

# Reexamining radiative decays of $1^{--}$ quarkonium into $\eta'$ and $\eta$

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Recently CLEO has studied the radiative decay of  $Y$  into  $\eta'$  and an upper limit for the decay has been determined. Confronted with this upper limit, most theoretical predictions for the decay fail. After briefly reviewing these predictions we reexamine the decay by separating the nonperturbative effect related to the quarkonium and that related to  $\eta'$  or  $\eta$ , in which the latter is parametrized by distribution amplitudes of gluons in  $\eta'$ . With this factorization approach we obtain theoretical predictions which are in agreement with experiment. Uncertainties in our predictions are discussed. The largest possible uncertainties are from the relativistic corrections for  $J/\psi$  and the value of the charm quark mass. We argue that the effect of these uncertainties can be reduced by using quarkonium masses instead of using quark masses. An example of the reduction is shown with an attempt to explain the violation of the famous 14% rule in radiative decays of charmonia.

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The gluon content of  $\eta$  and  $\eta'$  has been studied extensively in the literature. For example, recent works on the subject can be found in [1,2]. Radiative decays of  $1^{--}$  quarkonium into  $\eta(\eta')$  provide an ideal place to study this subject, because the decays are mediated by gluons and there is no complication of interactions between light hadrons. Recently, CLEO has studied the decay  $Y \rightarrow \gamma + \eta'$  and an upper limit has been determined [3]:

$$\text{Br}(Y \rightarrow \gamma + \eta') < 1.6 \times 10^{-5} \quad (1)$$

at 90% C.L. With this result most theoretical predictions deliver a branching ratio which is too large.

The radiative decay has been studied in different approaches. In [4] both the quarkonium and  $\eta(\eta')$  are taken to be nonrelativistic two-body systems; wave functions for these bound systems are introduced. The obtained branching ratio in this approach with a recent compilation of  $\alpha_s$  is  $(5 - 10) \times 10^{-5}$  [4,3] and is significantly larger than the upper limit. In [5] possible mixing between  $\eta(\eta')$  and  $\eta_b$  is assumed to be responsible for the decay; the branching ratio is obtained as  $6 \times 10^{-5}$ , which is also larger than the upper limit. In this approach it is possible to obtain  $\text{Br}(Y \rightarrow \gamma + \eta') \approx (1 \sim 3) \times 10^{-5}$  close to the upper bound [6].

The corresponding decay of  $J/\psi$  has been studied by saturating a suitable sum rule with  $J/\psi$  resonance and it has been shown that the decay is controlled by the  $U_A(1)$  anomaly [7]. The result of this study can be rewritten in the form:

$$\Gamma(J/\psi \rightarrow \gamma + \eta') = \frac{2^{11} \pi \alpha^3}{5^2 \times 3^{12}} \left(1 - \frac{m_{\eta'}^2}{4m_c^2}\right)^3 \times \frac{1}{m_c^4} \left| \langle 0 | \frac{3\alpha_s}{4\pi} G_{\mu\nu}^a \tilde{G}^{a,\mu\nu} | \eta' \rangle \right|^2 \Gamma^{-1}(J/\psi \rightarrow e^+ e^-), \quad (2)$$

where  $G^{a,\mu\nu}$  is the field strength tensor of the gluon and  $\tilde{G}^{a,\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\alpha\beta} G_{\alpha\beta}^a$ . In the above result we have neglected the binding energy of  $J/\psi$  and taken  $M_{J/\psi} = 2m_c$ , where  $m_c$

is the pole mass of  $c$  quark. If one can generalize the approach for the  $Y$  decays, one can obtain the ratio

$$R_{\eta'} = \frac{\text{Br}(Y \rightarrow \gamma + \eta')}{\text{Br}(J/\psi \rightarrow \gamma + \eta')} = \left[ \frac{\Gamma(J/\psi \rightarrow e^+ e^-)}{\Gamma(Y \rightarrow e^+ e^-)} \frac{\Gamma(J/\psi \rightarrow X)}{\Gamma(Y \rightarrow X)} \right] \frac{m_c^4}{m_b^4} \left( \frac{1 - \frac{m_{\eta'}^2}{4m_b^2}}{1 - \frac{m_{\eta'}^2}{4m_c^2}} \right)^3. \quad (3)$$

