Quantum dilaton gravity as a linear dilaton conformal field theory

M. Cadoni* and P. Carta†
Dipartimento di Fisica, Università di Cagliari and INFN Sezione di Cagliari, Cittadella Universitaria, 09042 Monserrato, Italy

S. Mignemi‡
Dipartimento di Matematica, Università di Cagliari, Viale Merello 92, 09123 Cagliari, Italy
and INFN Sezione di Cagliari, Cittadella Universitaria, 09042 Monserrato, Italy

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A model of matter-coupled gravity in two dimensions is quantized. The crucial requirement for performing the quantization is the vanishing of the conformal anomaly, which is achieved by tuning a parameter in the interaction potential. The spectrum of the theory is determined by mapping the model first onto a field theory with a Liouville interaction, then onto a linear dilaton conformal field theory. In the absence of matter fields, a pure gauge theory with a massless ground state is found; otherwise it is possible to minimally couple up to 11 matter scalar fields: in this case the ground state is tachyonic and the matter sector decouples, like the transverse oscillators in the critical bosonic string.

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I. INTRODUCTION

In recent years, many controversial features of the semiclassical and quantum theory of gravity have been studied using models in two spacetime dimensions. Two-dimensional (2D) theories of gravity are not only useful toy models but in some cases also have a direct physical meaning, since they may be obtained from dimensional reduction of higher-dimensional gravity theories.

Basically, one can approach 2D gravity from two different points of view: as the theory of 2D random surfaces (a bosonic string theory in noncritical dimensions) [1,2] or as an Einstein-like, Brans-Dicke theory of gravity (2D dilaton gravity) [3,4].

One crucial feature of 2D dilaton gravity is that it allows for interesting gravitational structures already at the classical level. The theory admits black hole solutions, which have been investigated both at the classical and semiclassical level [5]. Big progress in this direction has been achieved using the anti–de Sitter (AdS)/conformal field theory (CFT) correspondence in two dimensions [6–9]. The use of this correspondence made it possible to exactly reproduce the thermodynamical entropy of 2D black holes in terms of the degeneracy of states of a CFT.

It is obvious that one would like to go beyond the semiclassical approximation, in order to see how the results of the semiclassical approach are modified in a full 2D quantum theory of gravity. This program has been pursued using a variety of methods [10,11,28] but the results have been in some sense astonishing: the quantum theory ignores almost completely the richness of the structure of the classical and semiclassical theory. The spectrum of the model (which in some cases is that of a CFT) is characterized only by two quantum numbers, which can be identified with the black hole mass and the constant mode of the dilaton. This feature appears even more puzzling when compared with the results of the AdS_2/CFT_1 correspondence [6], which predicts a huge degeneracy of states accounting for the thermodynamical entropy of the black hole.

The purpose of this paper is to go a step further in this direction. We will quantize exactly a 2D dilaton gravity model. We will focus our analysis on matter-coupled dilaton gravity where the gravitational sector is given by

$$S=\frac{1}{k}\int d^2x\sqrt{-g}[\phi R+\lambda V(\phi)], \tag{1}$$

and we choose an exponential dilaton potential $V(\phi)=\text{const}\times e^{2\gamma\phi}$. We will approach the problem of the quantization of the model using nonperturbative methods. A natural way to introduce them is to relate the gravitational model to one of the several exactly solved quantum field theories in $1+1$ dimensions. The Liouville field theory (LFT) turns out to be, not surprisingly, the right choice. Fixing the diffeomorphism invariance of the action (1) and using suitable field redefinitions, we will show that the gravity model is classically equivalent to a LFT plus a decoupled free scalar with wrong sign kinetic energy.

We will then follow the common quantization procedure, imposing the positivity of quantum states energy (so, as usual in string theory, we will have ghosts in the spectrum). A consistent quantization can be performed provided the quantum anomaly vanishes. For this purpose, a simple mechanism may be used. It is well known that also at the classical level LFT has a central charge c_γ, depending on the parameter γ appearing in the potential $\exp(2\gamma\phi)$. In quantum LFT, c_γ is shifted by γ-dependent quantum corrections [12]. It will turn out that this shift is crucial: fixing γ properly, we can achieve a vanishing total anomaly and remove the obstruction to quantization. If it were not for this shift, any

*Electronic address: mariano.cadoni@ca.infn.it
†Electronic address: paolo.carta@ca.infn.it
‡Electronic address: smignemi@vaxca1.unica.it

dependence on the control parameter γ would be lost and a nonvanishing anomaly would be unavoidable.

