for a massless spinor particle, depend on the parameters that characterize the scalar-tensor domain wall.

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I. INTRODUCTION

Topological defects arise whenever a symmetry is spontaneously broken. They can be of various types according to the topology of the vacuum manifold of the field theory being under consideration. In this work, we will concentrate our attention on domain walls which are defects arising from a breaking of a discrete symmetry by means of a Higgs field [1–3].

Domain walls have been extensively studied in the literature. In particular, it was soon realized that they may lead to a cosmological catastrophe [2], even if they were produced in a late time phase transition [4]. From the gravitational point of view, an interesting feature of the wall's gravitational field is that its weak field approximation does not correspond to any exact static solution of the Einstein's equations, hence implying that they are gravitationally unstable [5]. In Ref. [6], a time-dependent metric was obtained and it was shown that observers experience a repulsion from the wall. Current-carrying walls and their cosmological consequences were also object of investigations. In Ref. [7], the internal structure of a surface current-carrying wall was studied and the internal quantities such as the energy per unit surface and the surface current were calculated numerically.

The above-mentioned features of a domain wall were analyzed in the framework of Einstein's theory of gravity. However, it has been argued that gravity may be described by a scalar-tensorial gravitational field, at least at sufficiently high energy scales. Indeed, a scalar field ϕ, which from now on we will call generically a *dilaton*, appears as a necessary partner of the graviton field $g_{\mu\nu}$ in all superstring models [8,9]. Topological defects of various types and their gravitational effects have already been studied in the framework of various low energy effective string models [10,11]. In what concernes the domain wall solutions, these configurations were the object of Ref. [12], in which the authors studied the properties of the wall's gravitational field in Brans-Dicke and in dilatonic gravities. In this class of solutions, the dilaton can couple to the matter potential forming the wall. It is shown that the dilaton's solution varies with the spatial distance from the wall giving rise to a defect called the "dilatonic domain wall."

The aim of this paper is twofold. First, we investigate the gravitational field of a domain wall in the context of a generalized scalar-tensor gravity. Second, we analyze how particles are affected by this particular gravitational field. The gravitational interaction on quantum mechanical systems has been studied by many authors [13]. For this purpose the Klein-Gordon and Dirac equations in covariant form have been used and solved in curved spacetimes. The search for these solutions is very interesting and may be accounted for by the scheme of unifying quantum mechanics and general relativity. As examples of works concerning this subject we can mention Audretsch and Schäfer [14] who presented a detailed analysis of the energy spectrum of the hydrogen atom in Robertson-Walker universes and Parker [15,16] who studied a one-electron atom in a curved spacetime.

In the present work, we are particularly interested in studying scalar and spinor particles in the spacetime of a scalar-tensorial domain wall. In Sec. II we derived the metric of a wall in the weak field approximation. We show that it depends on the wall's surface energy density σ and on two post-Newtonian parameters, G_0 and $\alpha^2(\phi_0)$. In Sec. III we solve the Klein-Gordon equation, we find the energy eigenvalues and we point out the dependence of the current on the parameters that characterize the scalar-tensor domain wall. In Sec. IV we first consider the Dirac equation for a massive spinor field. Then, we solve explicitly the Weyl equations for a massless spinor field and we determine the expression for the energy spectrum and for the current. Finally, in Sec. V, we present our concluding remarks.

*Email address: valdir@fisica.ufpb.br
†Email address: colatto@fis.unb.br
‡Email address: emilia@mat.unb.br
§Email address: rmuniz@ufba.br

II. THE METRIC OF A DOMAIN WALL IN THE WEAK-FIELD APPROXIMATION

In this section we will derive the metric of a domain wall in a low energy effective string model, in which the axion field is vanishing. This action is analogous to the class of scalar-tensor theories developed in Refs. [17] and, in the case of the scalar sector of the gravitational interaction, is massless. For technical purposes, it is better to work in the so-called Einstein (conformal) frame in which the kinematic terms of tensor and scalar fields do not mix. Then, a domain wall solution arises from the action

$$S=\frac{1}{16\pi G_*}\int d^4x\sqrt{-g}[R-2g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi] + \int d^4x\sqrt{-g}A^2(\phi)\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi-V(\Phi)\right], \quad (1)$$

