Limits on cosmological variation of strong interaction and quark masses from big bang nucleosynthesis, cosmic, laboratory and Oklo data

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Recent data on the cosmological variation of the electromagnetic fine structure constant from distant quasar (QSO) absorption spectra have inspired a more general discussion of the possible variation of other constants. We discuss the variation of strong scale and quark masses. We derive limits on their relative change from (i) primordial big bang nucleosynthesis, (ii) the Oklo natural nuclear reactor, (iii) quasar absorption spectra, and (iv) laboratory measurements of hyperfine intervals.

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I. INTRODUCTION

Time variation of the major constants of physics is an old and fascinating topic; its discussion by many great physicists—Dirac as the most famous example—has surfaced many times in the past. Recent attention to this issue was caused by astronomical data which seem to suggest a variation of the electromagnetic α at the 10^{-5} level for the time scale 10 bn years; see [1] (a discussion of other limits can be found in Ref. [2] and below). The issue discussed in this work is related to it, although indirectly. Instead of looking into atomic spectra and testing the stability of the electric charge, we will discuss possible variations of *nuclear* properties induced by a change in strong and weak scales. We will not go into a theoretical discussion of why such changes may occur and how they can be related to modification of the electromagnetic α. Our aim is to identify the most stringent phenomenological limitations on such a change, at (i) a time of the order of few minutes, when the big bang nucleosynthesis (BBN) took place, as well as (ii) at the time of the Oklo natural nuclear reactor (1.8 bn years ago), (iii) when quasar radiation has been absorbed in the most distant gas clouds (3–10 bn years ago), and (iv) at the present time.

Mentioning the relevant literature we start with the BBN limits on the electromagnetic α, obtained in [3]. The main results come from variation of late-time nuclear reactions. Because of the low temperatures and velocities involved at this stage, those reactions have quite a significant suppression due to Coulomb barriers, in spite of the fact that only $Z=1-3$ is involved. These limits are in the following range:

$$|\delta\alpha|^{BBN}/\alpha < 0.02. \tag{1}$$

In general, all models for time variations of electromagnetic-weak-strong interactions can be divided into two distinct classes, depending on whether it originates in (i) *infrared* or (ii) *ultraviolet*. The former approach ascribe variations to some hypothetical interaction of the corresponding gauge bosons with some matter in the universe, such as vacuum expectation values (VEVs) or ‘‘condensates’’ of some scalar fields. Those typically have zero momentum but can have cosmological time dependence. We will not discuss it: for recent examples and references see [4].

We would, however, mention a few details from two recent examples of the latter approach by Calmet and Fritzsch [5] and Langacker, Segre, and Strassler [6]. Their main assumption is that grand unification [7] of electromagnetic, weak, and strong forces holds *at any time*. Therefore, a relation between all three coupling constants exists: the truly modified two parameters are in this approach the *unification scale*[1] Λ_{GUT} and *the value of the unified coupling* α_{GUT} at this scale. Their time variation is assumed to propagate down the scales by the usual (unmodified) renormalization group.

If this assumption is correct, any variation of the electromagnetic α should be accompanied by a variation of strong and weak couplings as well. Specific predictions need a model; we will mention the one discussed in [6]. In it, the QCD scale Λ_{QCD} (determined as usual by a continuation of the running coupling constant into its—unphysical—Landau pole) is modified as follows:

$$\frac{\delta\Lambda_{QCD}}{\Lambda_{QCD}} \approx 34\frac{\delta\alpha}{\alpha}. \tag{2}$$

Another focus of our work is possible limits on cosmological modifications of *quark masses*. According to the standard model, they are related to the electroweak symmetry breaking scale, as well as to some Yukawa couplings h_i. In [6] running of those has been considered, with a (model-dependent) conclusion that the quark mass indeed may have a different (and stronger) change:

[1]One might think that if the grand unified theory (GUT) scale is used to set units, its variation would be impossible to detect without explicit measurements related to gravity. But it is not so, since the cosmological expansion itself (which is quite important for BBN) contains Newton’s constant (or the Plank mass) in the Hubble constant.

$$\frac{\delta m_q}{m_q}\approx 70\frac{\delta\alpha}{\alpha}. \tag{3}$$

Large coefficients in these expressions are generic for GUT and other approaches, in which modifications come from high scales: they appear because weak and strong couplings run more.

If the coefficients of such a magnitude are indeed there, at the BBN time of a few minutes the QCD scale and quark masses would be modified quite a bit, if the upper limit (1) were used on the right-hand side (RHS).

The type of questions we are trying to answer in this work are the following: Do we know whether it might or might not actually happened? Which simultaneous change of strong and weak interaction scales is or is not observable? What observables are the most useful ones for that purpose? What are the actual limits on their variation which can be determined from BBN and other cosmological and laboratory data?

Let us repeat that although we use the above-mentioned papers as a motivation, we do not rely on any particular model. Nevertheless, we will at the end of the paper return to these predictions in order to see whether our limits on time variation imply stronger or weaker effects than the electromagnetic ones.

II. ROLE OF HEAVY AND STRANGE QUARK MASSES IN HADRONIC AND NUCLEAR OBSERVABLES

Both papers just mentioned [5,6] argue for what we would call the *zeroth* approximation to QCD modification. It assumes that the QCD scale Λ_{QCD} is so dominant in all hadronic and nuclear phenomena that all dimensional parameters—hadronic masses, magnetic moments, energies of nuclear levels, etc.—are to a good approximation simply proportional to its respective powers.

If so, *any* time variation of the overall strong interaction scale would *not* change dimensionless quantities (such as mass ratios or g factors) which we can observe. The absolute scale of hadronic and nuclear spectroscopy may change, but even if we were able to observe it from cosmological distances, such a modification would easily be confused with the overall redshift.

Fortunately, this pessimistic situation[2] is in fact rather far from reality. Quark masses do play significant role in hadronic and nuclear physics, and if they have time modification different from that of Λ_{QCD}, those can be detected.