In a sense, our approach is similar to that used by David, Distler, and Kawai in a different context [2]. In Ref. [2], the Weyl symmetry is used to determine the coefficients of a free action (the exponential interaction is canceled by fixing the cosmological constant of the string). Conversely, in this paper we will fix the parameter γ appearing in the dilaton potential in order to ensure the conformal symmetry. It turns out that also in our case the whole theory, including matter fields, may be mapped onto a free field theory: a linear dilaton CFT (see, for instance, Ref. [13]) with energy-momentum tensor given by

$$T_{\pm\pm}=-\frac{1}{\alpha'}:\partial_\pm X^\mu\partial_\pm X_\mu:+V_\mu\,\partial_\pm^2X^\mu. \tag{2}$$

In our case, V_μ is determined by the dilaton gravity action.

The theory described by Eq. (2) is not suitable as a string theory since the Lorentz invariance in the target space is explicitly broken by V_μ. This is not important in our context, since internal symmetries of the fields are not relevant for us. The quantization of the theory then follows in a straightforward way.

The structure of the paper is the following. In Sec. II, we introduce the gravitational model we are going to investigate, we discuss the classical equivalence to constrained Louiville field theory plus a scalar free field, and we calculate the central charge as a function of the parameter γ. In Sec. III, we briefly recall some basic features of LFT and we map our model onto a linear dilaton CFT. In Sec. IV, the spectrum of the quantum theory is determined. Finally, in Sec. V, we state our conclusions.

II. THE GRAVITATIONAL MODEL

Let us consider a general dilaton gravity model with $N-2$ minimally coupled matter fields φ_i, $i=3,\dots,N$ ($N\geqslant 2, N<14$):

$$S=\frac{1}{k}\int d^2x\sqrt{-g}[\phi R+K(\phi)(\nabla_\mu\phi)(\nabla^\mu\phi)+\lambda W(\phi)+L(\nabla_\mu\varphi_i)(\nabla^\mu\varphi_i)], \tag{3}$$

where $W(\phi)$ (the dilaton potential) and $K(\phi)$ are functions of the dilaton and λ and L are coupling constants. At the classical level, whereas the matter part of the action is both diffeomorphism and Weyl invariant, the gravitational sector looks invariant only under diffeomorphisms. However, it has been shown that also the gravitational sector is classically Weyl invariant [14]. Using this symmetry, the action (3) can be transformed, by means of a dilaton-dependent Weyl rescaling of the metric, into the action (1), with $W(\phi)=\exp(\int d\phi K(\phi))V(\phi)$. Owing to the conformal anomaly, the extension of the Weyl symmetry at the quantum level is in general problematic. But not in our case, since we tune to zero the conformal anomaly. [For the dilaton gravity model with an exponential potential we are considering in this paper, the quantum equivalence between the model (3) (with $K=$const) and Eq. (1) can be also established showing that both can be mapped into the action (5), see below.]

We fix the diffeomorphism invariance of the action (1) by choosing the conformal gauge for the metric,

$$ds^2=e^{2\gamma\rho(x)}dx^+\,dx^-, \tag{4}$$

where γ is a real free parameter. A glance at the classical theory may be useful. For the moment we consider only the gravitational sector; matter fields will be reintroduced later. The equations of motion in the conformal gauge read

$$8\gamma e^{-2\gamma\rho(x)}\partial_+\partial_-\rho(x)=\lambda\frac{dV(\phi)}{d\phi},$$

$$\partial_+\partial_-\phi(x)-\frac{\lambda}{4}e^{2\gamma\rho(x)}V(\phi)=0,$$

$$\partial_+^2\phi-2\gamma\partial_+\rho(x)\partial_+\phi(x)=0,$$

$$\partial_-^2\phi-2\gamma\partial_-\rho(x)\partial_-\phi(x)=0,$$

where the last two equations are the constraints for the theory. Two choices for $V(\phi)$ lead to integrable models suitable for our purposes: $V(\phi)=\phi$ and $V(\phi)=\alpha e^{2\gamma\phi}+\beta e^{-2\gamma\phi}$ [15]. The first case was considered in Refs. [10,16]. Classical solutions of the latter have been discussed in [17]. Here we will focus on the second case. Later on, we will show that a quantum treatment of the model is not possible for generic values of α and β. We choose $\alpha=1$ and $\beta=0$. Setting $k=\pi\gamma$, $\lambda=\mu/(8\gamma)$, and performing the field redefinitions,