where $g_{\mu\nu}$ is a pure rank-2 metric tensor, R is the curvature scalar associated with it and G_* is some "bare" gravitational coupling constant. The second term in the right-hand side of Eq. (1) is the matter action representing a model of a real Higgs scalar field Φ and the symmetry breaking potential $V(\Phi)$ which possesses a discrete set of degenerate minima. Action (1) is obtained from the original action appearing in Refs. [17] by a conformal transformation (see, for instance, [18])

$$\tilde{g}_{\mu\nu}=A^2(\phi)g_{\mu\nu}, \quad (2)$$

where $\tilde{g}_{\mu\nu}$ is the physical metric which contains both scalar and tensor degrees of freedom, and by a redefinition of the quantities

$$G_*A^2(\phi)=\frac{1}{\tilde{\Phi}},$$

where $\tilde{\Phi}$ is the original scalar field, and

$$\alpha(\phi)\equiv\frac{\partial\ln A}{\partial\phi}=\frac{1}{[2\omega(\tilde{\Phi})+3]^{1/2}},$$

which can be interpreted as the (field-dependent) coupling strength between matter and the scalar field. We choose to leave $A^2(\phi)$ as an arbitrary function of the dilaton field.

In the Einstein frame, the field equations are written as follows:

$$R_{\mu\nu}=2\partial_\mu\phi\partial_\nu\phi+8\pi G_*\left(T_{\mu\nu}-\frac{1}{2}g_{\mu\nu}T\right)$$
$$\Box_g\phi=-4\pi G_*\alpha(\phi)T \quad (3)$$

where the energy-momentum tensor is obtained as

$$T_{\mu\nu}\equiv\frac{2}{\sqrt{-g}}\frac{\delta S_m}{\delta g_{\mu\nu}}.$$

In what follows, we will consider the solution of a domain wall in the yz plane in the weak-field approximation. Therefore, we will expand Eqs. (3) to first order in $G_*A^2(\phi_0)$ in such a way that

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$$
$$\phi=\phi_0+\phi_{(1)}$$
$$A(\phi)=A(\phi_0)[1+\alpha(\phi_0)\phi_{(1)}]$$
$$T^\mu_\nu=T^\mu_{(0)\nu}+T^\mu_{(1)\nu}. \quad (4)$$

In this approximation, $T^\mu_{(0)\nu}=A^2(\phi_0)\tilde{T}^\mu_{(0)\nu}$ is the energy-momentum tensor of a static domain wall with negligible width and lying in a yz plane. Therefore,

$$T^\mu_{(0)\nu}=A^2(\phi_0)\sigma\delta(x)\mathrm{diag}(1,0,1,1) \quad (5)$$

in the Cartesian coordinate system (t,x,y,z). The parameter σ is the wall's surface energy density. In our convention, the metric signature is -2.

Equations (3) in the linearized regime reduce to

$$\nabla^2h_{\mu\nu}=16\pi G_*\left(T_{(0)\mu\nu}-\frac{1}{2}\eta_{\mu\nu}T_{(0)}\right)$$
$$\nabla^2\phi_{(1)}=4\pi G_*\alpha(\phi_0)T_{(0)}. \quad (6)$$

Let us begin by solving the equation for the dilaton field $\phi_{(1)}$ in Eq. (6):

$$\nabla^2\phi_{(1)}=12\pi\sigma G_0\alpha(\phi_0)\delta(x)$$
$$\phi_{(1)}=6\pi\sigma G_0\alpha(\phi_0)|x|, \quad (7)$$

where $G_0\equiv G_*A^2(\phi_0)$.

Now, the linearized Einstein's equation in Eq. (6) with a source given by Eq. (5) are just the same as in Vilenkin's paper [5], except that in our case the metric is multiplied by the linearized factor $A^2(\phi)$. Therefore, we have (to first order in G_0):

$$ds^2=A^2(\phi_0)[1+4\pi\sigma G_0|x|(3\alpha^2(\phi_0)-1)]\times[dt^2-dx^2-dy^2-dz^2]. \quad (8)$$

The factor $A^2(\phi_0)$ appearing in the above expression can be absorbed by a redefinition of the coordinates (t,x,y,z). We finally, then, obtain

$$ds^2=(1+4D|x|)[dt^2-dx^2-dy^2-dz^2], \quad (9)$$

where $D\equiv\pi\sigma G_0(3\alpha^2-1)$.

