Loop quantum gravity corrections and cosmic ray decays

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(Received 20 November 2001; published 10 May 2002)

Loop quantum gravity effective theories are reviewed in the context of the observed Greizen-Zatsepin-Kuz'min limit anomaly and related processes. This is accomplished through a kinematical analysis of the modified threshold conditions for the decay reactions involved, arising from the theory. Especially interesting is the possibility of a helicity dependent violation of the limit, whose primary effect would be the observation of favored helicity states for highly energetic particles.

DOI: 10.1103/PhysRevD.65.103516 PACS number(s): 98.80.Hw, 04.60.Ds

I. INTRODUCTION

The vast void that still separates us from a definite version of a quantum theory of gravity, and the fact that several alleged versions of it are being proposed, has motivated the development of various semiclassical approaches. These approaches follow the form of effective theories which take into consideration matter-gravity couplings, such as described in a number of recent works [1–4], whose main results are the introduction of new terms in the equations of motion for the system described. An inevitable outcome of these works is the introduction of Lorentz invariance deformations (LID's) at the effective theory level. These deformations become manifest when one analyzes the dispersion relations for freely propagating particles, and may have notorious consequences in high energy phenomena.

In particular, both [3] and [4] are based on the loop quantum gravity (LQG) framework [5]. In these works, the effects of the loop structure of space, at the Planck level, are treated semiclassically through a coarse-grained approximation. An interesting feature of this kind of method is the appearance of a new length scale $\mathcal{L}$ (with $\mathcal{L} \gg$ Planck length l_p), such that for distances $d \ll \mathcal{L}$ the quantum loop structure of space is manifest, while for distances $d \geq \mathcal{L}$ the continuous flat geometry is regained. This scale gives us the hope of bringing the effects of quantum gravity to an observable level. A natural question thus arises. Are we actually observing these quantum gravity effects? To answer this question we are forced to go through the observations of the greatest energy registered.

The most energetic measured events are found in the form of ultrahigh energy cosmic rays (UHECR's) [6,7]. Such events (energies above 10^{20} eV) actually violate the theoretical threshold known as the Greisen-Zatsepin-Kuz'min (GZK) limit [8,9], according to which no extragalactic cosmic ray can exceed, in energy, the value of 5×10^{19} eV. This current limit takes into consideration the interaction of protons with photons from the cosmic microwave background radiation (CMBR). There have been different attempts to formulate a convincing explanation about why such energetic particles are reaching the Earth. In a purely theoretical fashion, perhaps the most interesting explanations are the manifestation of decaying magnetic monopoles [10], and the decay of superheavy relic particles [11]. Another more orthodox explanation can be found in the existence of Z bursts produced by collisions between ultrahigh energy neutrinos and cosmic relic neutrinos [12–14]. However, neither of these previous possibilities is fully satisfactory.

Another relevant observation is the detection of extragalactic multi-TeV photons from the BL Lac object known as Markarian (Mrk) 501 [15]. These detected photons have reached energies up to 20 TeV. Similar to the case of protons, these multi-TeV γ rays are subject to interaction with the far infrared background radiation (FIBR), setting a limit to the energy of the photons that can reach us. Initially, the collected data suggested a violation of this limit, although it has recently been stated that no such violation exists [18,19]. We adopt this last position.

In this paper we study the possible bounds on the length scale $\mathcal{L}$ emerging from the two observations mentioned above. This is accomplished through a kinematical analysis of the threshold conditions for the decays to be possible. In particular, since the GZK limit is broken, we assume that a reasonable explanation is found in the LID's offered by the theory (see [16–18] for other similar approaches). On that score, the LID manifestations will, in certain cases, depend on the difference between two LQG parameters, each one belonging to a different particle. For instance, as shown in [16], if the dispersion relation for a particle i is (from here on, $\hbar = c = 1$)

$$E_i^2 = A_i^2 p_i^2 + m_i^2 \tag{1}$$

(where E_i, p_i, and m_i are the energy, momentum, and mass of the ith particle, and A_i is a LID parameter that can be interpreted as the maximum velocity of the ith particle), then one can show that the mentioned thresholds can be substantially modified provided that the difference $\delta A = A_a - A_b$ is not null (a and b are two particles involved in the reaction leading to the mentioned threshold). Of course, this effect compromises the universality of the given parameters, namely, the fact that the A_i parameters—which eventually contain the information regarding the matter-gravity coupling—are not the same for all particles. In the case of the current LQG effective theories, these nonuniversal deviations could be understood as the manifestation of the breakup of classical symmetries, emerging as a consequence of the choice of the quantum gravity vacuum. In this way, the stan-

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dard model structure of different particles could appear through differentiated values for the parameters in question. In this respect, since we do not have detailed knowledge of the precise values of the correction parameters, we shall consider all possible scenarios for the mentioned observations.