$$\psi=2(\rho+\phi),\qquad \chi=2(\rho-\phi),$$

action (1) in the conformal gauge (4) becomes

$$S=\frac{1}{4\pi}\int dx^+\,dx^-\left(\partial_+\psi\,\partial_-\psi-\partial_+\chi\,\partial_-\chi+\frac{\mu}{4\gamma^2}e^{\gamma\psi}\right), \tag{5}$$

whereas the constraint equations are

$$T_{\pm\pm}=-\frac{1}{2}(\partial_\pm\psi)^2+\frac{1}{\gamma}\partial_\pm^2\psi-\left(-\frac{1}{2}(\partial_\pm\chi)^2+\frac{1}{\gamma}\partial_\pm^2\chi\right)=0. \tag{6}$$

The action (5) is invariant, up to boundary terms, under transformations of the conformal group in two dimensions, given in light-cone coordinates by $x^+\to w^+(x^+), x^-\to w^-(x^-)$. Under these transformations, the field χ transforms as a scalar, whereas ψ transforms as a Liouville field $\psi\to\psi+(1/\gamma)\ln[(dw^+/dx^-)(dw^-/dx^-)]$.

The physical content of the 2D field theory we have obtained can be immediately read from Eqs. (5) and (6). It is given by Liouville theory for the field ψ and one decoupled free scalar field χ with wrong sign kinetic energy. Both fields have an improvement term in the stress-energy tensor. Alternatively, one can see the theory as a conformally improved bosonic string in 2D target space, with one of the fields self-interacting. Of course this is just a way to describe our theory and clearly no Lorentz structure does exist in the "target space." The $N-2$ matter fields φ can be straightforwardly introduced in the theory by adding to the previous Lagrangian the term $\partial_+\varphi_i\,\partial_-\varphi_i$, $i=3,\ldots,N$, and to the stress-energy tensor the term $-(1/2)(\partial_\pm\varphi_i)^2$ [we fix for convenience in Eq. (3) $L=\gamma/8$].

The conformal anomaly $c(\gamma)$

The quantization of the model (5) will be performed using the conventional quantization scheme, which preserves the Weyl symmetry. In this way we will have a contribution to the anomaly both from the gravitational and matter field sectors. A consistent quantization will require a cancellation of the two contributions.

Two different approaches have been proposed in the literature to realize this cancellation: the 2D quantum gravity in the manner of David, Distler, and Kawai [2] and the string-inspired dilaton gravity of Cangemi, Jackiw, and Zwiebach [10].

In Ref. [2], the noncritical string theory is considered after Polyakov's famous paper on the geometry of the bosonic string [1]. The string in d dimensions is viewed as a model of d free bosons coupled to 2D quantum gravity. In the functional integral, it is assumed that the measures can all be made independent of the Liouville mode by a field transformation. The Jacobian so introduced is supposed to be the exponential of a Liouville action. From these assumptions follows the Weyl symmetry of the theory, and this is sufficient to determine the unknown parameters of the model. This approach is consistent for $d\leqslant 1$.

In Ref. [10], the authors considered a 2D dilaton gravity model (1) with $V(\phi)=\phi$. In the absence of matter fields, the crucial problem of the cancellation of the quantum anomaly has been solved by resorting to an unconventional quantization procedure. The action (1) is viewed as a theory of two free scalars with two constraints, where the kinetic energy terms of the two scalars have opposite sign. Usually both positive and negative kinetic energy scalars are quantized imposing positivity of the energy of the quantum states. This leads to negative norm (ghost) states for the scalar with wrong sign kinetic energy and to the same central charge $(+1)$ for both scalars. In Ref. [10], an opposite choice is made; positivity of the norm is required, leading to negative energy states. The two scalars contribute with opposite sign to the central charge and the total conformal anomaly cancels. Of course this method does not work when matter fields are present.