Logistically it is convenient to start with masses of heavy—c,b,t— quarks. They do play a role in the running of the strong charge, from high scales down, changing the beta function each time a corresponding mass scale is passed. However, as is well known, those effects can be readily absorbed in a redefinition of Λ_{QCD}.

If strong coupling is the same at some high normalization point M, between m_b and m_t, the corresponding relations between Λ_{QCD} with all experimental quarks and *without* heavy c,b,t quarks is as follows:

$$\Lambda_{without\ c,b,t}=\Lambda_{QCD}\left(\frac{M^2}{m_c m_b}\right)^{2/27}. \tag{4}$$

Note that rather small powers of the masses are involved in this relation. In effect, we indeed may pretend that c,b,t quarks do not exist at all, as far as basic hadronic or nuclear physics is concerned.

The situation is completely different with the next quark flavor we have to discuss, the *strange quark*. It is still true that if one fixes the strong coupling $\alpha_s(k)$ at some sufficiently high scale[3] and then considers the role of nonzero m_s in the perturbative beta function, the effect is negligible (since its scale is too low for QCD to be passed by).

But hadronic and nuclear masses and properties are not determined by perturbative diagrams, leading to a beta function: they are of course determined by a much more complicated *nonperturbative* dynamics. Although it is far from being completely understood, it is clear that it does indeed depend strongly on quark masses. In particular, the "strange part" of the vacuum energy density can be estimated, because the derivative of the vacuum energy,

$$\frac{\partial\epsilon_{vac}}{\partial m_s}=\langle 0|\bar{s}s|0\rangle\approx -1.4\ \ \mathrm{fm}^{-3}, \tag{5}$$

is known.[4] Thus the linear term in the strange part of the vacuum energy

$$\epsilon_s=m_s\langle\bar{s}s\rangle\approx -0.2\ \ \mathrm{GeV/fm^3}, \tag{6}$$

is not negligible compared to gluonic vacuum energy [8]

$$\epsilon_g=-\frac{(11/3)N_c-(2/3)N_f}{128\pi^2}\langle 0|(gG^a_{\mu\nu})^2|0\rangle \tag{7}$$

$$\approx -(0.5\text{–}1)\ \ \mathrm{GeV/fm^3}, \tag{8}$$

making 20%–40% of it. (The numerator in this expression is the familiar coefficient of the QCD beta function, with $N_c=3$ and $N_f=3$ being the number of colors and relevant flavors. It appears because this expression, known as the scale anomaly to lowest order, has the same origin as the beta function itself.)

[2]Motivated historically by the large number of colors limit or quenched lattice QCD. Both may be reasonable starting approximations, which are not expected to be really accurate.

[3]It is in fact done on the lattice, where k is the inverse lattice spacing, typically 2–3 GeV.

[4]For definiteness, this number had come from the QCD sum rules, which typically correspond to operator normalization at $\mu=1$ GeV. The lattice numbers are similar, but normalized at inverse lattice spacing, typically $\mu=2$ GeV. Anomalous dimension of this operator lead to small difference between two normalizations which we ignore here.

Furthermore, for the nucleon one finds that similar "strange fractions" of their masses are of the same magnitude, e.g.,[5]

$$\frac{\partial m_N}{\partial m_s} = \langle N|\overline{s}s|N\rangle \approx 1.5. \tag{9}$$

Putting into linear expansion the strange quark mass $m_s = 120–140$ MeV one finds that about 1/5 of the nucleon mass comes from the "strange term."

[Although in this paper we cannot go into discussion of why it is the case, let us make a small digression. First of all, the reader should not be confused with the fact that only a very small fraction of the *energy* of a fast moving nucleon is due to the strange sea, as experimentally measured partonic densities tell us. Such a drastic difference between vectorlike (or chiral-even) and scalarlike (or chiral-odd) operators is a very common feature of nonperturbative QCD. Its origin is related to the dominance of the instanton-induced 't Hooft interaction; see [10]. In short, it happens because topological tunneling events should necessarily involve *all* light fermion flavors.]

Returning to cosmology, we conclude that due to strange terms, any variation of quark masses would imply significant modification of hadronic masses and other properties. It remains a challenging task for model builders and lattice practitioners to establish to what extent the $O(m_s)$ part of hadronic masses is or is not universal.

In principle, what we would call the *most pessimistic scenario* is possible, in which Λ_{QCD} and m_s enter into *all* hadronic observables in one combination:

$$\Lambda_{eff} = \Lambda_{QCD} + K m_s, \tag{10}$$

where K is some *universal* constant. If such a scenario happens to be true, its time modification can indeed be neutralized by a change of units, since *experiments can only measure dimensionless ratios*.

In fact, when lattice practitioners express the obtained results in terms of so-called "physical units,"[6] the dependence on m_s indeed tends to become weaker.

However, at the moment this is just a hint, with an accuracy not better than, say, 10%, and there is no reason to expect this scenario to be the case. We mentioned such a pessimistic case provocatively, emphasizing that at the moment we lack a solid theory which would explain how any particular hadronic observable depends on m_s. In general, *variation of m_s alone* can noticeably influence strong interaction parameters, since different quantities in general depends differently on it. Let us give an example.

One of the most important quantity for astronomical and laboratory experiments is the magnetic moments of nuclei. For example, the ratio of hyperfine splitting to molecular rotational intervals is proportional to $\alpha^2 g$ where g is defined by the magnetic moment

$$\mu = \frac{g e \hbar}{2 m_p c}. \tag{11}$$

In zeroth approximation as well as in the most pessimistic scenario discussed above, only one-dimensional parameters (Λ_{QCD} and $\Lambda_{QCD} + K m_s$, respectively) exist, so a dimensionless g factor cannot have any time variation. But, we repeat, there is no general argument for such approximations to be accurate. The magnetic moment and the nucleon mass are not directly related to each other at the QCD level,[7] and the dimensionless derivative

$$\mu_s^N \equiv \frac{\partial(1/\mu_N)}{\partial m_s} \tag{12}$$

remains unknown even for the proton and neutron, to say nothing about the composite nuclei. If it is not the same as derivative (9) for the nucleon mass, any time variation of m_s/Λ would induce a variation of the nuclear g factors.

