Phenomenological analysis of lepton and quark mass matrices

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We propose a model in which all quark and lepton mass matrices originally have the same zero texture, namely, their (1,1), (1,3), and (3,1) components are zeros. For the neutrino mass matrix, we further impose symmetry between the second and the third generations. Then the neutrino mass matrix has maximal ν_μ-ν_τ mixing. Our model is consistent with all the neutrino oscillation experiments and also the experimental data in the quark sector. The neutrino mass matrix also can be well incorporated with the seesaw mechanism.

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The recent neutrino oscillation experiments [1] have brought us much knowledge of neutrino masses and lepton mixing. At this stage, the phenomenological construction of quark and lepton mass matrices might be an important step toward the understanding of high energy physics beyond the standard model. In our previous paper [2], we proposed a mass matrix model in which all quark and lepton mass matrices have the same texture (see also [3–9]):

$$M=\begin{pmatrix} 0 & A & 0 \\ A & B & C \\ 0 & C & D \end{pmatrix}. \tag{1}$$

That is M_u, M_d, and M_e [mass matrices of up quarks (u,c,t), down quarks (d,s,b), and charged leptons (e,μ,τ), respectively] have the same zero texture as follows:

$$M_u=\begin{pmatrix} 0 & A_u & 0 \\ A_u & B_u & C_u \\ 0 & C_u & D_u \end{pmatrix},$$

$$M_d=P_d\begin{pmatrix} 0 & A_d & 0 \\ A_d & B_d & C_d \\ 0 & C_d & D_d \end{pmatrix}P_d^\dagger,$$

$$M_e=P_e\begin{pmatrix} 0 & A_e & 0 \\ A_e & B_e & C_e \\ 0 & C_e & D_e \end{pmatrix}P_e^\dagger. \tag{2}$$

where $P_d \equiv \mathrm{diag}(e^{i\alpha_1},e^{i\alpha_2},e^{i\alpha_3})$, $\alpha_{ij}\equiv\alpha_i-\alpha_j$, and $P_e \equiv \mathrm{diag}(e^{i\beta_1},e^{i\beta_2},e^{i\beta_3})$, $\beta_{ij}\equiv\beta_i-\beta_j$ are the CP violating phase factors. In Ref. [2], we also assumed that the mass matrix M_ν of neutrinos (ν_e,ν_μ,ν_τ) has the same texture:

$$M_\nu=\begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & B_\nu & C_\nu \\ 0 & C_\nu & D_\nu \end{pmatrix}. \tag{3}$$

In this paper, however, motivated by the maximal atmospheric neutrino mixing [1], we further impose symmetry of M_ν under permutation of the second and third generations in place of the form of Eq. (3). That is, first let us decompose the M_ν of Eq. (3) into symmetric and antisymmetric parts with respect to the permutation of the second and third generations,

$$\begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & B_\nu & C_\nu \\ 0 & C_\nu & D_\nu \end{pmatrix}=\begin{pmatrix} 0 & \frac{1}{2}A_\nu & \frac{1}{2}A_\nu \\ \frac{1}{2}A_\nu & \frac{1}{2}(D_\nu+B_\nu) & C_\nu \\ \frac{1}{2}A_\nu & C_\nu & \frac{1}{2}(D_\nu+B_\nu) \end{pmatrix}$$
$$+\begin{pmatrix} 0 & \frac{1}{2}A_\nu & -\frac{1}{2}A_\nu \\ \frac{1}{2}A_\nu & -\frac{1}{2}(D_\nu-B_\nu) & 0 \\ -\frac{1}{2}A_\nu & 0 & \frac{1}{2}(D_\nu-B_\nu) \end{pmatrix}. \tag{4}$$

Then, by imposing the above symmetry on M_ν, we keep only the symmetric part,

$$M_\nu=\begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & B_\nu & C_\nu \\ 0 & C_\nu & D_\nu \end{pmatrix}_{2\leftrightarrow 3 sym.part}$$
$$=\begin{pmatrix} 0 & \frac{1}{2}A_\nu & \frac{1}{2}A_\nu \\ \frac{1}{2}A_\nu & \frac{1}{2}(D_\nu+B_\nu) & C_\nu \\ \frac{1}{2}A_\nu & C_\nu & \frac{1}{2}(D_\nu+B_\nu) \end{pmatrix}$$
$$\equiv\begin{pmatrix} 0 & Y_\nu & Y_\nu \\ Y_\nu & Z_\nu & W_\nu \\ Y_\nu & W_\nu & Z_\nu \end{pmatrix}. \tag{5}$$

The structure of Eq. (5) was previously suggested in Refs. [10–14] using the basis where the charged-lepton mass matrix is diagonal. For M_u, M_d, and M_e, we keep both the symmetric and antisymmetric parts. Equations (2) and (5) are our new proposed mass matrix forms for quarks and leptons.