In this paper, we are considering, as in Ref. [10], a dilaton gravity model, but we will use the conventional quantization procedure of Ref. [2]. Let us now evaluate the conformal anomaly for the model (5) as a function of γ. It is standard lore that a single scalar field φ with action $S=(1/8\pi)\int d^2x\sqrt{|g|}[\partial_\mu\varphi\,\partial^\mu\varphi+(2/\gamma)\varphi R]$ has a central charge $c_\pm=1\pm 12/\gamma^2$, depending on the sign of the kinetic energy term (the plus sign holds in the positive case). c_+ is the classical central charge for LFT $S=(1/4\pi)\int dx^+\,dx^-[\partial_+\psi\,\partial_-\psi+(\mu/4\gamma^2)\exp(\gamma\psi)]$. At the quantum level this charge is shifted [12],

$$1+\frac{12}{\gamma^2}\rightarrow 1+\frac{12}{\gamma^2}\left(1+\frac{\gamma^2}{2}\right)^2. \tag{7}$$

It should be noticed that choosing a dilaton potential $V(\phi)=\alpha e^{2\gamma\phi}+\beta e^{-2\gamma\phi}$, the central charges for ψ and χ (the ghost field) would be shifted by opposite amounts so that the total anomaly would no longer depend on γ. Therefore, there is no way to get a vanishing anomaly. A dilaton gravity model with such a potential cannot be, at least using our scheme, quantized.

Taking into account the shift (7), the well-known contribution from the reparametrization ghosts, and from the $N-2$ matter fields, the total central charge is

$$\begin{aligned} c_\psi+c_\chi+(N-2)-26 &= \left[1+\frac{12}{\gamma^2}\left(1+\frac{\gamma^2}{2}\right)^2\right] \\ &\quad +\left[1-\frac{12}{\gamma^2}\right]+(N-2)-26 \\ &= 3(\gamma^2-4)+N-2=c(\gamma). \end{aligned}$$

It follows that

$$c(\gamma)=0\Rightarrow\gamma=\pm\sqrt{\frac{14-N}{3}}. \tag{8}$$

In the following, only a positive solution will be considered. Since γ has to be real and nonvanishing, we find the upper bound $N<14$.

III. MAPPING ONTO A LINEAR DILATON CFT

We are now in a position to consistently quantize our model. We have already pointed out that our theory describes a Liouville field ψ, a decoupled free scalar field χ with wrong sign kinetic energy, and $N-2$ free scalar matter fields. The only (self-)interacting field is the Liouville field, whose energy momentum tensor is the same as a free field with a conformal improvement. From Eq. (6) it is easy to see that also the field χ has an improvement. The energy momentum tensor for the quantum theory with N fields is therefore given by

$$T_{\pm\pm}=-\frac{1}{2}(\partial_\pm X)^2+v_\mu\partial_\pm^2X^\mu,\qquad v_\mu=\left(\frac{1}{\gamma},\frac{Q}{2},\mathbf{0}\right), \tag{9}$$

where $\mu=0,1,\ldots,N-1$. X^0 is the field χ, X^1 is the Liouville field ψ, and the remaining are the matter fields. $Q=(2/\gamma+\gamma)$ and v_μ gives the conformal improvements. For the X^μ fields (the "target space") we are using the flat metric $\eta_{\mu\nu}=\mathrm{diag}(-1,1,\ldots,1)$. The energy-momentum tensor (9) can also be derived from the action

$$S=-\frac{1}{8\pi}\int d^2x\sqrt{-g}(\partial^aX^\mu\,\partial_aX_\mu-2v_\mu X^\mu R). \tag{10}$$

This action describes a linear dilaton CFT. This is a free field theory whereas the theory described by Eqs. (5) and (6) is not. However, both theories have the same energy-momentum tensor and this property can be used to determine the spectrum of our model. Before doing so, we will briefly remind the reader of some basic features of LFT [12,19], relying mainly on the picture proposed in Refs. [20,21] (see also Ref. [22]). In this approach, LFT is viewed as a mild generalization of the standard 2D CFT structure. The older approach to LFT (see, e.g., Ref. [18]) is much more involved and cannot be used for our purposes.

A. Liouville field theory

LFT is quantized as a CFT generalizing the celebrated framework of Belavin, Polyakov, and Zamolodchikov [23], (BPZ). The space of the states forms a representation of the Virasoro algebra but, in contrast with the minimal model (BPZ scheme), the set of representations is continuous. This is due to the noncompactness of the space where the zero mode of the theory takes values.

