σ meson in J/ψ decays

Wujun Huo, Xinmin Zhang, and Tao Huang
CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, People's Republic of China
and Institute of High Energy Physics, Academia Sinica, P.O. Box 918(4), Beijing 100039, People's Republic of China
(Received 13 December 2001; published 9 May 2002)

Recently the BES Collaboration found evidence for the existence of the σ meson in the process $J/\psi \to \sigma\omega \to \pi\pi\omega$ at the BEPC. In this paper we first discuss the relevant coupling $g_{\sigma\pi\pi}$ and show that the linear σ model gives rise to a reasonable description of the σ decay into π's; then we calculate the coupling constant $g^{th}_{J/\psi\sigma\omega}$ using the perturbative QCD technique and the light-cone wave functions of the σ and ω mesons. The results show that the theoretical value of $g^{th}_{J/\psi\sigma\omega}$ is within the range of the experimental value of $g_{J/\psi\sigma\omega}$.

DOI: 10.1103/PhysRevD.65.097505 PACS number(s): 13.87.Fh, 12.39.Hg, 13.85.Hd

I. INTRODUCTION

The Beijing Spectrometer (BES) Collaboration at the Beijing Electron-Positron Collider (BEPC) recently reported evidence for the existence of the σ particle in J/ψ decays. In the $\pi^+\pi^-$ invariant mass spectrum in the process of $J/\psi \to \pi^+\pi^-\omega$ they found a low mass enhancement, and the detailed analysis strongly favors $\mathcal{O}^{++}$ spin parity and a statistical significance for the existence of the σ particle of about 18σ [1]. The BES measured values of the σ mass and width are

$$m_\sigma = 390^{+60}_{-36}\ \text{MeV},$$

$$\Gamma_\sigma = 282^{+77}_{-50}\ \text{MeV}, \tag{1}$$

and the branching ratio is

$$\text{BR}(J/\psi \to \sigma\omega \to \pi^+\pi^-\omega) = (1.71 \pm 3.4 \pm 4.3)\times 10^{-3}. \tag{2}$$

The σ particle has been absent for many years from the Particle Data Group (PDG) data [2]; however, in recent years there has been a revival of interest in studying the light scalar-isoscalar meson, the σ particle, as a broad resonance [3] experimentally and theoretically. Direct experimental evidence for the σ meson has been reported recently by the Fermilab E791 Collaboration [4] in the D-meson decay process $D^+ \to \sigma\pi^+ \to 3\pi$. Theoretically, the σ meson can play important roles in some problems. In Ref. [5], the σ can be regarded as the dominant contribution to the $u\bar{u}+d\bar{d}$ current. Moreover, using the sigma mass and width from E791, some people [6] have investaged the σ contribution to $B \to 3\pi$ and found that it can explain the recent data from the CLEO and BABAR Collaborations.

In this paper, we study phenomenologically the decay process $J/\psi \to \sigma\omega \to \pi^+\pi^-\omega$ and discuss the couplings $g_{\sigma\pi\pi}$ and $g_{J/\psi\sigma\omega}$. By taking the meson wave functions of σ and ω to be similar to those of π and ρ, and using the perturbative QCD technique, we calculate the decay constant $g^{th}_{J/\psi\sigma\sigma}$. Our theoretical prediction is shown to be within the range of the experimental value of $g_{J/\psi\sigma\omega}$.