Using the experimental results for the widths in the brackets we obtain

$$R_{\eta'} \approx 6.6 \frac{m_c^4}{m_b^4} \left( \frac{1 - \frac{m_{\eta'}^2}{4m_b^2}}{1 - \frac{m_{\eta'}^2}{4m_c^2}} \right)^3. \quad (4)$$

The branching ratio of  $J/\psi$  has been measured and its value is  $(4.31 \pm 0.3) \times 10^{-3}$ . We take the quark masses as  $m_b = M_Y/2 \approx 5$  GeV and  $m_c = M_{J/\psi}/2 \approx 1.5$  GeV and obtain  $\text{Br}(Y \rightarrow \gamma + \eta') \approx 3.1 \times 10^{-4}$ , which is too large for the upper limit. However, the generalization of Eq. (2) to  $Y$  may not be correct. In the spirit of the approach the emitted gluons, which are converted into  $\eta'$ , are soft, while in  $Y$  decay the gluons are definitely hard. Because  $m_b$  is large, large perturbative and nonperturbative corrections are expected in the generalization of Eq. (2) for  $J/\psi$  to the case with  $Y$ ; also the scale dependence of the perturbative correction can be significant. Hence an accurate result cannot be made for  $Y$  without these corrections. Employing multipole expansion for the soft gluons one is also able to predict the decay of  $J/\psi$  [8].

From the above discussions one may conclude that the predictions based on QCD-inspired models or on sum rule are not compatible with the upper limit, or significant modifications are needed. It should also be noted that phenomenological models can have compatible predictions. In an extended vector-dominance model one indeed finds the

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branching ratio from  $5.3 \times 10^{-7}$  to  $2.5 \times 10^{-6}$  [9], but this model has no direct relation to QCD as the fundamental theory of strong interaction.

Decays of quarkonia were intensively studied in the 1980s. Now that our understanding of QCD has been greatly improved, a restudy of these decays is necessary to explain new experimental results like the upper bound in Eq. (1). On the other hand, there is a large data sample with  $5 \times 10^7 J/\psi$  events collected by the Beijing Spectrometer BES Collaboration [10]; a data sample with several billions  $J/\psi$  events is planned to be collected with the proposed BES III at Beijing Electron-Positron Collider (BEPC II) and with CLEO-C at a modified Cornell Electron Storage Ring (CESR) [10,11]. Furthermore, about  $4 \text{ fb}^{-1} b\bar{b}$  resonance data are planned to be taken at CLEO III in the year prior to conversion to low energy operation (CLEO-C) [11]. These data samples of quarkonia will allow us to study the decays, which have been observed before, with more accuracy, and also those decays which have not been observed. Therefore, experimental activities will bring more information about these decays and may also lead to new discoveries, e.g., discovery of glueball. In this Brief Report we present an approach based on QCD factorization to explain the experimental result from CLEO [3]. This approach was used for the radiative decay into the tensor meson  $f_2$  [12].

We consider the heavy quark limit, i.e.,  $m_c \rightarrow \infty$ ,  $m_b \rightarrow \infty$ . In the limit, a quarkonium system, taking  $Y$  as an example, can be taken as a bound state of  $b$  and  $\bar{b}$  quark which move with a small velocity  $v$ ; hence an expansion in  $v$  can be employed and nonrelativistic QCD (NRQCD) can be used to describe the nonperturbative effect related to  $Y$  [13]. The decay can be regarded as follows: the quarkonium will be annihilated into a real photon and gluons and the gluons will be subsequently converted into the meson  $\eta'$ . Also in the limit, the meson  $\eta'$  has a large momentum; this enables an expansion in twist to characterize the gluonic conversion into  $\eta'$  the conversion is then described by a set of distribution amplitudes of gluons. The large momentum of  $\eta'$  requires that the gluons should be hard, hence the emission of the gluons can be handled by perturbative theory. The above discussion implies that we may factorize the decay amplitude into three parts: the first part consists of matrix elements of NRQCD representing the nonperturbative effect related to  $Y$ , the second part consists of some distribution amplitudes, which are for the gluonic conversion into  $\eta'$ , and the third part consists of some coefficients, which can be calculated with perturbative theory for the  $b\bar{b}$  pair annihilated into gluons and a real photon. In this Brief Report we show that the contribution of twist-2 operators are suppressed by  $m_{\eta'}^2$ . This indicates that a complete QCD analysis should include contributions from twist-4 operators. However, without such a complete analysis one still can make some predictions like the branching ratio given in Eq. (1).