A two-dimensional CFT is characterized by a correspondence between fields, i.e., local operators, and states [24]. The primary fields V_α acting on the $SL(2,C)$ invariant vacuum $|0\rangle$ generate the highest weight states of the Virasoro algebra. The descendant fields (states) are defined by the action of the energy-momentum tensor $T(w)$ (Virasoro modes L_n) on the primary fields (highest weight states). The theory is fully specified by vacuum expectation values (VEV) of the form

$$\langle 0|\prod_{p=1}^{N} T(w_p)\prod_{q=1}^{M} \overline{T}(\overline{w_q})\prod_{r=1}^{R} V_{\alpha_r}(z_r,\overline{z_r})|0\rangle. \tag{11}$$

LFT fits into this general scheme but there are some subtleties. If the central charge is given by $c=1+3Q^2$ (in our case $Q=2/\gamma+\gamma$), then $e^{\alpha\phi}$ are spinless primary fields with conformal dimension

$$\Delta(e^{\alpha\phi})=\frac{1}{2}\alpha(Q-\alpha). \tag{12}$$

The correspondence exists only in the region

$$0<\mathrm{Re}(\alpha)\leqslant\frac{Q}{2}.$$

A scalar product can be defined for states $|\alpha\rangle$, defined as usual by $\lim_{z\to 0}V_\alpha(z)|0\rangle$, if α is given by

$$\alpha=\frac{Q}{2}+iP,$$

where P is real (we can take $P>0$). As a consequence, using Eq. (12) we find that a state $|\alpha\rangle$ has a conformal dimension $\Delta(P)=Q^2/8+P^2/2$. The Hilbert space theory is given by

$$\bigoplus_{P>0}\mathrm{Vir}_{\Delta(P)}\otimes\mathrm{Vir}_{\overline{\Delta}(P)}, \tag{13}$$

where $\oplus_{P>0}$ is the direct sum over $P>0$ and $\mathrm{Vir}_{\Delta(P)}$ are irreducible representations of the Virasoro algebra of highest weight $\Delta(P)$.

From Eq. (13) it is evident that, paradoxically, the vacuum $|0\rangle$ does not belong to the Hilbert space. Nevertheless the state-operator correspondence and VEV (11) still make sense. Owing to the CFT structure of LFT it is in principle possible to reconstruct all correlation functions starting from the three-point function, whose exact expression is given in Ref. [20]. VEV are obtained by suitable analytical continuations summing over intermediate states [22].

B. The quantization of the linear dilaton CFT

We have already shown that our dilaton gravity model has the same energy-momentum tensor of a linear dilaton CFT. The spectrum of the theory can be found by quantizing a sort of bosonic string with N bosons X^μ. The energy-momentum tensor and the action are given by Eqs. (9) and (10). We can simply follow the steps that are usual for the critical bosonic string, taking into account (i) the presence of a conformal improvement and (ii) the previously discussed structure of LFT. Concerning point (ii), we take a ground state (oscillatory vacuum) $|p;0\rangle$ of momentum p with component $(p^1)^2>Q^2/8$.

We follow the conventions of Ref. [25]. The worldsheet is parametrized by (τ,σ), $-\infty<\tau<\infty$, $0\leqslant\sigma\leqslant\pi$ and we use periodic boundary conditions. The modes for right (α_n^μ) and left $(\tilde{\alpha}_n^\mu)$ movers are independent. The commutation relations are as usual $[\alpha_m^\mu,\alpha_n^\nu]=[\tilde{\alpha}_m^\mu,\tilde{\alpha}_n^\nu]=m\,\eta^{\mu\nu}\delta_{m+n}$. The Virasoro operators for $m\neq 0$ are given by

$$L_m=\frac{1}{2}\sum_q \alpha_{m-q}\cdot\alpha_q-imv\cdot\alpha_m. \tag{14}$$

Here and in the following, the expressions for the left mode operators are obtained substituting $\alpha_n\to\tilde{\alpha}_n$. The normal ordered expression for L_0 is

$$L_0=\frac{p^2}{8}+\sum_{n=1}^{\infty}\alpha_{-n}\cdot\alpha_n.$$

The Hamiltonian is $H=L_0+\tilde{L}_0$. The normal ordering constant a in the mass-shell conditions $(L_0-a)|\phi\rangle=(\tilde{L}_0-a)|\phi\rangle=0$ and the conformal anomaly are determined in a well-known way. From the relations for integer m,

$$\frac{1}{12}[(N+12v^2)m^3-Nm]+2ma$$

$$+\frac{1}{6}(m-13m^3)=0,$$

we read

$$v^2=\frac{26-N}{12},\quad a=\frac{N-2}{24}. \tag{15}$$

As expected, the first equation above is the same as Eq. (8). From Eq. (15), it follows immediately that in the absence of matter fields ($N=2$) the ground state is massless ($a=0$). If matter fields are present ($3\leqslant N<14$), the ground state is tachyonic ($a>0$).