This is the line element corresponding to a domain wall in the framework of scalar-tensor gravity in the weak-field approximation. The geometry given by Eq. (9) is only valid for $D|x|\ll 1$.

III. KLEIN-GORDON EQUATION IN SCALAR-TENSOR DOMAIN WALL

Let us consider a scalar quantum particle embedded in a classical background gravitational field. Its behavior is described by the covariant Klein-Gordon equation

$$\left[\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)+m^2\right]\psi=0, \tag{10}$$

where m is the mass of the particle, g is the determinant of the metric tensor $g_{\mu\nu}$ and we are considering a minimal coupling.

In the space-time of a scalar-tensor domain wall given by metric (9), Eq. (10) takes the form

$$\left[\frac{1}{1+4D|x|}\left(\partial_t^2-\partial_x^2-\partial_y^2-\partial_z^2-\frac{4D}{1+4D|x|}\frac{d|x|}{dx}\partial_x\right)+m^2\right]\psi =0. \tag{11}$$

Multiplying this equation by $1+4D|x|$ and neglecting terms of order D^2 and up (because we are working in the weak field aproximation), we get

$$\left[\partial_t^2-\partial_x^2-\partial_y^2-\partial_z^2-4D\frac{d|x|}{dx}\partial_x+m^2(1+4D|x|)\right]\psi=0. \tag{12}$$

Since Eq. (12) is invariant under the transformation $x\rightarrow -x$, we shall restrict the allowed values of x to the interval $x>0$. Then, we have

$$[\partial_t^2-\partial_x^2-\partial_y^2-\partial_z^2-4D\partial_x+m^2(1+4Dx)]\psi=0. \tag{13}$$

Let us assume that

$$\psi(t,x,y,z)=e^{-i(Et-k_y y-k_z z)}X(x), \tag{14}$$

where E, k_y and k_z are constants. If we substitute relation (14) into Eq. (13), we obtain

$$\frac{d^2X(x)}{dx^2}+4D\frac{dX(x)}{dx} +[E^2-k_y^2-k_z^2-m^2(1+4Dx)]X(x)=0, \tag{15}$$

whose general solution is

$$\begin{aligned}X(x)=\{&C_1\text{Airy Ai}[-(-1)^{1/3}2^{-4/3}(m^2D)^{-2/3}\\ &\times(m^2-E^2+k_y^2+k_z^2+4m^2Dx+4D^2)]\\ &+C_2\text{Airy Bi}[-(-1)^{1/3}2^{-4/3}(m^2D)^{-2/3}\\ &\times(m^2-E^2+k_y^2+k_z^2+4m^2Dx+4D^2)]\}e^{-2Dx},\end{aligned} \tag{16}$$

with functions Airy Ai(x) and Airy Bi(x) being the Airy functions and C_1 and C_2 are integration constants. Note that the arguments of the Airy functions have only terms of order less than D^2. Neglecting terms of order $\geqslant D^2$, we get

$$\begin{aligned}X(x)\simeq\{&C_1\text{Airy Ai}[-(-1)^{1/3}2^{-4/3}(m^2D)^{-2/3}\\ &\times(m^2-E^2+k_y^2+k_z^2+4m^2Dx+4D^2)]\\ &+C_2\text{Airy Bi}[-(-1)^{1/3}2^{-4/3}(m^2D)^{-2/3}\\ &\times(m^2-E^2+k_y^2+k_z^2+4m^2Dx+4D^2)]\}(1-2Dx),\end{aligned} \tag{17}$$

In order to determine the bound states energies we must require periodicity conditions in directions y and z, with periods L_y and L_z, respectively, supplemented by the boundary conditions that the solution vanishes at $x=a$ and $x=b$, with $a<b$ and such that $Da\ll 1$ and $Db\ll 1$. These boundary conditions can be expressed as

$$\begin{aligned}&\psi(t,x,y,z)=\psi(t,x,y+L_y,z)\\ &\psi(t,x,y,z)=\psi(t,x,y,z+L_z)\\ &\psi(t,a,y,z)=\psi(t,b,y,z)=0.\end{aligned} \tag{18}$$