We consider the decay of  $Y$ :  $Y \rightarrow \gamma(q) + \eta'(k)$ , where the momenta are given in the brackets. We take a light-cone coordinate system, in which the momentum  $k$  of  $\eta'$  is  $k^\mu = (k^+, k^-, 0, 0)$ . We consider the contribution from emission

of two gluons, and assume a factorization can be performed. Then the  $S$ -matrix can be written as

$$\begin{aligned} \langle \gamma \eta' | S | Y \rangle &= -i \frac{1}{2} e Q_b g_s^2 \varepsilon_\rho^* \int d^4 x d^4 y d^4 z d^4 x_1 d^4 y_1 \\ &\times e^{iq \cdot z} \langle \eta' | G_\mu^a(x) G_\nu^b(y) | 0 \rangle \\ &\times \langle 0 | \bar{b}_j(x_1) b_i(y_1) | Y \rangle \\ &\times M_{ji}^{\mu\nu\rho,ab}(x, y, x_1, y_1, z), \end{aligned} \quad (5)$$

where  $M_{ji}^{\mu\nu\rho,ab}(x, y, x_1, y_1, z)$  is a known function,  $i$  and  $j$  stand for Dirac and color indices,  $a$  and  $b$  is the color of gluon field,  $b(x)$  stands for the Dirac field of  $b$  quark,  $\varepsilon^*$  is the polarization vector of the photon, and  $Q_b$  is the charge fraction of the  $b$  quark in unit  $e$ . The above equation can be generalized to emission of an arbitrary number of gluons. Using the fact that  $b$  or  $\bar{b}$  quark moves with a small velocity  $v$ , the matrix element with the Dirac fields can be expanded in  $v$ . We obtain

$$\begin{aligned} \langle 0 | \bar{b}_j(x) b_i(y) | Y \rangle &= -\frac{1}{6} (P_+ \gamma^\dagger P_-)_{ij} \langle 0 | \chi^\dagger \sigma' \psi | Y \rangle \\ &\times e^{-ip \cdot (x+y)} + \mathcal{O}(v^2), \end{aligned} \quad (6)$$

where  $\chi^\dagger(\psi)$  is the NRQCD field for  $\bar{b}(b)$  quark and  $P_\pm = (1 \pm \gamma^0)/2$ ,  $p^\mu = (m_b, 0, 0, 0)$ , where  $m_b$  is the pole mass of the  $b$  quark. In Eq. (6) we do not count the power of  $v$  for quark fields because this power is the same for every term in the expansion of Eq. (6). With this in mind the leading order of the matrix element is then  $\mathcal{O}(v^0)$ ; we will neglect the contribution from higher orders and the momentum of  $Y$  is then approximated by  $2p$ . It should be noted that effects at higher order of  $v$  can be added with the expansion in Eq. (6).

For the matrix element with gluon fields we observe that the  $x$  dependence of the matrix element is controlled by different scales: the  $x^-$  dependence is controlled by  $k^+$ , while the  $x^+$  and  $x_T$  dependence is controlled by the scale  $\Lambda_{QCD}$  or  $k^-$ , which are small in comparison with  $k^+$ . Because of these small scales we can expand the matrix element in  $x^+$  and in  $x_T$ . With this expansion we obtain the result for the Fourier transformed matrix element:

$$\begin{aligned} &\int d^4 x e^{-iq_1 \cdot x} \langle \eta' | G^{a,\mu}(x) G^{b,\nu}(0) | 0 \rangle \\ &= \frac{1}{8} \delta^{ab} (2\pi)^4 \delta(q_1^-) \delta^2(q_{1T}) \frac{1}{2k^+ x_1 (x_1 - 1)} \\ &\times e^{\mu\nu} F_{\eta'}(x_1) + \dots, \end{aligned} \quad (7)$$

$$e^{\mu\nu} = \varepsilon^{\mu\nu\alpha\beta} l_\alpha n_\beta, \quad l^\mu = (1, 0, 0, 0),$$

$$n^\mu = (0, 1, 0, 0), \quad q_1^+ = x_1 k^+,$$

$$\begin{aligned} F_{\eta'}(x_1) &= \frac{1}{2\pi k^+} \int dx^- e^{-ix_1 k^+ x^-} \langle \eta'(k) | G^{a,+ \mu}(x^-) \\ &\times G^{a,+ \nu}(0) | 0 \rangle e_{\mu\nu}, \end{aligned} \quad (8)$$