Finally, we must mention the fact that, in general, the presence of LID's forces us to consider the appearance of a preferred reference system. In the case of the LQG corrections that we will consider, the dispersion relations are valid only in an isotropic system. For this reason, we shall naturally assume that this preferred system is the CMBR comoving reference frame and, consequently, the threshold conditions for the different decays should be considered keeping this in mind.

II. DISPERSION RELATIONS FROM LOOP QUANTUM GRAVITY

Here we present the main results from [3] and [4] relative to the modifications of the dispersion relations of freely propagating neutrinos (more precisely, Majorana fermions) and photons. We shall assume that the results for Majorana fermions can be extended to fermions in general. This assumption relies on the fact that no substantial departure from the original methods would be expected for the general case, since the only difference is that for Majorana fermions one must impose the reality condition on the field equations. Of course, one could expect that in the case of more general fermions there would appear more corrective terms. Nevertheless, from the symmetry arguments found in [3], we should not expect new $\mathcal{L}$ and l_p dependent corrections different from those that already appear in the present theory.

The appearance of the length scale $\mathcal{L}$ deserves special attention.

A. Fermions

For Majorana fermions [3], the dispersion relation is given by

$$E_{\pm}^2=\left(Ap\pm\frac{B}{2\mathcal{L}}\right)^2+m^2(\alpha\pm\beta p)^2 \tag{2}$$

where

$$\begin{aligned} A&=\left(1+\kappa_1\frac{l_p}{\mathcal{L}}+\kappa_2\left(\frac{l_p}{\mathcal{L}}\right)^2+\frac{\kappa_3}{2}l_p^2p^2\right),\\ B&=\left(\kappa_5\frac{l_p}{\mathcal{L}}+\kappa_6\left(\frac{l_p}{\mathcal{L}}\right)^2+\frac{\kappa_7}{2}l_p^2p^2\right),\\ \alpha&=\left(1+\kappa_8\frac{l_p}{\mathcal{L}}\right),\\ \beta&=\frac{\kappa_9}{2}l_p. \end{aligned} \tag{3}$$

In these expressions, $E_{\pm}$ is the energy of the fermionic particle of mass m and momentum p, and the κ_i are unknown adimensional parameters of order 1. The $\pm$ signs stand for the helicity of the propagating fermion. It should be stressed that the terms associated with B and β, and which are precisely causing the $\pm$ signs, are both parity and CPT odd (in fact, the equations of motion are invariant under charge conjugation and time reversal operations).

In what follows, it will be sufficient to consider

$$E_{\pm}^2=A^2p^2+\kappa_3l_p^2p^4\pm\kappa_5\frac{l_p}{\mathcal{L}^2}|p|+m^2+\frac{1}{4}\left(\kappa_5\frac{l_p}{\mathcal{L}^2}\right)^2, \tag{4}$$

where now $A=1+\kappa_1 l_p/\mathcal{L}$ and κ_1, κ_3, and κ_5 are of order 1. For simplicity, let us write (with $\eta=\kappa_3l_p^2$ and $\lambda=\kappa_5l_p/2\mathcal{L}^2$)

$$E_{\pm}^2=A^2p^2+\eta p^4\pm2\lambda p+m^2, \tag{5}$$

where we have absorbed the quadratic term in κ_5 into the mass. As we have said, the basis of the present work relies on the assumption that Eq. (2) is a valid expression for fermionic particles in general. In particular, we will adopt the expression (5) for electrons, protons, and Δ particles.

B. Photons

For photons [4], the dispersion relation is

$$E_{\pm}=p[A_{\gamma}-\theta_3(l_pp)^2\pm\theta_8l_pp], \tag{6}$$

where

$$A_{\gamma}=1+\kappa_{\gamma}\left(\frac{l_p}{\mathcal{L}}\right)^{2+2\Upsilon}. \tag{7}$$

In the previous expressions, $E_{\pm}$ and p are the energy and momentum of the photon, while κ_{γ} and θ_i are adimensional parameters of order 1, and Υ is a free parameter that, for the moment, still needs interpretation (it should be noted that the presence of the Υ parameter in the fermion dispersion relation was not considered in [3]). For simplicity we shall consider only the possibilities $\Upsilon=-1/2$, 0, 1/2, 1, etc., in such a way that $A_{\gamma}\sim1+\mathcal{O}[(l_p/\mathcal{L})^n]$, with $n=2+2\Upsilon$ a positive integer. With this assumption, we will be able to find a tentative value for Υ, through the bounding of the lower order correction of $\delta A\sim\mathcal{O}[(l_p/\mathcal{L})^n]$ (where $\delta A=A_{\gamma}-A_a$, a denoting another particle).