The role of light quark masses is another issue, and a part of magnetic moments related to the contribution of the so-called "pion cloud" will be briefly discussed in Sec. XI.

In summary, any dimensionless ratios should be viewed as a function of the ratio

$$g(t) = g\left(\frac{m_s(t)}{\Lambda_{QCD}(t)}\right), \tag{13}$$

which for small variations is reduced to partial derivatives such as mentioned above.

III. ROLE OF LIGHT QUARK MASSES

Unlike the strange quark mass, we have a more solid theory explaining what the effect of a change of *light quark masses* m_u, m_d relative to that of the strong scale can be. The main focus of this work is this particular variation, which at the end we will be able to constrain rather well.

[5]The reader may find a discussion of the phenomenological situation in the first paper from [9], while the second contains lattice calculations of this quantity. For reference, their conclusion is that the (RHS) of Eq. (9) is 1.53 ± 0.07, with the errors being statistical only.

[6]Those units are defined by some nonperturbative observable such as (i) ρ-meson mass or (ii) the string tension, or (iii) the force between two pointlike charges at a fixed distance. All of those are measured on the lattice, with whatever quark masses one wants to have, and then put equal to its observable value in the real world *by decree*.

[7]For example, in the nonrelativistic quark model the nucleon mass is approximately 3 times the constituent quark mass, and the quark magnetic moment is given by the "quark magneton." However, even in this model there is also a binding energy and other corrections. A constituent quark itself is a complicated composite object, so one should not expect its magnetic moment to be exactly equal to the Dirac value related to its mass.

Since the pion is a Goldstone boson, its mass scales as a geometric mean between weak and strong scales[8] [11]

$$m_\pi^2 \sim (m_u + m_d)\Lambda_{QCD}. \tag{14}$$

Therefore the appropriate parameter characterizing the relative change of the pion mass ratio to the strong scale can be defined as

$$\delta_\pi = \delta\left(\frac{m_\pi}{\Lambda_{QCD}}\right) \Bigg/ \left(\frac{m_\pi}{\Lambda_{QCD}}\right) = \frac{1}{2}\,\delta\left(\frac{m_q}{\Lambda_{QCD}}\right) \Bigg/ \left(\frac{m_q}{\Lambda_{QCD}}\right). \tag{15}$$

Because the pion mass determines the range of nuclear forces, its modification leads directly to changes in nuclear properties. The main question we would like to study is how such a change in the pion mass relative to that of other hadrons, described by a nonzero δ_π, is limited at the cosmological time when primordial nucleosynthesis took place.

The first limit on the relative change of quark masses has been put in [6],

$$-0.1 < \delta\left(\frac{m_n - m_p}{T_\nu}\right) < 0.02, \tag{16}$$

where both numerical values come from the observational uncertainty of He^4 production, and T_ν is the freeze-out temperature for neutrino-induced weak processes. This effect is, however, not very restrictive for the following reasons.

(i) The sensitivity of He^4 production to any variation is itself not very impressive. (Below we will discuss *d* or Li^7 yields which may vary by orders of magnitude and is much more sensitive.)

(ii) As correctly explained in [6], T_ν scales as

$$T_\nu \sim v^{4/3}/M_P^{1/3} \tag{17}$$

as a function of weak scale (Higgs VEV) v and Planck mass, while the $m_n - m_p$ variation is expected to come primarily from $\delta(m_d - m_u)$, with a smaller electromagnetic correction. Therefore, both numerator and denominator in Eq. (16) mostly reflect the same physics and thus a large portion of the modifications, if they exist, would tend to cancel in this combination.

IV. CRUDE LIMITS ON SCALE VARIATIONS AT BBN TIME

Before we come to specifics, let us explain our basic philosophy in the selection of the observables. To get the maximal sensitivity, one has to focus on phenomena which may vary by very large factors. Basically, there are two sources for that inside the dynamics which drives BBN. One is the *Boltzmann factor*, which may at late times reach ten orders of magnitude or more; the other is the *Gamow factor* due to Coulomb barriers, which may reach three to four orders of magnitude. So, naturally, production of the heaviest primordial nuclides—especially Li^7, which is sensitive to both of those—would be a most promising place to look.

We start, however, with a preliminary discussion of possible drastic changes when δ_π is of the order of several percent, and then return to more delicate limits on the scale variation.

Nucleosynthesis starts with two-nucleon states. Standard nuclear forces produce one bound state—the deuteron, pn with $T=0$, $J=1$—and a virtual triplet of states—pp, pn, nn with $T=1$, $S=L=J=0$.

If the pion was lighter at BBN time relative to its present value, it leads to better binding. Rather dramatic effects should have occurred if the virtual states unbound by standard nuclear forces were bound.

Specifically, the binding of a *(pp)* state, with its subsequent beta decay into the deuteron, could eliminate free protons, in obvious contradiction with observations [12,13]. Conditions for a binding of pn and nn states have been studied in [13,14]. These states may add new paths to nucleosynthesis.

Note, also, that if the pion was so much heavier than now that the deuteron gets unbound, no primordial nucleosynthesis could possibly proceed at all. We will not discuss those in this paper, as they are superseded by modification of the deuteron discussed below.

As the only exception to this rule, we briefly discuss another important bottleneck on the way toward heavy nuclides, the absence of $A=5$ bound states. We have shown below that with $\delta_\pi \approx < -0.052$ it can be bridged in a modified world.

V. DEFINING THE TWO MOST IMPORTANT TEMPERATURES OF BBN

As we will argue throughout this work that the most sensitive parameter in BBN remains the binding energy of the deuteron $|E_d|$ (E_d is negative), we provide a (very brief) account of BBN, with emphasis on the role of deuterons. For details of the evolution and the role of all processes involved the reader should consult appropriate reviews, e.g., [15,16].