where  $\varepsilon^{\mu\nu\alpha\beta}$  is totally antisymmetric with  $\varepsilon^{0123} = 1$ .  $F_{\eta'}(x_1)$  is the distribution amplitude characterizing the conversion of two gluons into  $\eta'$  and it is defined with twist-2 operators in

the light-cone gauge, where the two gluons carry the momentum  $x_1 k^+$  and  $(1-x_1)k^+$ , respectively. In other gauges a gauge link should be supplied in Eq. (8) to maintain the gauge invariance. It should be noted that there is no simple relation between  $F_{\eta'}(x_1)$  and the gluonic matrix element in Eq. (2). The  $\dots$  in Eq. (7) stands for contributions from higher twist. The next-to-leading twist is 4. With the above results we obtain the  $S$ -matrix element with the twist-2 contribution in the limit  $m_{\eta'} \rightarrow 0$ :

$$\begin{aligned} \langle \gamma \eta' | S | Y \rangle &= \frac{-i}{48} e Q_b g_s^2 (2\pi)^4 \delta^4(2p-k-q) \\ &\times \varepsilon_\rho^* \langle 0 | \chi^\dagger \sigma^l \psi | Y \rangle e^{l\rho} \frac{1}{m_b^4} m_{\eta'}^2 \int dx_1 \frac{1-2x_1}{x_1(1-x_1)^2} F_{\eta'}(x_1) \\ &\times \left( 1 + \mathcal{O}\left(\frac{m_{\eta'}^2}{m_b^2}\right) \right). \end{aligned} \quad (9)$$

It shows that the twist-2 contribution is suppressed by  $m_{\eta'}^2$ . In the twist expansion the light hadron mass  $m_{\eta'}$  should be taken as a small scale as  $\Lambda_{QCD}$ ; hence the contribution is proportional to  $\Lambda_{QCD}^2$ . This implies that a complete analysis at the leading order should include not only this contribution but also twist-4 contributions, in which one needs to consider the contributions from emission of 2, 3 and 4 gluons. This is too complicated to be done here. However, without a complete analysis we can always write the result of a complete analysis as

$$\begin{aligned} \langle \gamma \eta' | S | Y \rangle &= \frac{-i}{48} e Q_b g_s^2 (2\pi)^4 \delta^4(2p-k-q) \\ &\times \varepsilon_\rho^* \langle 0 | \chi^\dagger \sigma^l \psi | Y \rangle e^{l\rho} \frac{1}{m_b^4} g_{\eta'}, \end{aligned} \quad (10)$$

where the parameter  $g_{\eta'}$  has a dimension 3 in mass. This parameter is a sum of the twist-2 contribution in the second line of Eq. (9) and the twist-4 contributions which need to be analyzed. The parameter characterizes the conversion of gluons into  $\eta'$  and it does not depend on properties of  $Y$ . The origin of the factor  $m_b^4 = m_b^2 \cdot m_b^2$  is as follows: one  $m_b^2$  comes from the perturbative part; another  $m_b^2$  reflects the fact that the contribution of twist-2 operators is proportional to  $m_{\eta'}^2$  and contributions of twist-4 are proportional to  $\Lambda_{QCD}^2$ . It is interesting to note that this power behavior is also obtained in [14]; in contrast, it is also pointed out in [14] that this behavior holds by taking  $m_c$  and  $m_b$  as light quark masses; in the heavy quark limit this behavior does not hold.

The above result may also be generalized for the decay of  $J/\psi$ . One may question whether the twist expansion may be applicable for the  $J/\psi$  decay or not, because  $m_c$  is not large enough. The twist expansion means a collinear expansion of momenta of partons in  $\eta'$ ; components of these momenta have the order of  $\mathcal{O}(k^+)$ ,  $\mathcal{O}(k^-)$ ,  $\mathcal{O}(\Lambda_{QCD})$ ,  $\mathcal{O}(\Lambda_{QCD})$ . Hence the expansion parameters are  $k^-/k^+ = m_{\eta'}^2/M_{J/\psi}^2 \approx 0.1$ ,  $\Lambda_{QCD}/k^+ \approx 0.2$ , where we have taken  $\Lambda_{QCD} \approx 400$  MeV. These estimations show that the twist expansion