As before, we note the presence of the $\pm$ signs which denote the helicity dependence of the photon. To the order of interest, Eq. (6) can be written

$$E_{\pm}^2=p^2[A_{\gamma}^2\pm2\,\theta_{\gamma}(l_pp)]. \tag{8}$$

Notably, Eq. (8) is essentially the same result that Gambini and Pullin [2] obtained for the photon's dispersion relation, with the difference that they have $A_{\gamma}=1$ and therefore the semiclassical scale $\mathcal{L}$ is absent.

A similar contribution was also suggested by Ellis *et al.* [21,22] (in this case, without helicity dependence). They found

$$E^2 = p^2[1 - 2M_D^{-1}p], \tag{9}$$

where M_D is a mass scale coming from D-brane recoil effects for the propagation of photons in vacuum. When gamma ray burst (GRB) data are analyzed to restrict M_D [22], the following condition arises:

$$M_D \gtrsim 10^{24} \ \text{eV}. \tag{10}$$

For the photon dispersion relation that we are currently considering, Eq. (10) can be interpreted as the bound $\theta_\gamma \lesssim 10^4$. Since θ_γ is an adimensional parameter of order 1, expression (8) is still a permitted dispersion relation, as far as GRB's are concerned. We shall soon see other possibilities to contrast with a θ_γ like term.

III. KINEMATICAL APPROACH

A decay reaction is kinematically allowed when, for a given value of the total momentum $\vec{p}_0 = \Sigma_{initial}\vec{p} = \Sigma_{final}\vec{p}$, one can find a total energy value E_0 such that $E_0 \geqslant E_{min}$. Here E_{min} is the minimum value that the total energy of the decaying products can acquire, for a given total momentum $\vec{p}_0$. To find E_{min} for the dispersion relations under consideration, it is enough to take the individual decay product momenta to be collinear with respect to the total momentum $\vec{p}_0$ and with the same direction. To see this, it is enough to vary E_0 with the appropriate restrictions:

$$E_0 = \sum_i E_i(|p_i|) + \xi_j\left(p_0^j - \sum_i p_i^j\right), \tag{11}$$

where ξ_j are Lagrange multipliers, the i index specifies the ith particle, and the j index the jth vectorial component of the different quantities. Doing the variation, we obtain

$$\frac{\partial E_i}{\partial p_i^j} \equiv v_i^j = \xi_j . \tag{12}$$

That is to say, the velocities of all product particles must be equal to ξ. Since the dispersion relations that we are treating are monotonically increasing in the range of momenta $p > \lambda$, this result means that the momenta can be taken as collinear and with the same direction as $\vec{p}_0$.

In the present work, we will focus on those cases in which two particles (say a and b) collide, and later decay. For the present, these particles will have momenta $\vec{p}_a$ and $\vec{p}_b$, respectively, and a total momentum $\vec{p}_0$. Nevertheless, the total energy of the system will depend only on $|p_a|$ and $|p_b|$. Therefore, to get the threshold condition for the mentioned process, we must find the maximum possible total energy E_{max} of the initial configuration, given $|p_a|$ and $|p_b|$. For this, let us fix $\vec{p}_a$ and vary the direction of $\vec{p}_b \equiv |p_b|\hat{n}$ in

$$E_0 = E_a(\vec{p}_0 - |p_b|\hat{n}) + E_b(|p_b|) + \chi(\hat{n}^2 - 1). \tag{13}$$

Varying Eq. (13) with respect to $\hat{n}$ (χ is a Lagrange multiplier), we find

$$\hat{n}^i = \frac{v_a^i |p_b|}{2\chi}. \tag{14}$$

In this way we obtain two extremal situations $\chi = \pm v_a |p_b|/2$, or simply

$$\hat{n}^i = \pm \frac{v_a^i}{v_a}. \tag{15}$$

A simple inspection shows that for the dispersion relations that we are considering the maximum energy is given by $\hat{n}^i = -v_a^i/v_a$, or, in other words, when frontal collision occurs.

Summarizing, the threshold condition for a two-particle (a and b) collision and posterior decay can be expressed through the following requirements:

$$E_a + E_b \geqslant \sum_{final} E_f \tag{16}$$

with all final particles having the same velocity, and

$$p_a - p_b = \sum_{final} p_f, \tag{17}$$

where the sign of the momenta $\Sigma_{final} p_f$ is given by the direction of the highest momentum magnitude of the initial particles. A more detailed treatment can be found in [16].

As a final remark for this section, under certain circumstances (for example, some special choice of the LID parameters) the condition $v_i^j = \xi_j$ could give more than one solution for the threshold-condition configuration. In fact, as noted in [20], for a reaction where two identical particles are the decaying products, it is possible to find configurations where the momenta of these particles are distributed asymmetrically within them. However, for the present work, these effects can be neglected since they give contributions to the threshold conditions that are smaller than those which we will consider.