IV. THE SPECTRUM OF THE MODEL

Let us now construct explicitly the spectrum of our model. We will first consider the case in which matter fields are present, $N\geqslant 3$. In this case, we can generate the spectrum and prove the unitarity of the theory, using a spectrum-generating algebra. This can be done adapting to our case the Brower construction [26], a version of the covariant formalism of Del Giudice, Di Vecchia, and Fubini [27] (see also Ref. [25]) used to prove the no-ghost theorem for the bosonic string.

We consider only the right-movers sector (our results can be immediately extended to the left-movers sector). We want to construct a spectrum-generating set of operators A_n^μ commuting with the Virasoro generators L_m. Starting from the right-moving solution at $\sigma=0$,

$$X_R^\mu(\tau)=\frac{1}{2}x^\mu+\frac{1}{2}\tau p^\mu+\frac{1}{2}i\sum_{n\neq 0}\frac{\alpha_n^\mu}{n}e^{-2in\tau},$$

we first construct primary fields of conformal dimension 1 from the vertex operators $V(k,\tau)=:\lambda\cdot\dot{X}_R\exp(ikX_R):$, where λ is a proper polarization vector. Using Eq. (14), we find

$$\begin{aligned}[L_m,:\lambda\cdot\dot{X}_R e^{ikX_R(\tau)}:]&=e^{2im\tau}\left[-\frac{i}{2}\frac{d}{d\tau}+m\left(1+\frac{k^2}{2}-ik\cdot v\right)\right]:\lambda\cdot\dot{X}_R e^{ikX_R}:\\&\quad+e^{2im\tau}m^2\left(\frac{k\cdot\lambda}{2}-iv\cdot\lambda\right):e^{ikX_R(\tau)}:\end{aligned}\tag{16}$$

Let us now take k lightlike and orthogonal to v. This is always possible since from Eq. (15) it follows that v is spacelike. It is convenient to use light-cone coordinates $X^\mu=(X^+,X^-,X^i)$ with $X_\mu Y^\mu=-X^+Y^--X^-Y^++X^iY^i$. By means of a Lorentz rotation, we are free to take $v_\mu=(0,0,v,0,\ldots,0)$, where $v_\mu v^\mu=v^2$. The kinematical setup is fixed as follows. The ground state momentum p_0^μ in $|p_0;0\rangle$ can be chosen such that $p_0^\mu=(-2,0,\beta,\ldots)$, $\beta^2=8a$. The k in $V(k,\tau)$ is $k_n=(0,-2n,\mathbf{0})$ for integer n. $k_n^2=0$, $k_n^\mu v_\mu=0$. At the level n the mass-shell condition is satisfied: $(p+k_n)^2+8(n-a)=0$.

Let us first construct the operators that generate states describing excitations of the matter fields ($N\geqslant 4$). If λ^i is a vector pointing in the i direction, we can find $N-4$ operators,

$$V^i(k_n,\tau)=:\lambda^i\cdot\dot{X}_R e^{ik_nX_R}:,\quad i\geqslant 4,\tag{17}$$

which, from Eq. (16), satisfy the commutation relations

$$[L_m,V^i(k_n,\tau)]=e^{2im\tau}\left(-\frac{i}{2}\frac{d}{d\tau}+m\right)V^i(k_n,\tau).\tag{18}$$

The operators of the spectrum-generating algebra A_n^μ are easily found to be

$$A_n^\mu=\frac{1}{\pi}\int_0^\pi d\tau\,V^\mu(k_n,\tau).$$

Due to the kinematical setup, the vertex operators V^μ are periodic with period π. It follows that for $i\geqslant 4$, $[L_m,A_n^i]=0$.

For $i=3$, a compensating term has to be added to the expression (17), since $\lambda^3\cdot v\neq 0$ [see Eq. (16)]. The form of this term is well known [26]. If $\hat{v}$ is the unit vector in the direction v_μ, we have

$$V^3(k_n,\tau)=:\hat{v}\cdot\dot{X}_R e^{ik_nX_R}:+v\frac{d}{d\tau}(\ln k_n\cdot\dot{X}_R)e^{ik_nX_R},$$

so that V^3 satisfies Eq. (18) and consequently A_n^3 commutes with L_m.