In this paper, in order to get the neutrino mass eigenvalues, we assign

$$Y_\nu = \pm\sqrt{\frac{m_1 m_2}{2}}, \quad Z_\nu = \frac{1}{2}(m_3 + m_2 - m_1),$$

$$W_\nu = -\frac{1}{2}(m_3 - m_2 + m_1), \tag{6}$$

where m_1, m_2, and m_3 are the neutrino masses. Then the M_ν is diagonalized by an orthogonal matrix O_ν as $O_\nu^T M_\nu O_\nu = \mathrm{diag}(-m_1, m_2, m_3)$ with

$$O_\nu = \begin{pmatrix} \mp\sqrt{\frac{m_2}{(m_2+m_1)}} & \pm\sqrt{\frac{m_1}{(m_2+m_1)}} & 0 \\ \sqrt{\frac{m_1}{2(m_2+m_1)}} & \sqrt{\frac{m_2}{2(m_2+m_1)}} & -\frac{1}{\sqrt{2}} \\ \sqrt{\frac{m_1}{2(m_2+m_1)}} & \sqrt{\frac{m_2}{2(m_2+m_1)}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{7}$$

Note that the elements of O_ν are independent of m_3 because of the above structure of M_ν.

Next let us discuss the mass matrix M_e of the charged leptons. Using the type I set discussed in Ref. [2], we assign $A_e = \sqrt{m_e m_\mu m_\tau/(m_\tau - m_e)}$, $B_e = m_\mu$, $C_e = \sqrt{m_e m_\tau (m_\tau - m_\mu - m_e)/(m_\tau - m_e)}$, and $D_e = m_\tau - m_e$. Here m_e, m_μ, and m_τ are the charged-lepton masses. Then M_e becomes

$$M_e \simeq P_e \begin{pmatrix} 0 & \sqrt{m_e m_\mu} & 0 \\ \sqrt{m_e m_\mu} & m_\mu & \sqrt{m_e m_\tau} \\ 0 & \sqrt{m_e m_\tau} & m_\tau - m_e \end{pmatrix} P_e^\dagger$$

$$(\text{for } m_\tau \gg m_\mu \gg m_e) \tag{8}$$

M_e is diagonalized by $P_e O_e$. Here the orthogonal matrix O_e which diagonalizes $P_e^\dagger M_e P_e$ as $O_e^T P_e^\dagger M_e P_e O_e = \mathrm{diag}(-m_e, m_\mu, m_\tau)$ is obtained, for $m_\tau \gg m_\mu \gg m_e$, as

$$O_e \simeq \begin{pmatrix} 1 & \sqrt{\frac{m_e}{m_\mu}} & \sqrt{\frac{m_e^2 m_\mu}{m_\tau^3}} \\ -\sqrt{\frac{m_e}{m_\mu}} & 1 & \sqrt{\frac{m_e}{m_\tau}} \\ \sqrt{\frac{m_e^2}{m_\mu m_\tau}} & -\sqrt{\frac{m_e}{m_\tau}} & 1 \end{pmatrix}. \tag{9}$$

In this case, the Maki-Nakagawa-Sakata (MNS) [15] lepton mixing matrix U is given by

$$U = P_l^\dagger P_e^\dagger O_e^T P_e O_\nu P_l = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}, \tag{10}$$

where $P_l = \mathrm{diag}(i,1,1)$ is included to have positive neutrino masses. The $P_l^\dagger P_e^\dagger$ factor leads U to a form whose diagonal elements are real to a good approximation. We obtain the expressions for some elements of U by keeping terms up to order of $\sqrt{m_e/m_\mu}$ as follows:

$$U_{e1} \simeq \mp\sqrt{\frac{m_2}{(m_2+m_1)}} - \sqrt{\frac{m_1}{(m_2+m_1)}}\sqrt{\frac{m_e}{m_\mu}} e^{-i\beta_{12}},$$

$$U_{e2} \simeq -i\left(\pm\sqrt{\frac{m_1}{(m_2+m_1)}} - \sqrt{\frac{m_2}{(m_2+m_1)}}\sqrt{\frac{m_e}{m_\mu}} e^{-i\beta_{12}}\right),$$

$$U_{e3} \simeq -i\frac{1}{\sqrt{2}}\sqrt{\frac{m_e}{m_\mu}} e^{-i\beta_{12}}. \tag{11}$$

Here we neglect terms of order of $\sqrt{m_e^2/m_\mu m_\tau}$. In the leading order with respective to m_e/m_μ, we have

$$U \simeq \begin{pmatrix} \mp\sqrt{\frac{m_2}{(m_2+m_1)}} & \pm(-i)\sqrt{\frac{m_1}{(m_2+m_1)}} & -i\frac{1}{\sqrt{2}}\sqrt{\frac{m_e}{m_\mu}} e^{-i\beta_{12}} \\ i\sqrt{\frac{m_1}{2(m_2+m_1)}} & \sqrt{\frac{m_2}{2(m_2+m_1)}} & -\frac{1}{\sqrt{2}} \\ i\sqrt{\frac{m_1}{2(m_2+m_1)}} & \sqrt{\frac{m_2}{2(m_2+m_1)}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{12}$$