IV. DECAY REACTIONS

Using the methods described in the last section, we can find the threshold conditions for the decay reactions leading to the theoretical limits for cosmic rays. These thresholds will present some consequential modifications due to the parameters of the theory. Here we examine the possible bounds on these parameters. Let us start with the observations coming from multi-TeV γ rays.

A. Pair decay $\gamma + \gamma_\epsilon \rightarrow e^- + e^+$

Multi-TeV photons are subject to interactions with the FIBR through the process $\gamma + \gamma_\epsilon \rightarrow e^- + e^+$, where γ_ϵ is a soft photon from the FIBR. For this reaction to occur, the following threshold condition must be satisfied

$$E_\gamma + \omega \geqslant E_{e^+} + E_{e^-} \tag{18}$$

with

$$p_\gamma - k = p_{e^+} + p_{e^-}. \tag{19}$$

In the above expressions, ω and k are the energy and momentum of the target photon from the FIBR. Since the energy of these photons does not significantly exceed the eV range, we will consider for these the usual dispersion relation $\omega = k$. The above equations can be reexpressed as

$$E_\gamma^2 + 2\omega E_\gamma \geqslant E_{e^+}^2 + E_{e^-}^2 + 2E_{e^+}E_{e^-} \tag{20}$$

and

$$p_\gamma^2 - 2kp_\gamma = p_{e^+}^2 + p_{e^-}^2 + 2p_{e^+}p_{e^-}, \tag{21}$$

where we have neglected the quadratic terms in the FIBR quantities. An important property of the field equations from which the fermion dispersion relation comes is that they are charge conjugation invariant. Therefore we can take for both electron and positron the same dispersion relation with the same sign conventions. Furthermore, an analysis of conservation of angular momenta shows that both helicities are equally probable for the emerging pair; hence for the right hand side of Eq. (20) we must ensure that the energy of both electron and positron, is the minimum possible. For this reason, we must use $E_{e^+} = E_{e^-} = E_{(-)}$, where $E_{(-)}$ is defined as

$$E_{(-)}^2 \equiv A^2p^2 + \eta p^4 - 2|\lambda|p + m^2. \tag{22}$$

Physically, this condition means that the helicity state of less energy is the one that sets the threshold condition. With this consideration, we are left with

$$E_\gamma^2 + 2\omega E_\gamma \geqslant 4E_{(-)}^2. \tag{23}$$

From the dispersion relations (4) and (8) we can write the last equation as

$$\begin{aligned} p_\gamma^2[A_\gamma^2 + (\pm)_\gamma 2\theta_\gamma(l_p p_\gamma)] + 2\omega E_\gamma \\ \geqslant 4[A_e^2 p_e^2 - 2|\lambda|p_e + m_e^2]. \end{aligned} \tag{24}$$

Here, $(\pm)_\gamma$ stands for the incident photon helicity. Note that we have neglected the terms related to η; these terms will become important when we study other reactions. Replacing the momentum conservation, we obtain

$$\begin{aligned} p_\gamma^2(A_\gamma^2 - A_e^2) + (\pm)_\gamma 2\theta_3 l_p p_\gamma^3 \\ + 2(\omega E_\gamma + p_\gamma k A_e^2) + 8|\lambda|p_e \geqslant 4m_e^2. \end{aligned} \tag{25}$$

To the order under consideration we can replace the p's by E's. Additionally, we can use $2E_e \simeq E_\gamma$,

$$\begin{aligned} E_\gamma^2(A_\gamma^2 - A_e^2) + (\pm)_\gamma 2\theta_\gamma l_p E_\gamma^3 + 4\omega E_\gamma + 4|\lambda|E_\gamma \\ \geqslant 4m_e^2. \end{aligned} \tag{26}$$

Now, note that in the absence of quantum gravity corrections we would have the usual threshold condition

$$E_\gamma \geqslant \frac{m_e^2}{\omega}; \tag{27}$$

therefore, to contrast the new terms, we compare them with the quantity $4m_e^2$ in the right side of the inequality (26).

Following [19], no LID's should be inferred from the analysis of the data from the observed Markarian Blazar Mrk 501. This imposes strong bounds on our parameters and, in particular, it means that any modified term must be less than $4m_e^2$ up to photons of energy ~ 20 TeV. In the first place, let us see the A terms:

$$E_\gamma^2|A_\gamma^2 - A_e^2| \cong 2E_\gamma^2|A_\gamma - A_e| \leqslant 4m_e^2. \tag{28}$$