Let us now construct the operators A_n^+ and A_n^-, which generate states describing excitations in the $\pm$ directions. From the equation $V^+(k_n,\tau)=\dot{X}_R^+\exp(ik_nX_R)$, it follows that A_n^+ is trivial, $A_n^+=(1/\pi)\int_0^\pi V^+(k_n,\tau)=p^+/2=-\delta_n$. On the other hand, V^- can be defined as

$$V^-(k_n,\tau)=:\dot{X}_R^- e^{ik_nX_R}:-\frac{1}{2}in\frac{d}{d\tau}(\ln k_n\cdot\dot{X}_R)e^{ik_nX_R}.$$

We have completed the construction of the spectrum-generating operators A_n^μ. They satisfy the algebra,

$$\begin{aligned}&[A_m^i,A_n^j]=m\,\delta^{ij}\delta_{n+m},\quad i,j\geqslant 3,\\&[A_m^-,A_n^j]=-nA_{n+m}^i,\quad i>3,\\&[A_m^-,A_n^3]=-nA_{n+m}^3-ivn^2\delta_{n+m},\\&[A_m^-,A_n^-]=(m-n)A_{m+n}^-+2m^3\delta_{n+m}.\end{aligned}\tag{19}$$

Notice that the matter fields act as transverse "string" oscillators. As usual, instead with A_n^- it is convenient to work with the operators

$$\tilde{A}_n^-=A_n^--\frac{1}{2}\sum_{p=-\infty}^{\infty}\sum_{i=1}^{N-2}:A_{n-p}^iA_p^i:+\delta_n\frac{\beta^2}{8}.$$

The operators $\tilde{A}_n^-$ commute with A_m^i. By definition, they annihilate the oscillator vacuum,

$$\tilde{A}_0^-|p_0;0\rangle=0.\tag{20}$$

Furthermore, $\tilde{A}_n^-$ obey a Virasoro algebra:

$$[\tilde{A}_m^-,\tilde{A}_n^-]=(m-n)\tilde{A}_{m+n}^-.$$

We have successfully completed our task of finding the spectrum of the model we are considering. From the algebra (19) it follows that for $N\geqslant 3$ the spectrum is that of a bosonic string oscillating in a target space with the $N-2$ transverse direction. The gravitational sector is decoupled from the matter field sector. Equation (20) implies that the states of this sector have zero norm, as expected for pure gauge states.

Until now we have considered only the case in which matter fields are present ($N \geqslant 3$). If matter fields are absent ($N=2$), we cannot use the previously explained construction. However, in this case all the excitations are pure gauge and it is evident that the Hilbert space consists of the ground state whereas all the excited states have zero norm. This conclusion is consistent with the results for pure (Jackiw-Teitelboim) dilaton gravity obtained in Ref. [10], using a different approach.

V. CONCLUSIONS

In this paper, we have consistently quantized a model of matter-coupled dilaton gravity in two dimensions with the exponential dilaton potential. A vanishing conformal anomaly has been achieved by tuning a parameter in the dilaton potential. The quantization has been performed by mapping the theory first onto a field theory with a Liouville interaction and then onto a linear dilaton CFT. The spectrum has been determined in a straightforward way, analogous to that used for the bosonic string in critical dimensions.

We have found that the ground state is tachyonic (or massless in the absence of matter). The spectrum has two decoupled sectors: the gravitational sector made of pure gauge, zero norm states and the matter field sector describing transverse physical excitations. This result confirms previous results [10] about quantization of pure dilaton gravity models.

The theory has a free field spectrum but it is not trivial, since as far as correlations are concerned, it has at least the same complexity as LFT. With our approach, establishing the equivalence with a sort of critical string, we have succeeded in what seems difficult using other methods: the quantization of matter-coupled dilaton gravity.

As a final point, we observe that the rich structure exhibited by the semiclassical analysis of 2D dilaton gravity has disappeared. The requirement of a vanishing anomaly, i.e., the criticality of the theory, washes out the semiclassical structures. The gap between the semiclassical and the quantum theory must still be filled. We believe this can be done only going off-criticality. In this way, the theory would be equivalent to a noncritical string and, needless to say, would present severe difficulties.