is also a good approximation for the  $J/\psi$  decay. There are also other possible large uncertainties, due to effects from higher orders of  $v$  and  $\alpha_s$ . These can be eliminated partly by building the ratio  $\Gamma(J/\psi(Y) \rightarrow \gamma \eta')/\Gamma(J/\psi(Y) \rightarrow \text{light hadrons})$ . Theoretical prediction for this ratio will have less uncertainties than the width, because corrections from higher orders of  $v$  and  $\alpha_s$  are canceled at a certain level. Using experimental data for  $\text{Br}(J/\psi(Y) \rightarrow \text{light hadrons})$  we can predict the branching ratio. With this consideration we rewrite the ratio defined in Eq. (3) as  $R_{\eta'} = (\text{Br}(Y \rightarrow \text{light hadrons})/\text{Br}(J/\psi \rightarrow \text{light hadrons})) r_{\eta'}$ ,  $r_{\eta'} = \Gamma(Y \rightarrow \gamma \eta')/\Gamma(Y \rightarrow \text{light hadrons})/\Gamma(J/\psi \rightarrow \gamma \eta')/\Gamma(J/\psi \rightarrow \text{light hadrons}) \approx (Q_b^2 m_c^6/Q_c^2 m_b^6) (\alpha_s(m_c)/\alpha_s(m_b))$ , where leading order results for the decay widths are used for  $r_{\eta'}$ . Using the experimental results for the branching ratios of decays into light hadrons, we obtain  $R_{\eta'} = \text{Br}(Y \rightarrow \gamma + \eta')/\text{Br}(J/\psi \rightarrow \gamma + \eta') \approx 1.31 (Q_b^2 m_c^6/Q_c^2 m_b^6) (\alpha_s(m_c)/\alpha_s(m_b))$ . This is the result at the leading order of  $\Lambda$ , where  $\Lambda$  is  $\Lambda_{QCD}$  or  $m_{\eta'}$ , and the dependence of the renormalization scale in gluonic distribution amplitudes is neglected. The dependence may be extracted from the study in [15]. By taking  $\alpha_s(m_c) \approx 0.3$  and  $\alpha_s(m_b) \approx 0.18$  we obtain  $R_{\eta'} \approx 3.9 \times 10^{-4}$ . With the experimental value of  $\text{Br}(J/\psi \rightarrow \gamma + \eta')$  we obtain the branching ratio

$$\text{Br}(Y \rightarrow \gamma + \eta') \approx 1.7 \times 10^{-6}. \quad (11)$$

This value is much smaller than the values obtained with other approaches and it is in consistency with the upper limit. Similarly we also obtain

$$\text{Br}(Y \rightarrow \gamma + \eta) \approx 3.3 \times 10^{-7}. \quad (12)$$

It should be emphasized that our results obtained in the above equations are not based on any model; corrections to these results can be systematically added in the framework of QCD. The possibly largest uncertainties in our results are from relativistic corrections for  $J/\psi$  decays and, from the uncertainty of the value of the charm quark mass, each of them can be at the level of 50%. For  $Y$  the relativistic correction is expected to be small, because the  $b$  quark inside  $Y$  moves with a small velocity,  $v^2 \approx 0.1$ , while for charmonia the  $c$  quark inside a charmonium moves with a velocity, which is estimated to be  $v^2 \approx 0.3$  or larger. This large value of  $v^2$  may lead to a large relativistic correction. Taking these into account, our prediction in Eqs. (11) and (12) can be close to the experimental bound in Eq. (1). However, these largest uncertainties may be reduced by using hadron masses, i.e., using  $2m_c = M_{J/\psi}$ . This possibility is based on the result for relativistic correction in [16] and on the observation that the violation of the famous 14% rule may be reduced in this way. If one analyzes the correction at the next-to-leading order of  $v$  for decays of  $1^{--}$  quarkonia, one obtains that the correction is proportional to a NRQCD matrix element defined in [13]. This matrix element represents the relativistic correction. In [16] it is shown that this matrix element is proportional to the binding energy, i.e., to  $M_{J/\psi} - 2m_c$  for  $J/\psi$  and to  $M_{\psi'} - 2m_c$  for  $\psi'$ , respectively. If we use  $2m_c = M_{J/\psi}$  for  $J/\psi$  decays and  $2m_c = M_{\psi'}$  for  $\psi'$  de-

cays, respectively, the relativistic correction disappears formally, but it is actually included by using hadron masses. However, it should be noted that this should be regarded as a phenomenological estimation; a detailed analysis and an precise determination of quark masses is needed to study the correction in a consistent way.