So it follows that

$$|\delta A| \leqslant 2\frac{m_e^2}{E_\gamma^2}. \tag{29}$$

Evaluating with $E_\gamma \sim 20$ TeV, we obtain $|\delta A| \leqslant 1.3\times 10^{-15}$. If we assume that the adimensional parameters are of order 1, and take for Υ the value $\Upsilon = -1/2$ [so that $\delta A = \mathcal{O}(l_p/\mathcal{L})$], we can estimate the following bound for $\mathcal{L}$:

$$\mathcal{L} \gtrsim 6.4\times 10^{-14}\ \ \mathrm{eV}^{-1}. \tag{30}$$

Nevertheless, typical values for the LID parameter difference $|\delta A|$ are below 10^{-22} [16]. This in turn imposes a new bound $\mathcal{L} \gtrsim 8.3\times 10^{-7}$ eV^{-1} (or $\mathcal{L} \gtrsim 10^{-11}$ cm) which is nearly in the range of nuclear physics. Since there is no evidence that space manifests its loop structure at this scale, we interpret this result to mean that $\delta A = \mathcal{O}(l_p^2/\mathcal{L}^2)$ (that is, the universality is broken at most in second order in the ratio $l_p/\mathcal{L}$). With this last assumption we obtain a favored $\Upsilon = 0$ value, and the bound

$$\mathcal{L} \gtrsim 8.3\times 10^{-18}\ \ \mathrm{eV}^{-1}. \tag{31}$$

This is by far a more reasonable bound for $\mathcal{L}$.

In the second place, we have the θ_γ term (recall that this term involves the photon helicity dependence). Imposing the same kind of constraint with photons of energy $E_\gamma \sim 20$ TeV, we obtain

$$|\theta_\gamma| \lesssim 0.8. \tag{32}$$

This is not a serious bound on the parameter θ_γ. In any case, if $|\theta_\gamma| \gtrsim 1$ then the observed photons from Mrk 501 should have a preferred helicity (this particular helicity will depend on the sign of θ_γ). Furthermore, since θ_γ is assumed to be a parameter of order 1, expression (32) tells us that more energetic photons than those we are considering (energies ~ 20 TeV) should appear with this preferred helicity.

Finally, there remains the term involving the λ parameter for electrons. For this, we obtain

$$|\kappa_5|\frac{l_p}{\mathcal{L}^2} \leqslant 2.6\times 10^{-2}\ \ \mathrm{eV}, \tag{33}$$

or, assuming that κ_5 is of order 1,

$$\mathcal{L} \gtrsim 5.7\times 10^{-14} \ \ \mathrm{eV}^{-1}. \tag{34}$$

B. Proton decay $p+\gamma\rightarrow\Delta$

The main reaction leading to the GZK limit is the resonant $\Delta(1232)$ decay $p+\gamma\rightarrow\Delta$. The threshold condition is

$$E_p+\omega \geqslant E_\Delta \tag{35}$$

with

$$p_p-k=p_\Delta. \tag{36}$$

Here $E_\Delta^2=A^2p^2+\eta p^4-2|\lambda|p+m^2$, that is to say, the minimum possible value for the energy of the emerging Δ. With some algebraic manipulation we can find

$$2\,\delta A E_p^2+\delta\eta E_p^4+2[(\pm)_p\lambda_p+|\lambda_\Delta|]E_p+4\omega E_p \geqslant M_\Delta^2-M_p^2, \tag{37}$$

where $\delta A=A_p-A_\Delta$ and $\delta\eta=\eta_p-\eta_\Delta$. Additionally, $(\pm)_p$ refers to the incident proton helicity. Note that in the absence of LQG modifications the threshold condition becomes

$$E_p \geqslant \frac{M_\Delta^2-M_p^2}{4\omega}. \tag{38}$$

Since we do not have a detailed knowledge of the deviation parameters, we take account of them independently. Naturally, there will always exist the possibility of having an adequate combination of these parameter values that could affect the threshold condition simultaneously. However, as we will soon see, each one of these parameters will be significant in different energy ranges.

Let us start by considering the terms involving A:

$$2\,\delta A E_p^2+4\omega E_p \geqslant M_\Delta^2-M_p^2. \tag{39}$$

For this inequality it is easy to see that, for a given value of ω, the reaction is kinematically precluded for all E, if

$$A_\Delta-A_p > \frac{2\omega^2}{M_\Delta^2-M_p^2} \simeq 1.7\times 10^{-25}[\omega/\omega_0]^2, \tag{40}$$

where $\omega_0=2.35\times 10^{-4}$ eV is the kT energy (with $T=2.73$ K) of the CMBR thermal distribution. For all purposes the GZK limit is forbidden for CMBR photons if we take $\omega\simeq\omega_0$. Incidentally, assuming that the adimensional parameters are of order 1 and that—as previously asserted—the nonuniversal deviation of A is at most of second order in $l_p/\mathcal{L}$, we obtain

$$\mathcal{L} \lesssim 2\times 10^{-16} \ \ \mathrm{eV}^{-1}. \tag{41}$$

Of more relevance than the A terms (as we will verify) are the η related ones. Here we have

$$\delta\eta E_p^4+4\omega E_p \geqslant M_\Delta^2-M_p^2. \tag{42}$$

In this case the condition is independent of $\mathcal{L}$ and it depends strictly on the difference $\delta\eta$. For this, the reaction is forbidden if

$$(\eta_\Delta-\eta_p) > \frac{27\omega^4}{(M_\Delta^2-M_p^2)^3} \simeq 3.2\times 10^{-67}[\omega/\omega_0]^4 \ \ \mathrm{eV}^{-2}. \tag{43}$$