These boundary conditions lead us to the following results:

$$\begin{aligned}X(x)=C_1\Bigg\{&\text{Airy Ai}[-(-1)^{1/3}2^{-4/3}(m^2D)^{-2/3}(m^2-E^2+k_y^2+k_z^2+4m^2Dx+4D^2)]\\ &-\frac{\text{Airy Ai}[-(-1)^{1/3}2^{-4/3}(m^2D)^{-2/3}(m^2-E^2+k_y^2+k_z^2+4m^2Da+4D^2)]}{\text{Airy Bi}[-(-1)^{1/3}2^{-4/3}(m^2D)^{-2/3}(m^2-E^2+k_y^2+k_z^2+4m^2Da+4D^2)]}\text{Airy Bi}[-(-1)^{1/3}2^{-4/3}(m^2D)^{-2/3}\\ &\times(m^2-E^2+k_y^2+k_z^2+4m^2Dx+4D^2)]\Bigg\}(1-2Dx),\end{aligned} \tag{19}$$

where

$$k_y=\frac{2\pi n_y}{L_y},\quad n_y=0,\pm 1,\pm 2,\ldots \tag{20}$$

$$k_z=\frac{2\pi n_z}{L_z},\quad n_z=0,\pm 1,\pm 2,\ldots, \tag{21}$$

and

$$X(b)=0. \tag{22}$$

The boundary condition given by Eq. (22) determines the energy levels of the particle in the stationary state in the region under consideration. It should be stressed that in order to solve this problem we must impose that $|m^2-E^2+k_y^2+k_z^2+4m^2Dx|\gg|(m^2D)^{2/3}|$, in which case the absolute value of the argument of Airy's functions are much greater than unity, which allows us to use the following assymptotic expansions [19]:

$$\text{Airy Ai}(z)\sim\frac{1}{2\sqrt{\pi}}z^{-1/4}e^{-\xi}\sum_{k=0}^{\infty}(-1)^k c_k\xi^{-k}$$

$$\text{Airy Bi}(z)\sim\frac{1}{\sqrt{\pi}}z^{-1/4}e^{\xi}\sum_{k=0}^{\infty}c_k\xi^{-k} \tag{23}$$

where

$$\xi=\frac{2}{3}z^{3/2}$$

$$c_0=1,\quad c_k=\frac{(2k+1)(2k+3)\cdots(6k-1)}{216^k k!}. \tag{24}$$

In this approximation, Eq. (22) can be written as

$$e^{-(2/3)W^{3/2}(b)}\sum_{k=0}^{\infty}(-1)^k c_k\left[\frac{2}{3}W^{3/2}(b)\right]^{-k}-e^{-(4/3)W^{3/2}(a)}\times\frac{\sum_{k=0}^{\infty}(-1)^k c_k\left[\frac{2}{3}W^{3/2}(a)\right]^{-k}}{\sum_{k=0}^{\infty}c_k\left[\frac{2}{3}W^{3/2}(a)\right]^{-k}}e^{2/3W^{3/2}(b)}\times\sum_{k=0}^{\infty}c_k\left[\frac{2}{3}W^{3/2}(b)\right]^{-k}=0, \tag{25}$$

where $W(x)=-(-1)^{1/3}2^{-4/3}(m^2D)^{-2/3}(m^2-E^2+k_y^2+k_z^2+4m^2Dx+4D^2)$.

Considering only the first two terms of the summation and neglecting terms of order $\geq D^2$, we find

$$e^{i2\sqrt{E^2-k_y^2-k_z^2-m^2}(b-a)}\left[1-i\frac{4m^2D(b^2-a^2)}{\sqrt{E^2-k_y^2-k_z^2-m^2}}\right]=1. \tag{26}$$

Now, let us take $E^2-k_y^2-k_z^2-m^2>0$, then we can rewrite Eq. (26) as a system of equations involving its real and imaginary parts as follows:

$$1+\frac{4m^2D(b^2-a^2)}{\sqrt{E^2-k_y^2-k_z^2-m^2}}\tan[2\sqrt{E^2-k_y^2-k_z^2-m^2}](b-a)=\sec[2\sqrt{E^2-k_y^2-k_z^2-m^2}](b-a), \tag{27}$$

$$\tan[2\sqrt{E^2-k_y^2-k_z^2-m^2}](b-a)=\frac{4m^2D(b^2-a^2)}{\sqrt{E^2-k_y^2-k_z^2-m^2}}. \tag{28}$$