When the basic reaction producing deuterons $p+n \rightarrow d + \gamma$ is in equilibrium at high enough *T*, the density of deuterons relative to photons is of the order of

$$\frac{n_d}{n_\gamma} \sim \eta^2 \exp(|E_d|/T). \tag{18}$$

Here $\eta \approx 3\times 10^{-10}$ is the famous primordial baryon-to-photon ratio. Although it remains unknown what created it and even at which stage of the bing bang it happened, we will assume that it happens early and do not consider its variations.

So the density of deuterium remains negligible small until the Boltzmann factor (needed for the photons to split *d*)

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[8] The QCD anomalous dimension of the quark mass is canceled by the opposite one of the quark condensate, in the Gell-Mann–Oakes–Renner (GOR) relation: thus quark masses here are meant with the dependence on the normalization point removed. The same remark should also be made about the term with the strange quark. (We thank T. Dent, who reminded us that this comment is needed.)

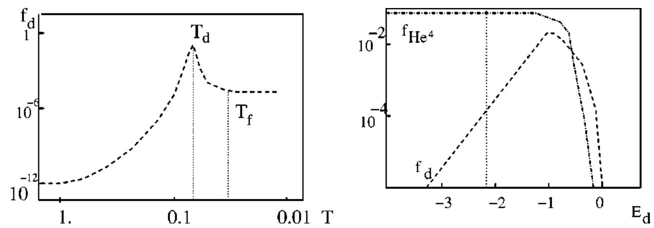

FIG. 1. (a) Schematic dependence of the deuterium mass fraction f_d on temperature T (MeV). (b) Schematic dependence of the deuterium and He^4 mass fractions on the binding energy of the deuteron E_d (MeV). The vertical dotted line indicates its experimental value, -2.2 MeV.

helps. In particular, when this factor is so large than it can compensate one of the η, the deuteron fraction may be comparable to that of p,n. This condition determines the first crucial temperature value, to be denoted by T_d:

$$\eta \exp(|E_d|/T_d) \sim 1. \tag{19}$$

Note that T_d will be modified below together with the value of E_d. At standard BBN parameters it is about 70 keV, at which n_d reaches its maximum; see Fig. 1(a).

What happens after *d* reaches this maximum, is significant *reduction* of the density of *d* (and other species) due to several reactions leading into the best bound light nuclei He^4. At such low T the equilibrium configuration would wipe out all of other nuclides. Indeed, an advantage in the binding of He^4 by more than 20 MeV is that the Boltzmann factor is enormous.

However, the universe expansion does not allow to reach such equilibrium. Its rate leads to the *final freeze-out* of all production reactions, the stage when reaction rates and the Hubble expansion rates are comparable. The Hubble rate, according to one of the Friedman equations for a flat universe and zero cosmological constant (not important so early anyway), is

$$H^2 \equiv (\dot{R}/R)^2 = (8\pi/3) G_N \epsilon, \tag{20}$$

where G_N and ϵ are Newton's constant and the matter energy density. Ignoring numerical constants, $H \sim T^2/M_P$, with M_P being the Planck mass, and ignoring the T dependence of the reaction rate itself, $\langle \sigma v \rangle_T$, one can obtain the freeze-out temperature

$$T_f \sim 1/(\eta M_P \langle \sigma v \rangle_{Tf}). \tag{21}$$

Note how the small parameter η fights with large M_P. For standard BBN it is $T_f \approx 35$ keV.

VI. HOW VARIATION OF THE DEUTERON BINDING AFFECTS BBN

Now we can proceed to discussion of how *modification* of the fundamental interactions would affect the BBN yields.

Let us start with *the most pessimistic case* when *both* the strong scale and quark masses are modified *identically*, so that this change can be eliminated from the discussion by simply using modified units. In such units all reactions rates, bindings, and T_d would be the same as in standard BBN. The only difference appears in the freeze-out time and temperature T_f. The reason is, as we changed the units to adjust to time varying scales, the value of the Planck mass entering relation (21) is modified instead.

We can estimate crudely the effect of that modification as follows. At the second stage of BBN, $T_d < T < T_f$, a decrease of d and Li^7 is roughly power like:

$$f^7_{d,\mathrm{Li}} \sim T^{-a}, \quad a = 6-7. \tag{22}$$

The fall is by about two orders of magnitude. Using this trend we conclude that modification of their yields within a factor of 2 (the magnitude of current error bars) corresponds to a certain change in T_f, which implies the following limits on variation of the strong scale (relative to gravity):

$$\left(\frac{\delta(\Lambda_{QCD}/M_P)}{(\Lambda_{QCD}/M_P)} \right) < 0.1. \tag{23}$$

We now switch to a discussion of the *relative* change between quark masses and hadronic scale, leading to modification of the pion-induced forces and, consequently, of the deuteron binding energy. We will look at modification of T_d, which is so important for BBN final observable yields.

A qualitative dependence of the yields of different species on the magnitude of the deuteron binding is schematically shown in Fig. 1(b). As T_d is allowed to vary, the first thing to note is that there exists an *optimum* for *d* production corresponding to the case when two crucial temperatures are close to each other:

$$T_f \approx T_d. \tag{24}$$

It is easy to see from the expressions given above that it happens if the deuteron binding is around $E_d \approx -1$ MeV and its fraction reaches at this time about a percent level.

When the value of T_d reduces further, so that the relation between the two temperatures is inverted and T_f *becomes smaller than* T_d, the stage at which d and other light nuclei are "eaten" by He^4 is no longer present. All heavier nuclides, t,He^3, He^4, etc., reach their final yields from below, basically tracing the deuteron production. There is no time for He^4 to grab most of the available neutrons anymore, and its final yield starts declining [see Fig. 1(b)]. Finally, when deuterons become nearly unbound, all of the primordial nucleosynthesis is wiped out altogether.