Recalling that $\eta=\kappa_3 l_p^2$, Eq. (43) tells us that it is enough to have $|\kappa_3|>5\times 10^{-11}$ with $\kappa_{3\Delta}>\kappa_{3p}$ for the reaction to be precluded. Since we are assuming that $\mathcal{O}(\kappa_3)=1$, this result shows us that the presence of a non-null $\delta\eta<0$ ensures the GZK violation effect.

In view of the possibilities $\delta\eta=0$ and $\delta A=0$, we must consider the λ dependent terms

$$2[(\pm)_p\lambda_p+|\lambda_\Delta|]E_p+4\omega E_p \geqslant M_\Delta^2-M_p^2. \tag{44}$$

This last expression is very interesting faced with the fact that its terms are helicity dependent. In this case, the reaction is more sensitive to the energy of the target photon. For instance, if ω is such that

$$(\pm)_p\lambda_p+|\lambda_\Delta|+2\omega \leqslant 0, \tag{45}$$

the reaction will be forbidden. Of course, this situation will depend on the helicity configuration of the incident proton. For example, if

$$|\lambda_p| \geqslant |\lambda_\Delta|+4.7\times 10^{-4}[\omega/\omega_0] \ \ \mathrm{eV}, \tag{46}$$

the reaction will also be forbidden at least for one proton helicity. Indeed, if $|\kappa_{5p}|-|\kappa_{5\Delta}|\gtrsim 1$ (recall that $\lambda=\kappa_5 l_p/2\mathcal{L}^2$) the threshold condition is dominated by the λ_p term:

$$|\lambda_p| \gtrsim 4.7\times 10^{-4}[\omega/\omega_0] \ \ \mathrm{eV}. \tag{47}$$

This imposes a new bound on the parameters of the theory,

$$\mathcal{L} \lesssim 3\times 10^{-13} \ \ \mathrm{eV}^{-1}. \tag{48}$$

On the other hand, if $|\kappa_{5\Delta}|-|\kappa_{5p}|\gtrsim 1$, the conservation of angular momentum always allows the reaction, and no GZK violation is obtained. Although for this effect to be noticeable we must demand universality of both A and η (at least for these hadronic particles), instead of λ, since Δ's and protons have different spins, we cannot discard this possibility.

C. Photo-pion production $p+\gamma\rightarrow p+\pi$

The next relevant reaction leading to the GZK threshold is the nonresonant photopion production $p+\gamma\rightarrow p+\pi$. Since the pion is a spin 0 particle, we may assume that, to the order considered for Eq. (4), the relevant dispersion relation is

$$E^2=A_\pi^2p^2+\eta_\pi p^4+m_\pi^2, \tag{49}$$

where $A_\pi=1+\kappa_\pi(l_p^2/\mathcal{L}^2)$ (recall that we must have $\delta A\sim l_p^2/\mathcal{L}^2$). As in the other cases, the threshold condition will be given by

$$E_p+\omega \geqslant \overline{E}_p+E_\pi \tag{50}$$

with

$$p_p-k=\overline{p}_p+p_\pi, \tag{51}$$

where $\overline{E}_p$ and $\overline{p}$ refer to the emerging proton. In analogy with the Δ decay, for this threshold condition we must put $\overline{E}_p^2=A_p^2\overline{p}^2+\eta\overline{p}^4-2|\lambda_p|\overline{p}+m_p^2$.

With a little amount of algebra we are able to find

$$2\,\delta A E_\pi^2+\left(\delta\eta+3\,\eta_p\frac{M_p(M_p+M_\pi)}{M_\pi^2}\right)E_\pi^4+4E_\pi\omega$$
$$+2E_\pi(|\lambda|\pm\lambda)\geqslant\frac{M_\pi^2(2M_p+M_\pi)}{M_p+M_\pi}, \tag{52}$$

where $\delta A=A_p-A_\pi$ and $\delta\eta=\eta_p-\eta_\pi$. In the last expression, $\pm$ refers to the helicity of the incoming proton. Since there will necessarily be an incident proton helicity that can minimize this term, we can take for the threshold condition

$$2E_\pi(|\lambda|\pm\lambda)=0. \tag{53}$$

With this consideration in mind, we get

$$2\,\delta A E_\pi^2+(\delta\eta+168\,\eta_p)E_\pi^4+4E_\pi\omega$$
$$\geqslant\frac{M_\pi^2(2M_p+M_\pi)}{M_p+M_\pi}. \tag{54}$$