As the function $\tan(x)$ is of order D, we can make the assumption that $\tan(x)\simeq x$. Then, Eq. (28) results in

$$2\sqrt{E^2-k_y^2-k_z^2-m^2}(b-a)\simeq\frac{4m^2D(b^2-a^2)}{\sqrt{E^2-k_y^2-k_z^2-m^2}}+n\pi,\quad n=0,\pm 1,\pm 2,\ldots. \tag{29}$$

Equation (27) is assured if n is even. Then, we have

$$2\sqrt{E^2-k_y^2-k_z^2-m^2}(b-a)\simeq\frac{4m^2D(b^2-a^2)}{\sqrt{E^2-k_y^2-k_z^2-m^2}}+2n\pi,\quad n=0,\pm 1,\pm 2,\ldots. \tag{30}$$

From the previous equation we get, finally, that

$$E^2=m^2+k_y^2+k_z^2+\frac{n^2\pi^2}{(b-a)^2}+\left(2+\frac{n\pi}{b-a}\right)m^2D(b+a),\quad n=1,2,\ldots. \tag{31}$$

It is worth noting that the presence of the wall increases the energy eigenvalues with parameters[1] σ, $\alpha(\phi_0)$ and G_0 and that for $D=0$ (absence of the wall) we recover the result corresponding to the Minkowski spacetime as it should be.

In what concerns the current associated with the scalar field given by

$$J^\mu=\frac{i\sqrt{-g}}{2m}(\psi^*\partial^\mu\psi-\psi\partial^\mu\psi^*), \tag{32}$$

[1]Just as a reminder for the reader, $\sigma,\alpha(\phi_0)$ and G_0 are the wall's surface energy density, the coupling strength between the wall and the dilaton, and the effective gravitational constant, respectively.

it is clear that the current depends on the parameters that characterize the scalar-tensor domain wall through $\sqrt{-g}$ and the solution ψ of the Klein-Gordon equation.

IV. DIRAC EQUATION IN A SCALAR-TENSOR DOMAIN WALL

Now, let us consider a spinor particle embedded in a classical gravitational field. The covariant Dirac equation governing the particle in a curved spacetime for a spinor Ψ may be written as

$$[i\gamma^{\mu}(x)\partial_{\mu}+i\gamma^{\mu}(x)\Gamma_{\mu}-m]\Psi(x)=0, \tag{33}$$

where $\gamma^{\mu}(x)$ are the generalized Dirac matrices and are given in terms of the standard flat space Dirac matrices $(\gamma^{(a)})$ as

$$\gamma^{\mu}(x)=e^{\mu}_{(a)}(x)\gamma^{(a)}, \tag{34}$$

where $e^{\mu}_{(a)}(x)$ are tetrad components defined by

$$e^{\mu}_{(a)}(x)e^{\nu}_{(b)}(x)\eta^{(a)(b)}=g^{\mu\nu}. \tag{35}$$

The product $\gamma^{\mu}(x)\Gamma_{\mu}$ that appears in Dirac equation can be written as

$$\gamma^{\mu}(x)\Gamma_{\mu}=\gamma^{(a)}[A_{(a)}(x)+i\gamma^{(5)}B_{(a)}(x)], \tag{36}$$

with $\gamma^{(5)}=i\gamma^{(0)}\gamma^{(1)}\gamma^{(2)}\gamma^{(3)}$ and $A_{(a)}$ and $B_{(a)}$ are given by

$$A_{(a)}=\frac{1}{2}(\partial_{\mu}e^{\mu}_{(a)}+e^{\rho}_{(a)}\Gamma^{\mu}_{\rho\mu}) \tag{37}$$

and

$$B_{(a)}=\frac{1}{2}\epsilon_{(a)(b)(c)(d)}e^{(b)\mu}e^{(c)\nu}\partial_{\mu}e^{(d)}_{\nu}, \tag{38}$$

where $\epsilon_{(a)(b)(c)(d)}$ is the completely antisymmetric fourth-order unit tensor.