II. COUPLING CONSTANTS $g_{\sigma\pi\pi}$ AND $g_{J/\psi\sigma\omega}$

Given the data on the σ mass and width measured by the BES Collaboration in Eqs. (1) and (2) we study the coupling constants $g_{\sigma\pi\pi}$ and $g_{J/\psi\sigma\omega}$. For a two-body decay of particle X into final states X_1 and X_2 the decay width is given by

$$\Gamma(X \to X_1X_2) = \frac{|\mathbf{p}|}{32\pi^2 M_X^2}\int |\mathcal{M}|^2 d\Omega = \frac{|\mathbf{p}|}{8\pi M_X^2}|\mathcal{M}|^2, \tag{3}$$

where $|\mathbf{p}|$ is the three-momentum of the final state in the center-of-mass (c.m.) frame. For $\sigma \to \pi^+\pi^-$, the σ meson decays 100% into $\pi\pi$. Furthermore, isospin conservation requires the final states be charged pions for two-thirds of the time. So we have

$$\frac{2}{3}\Gamma^0_\sigma = g^2_{\sigma\pi\pi}\frac{1}{8\pi m_\sigma^2}\sqrt{\frac{m_\sigma^2}{4} - m_\pi^2}. \tag{4}$$

Using BES data on the σ's mass and width, and the experimental value of the π mass, we obtain

$$g_{\sigma\pi\pi} = 2.0^{+0.30}_{-0.19}\ \text{GeV}. \tag{5}$$

This number is surprisingly consistent with the theoretical value of the linear sigma model [7]:

$$g^{linear\ \ \sigma}_{\sigma\pi\pi} = \frac{\sqrt{2}m_\sigma^2}{f_\pi} = 1.80^{+0.50}_{-0.30}\ \text{GeV}, \tag{6}$$

where f_π is the pion decay constant. In Eq. (6) the σ mass is taken from the BES measurement in Eq. (1).

To get the phenomenological value of $g_{J/\psi\sigma\omega}$, we use the full three-body decay width (for example, see [7]):

$$\Gamma(J/\psi\to\sigma\omega\to\pi^+\pi^-\omega)$$
$$=\frac{1}{2}\frac{1}{2m_{J/\psi}}\ g^2_{J/\psi\sigma\omega}\ g^2_{\sigma\pi\pi}$$
$$\times\int_{4m_\omega^2}^{(m_{J/\psi}-m_\omega)^2}\frac{d\chi^2}{2\pi}\frac{1}{8\pi}\lambda^{1/2}\left(1,\frac{\chi^2}{m^2_{J/\psi}},\frac{m^2_\omega}{m^2_{J/\psi}}\right)$$
$$\times\frac{1}{8\pi}\lambda^{1/2}\left(1,\frac{m_\pi^2}{\chi^2},\frac{m_\pi^2}{\chi^2}\right)\frac{1}{(\chi^2-m_\sigma^2)^2+\Gamma_\sigma(\chi)^2m_\sigma^2},\tag{7}$$

where the factor $1/(8\pi)\times\lambda^{1/2}$ is the phase space integral of the corresponding two-body decay subprocess and

$$\lambda(a,b,c)=a^2+b^2+c^2-2ab-2bc-2ca.\tag{8}$$

Also in Eq. (7)

$$\Gamma_\sigma(\chi)\equiv\Gamma_\sigma^0\times(m_\sigma/\chi)[p^*(\chi)/p^*(m_\sigma)]\tag{9}$$

is the comoving resonance width where $p^*(\chi)=\sqrt{\chi^2/4-m_\pi^2}$ and $\Gamma_\sigma^0=370^{+60}_{-36}$ MeV is the experimental value in Eq. (1).

With the experimental values of $g_{\sigma\pi\pi}$ in Eq. (5) and the branching ratio in Eq. (2), solving Eq. (7) gives rise to

$$g^{expt}_{J/\psi\sigma\omega}=7.3^{+2.6}_{-1.9}\ \text{MeV}.\tag{10}$$

III. $g^{th}_{J/\psi\sigma\omega}$ CALCULATED BY PERTURBATIVE QCD

In this section we follow closely the calculation of the exclusive decays of Υ in Ref. [8] to study the decay of $J/\psi\to\sigma\omega$. The decay amplitude consists of two parts. One is the hard decay amplitude of the three-gluon modes and the another consists of the bound-state matrix elements of outgoing mesons.