As before, let us consider the modifications separately. If δA were the dominant term, we would have to consider

$$2\,\delta A E_\pi^2+4E_\pi\omega\geqslant\frac{M_\pi^2(2M_p+M_\pi)}{M_p+M_\pi}; \tag{55}$$

consequently, the violation condition would be

$$A_\pi-A_p>\frac{2\,\omega^2(M_p+M_\pi)}{M_\pi^2(2M_p+M_\pi)}\simeq 3.3\times10^{-24}[\omega/\omega_0]^2. \tag{56}$$

Using $\delta A\sim l_p^2/\mathcal{L}^2$, this result can be understood as

$$\mathcal{L}\lesssim 4.6\times10^{-17}\ \ \mathrm{eV}^{-1}. \tag{57}$$

Let us now consider the η terms. For these we have a violated threshold if

$$-\delta\eta-168\,\eta_p>27\omega^4\left(\frac{M_p+M_\pi}{M_\pi^2(2M_p+M_\pi)}\right)^3$$
$$\simeq 2.2\times10^{-63}[\omega/\omega_0]^4\ \ \mathrm{eV}^{-2}. \tag{58}$$

Since $\mathcal{O}(\delta\eta)\simeq\mathcal{O}(\eta_p)$ (when $\delta\eta\neq0$), let us assume that the η_p term dominates. In this case, for the threshold condition to be violated we just require $|\eta|>1.3\times10^{-65}\ \ \mathrm{eV}^{-2}$ with η negative. Recalling that $\eta=\kappa_3 l_p^2$ with κ_3 of order 1, this condition can be well read as $|\kappa_3|\geqslant1.9\times10^{-9}$. Hence, if κ_3 is not strictly zero, this term can cause the GZK limit violation, as far as photopion production is concerned. Finally, if η is null, the next relevant terms will be the λ helicity dependent ones. But, by angular momentum conservation, there will always be an emergent proton helicity that cancels them; hence these terms cannot forbid the reaction.

V. CONCLUSIONS

We have seen how the introduction of modifications from loop quantum gravity can affect and explain the anomalies observed in highly energetic phenomena such as cosmic rays. In particular, the notable appearance of helicity dependent decays could be a special footprint of this kind of effective theory.

Provided that the difference δA between A_γ and A_e does not affect the observations of the arrival of multi-TeV photons (as the ultimate analysis shows), we have the strong possibility granted by Eq. (8),

$$E_\pm^2=p^2[A_\gamma^2\pm2\,\theta_\gamma(l_p p)],$$

to be on the edge of observing polarized multi-TeV photons. Briefly, the actual universe could be transparent for one helicity state (while not for the other), nearly over the TeV range. The specific helicity necessarily depends on the sign of θ_γ and for the moment no related observations can decide this sign.

Likewise, there also is the possibility that we have been observing polarized protons in the form of GZK limit violating events. For these helicity effects to take place, it is necessary that both A and η be universal parameters as opposed to λ, which would need to respect Eq. (46). This last assumption appears to be a little forced. Nevertheless, faced with the fact that these terms depend on the parity and CPT violation structure of the theory and hence the helicity degeneracy of states is broken, we must take this possibility seriously. For instance, it is enough to note the great difference that, in what concerns these helicity terms, must exist between particles of spin zero and fractional spin.

Summarizing, the GZK limit can be violated by A, η, and λ in three different ways. First, by having a nonuniversal A parameter up to second order in $l_p/\mathcal{L}$ ($\Upsilon=0$ in the case of photons) in such a way that $A_p<A_\Delta$ and $A_p<A_\pi$; in this case, from the bounds (31) and (57), the favored range for $\mathcal{L}$ is

$$4.6\times10^{-17}\ \ \mathrm{eV}^{-1}\gtrsim\mathcal{L}\gtrsim8.3\times10^{-18}\ \ \mathrm{eV}^{-1}. \tag{59}$$

Note, however, that this possibility necessarily excludes the existence of a λ term in the dispersion relations for fermions, since there would be fermions having velocities in the opposite direction from that of the momentum (up to $p=\lambda\simeq$ keV). This last assumption, at the same time, has the consequence that no parity violation (and therefore CPT violation) should be present in the fermionic part of the theory, at the level discussed.

Secondly, by having $\eta_p<\eta_\Delta$ with η a negative parameter; this case is more interesting since it fixes the sign of η

and, consequently, its effects could be studied in other high energy reactions with at least a little more knowledge of these corrections.