The famous 14% rule is derived simply by taking leading order results for decays. In our case we have

$$\frac{\text{Br}(\psi' \rightarrow \gamma \eta')}{\text{Br}(J/\psi \rightarrow \gamma \eta')} = \frac{\text{Br}(\psi' \rightarrow e^+ e^-)}{\text{Br}(J/\psi \rightarrow e^+ e^-)} = 0.147 \pm 0.023, \quad (13)$$

where the number is estimated with experimental results of leptonic decay widths. This result is theoretically expected not only for radiative decays into any light hadron, but also for hadronic decays; this is the so-called 14% rule. However, this rule is significantly violated; one of the violations is the well known  $\rho\pi$  puzzle. A possible explanation and useful references can be found in [17]. The experimental result made by BES [18] indicates that the rule is also violated in our case:  $\text{Br}(\psi' \rightarrow \gamma \eta')/\text{Br}(J/\psi \rightarrow \gamma \eta') = 0.036 \pm 0.009$ . This value is only fourth of the expected. It should be noted that corrections from higher orders of  $\alpha_s$  are canceled in the ratios in Eq. (12); the theoretical uncertainties come from effects of higher orders in  $v$  in Eq. (6) and higher twists. In the case of  $1^{--}$  quarkonia, the correction from the next-to-leading order of  $v$  is the relativistic correction, whose effect is expected to be significant for charmonia. As discussed before, this correction may be estimated by replacing  $m_c$  with the half of the mass of quarkonium, i.e., we use  $2m_c = M_{J/\psi}$  for the  $J/\psi$  decays and  $2m_c = M_{\psi'}$  for  $\psi'$  decays. With this replacement and with our result in Eq. (10), the ratio in Eq. (13) is modified as  $\text{Br}(\psi' \rightarrow \gamma \eta')/\text{Br}(J/\psi \rightarrow \gamma \eta') = (M_{J/\psi}^6/M_{\psi'}^6) (\text{Br}(\psi' \rightarrow e^+ e^-)/\text{Br}(J/\psi \rightarrow e^+ e^-)) = 0.0512 \pm 0.0080$ . This result shows that the relativistic correction is indeed significant. With the replacement the pre-

dicted ratio is much closer to the experimental result than that of the 14% rule and the two largest uncertainties are reduced in the prediction. However, this is a naive estimation; a detailed study is needed and is in progress [19]. With this case we can expect that the two largest uncertainties are also reduced in our predictions in Eqs. (11) and (12) because we have used  $2m_b = M_\Upsilon$  and  $2m_c = M_{J/\psi}$ .

It is also interesting to look at decays into  $\rho\pi$ . In this decay one of the final hadrons is produced at the level of twist-2 and another is at the level of twist-3 [20]. With this fact and with the replacement the rule is modified as  $Q_{\rho\pi} = \text{Br}(\psi' \rightarrow \rho\pi)/\text{Br}(J/\psi \rightarrow \rho\pi) = (M_{J/\psi}^8/M_{\psi'}^8) (\text{Br}(\psi' \rightarrow e^+ e^-)/\text{Br}(J/\psi \rightarrow e^+ e^-)) = 0.036 \pm 0.006$ . With the modification the rule is changed significantly. The above result also holds for decays into  $K^*K$ . Although the ratio is reduced, it is still in conflict with experimental results. In [21] it is found that  $Q_{\rho\pi} < 0.006$  and  $Q_{K^*K} < 0.64$ . Recent data from BES gives  $Q_{\rho\pi} < 0.0022$  and  $Q_{K^*K} = 0.019 \pm 0.007$  [22]. However, the predictions are closer to experiment than those in Eq. (13). One should also keep in mind that these decays are more complicated than radiative decays discussed before, because the final state consists of two light hadrons.

To summarize, we have presented a QCD-factorization approach for radiative decays of  $1^{--}$  quarkonium into  $\eta(\eta')$ ; the result is consistent with the experimental result made by CLEO. On the other hand, most of the theoretical results are not compatible with the upper limit. A possible explanation for the violation of the 14% in our case is given. With this explanation we show that the effect of relativistic corrections and that due to uncertainty of the quark mass can be reduced by using quarkonium masses and uncertainties in our predictions may be not as large as those usually expected.

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