In general the decay amplitude of $J/\psi\to\sigma\omega$ can be written as

$$\mathcal{M}=\Psi_{J/\psi}(0)\int_{-1}^{1}dx\int_{-1}^{1}dy\,\phi_\sigma^*(x,M_c^2)$$
$$\times\phi_\omega^*(y,M_c^2)T_h(x,y,M_c^2),\tag{11}$$

where $x=x_1-x_2, y=y_1-y_2$, and x_i, y_i are the constituents' fractional longitudinal momenta which satisfy $\Sigma x_i=1$ and $\Sigma y_i=1$. In Eq. (11) $\Psi_{J/\psi}(0)$ is a nonrelativistic approximate wave function of the J/ψ meson, $\phi_\sigma(x,M_c^2)$ and $\phi_\omega(y,M_c^2)$ are the distribution amplitudes of the σ and ω wave functions, respectively, and M_c is the charm quark mass.

$T_h(x,y,M_c^2)$ is the hard decay amplitude of the charm quark pairs into two light quark-antiquark pairs, which is defined by

$$T_h(x,y,M_c^2)=\int\frac{d^4l}{(2\pi)^4}\mathcal{T}_h(x,y,l,M_c^2).\tag{12}$$

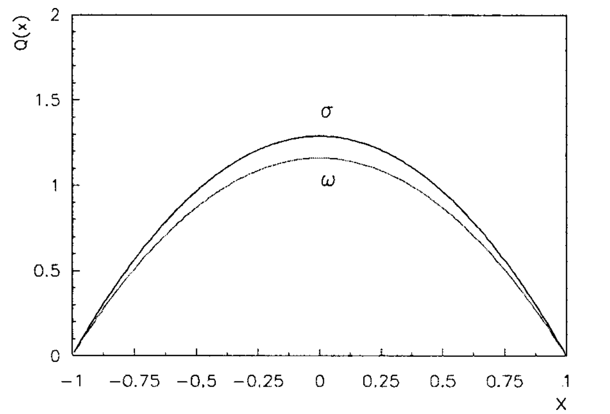

FIG. 1. Feynmann diagram for the hard three-gluon decay. There are 12 diagrams in total; however, just one of them is shown.

There are 12 Feynmann diagrams shown in Fig. 1 contributing to T_h, which are all explicitly shown in Ref. [8]. Using the Landau rules, it is found that both infrared and collinear divergences exist in every diagram. Fortunately, through the use of color neutrality and collinear Ward identities, one can prove that, on summing up all of the diagrams, the divergences cancel [8].

For quarkonia J/ψ, the nonrelativistic approximate wave function $\Psi_{J/\psi}(0)$ can be determined by the decay $J/\psi\to e^+e^-$,

$$\Gamma(J/\psi\to e^+e^-)=\frac{16\pi\alpha_s^2e^2}{M^2_{J/\psi}}|\Psi(0)|^2.\tag{13}$$

According to Brodsky, Huang, and Lepage [9], the light-cone wave function of a hadron is essentially determined by the off-shell energy variable. Thus, for the light scalar meson σ, we assume the light-cone wave function to be the same as that of the pion [10]:

$$\psi_\sigma(x_i,\mathbf{k}_\perp)=A_\sigma\exp\left[-b_\sigma^2\left(\frac{\mathbf{k}_\perp^2+m_u^2}{x_1}+\frac{\mathbf{k}_\perp^2+m_u^2}{x_2}\right)\right],\tag{14}$$

where $\mathbf{k}_\perp$ is the relative transverse momentum of the final meson, m_u is the u quark mass, and x_i are the constituent's fractional longitudinal momenta. A_σ and b_σ are two free parameters which are taken to be $A_\sigma=A_\pi\approx32\ \text{GeV}^{-1}$ and $b_\sigma^2=b_\pi^2\approx0.84\ \text{GeV}^{-2}$ [10]. For the wave function of the vector meson ω, we assume it to be the same as that of the ρ meson [11]:

$$\psi_\omega(y_i,\mathbf{k}_\perp)=A_\omega\exp\left[-b_\omega^2\left(\frac{\mathbf{k}_\perp^2+m_u^2}{y_1}+\frac{\mathbf{k}_\perp^2+m_u^2}{y_2}\right)\right],\tag{15}$$

where A_ω and b_ω are taken to be $A_\omega=A_\rho\approx30\ \text{GeV}^{-1}$ and $b_\omega^2=b_\rho^2\approx0.55\ \text{GeV}^{-2}$, which can be determined from two constraints [10,11].

The distribution amplitude ϕ in Eq. (11) is defined as

$$\phi(x_1,x_2,M_c^2)=\int_0^{M_c^2}\frac{dk_\perp^2}{16\pi^2}\psi(x_1,x_2,k_\perp^2).\tag{16}$$

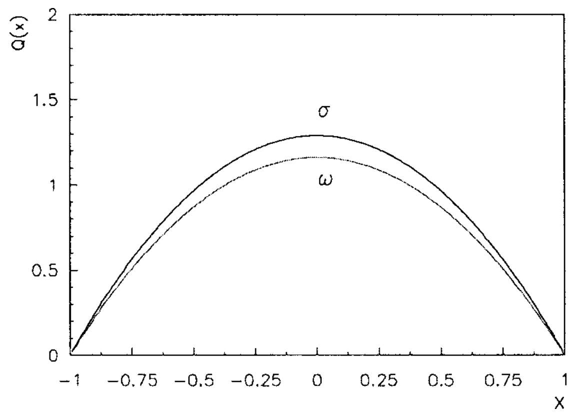

FIG. 2. Distribution amplitudes $\phi_\sigma(x)$ and $\phi_\omega(x)$ of the σ and ω mesons. The solid curve $\phi_\sigma(x)$ is determined by Eq. (17) with $A_\sigma = 32\ \text{GeV}^{-1}$ and $b_\pi^2 = 0.84\ \text{GeV}^{-2}$ and the dashed curve $\phi_\omega(x)$ is determined by Eq. (18) with $A_\omega = 30\ \text{GeV}^{-1}$ and $b_\rho^2 = 0.55\ \text{GeV}^{-2}$.

Making use of the wave functions of σ and ω in Eqs. (14) and (15), we have

$$\phi_\sigma(x,M_c^2) = \frac{A_\sigma(1-x^2)}{64\pi^2 b_\sigma^2}\left[\exp\left(-4b_\sigma^2\frac{m_u^2}{1-x^2}\right) - \exp\left(-4b_\sigma^2\frac{M_c^2+m_u^2}{1-x^2}\right)\right], \tag{17}$$

$$\phi_\omega(y,M_c^2) = \frac{A_\omega(1-y^2)}{64\pi^2 b_\omega^2}\left[\exp\left(-4b_\omega^2\frac{m_u^2}{1-y^2}\right) - \exp\left(-4b_\omega^2\frac{M_c^2+m_u^2}{1-y^2}\right)\right], \tag{18}$$

where we have used $x = x_1 - x_2, y = y_1 - y_2, \Sigma x_i = 1$, and $\Sigma y_i = 1$. In deriving the integration of Eq. (16), we have ignored the QCD evolution of the distribution amplitude since the charm quark M_c is not large and the evolution effects are not significant in this energy range. Thus the distribution amplitude is essentially determined by the nonperturbative model [10]. In Fig. 2 we show the distribution amplitudes of σ,ω.

With the values of the parameters $A_\pi, b_\pi^2, A_\rho, b_\rho^2$ given above, we obtain the coupling constant $g_{J/\psi\sigma\omega}^{th}$ responsible for the decay $J/\psi \to \sigma\omega$,

$$g_{J/\psi\sigma\omega}^{th} = 10.7\ \text{MeV}. \tag{19}$$

One can see that our theoretical prediction for $g_{J/\psi\sigma\omega}^{th}$ above agrees within the errors with the experimental value $g_{J/\psi\sigma\omega}^{expt}$ in Eq. (10).