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[1] A.M. Polyakov, Phys. Lett. **103B**, 207 (1981).
[2] F. David, Mod. Phys. Lett. A **3**, 1651 (1988); J. Distler and H. Kawai, Nucl. Phys. **B321**, 509 (1989).
[3] T. Banks and M. O'Loughlin, Nucl. Phys. **B362**, 649 (1991); R.B. Mann, Phys. Rev. D **47**, 4438 (1993).
[4] R. Jackiw, *Quantum Theory of Gravity*, edited by S. Christensen (Hilger, Bristol, UK, 1984); C. Teitelboim, *ibid.*
[5] D. Louis Martinez and G. Kunstatter, Phys. Rev. D **49**, 5227 (1994); C.G. Callan, S.B. Giddings, J.A. Harvey, and A. Strominger, *ibid.* **45**, R1005 (1992); A. Bilal and C.G. Callan, Nucl. Phys. **B394**, 73 (1993); M. Cadoni, Phys. Rev. D **53**, 4413 (1996).
[6] M. Cadoni and S. Mignemi, Phys. Rev. D **59**, 081501 (1999); Nucl. Phys. **B557**, 165 (1999).
[7] M. Cadoni and M. Cavaglià, Phys. Lett. B **499**, 315 (2001); Phys. Rev. D **63**, 084024 (2001).
[8] A.J.M. Medved, ''Quantum Corrected Cardy Entropy For Generic (1+1)-Dimensional Gravity,'' hep-th/0201079.
[9] M. Cadoni, P. Carta, D. Klemm, and S. Mignemi, Phys. Rev. D **63**, 125021 (2001).
[10] D. Cangemi, R. Jackiw, and B. Zwiebach, Ann. Phys. (N.Y.) **245**, 408 (1996).
[11] M. Cavaglià, V. De Alfaro, and A.T. Filippov, Phys. Lett. B **424**, 265 (1998).
[12] T.L. Curtright and C.B. Thorn, Phys. Rev. Lett. **48**, 1309 (1982).
[13] J. Polchinski, *String Theory* (Cambridge University Press, Cambridge, UK, 1998), Vol 1, p. 49.
[14] M. Cadoni, Phys. Lett. B **395**, 10 (1997); M. Cavaglià, Mod. Phys. Lett. A **15**, 2113 (2000).
[15] A.T. Filippov, Int. J. Mod. Phys. A **12**, 13 (1997).
[16] R. Jackiw, Theor. Math. Phys. **9**, 404 (1992).
[17] J. Cruz, A. Fabbri, D.J. Navarro, and J. Navarro-Salas, Phys. Rev. D **61**, 024011 (2000).
[18] N. Seiberg, Prog. Theor. Phys. Suppl. **102**, 319 (1990).
[19] E. Braaten, T.L. Curtright, and C.B. Thorn, Phys. Lett. **118B**, 115 (1983); Ann. Phys. (N.Y.) **147**, 365 (1983); J.-L. Gervais and A. Neveau, Nucl. Phys. **B328**, 125 (1984); **B328**, 396 (1989).
[20] H. Dorn and H.J. Otto, Nucl. Phys. **B429**, 375 (1994).
[21] A.B. Zamolodchikov and Al.B. Zamolodchikov, Nucl. Phys. **B477**, 577 (1996).
[22] J. Teschner, Class. Quantum Grav. **18**, R153 (2001).
[23] A.A. Belavin, A.M. Polyakov, and A.B. Zamolodchikov, Nucl. Phys. **B241**, 333 (1984).
[24] A standard textbook reference is P. Di Francesco, P. Mathieu, and D. Sénéchal, *Conformal Field Theory* (Springer-Verlag, New York, 1997).
[25] M.B. Green, J.H. Schwarz, and E. Witten, *Superstring Theory* (Cambridge University Press, Cambridge, UK, 1987).
[26] R.C. Brower, Phys. Rev. D **6**, 1655 (1972); R.C. Brower and P. Goddard, Nucl. Phys. **B40**, 437 (1972).
[27] E. Del Giudice and P. Di Vecchia, Nuovo Cimento Soc. Ital. Fis., A **5A**, 90 (1971); E. Del Giudice, P. Di Vecchia, and S. Fubini, Ann. Phys. (N.Y.) **70**, 378 (1972).
[28] W. Kummer, H. Liebl, and D.V. Vassilevich, Nucl. Phys. **B493**, 491 (1997).