As the data tell us that the *d* fraction is at the level 10^{-4}–10^{-5}, BBN happened clearly away from the maximum. Furthermore, as the He^4 fraction is large, we are definitely at the left branch of the curve in Fig. 1(b). This shows exponential growth, and thus sensitivity of the results to the variation of E_d and T_d can be estimated from its slope, which we obtained from [16]. We conclude from this estimate that a change of the *d* yield by a factor of 2 corresponds to a constraint on the relative variation of the deuteron binding by

$$\left(\frac{\delta|E_d|}{|E_d|}\right)_d < 0.075. \tag{25}$$

VII. FURTHER LIMITATIONS AND THE COULOMB BARRIERS

Now we can discuss the refinement of the qualitative picture described above, in which the absolute change of the strong scale has been eliminated by a change of units, and the only weak effect included has been a quark mass. Now we let the electromagnetic effects come into the game, in their simplest and most important (exponential) form. We mean the Coulomb barriers, described by the well-known Gamow factors

$$f_{Gamow} = \frac{2\pi e^2 Z_1 Z_2}{v} \exp\left(-\frac{2\pi e^2 Z_1 Z_2}{v}\right). \tag{26}$$

Here v is the relative velocity, which scales as $(T/m)^{1/2}$. As explained above, the relevant T varies in between T_d and T_f. As the former one is modified, together with E_d, the Gamow factors change accordingly.

For standard BBN we estimated that the product of Gamow factors of reactions leading to Li^7 is about $\exp(14v_0/v)$, where v_0 is the unmodified value.[9] So, assuming that experimental data on Li^7 are restricted within a factor of 2, we see that only a variation of

$$\left(\frac{|\delta E_d|}{E_d}\right)_{Li^7} < 0.1 \tag{27}$$

can be tolerated. It leads to a limit on δ_π comparable to the one obtained above from the deuteron yield.

VIII. MODIFIED DEUTERON

In this section we try to relate the change of the deuteron binding energy, δE_d, to modification of the fundamental parameter δ_π introduced above.

In general, it is a very nontrivial dynamical issue, which is far from being really understood. We have discussed above to which extent the QCD vacuum energy (6) and the nucleon mass (9) depends on the quark masses. Maybe one day such information will be available from lattice QCD for nuclear forces as well, but right now we do not have it and have to rely on model-dependent potentials.

Of course, one can identify single-pion exchange forces. Especially for the deuteron channel, those lead to well-known tensor forces producing deuteron quadrupole moments. A textbook one-pion exchange corresponds to the following potential:

$$V_{1\pi} = \frac{f^2}{\tilde{m}_\pi^2}(\vec{\tau}_1\cdot\vec{\tau}_2)(\vec{\sigma}_1\cdot\vec{\partial})(\vec{\sigma}_2\cdot\vec{\partial})\frac{\exp(-m_\pi r)}{r}, \tag{28}$$

where $f^2 = 0.08$. The pion mass in denominator has a tilde: it indicates that this mass has been put there by hand for normalization purposes, and unlike masses in other places, it will *not* have any variations. (It has been put there in order to cancel the pion mass squared coming from differentiation of the exponent, which we will evaluate first.)

When the variation of the pion mass is sufficiently small, the variation is simply given in the first order of perturbation theory, $\delta E_d = \langle 0|\delta V|0\rangle$. For any weakly bound state the wave function *outside the potential range* can be approximated by the simple expression

$$\psi_0(r) = \sqrt{\frac{\kappa}{2\pi}}\exp(-\kappa r)/r, \tag{29}$$

where the parameter κ is related to the binding energy by $E_d = \kappa^2/m_N$. If we use this expression until the core size, we can easily evaluate it:

$$\frac{\partial E_d}{\partial m_\pi} = -2f^2\frac{\kappa}{m_\pi}\left[2\ln\frac{1}{(2\kappa+m_\pi)r_0} - \frac{m_\pi}{2\kappa+m_\pi}\right] \approx -0.05. \tag{30}$$

We therefore get, from these terms,

$$\frac{\delta|E_d|}{|E_d|} \approx 3\,\delta_\pi. \tag{31}$$

The sign tells us that reducing the pion mass we reduce the binding: this counterintuitive result comes from the m_π^2 in the preexponent.

One more one-pion-exchange term we have ignored is the one in which two derivatives act on $1/r$: this leads to the delta function. Naively it does not contribute since at $r=0$ the wave function is vanishingly small due to the repulsive core. This can be cured by an account of the finite size of the nucleon.

However, the main problem with the perturbative calculation outlined above is that the one-pion-exchange terms are actually several times smaller compared to the phenomenological potentials needed to reproduce NN scattering and the deuteron binding. Furthermore, the phenomenological potentials which fit scattering and d data actually ascribe most of the potential to two- and even three-pion exchanges. As an example, we used the well-known work by Hamada and Johnston [17], which has the central potential in the deuteron channel in the form of subsequent pion exchanges. We have only modified the pion mass in the exponents:

$$V(r) = V_c Y(x)[1 + a_c Y(x) + b_c Y(x)^2], \tag{32}$$

$$Y(x) = \frac{e^{-x(1+\delta_\pi)}}{x}, \tag{33}$$

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[9] We have also checked that this estimate agrees with the results by [3] of the dependence of its yield on α, which comes from running a compete BBN code.

where $x = r\tilde{m}_\pi$ is distance in units of *unmodified* pion mass, and $a_c = 6$, $b_c = -1$ [17]. The potential also has an infinite repulsive core with radius 0.4 fm.

We have ignored tensor and spin-orbit potentials as having a minor effect on the deuteron binding, adjusted the overall coefficient to have experimental binding energy -2.2 MeV. After that, we have modified the pion mass in Eq. (32), and calculated the binding energy by solving the *s*-wave Schrödinger equation. For the small pion modification we need, we found a good linear dependence with the following coefficient:

$$\frac{\delta|E_d|}{|E_d|} \approx -18\delta_\pi. \tag{34}$$

We have presented these two estimates as reasonable *minimal and maximal bounds* on this derivative. Presumably the true value is somewhere between 3 and -18.