Thirdly, the already mentioned possibility of a helicity dependent violation of the limit needs, as in the previous case, a negative and universal η. The reason for this exotic combination of parameters is that for the photopion production to be forbidden it is only necessary to have a negative η, while for the resonant Δ decay, the η sign is not sufficient. For this last effect to take place, the length scale $\mathcal{L}$ needs to satisfy [from the bounds (34) and (48)]

$$3\times 10^{-13}\ \mathrm{eV}^{-1} \gtrsim \mathcal{L} \gtrsim 5.7\times 10^{-14}\ \mathrm{eV}^{-1}. \tag{60}$$

It is worth noting that the helicity dependent effects tend to favor a length scale around $\sim 2\times 10^{-13}\ \mathrm{eV}^{-1}$ or, if we prefer, a mass scale in the TeV range. This is the same tentative range found in other work related to gravity [23]. For example, recent work on compactification of extra dimensions [24,25] shows the possibility of defining a mass scale in the TeV range and, as commonly emphasized, it is on the edge of actual empirical observations [26]. A length scale range like Eq. (60) gives a λ value of

$$\lambda \sim 2.5\times 10^{-3}\ \mathrm{eV}. \tag{61}$$

As was noted in [27], a dispersion relation of the type

$$E^2 = p^2 + \lambda p + m^2, \tag{62}$$

with a value of $\lambda \geqslant 10^{-7}$ eV, should be discarded because of the extremely sensitive measurements made of the Lamb shift. However, in the present framework, since the Lamb shift depends primarily on details of the interaction between electrons and photons, we are compelled to wait for a complete interaction picture of the effective LQG theories to say something about the symmetries involved in a low energy effect like that. In this sense, our development is strictly valid for analyses made on asymptotically free particle states (as in the present work), where the effects of interactions are taken to be negligible, and kinematical considerations are valid.

Future experimental developments like the Auger array, the Extreme Universe Space Observatory (EUSO), and orbiting wide-angle light (OWL) collectors satellite detectors, will increase the precision and phenomenological description (such as a favored proton helicity) of these UHECR's.

Other related bounds for these parameters can also be established. Such is the case of gamma ray burst observations which could give sensitive results for the δA difference between photons and neutrinos [3] (see the Appendix), or neutrino oscillations, in which the universality between the different neutrino flavor LID parameters can be measured [28].

ACKNOWLEDGMENTS

The authors are grateful to S. Liberati and A. Ringwald for calling to our attention Refs. [20] and [12,14], and also to S. Gálvez for his help with the betterment of the text paper. The work of J.A. was partially supported by Fondecyt 1010967. The work of G.P. was partially supported by CONICYT.

APPENDIX: TIME DELAY BETWEEN PHOTONS AND NEUTRINOS FROM GRB

The prediction of 10^{14}–10^{19} eV neutrino bursts generated in GRB events [29,30], opens the interesting possibility of observing a time delay between the arrival of photons and neutrinos. For instance, taking into account the range predicted in Eq. (59)—which gives an A difference of $\delta A \approx 10^{-22}$—the time delay from a typical source at 40 Mpc in a flat Friedmann–Robertson-Walker (FRW) universe, should be $\delta t \sim 10^{-6}$ s. This result may be compared with the corresponding one from [3], where, for the same distance, it is found that $\delta t \sim 0.4\times 10^{2}$ s. The great discrepancy can be understood not only on the ground of having different magnitude and expressions for δA in terms of the $\mathcal{L}$ parameter, but also by the fact that in [3] the length scale $\mathcal{L}$ was taken to be a mobile scale which sets a cutoff value for the momenta involved ($\mathcal{L} \lesssim 1/p$) given the specific physical situation. In this paper we have considered that $\mathcal{L}$ is a universal length scale. From this point of view, since the length scale is not mobile, we have included the possibility $p > 1/\mathcal{L}$ (in which the LQG structure of the region $l_p \lesssim d \lesssim \mathcal{L}$ is present through its effects), and therefore the actual results are different.

For completeness, let us show the time delay contributions (in a flat FRW universe) from the most significant terms of the dispersion relation for neutrinos. These delays are considered with respect to the arrival of photons with a conventionally rescaled $A_\gamma \equiv 1$.

A term:

$$\delta t_A = \frac{2|\delta A|}{H_0}[1-(1+z)^{-1/2}]. \tag{A1}$$

η_ν term:

$$\delta t_\eta = \frac{|\eta_\nu| p_0^2}{H_0}[(1+z)^{3/2}-1]. \tag{A2}$$

Additionally, there will be a time delay between photons of different helicities [4] [to follow the later convention, we take $(v_+ + v_-)/2 = A_\gamma \equiv 1$, where $v_\pm = A_\gamma \pm 2\theta_\gamma l_p p$]

θ_γ term:

$$\delta t_\pm = \frac{8|\theta_\gamma| l_p p_0}{H_0}[(1+z)^{1/2}-1]. \tag{A3}$$

In Eqs. (A1), (A2), and (A3), p_0 is the momentum (or energy) of the arriving particles, H_0 is the Hubble constant, and z is the source redshift. The above results can be used to analyze the GRB spectral structure in more detail and give additional bounds to the current parameters. Present observations cannot give such bounds.

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