IV. CONCLUSION

In this paper we have used the new BES experimental evidence for the σ particle and studied its properties in the process $J/\psi \to \sigma\omega \to \pi\pi\omega$. We have obtained the phenomelogical values of the coupling constants $g_{\sigma\pi\pi}$ and $g_{J/\psi\sigma\omega}$, and the theoretical prediction of $g_{\sigma\pi\pi}^{linear\ \sigma}$ in the linear σ model. We show that the linear σ model gives rise to a reasonable description of the σ decay into π's. We have calculated $g_{J/\psi\sigma\omega}$ by perturbative QCD and in this approach we take the wave functions of the σ and ω to be similar to those of π and ρ. Our theoretical predictions and the experimental value of $g_{J/\psi\sigma\omega}$ are shown to be consistent.

ACKNOWLEDGMENTS

One of the authors (W.J.H.) acknowledges support from the Chinese Postdoctoral Science Foundation and CAS K.C. Wong Postdoctoral Research Award Fund. We thank Zhi-Peng Zheng and N. Wu for useful discussions. This work was supported in part by the NSF of China.

[1] N. Wu, hep-ex/0104050.
[2] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).
[3] S. Ishida, M. Ishida, H. Takahashi, T. Ishida, K. Takamatsu, and T. Tsuru, Prog. Theor. Phys. **95**, 745 (1996); N.A. Törnqvist, hep-ph/0008135; V.E. Markushin, Z. Xiao, and H.Q. Zheng, Nucl. Phys. **A695**, 273 (2001); H. Jin and X. Zhang, hep-ph/9805412; N.A. Törnqvist, and A.D. Polasa, Frascati Phys. Ser. **A20**, 385 (2000); M. Ishida, hep-ph/0012325.
[4] E791 Collaboration, E.M. Aitala *et al.*, Phys. Rev. Lett. **86**, 770 (2001); R. Gatto, G. Nardulli, A.D. Polosa, and N.A. Tornqvist, Phys. Lett. B **494**, 168 (2000); E791 Collaboration, C. Gobel, hep-ex/0012009.
[5] S.N. Cherry and M.R. Pennington, hep-ph/0111158.
[6] A. Deandrea and A. Polosa, Phys. Rev. Lett. **86**, 216 (2001); S. Gardner and U. Meissner, Phys. Rev. D **65**, 094004 (2002).
[7] C. Dib and R. Rosenfeld, Phys. Rev. D **63**, 117501 (2001).
[8] S.C. Chao, Nucl. Phys. **B195**, 381 (1982).
[9] S.J. Brodsky, T. Huang, and G.P. Lepage, in *Particles and Fields*, edited by A. Z. Capri and A.N. Kamal (Plenum, New York, 1983), Vol. 2, p. 143; T. Huang, in Proceedings of the XXth International Conference on High Energy Physics, Madison, 1980, p. 1000.
[10] X.-H. Guo and T. Huang, Phys. Rev. D **43**, 2931 (1991); T. Huang, in *Proceedings of the International Symposium on Particles and Nuclear Physics*, Beijing, China, 1985, edited by N. Hu and C.S. Wu (World Scientific, Singapore, 1987); in *Second Asia Pacific Physics Conference, Proceedings*, Bangalore, India, 1986, edited by S. Chandrasekhar (World Scientific, Singapore, 1987); F.G. Cao and T. Huang, Commun. Theor. Phys. **27**, 217 (1997).
[11] V.L. Chernyak and A.R. Zhitnisky, Prog. Theor. Phys. **112**, 173 (1984); X.D. Xang, X.N. Wang, and T. Huang, Phys. Rev. D **35**, 1013 (1987).