In the spacetime of a scalar-tensor domain wall given by metric (9), let us choose the following set of tetrads:

$$e^{(a)}_{\mu}=[1+4D|x|]^{1/2}\delta^{a}_{\mu}, \tag{39}$$

which implies that

$$\gamma^{\mu}=[1-2D|x|]\gamma^{(\mu)}, \tag{40}$$

in which we have neglected terms of order $\geqslant D^2$.

Computing the expressions for $A_{(a)}$ and $B_{(a)}$ and putting these results into Eq. (36) and neglecting terms of order $\geqslant D^2$, we get

$$\gamma^{\mu}(x)\Gamma_{\mu}=3D\gamma^{(1)}. \tag{41}$$

Now, using Eqs. (40) and (41), the Dirac equation (33) in the spacetime of a scalar-tensor domain wall reads

$$\{i\gamma^{(\mu)}\partial_{\mu}+i3D\gamma^{(1)}-[1+2Dx]m\}\Psi(x)=0, \tag{42}$$

in which we neglected terms of order $\geqslant D^2$ and considered only the interval $x>0$. In order to determine the solutions in the spinoral case, let us choose the following representation of Dirac matrices:

$$\gamma^{(0)}=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},$$

$$\gamma^{(i)}=\begin{pmatrix}0 & \sigma_i\\ -\sigma_i & 0\end{pmatrix}, \quad i=1,2,3 \tag{43}$$

where σ_i $(i=1,2,3)$ are the usual Pauli matrices.

Since we are interested in the qualitative behavior of the particle with respect to the parameters that define the wall, we will simplify our analysis considering the solution of the Dirac equation corresponding to the massless spinor particle in which case Eq. (42) reduces to

$$\{i\gamma^{(\mu)}\partial_{\mu}+i3D\gamma^{(1)}\}\Psi(x)=0, \tag{44}$$

and is supplemented by the helicity condition

$$(1+\gamma^{5})\Psi(x)=0, \tag{45}$$

where

$$\gamma^{5}=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}.$$

Equations (44) and (45) are the Weyl equations for a massless spin-$\frac{1}{2}$ particle. The condition given by Eq. (45) implies that the four-spinor $\Psi(x)$ is such that

$$\Psi(x)=\begin{pmatrix}\Psi_1(x)\\ \Psi_2(x)\end{pmatrix},$$

with $\Psi_1(x)=\Psi_2(x)$.

A suitable set of solutions of Weyl's equations is of the form

$$\Psi(t,x,y,z)=\begin{pmatrix}\mathbf{u}(x)\\ \mathbf{u}(x)\end{pmatrix}e^{-i(Et-k_y y-k_z z)}, \tag{46}$$

where

$$\mathbf{u}(x)=\begin{pmatrix}u_1(x)\\ u_2(x)\end{pmatrix}.$$

If we substitute relation (46) into Eq. (44), we obtain

$$\begin{aligned} i\frac{du_1(x)}{dx}-(k_y-3D)u_1(x)+(E+k_z)u_2(x)=0\\ i\frac{du_2(x)}{dx}+(k_y+3D)u_2(x)+(E-k_z)u_1(x)=0.\end{aligned} \tag{47}$$

Neglecting terms of order D^2 and up, the set of Eqs. (47) can be written as

$$\frac{d^2u_1(x)}{dx^2}+6D\frac{du_1(x)}{dx}+(E-k_y^2-k_z^2)u_1(x)=0,$$
$$\frac{d^2u_2(x)}{dx^2}+6D\frac{du_2(x)}{dx}+(E-k_y^2-k_z^2)u_2(x)=0, \tag{48}$$

whose general solution reads

$$\mathbf{u}(x)=\mathbf{C_1}e^{(-3D+i\sqrt{E^2-k_y^2-k_z^2})x}+\mathbf{C_2}e^{(-3D-i\sqrt{E^2-k_y^2-k_z^2})x}, \tag{49}$$

where $\mathbf{C_1}$ and $\mathbf{C_2}$ are constant bispinors. Neglecting terms of order D^2 and up, we get

$$\mathbf{u}(x)=(1-3Dx)(\mathbf{C_1}e^{i\sqrt{E^2-k_y^2-k_z^2}x}+\mathbf{C_2}e^{-i\sqrt{E^2-k_y^2-k_z^2}x}). \tag{50}$$

Again, we notice that the solutions depend on σ, α and G_0, as expected.