The issue has been discussed in the literature. A general discussion of how all nuclear physics would change if quark masses, the number of flavors, or even the number of colors were modified can be found in [18]. Probably the latest paper addressing deuteron binding with modern methods is Ref. [19]. Like us, these authors have shown examples producing opposite sign of the derivative and conclude that, strictly speaking, neither the magnitude nor even the sign can be definitely obtained at this time. Further extensive lattice and chiral perturbation theory studies are needed.

If the derivative is small near zero, our arguments lose their weight. However, it would be very unnatural to obtain a small value of this derivative. The attractive and repulsive parts of the potential have already conspired to obtain the deuteron at the binding edge. *One more fine-tuning*, exactly at the physical quark mass values (which does not have any special meaning from the QCD point of view), is very unlikely to happen.

IX. BINDING OF He^5

We now return to the $He^4 + n$ channel mentioned above and evaluate which BBN limits on modification of the pion mass it will produce.

In this channel there is a $p_{3/2}$ resonance at energy 0.77 MeV. If reduction of the pion mass can make it bound, it will drastically enhance production of Li. Without bound He^5 BBN has to jump over $A = 5$, e.g., by a reaction $He^4 + t$, which is impaired by the Coulomb barrier as well as by a very low concentration of *t*.

We now study the sensitivity of He^5 binding to modifications of the nuclear potential. Before we discuss the calculations, let us make a couple of qualitative points. First of all, this case is more complicated compared to the deuteron, because we now discussed very weakly bound states, and so no simple linear dependence on δ_π is expected. Furthermore, let us point out an interesting quantum-mechanical distinction between low-lying levels with zero and nonzero angular momentum *l*. The wave function at large *r*, outside of the interaction range, for the zero-energy level can easily be found from the Schrödinger equation

$$\psi(r, E=0) \sim r^{-(1+l)}. \tag{35}$$

In the *s*-wave equation, $l = 0$, it is unacceptable because such a wave function is a nonnormalizable solution. In other words, keeping $\exp(-\kappa r)$ in Eq. (29) is crucial, no matter how small κ is. It is no longer so for $l = 1$: the tail of the wave function in question is now normalizable. In short, for $l > 0$ the centrifugal barrier keeps particles inside the attractive potential, contrary to the $l = 0$ states. This effect makes $l > 0$ states more sensitive to a change of the potential.

We have modeled the potential of the interaction between a neutron and He^4 in the following form:

$$V(r) = Q\frac{1 + w(r/c)^2}{\exp[(r-c)/\Delta] + 1}, \tag{36}$$

where $w = 0.445$, $\Delta = 0.327$ fm. If $c = 1.01$ fm, it is a good fit to the experimental density distribution in He^4 [20]. In the potential we simply changed it to $c = 1.01\ \text{fm} + 1/m_\pi$, with variable pion mass. The depth of the potential Q has been tuned to reproduce the position of the above-mentioned resonance; it gave $Q = -32.6$ MeV. After that we start changing the pion mass until this level becomes bound, $E = 0$. This happens at the magnitude of the pion mass modification:

$$\delta_\pi^{He^5} > -0.052. \tag{37}$$

If that would happen at BBN time, the yields of Li would be dramatically enhanced, by orders of magnitude, contrary to observations. (This is why it is written as an inequality.)

This value can be compared to the natural small parameter of the virtual He^5 level, $E_{res}/V \sim 0.02$. The needed shift is somewhat larger because, in spite of the general argument given above, its wave function is spread to large *r*; see Fig. 2.

Note that this limit (37) would become an order of magnitude weaker if we base our consideration on the direct effect of the one-pion exchange, Eq. (28)—see the discussion in the previous section. For He^5 there is an additional suppression because the average value of the potential (28) over a closed 1*s* shell is zero (the effect, however, appears due to the exchange interaction and correlation corrections).

As a result, we conclude that the limit from the He^5 binding cannot compete with that from the deuteron modification in its importance in BBN.

The next bottleneck, effectively blocking synthesis of heavy elements, is the gap at $A = 8$ nuclei, which too readily decay into two alpha particles. Although we have not investigated this case, we do not expect it to beat the limits related to deuteron modification as well.

X. LIMITATIONS FROM OKLO DATA

Finally, let us deviate from our discussion of BBN to a related subject: namely, similar limits following from data on the natural nuclear reactor in Oklo active about 2 bn years ago. The most sensitive phenomenon (used previously for limits on the variations of the electromagnetic α) is the disappearance of certain isotopes (especially Sm^{149}) possessing a neutron resonance close to zero [21]. Today the lowest

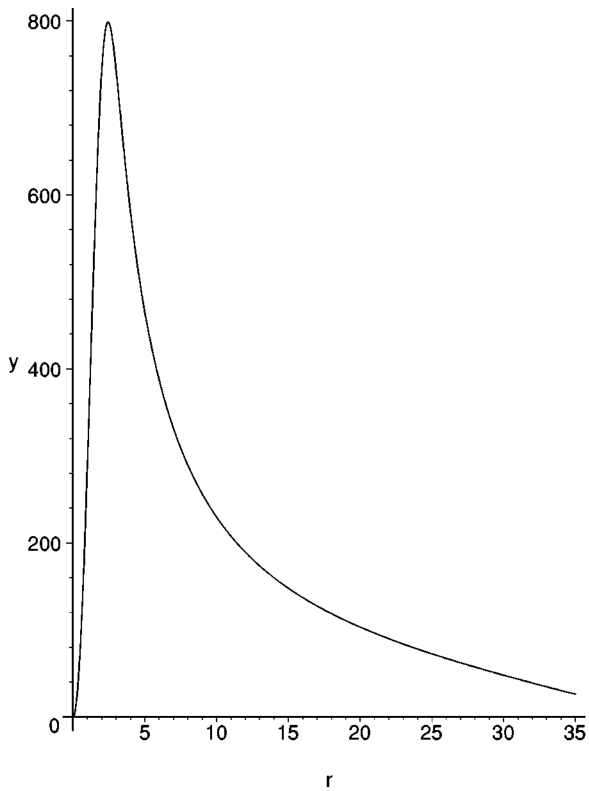

FIG. 2. The wave function for zero-energy neutrons in He^5 (arbitrary normalization) versus the distance *r*, in fm. Note that it is normalizable, although it does spread to rather large *r*.