Now, let us compute the current j^μ, which is defined by

$$j^\mu=\bar{\Psi}\gamma^\mu\Psi. \tag{51}$$

Using Eq. (40), the expression for the current in the approximation considered turns into

$$j^\mu=(1-4Dx)\Psi^\dagger\gamma^{(0)}\gamma^{(\mu)}\Psi. \tag{52}$$

Substituting Eqs. (46) and (50) into Eq. (52) and considering, for simplicity, the case in which $\mathbf{C_2}=\mathbf{0}$, we get

$$j^\mu=(1-10Dx)(\mathbf{C_1}^\dagger\,\mathbf{C_1}^\dagger)\gamma^{(0)}\gamma^{(\mu)}\begin{pmatrix}\mathbf{C_1}\\ \mathbf{C_1}\end{pmatrix}. \tag{53}$$

In order to obtain the energy spectrum, let us consider the same boundary conditions given by Eq. (18) for the scalar field. Thus, we have

$$\Psi(t,x,y,z)=\Psi(t,x,y+L_y,z)$$
$$\Psi(t,x,y,z)=\Psi(t,x,y,z+L_z)$$
$$\Psi(t,a,y,z)=\Psi(t,b,y,z)=0. \tag{54}$$

Analogously to the case of a scalar field, from these boundary conditions we get

$$\mathbf{u}(x)=\mathbf{C_1}e^{i\sqrt{E^2-k_y^2-k_z^2}x}(1-e^{-2i\sqrt{E^2-k_y^2-k_z^2}(x-a)}) \tag{55}$$

where

$$k_y=\frac{2\pi n_y}{L_y},\quad n_y=0,\pm1,\pm2,\ldots \tag{56}$$

$$k_z=\frac{2\pi n_z}{L_z},\quad n_z=0,\pm1,\pm2,\ldots, \tag{57}$$

and

$$\mathbf{u}(b)=0. \tag{58}$$

The energy levels arrive from Eq. (58) and are given by

$$E^2=k_y^2+k_z^2+\frac{n^2\pi^2}{(b-a)^2}. \tag{59}$$

Note that the energy spectrum is the same as in the flat Minkowski spacetime case. This result comes from the fact that the spinor field is massless. The same coincidence occurs in the case of a massless scalar field.

From previous results we conclude that the current differs from that in the Minkowski spacetime by the terms containing the parameters σ, α and G_0 used to describe the scalar-tensor domain wall and tends to the corresponding result in Minkowski spacetime in the absence of the wall as it should be.

V. CONCLUDING REMARKS

Recently there has been growing interest in domain walls as brane world scenarios and also in scalar-tensor theories of gravity due to its possible role in the understanding of the physics of the early Universe when topological defects like domain walls were formed. At that time the dilaton fields as well as the topological defect, such as a scalar-tensor domain wall, were, certainly, very relevant. These points constitute the main motivation for this work.

A scalar or spinor particle placed in the spacetime of a scalar-tensor domain wall is perturbed by this background due to the geometrical and topological features of the spacetime under consideration. In other words, the dynamic of atomic systems is determined by the curvature at the position of the system and also by the topology of the background spacetime.

Summarizing our conclusions we can say that the metric of a scalar-tensor domain wall depends on the wall's surface energy density σ and on two post-Newtonian parameters $\alpha(\phi_0)$ and G_0. The solutions for the scalar and spinor cases differ from the flat Minkowski spacetime case by the presence of these parameters. The presence of the wall shift the energy levels and alters the current in the scalar case as compared with the flat spacetime. In the massless spinor particle case, there is no shift in the energy spectrum, but the current is altered by the presence of the scalar-tensor domain wall.

Finally, it is worth commenting that the study of a quantum system in a gravitational field such as, for example, the one considered in this paper, may shed some light on the problems of combining quantum mechanics and gravity. On the other hand, the investigation of topological defects in the framework of general scalar-tensor theories seems to be important in order to understand the role played by these strutures in this general context.

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