resonance energy is only $E_0 = 0.0973 \pm 0.0002$ eV is larger compared to its width, so the neutron capture cross section $\sigma \sim 1/E_0^2$. The data constrain the ratio of this cross section to the nonresonance one. It therefore implies[10] that these data constrain the variation of the following ratio, $\delta(E_0/E_1)$, where $E_1 \sim 1$ MeV is a typical single-particle energy scale, which may be viewed as the energy of some one-body ''doorway'' state.

A generic expression for the level energy in terms of fundamental parameters of QCD can be written as follows:

$$E_i = A_i \Lambda_{QCD} + B_i m_q + C_i \alpha \Lambda_{QCD} + D_i (m_q \Lambda_{QCD})^{1/2} + \cdots, \quad (38)$$

where A_i, B_i, C_i, D_i are some coefficients. The first term is the basic QCD term, while others are corrections due to quark mass, pions, and electromagnetism. Without B_i, D_i terms one can see that in the E_0/E_1 ratio the QCD scale drops out, confirming our general statement above, that in such kinds of approximations the variation of Λ_{QCD} itself cannot be seen.

The sum is very small for E_0, just because we deliberately picked up the lowest resonance, but (and this is our main point) there is no reason to expect *each* term to be especially small. For example [21] the electromagnetic term is about 1 MeV.

Let us estimate what variation of the resonance energy would result from a modification of the pion mass. As we did above for the deuteron, we assume that the main effect comes from increase of the radius R of the nuclear potential well. The energy of the resonances $E_i = E_{excitation} - S_n$ consists of the excitation energy of a compound nucleus minus the neutron separation energy S_n. This, in turn, is a depth of the potential well V minus the neutron Fermi energy ϵ_F, $S_n = V - \epsilon_F$. The latter scales like $1/R^2$ if the radius of the well is changed. The kinetic part of the excitation energy $E_{excitation}$ scales in the same way. Adding both, one gets shift of the resonance

$$\delta E_i = -(\epsilon_F + E_{excitation}) \frac{2\,\delta R}{R}. \quad (39)$$

For resonance near zero the combination in brackets is approximately $V \sim 50$ MeV. If $R = 5$ fm$+1/m_\pi$, then $\delta R/R = -\delta m_\pi/(R m_\pi^2)$:

$$\left| \frac{\delta E_i}{E_i} \right| = 3 \times 10^8 |\delta_\pi| < 0.2. \quad (40)$$

The RHS above comes from the observational limits claimed in [21]. The resulting limitation on pion modification at time ≈ 1.8 bn years ago is

$$\delta_\pi^{Oklo} < 7 * 10^{-10}. \quad (41)$$

Note that the authors of the last work in [21] found also the nonzero solution $\delta E_i / E_i = -1 \pm 0.1$. This solution corresponds to the same resonance moved below the thermal neutron energy. In this case $\delta_\pi \approx -4 \times 10^{-9}$. In principle, the total number of the solutions can be very large since Sm^{149} nucleus has millions of resonances and each of them can provide two new solutions (thermal neutron energy on the right tale or left tale of the resonance). However, these extra solutions are probably excluded by the measurements of the neutron capture cross-sections for other nuclei since no significant changes have been observed there also; see [21].

As in the cases of the deuteron and He^5 binding one may argue that the limits on δ_π presented above should be weaker by an order of magnitude since the direct contribution of the one-pion exchange, Eq. (28), to the energy is small. To clarify this point it may be useful to perform a numerical calculation of this contribution to the neutron separation energy in Sm^{150} and He^5.

XI. LIMITS FROM ASTROPHYSICAL AND LABORATORY MEASUREMENTS

Comparison of the atomic H 21 cm (hyperfine) transition with molecular rotational transitions gave the following limits on $Y \equiv \alpha^2 g_p$ [22]: $\delta Y/Y = (-0.20 \pm 0.44) \times 10^{-5}$ for redshift $z = 0.2467$ and $\delta Y/Y = (-0.16 \pm 0.54) \times 10^{-5}$ for $z = 0.6847$. The second limit corresponds to roughly $t = 6$ bn years ago.

[10]Of course, under the assumption that the *same* resonance was the lowest one at the time of the Oklo reactor.

As we have already discussed above, only in the *most pessimistic scenario* do all strong interaction phenomena depend on only one parameter, e.g., $\Lambda_{eff}=\Lambda_{QCD}+Km_s$; its time variation cannot change dimensionless quantities like the proton magnetic g factor g_p. If so, the limits given above are just limits on variation of α. However, in general there is no reason to think this to be the case, and one may wonder which limits can be put from these data on a cosmological variation of m_s/Λ_{QCD}.

Another issue here is a contribution proportional to the light quark masses m_u, m_d. Let us first make a qualitative point, suggesting that their role in magnetic moments is expected to be *larger* than in hadronic masses. Hadrons are surrounded by the so-called "pion cloud," which have small virtual momenta p. Masses depends on it in the form $\sqrt{p^2+m_\pi^2}$ while magnetic moments have $\vec{p}\times\vec{r}$, r being the distance from the center. Small masses are partly compensated by large r in the latter but not former case.

Model-dependent estimates support this idea. The magnitude of the effect varies a lot between models, and as an example we use rather the conservative treatment by Sato and Sawada [23]. It can be seen as a minimal estimate: they identified the contribution of small virtual momenta $p<\Lambda$ by using form factors $\sim\Lambda^2/(\Lambda^2+p^2)$, and have shown that a consistent picture for p,n,d and hyperon magnetic moment emerges if the cutoff is $\Lambda\sim m_\pi$. Furthermore, this contribution is shown to be basically proportional to $\Lambda^2\sim m_\pi^2\sim m_q$. They found that the cloud contribution is about 1% of the proton magnetic moment, but 7.3% for the neutron (to be compared to the 1% level for the masses).

We conclude that at least due to the pion cloud effect one should expect that the gyromagnetic ratios g for nuclei have a term proportional to m_{light}/Λ_{QCD} contributing of the order of several percent. Combining it with the contribution proportional to m_s/Λ_{QCD} (which we hope does not exactly conspire with the effect on masses to cancel completely), we expect that the overall effect at the level of 1/10 of g is of this origin.

Assuming that it has such a magnitude, that a variation of alpha and m_q/Λ_{QCD} does not conspire to produce observed zero, and a simplest linear dependence

$$g_p=g_p(m_q=0)\left(1+q\,\frac{m_q}{\Lambda_{QCD}}\right), \qquad (42)$$

we may interpret the above-mentioned limits as the following on variation of this ratio:

$$\left|\delta\left(\frac{m_q}{\Lambda_{QCD}}\right)\Big/\left(\frac{m_q}{\Lambda_{QCD}}\right)\right|<10^{-4}. \qquad (43)$$

This should be compared with the limit on $X\equiv\alpha^2 g_p m_e/m_p$ [24] $\delta X/X=(0.7\pm1.1)\times10^{-5}$ for $z=1.8$. This limit was interpreted as a limit on variation of α or m_e/m_p. It also can be viewed as a limit on variation of m_q/Λ_{QCD}. Although few times weaker ($\sim2\times10^{-4}$) than the limit (43), it corresponds to a higher redshift.

The limits on variation of m_q/Λ_{QCD} can also be obtained from laboratory measurements of *ratios of hyperfine splittings*. By comparison the rates of two clocks based on different atoms, say, H and Hg^+ [25], we compare[11] g factors of quite different nuclei [26]. In terms of the standard shell model description, this gives the ratio of proton and neutron spin g factors. Other examples, such as using the hyperfine transition in Cs as a frequency standard, would also involve the orbital g factor, with $g_l=1$. In principle, corrections to the shell model include "exchange currents" which also ontribute to magnetic moments. All of the above may have different dependence on m_q/Λ_{QCD}, and we conclude that

$$\frac{d}{dt}\ln\frac{A_1}{A_2}=K\frac{d}{dt}\ln\frac{m_q}{\Lambda_{QCD}}, \qquad (44)$$

where A_1, A_2 are hyperfine structure constants of different atoms and where K, a combination of derivatives (9), (12), remains unknown but can be as big as 1/10 or even larger. Using such tentative value and H, Cs, and Hg^+ measurements [25,27], we obtain the limit on variation of m_q/Λ_{QCD} of about 5×10^{-13} per year.

XII. SUMMARY

Combining our strongest limits on the deuteron binding, from deuteron, Eq. (25), and Li^7, Eq. (27), corresponds to variation of their production by a factor of 2, with a relation between modification of the deuteron binding and modification of the pion mass, Eq. (34). Both effects suggest about the same BBN limit on modification of the pion mass *relative to the strong interaction scale* Λ_{QCD}.

Using Eq. (34) we obtain

$$|\delta_\pi|_{BBN}<0.005. \qquad (45)$$

Equation (31) provides a more conservative limit

$$|\delta_\pi|_{BBN}<0.03, \qquad (46)$$

and we think the true limit is somewhere in between.

We have investigated other effects, such as binding of He^5 or pp,nn,np ($S=0$) states, but found that in these cases the needed pion modification about an order of magnitude larger. (It is expected, since all these states are more loosely bound than the deuteron.) If m_s modification relative to the strong scale are as large as our limit on light quark modification just mentioned, it means that the nucleon mass can be modified within ±2 MeV due to the strange term. Note also that our limit on quark mass modification is stronger than the limit [6] coming from the proton-neutron mass difference (16).

We also pointed out significantly weaker limits on a *simultaneous* modification of the strong scale and m_q scale at the same rate, relative to the gravity scale:

$$\frac{\delta(\Lambda_{QCD}/M_P)}{(\Lambda_{QCD}/M_P)}<0.1. \qquad (47)$$

[11]Note that this comparison gives limits on variation of α only due to relativistic corrections to Hg^+.

Limits on a variation of the quark mass relative to strong scale at the 10^{-4} level 3–10 bn years ago follows from observations of distant objects (43), while at the time 1.8 bn years ago the Oklo data lead to even better limits, at the 10^{-8}–10^{-9} level.

Although there is no general relation between variation of weak, strong, and electromagnetic constants, as we mention in the Introduction it is implied by grand unification [5,6]. If one uses those (2),(3), one finds that all our limits on relative weak and strong modification are *much more restrictive* than the corresponding limits on the modification of the electromagnetic α. In the case of astronomical observations, in which variation of the alpha seems to be seen, one may either soon find the variation of g factors or rule out relations between couplings based on the grand unification idea.

Finally, let us emphasize that our discussion is semiqualitative in many aspects, and a lot of quantitative work remains to be done. Theorywise, the most straightforward thing to do is to add modifications directly into the BBN code, and get more quantitative limits. Although there seems to be no particular problem with the standard BBN at the moment, it is still true that the calculated yields and observations typically differ by one to two standard deviations [15]. Therefore it seem to be worthwhile to make a global fit to data with unrestricted modification parameters (like our δ_π) and see whether a zero value would or would not be the best one.

Another challenge to the theory, probably mostly lattice simulations, is to clarify the issue of the dependence of various hadronic parameters on the strange quark mass m_s, especially how universal are the derivatives like Eq. (9) for all hadrons.

Experimental laboratory work and astronomical observations of distant objects can significantly enhance the limits available today, hopefully with a nonzero effect eventually observed.

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