From Eq. (12), we obtain a maximal atmospheric neutrino mixing angle $\theta_{23} \simeq \pi/4$. The averaged neutrino mass $\langle m_\nu \rangle \equiv |\sum_{j=1}^3 U_{ej}^2 m_j|$ defined from the neutrinoless double beta decay [16] is also obtained from Eq. (11), as

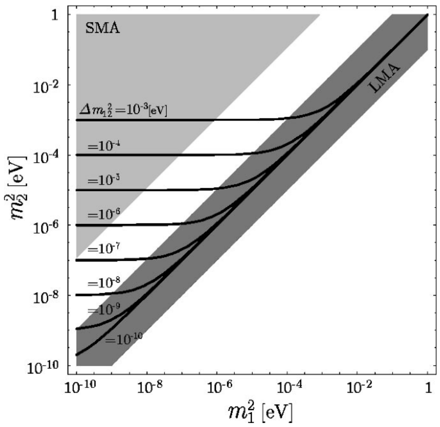

FIG. 1. The allowed regions of m_2^2 and m_1^2. The dark- and light-shaded regions are allowed for LMA and SMA solutions, Eqs. (18) and (19), respectively. The curve passing inside those shaded regions is allowed for each fixed Δm_{12}^2.

$$\langle m_\nu \rangle = \left| \left(\frac{m_3}{2} + m_2 - m_1 \right) \frac{m_e}{m_\mu} \mp 2\sqrt{m_1 m_2}\sqrt{\frac{m_e}{m_\mu}} e^{i\beta_{12}} \right|. \tag{13}$$

It should be noted that m_1, m_2, and m_3 remain free parameters and can be fixed from the solar and atmospheric neutrino data $|U_{e2}|^2$, $\Delta m_{\mathrm{solar}}^2$, and $\Delta m_{\mathrm{atm}}^2$ as well as the averaged neutrino mass from neutrinoless double beta decay $\langle m_\nu \rangle$. From Eqs. (11) and (13), by varying the unknown CP violating phase β_{12}, we obtain the following constraints on $|U_{e2}|^2$ and $\langle m_\nu \rangle$:

$$\left| \sqrt{\frac{m_1}{m_2+m_1}} - \sqrt{\frac{m_2}{m_2+m_1}}\sqrt{\frac{m_e}{m_\mu}} \right|^2 \leqslant |U_{e2}|^2 \leqslant \left| \sqrt{\frac{m_1}{m_2+m_1}} + \sqrt{\frac{m_2}{m_2+m_1}}\sqrt{\frac{m_e}{m_\mu}} \right|^2, \tag{14}$$

$$\left| \left(\frac{m_3}{2} + m_2 - m_1 \right) \frac{m_e}{m_\mu} - 2\sqrt{m_1 m_2}\sqrt{\frac{m_e}{m_\mu}} \right| \leqslant \langle m_\nu \rangle \leqslant \left| \left(\frac{m_3}{2} + m_2 - m_1 \right) \frac{m_e}{m_\mu} + 2\sqrt{m_1 m_2}\sqrt{\frac{m_e}{m_\mu}} \right|. \tag{15}$$

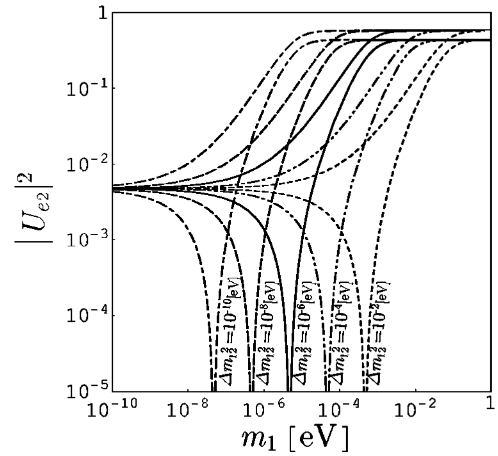

FIG. 2. The allowed regions of $|U_{e2}|^2$ and m_1. The allowed regions are inside the boundary curves for each fixed Δm_{12}^2.

In the following discussion we consider the normal mass hierarchy $\Delta m_{23}^2 > 0$ for the neutrino mass. The case of the inverse mass hierarchy $\Delta m_{23}^2 < 0$ is quite similar to that of the normal mass hierarchy. Substituting $m_e = 0.51$ MeV and $m_\mu = 106$ MeV into Eq. (11), we predict

$$|U_{e3}|^2 = 0.0024, \tag{16}$$

which is consistent with the experimental constraint $|U_{e3}|^2_{\mathrm{exp}} < 0.03$ obtained from the CHOOZ [17], solar, and atmospheric neutrino experiments [1]. Let us estimate the neutrino mass m_i by fitting the experimental data. From the solar neutrino experiment, we have

$$\Delta m_{12}^2 = m_2^2 - m_1^2 = \Delta m_{\mathrm{solar(MSW)}}^2 = 1.0 \times 10^{-5} \ \mathrm{eV}^2, \tag{17}$$

$$0.3 \leqslant |U_{e2}|^2 \leqslant 0.7 \quad \text{for LMA MSW}, \tag{18}$$

$$1 \times 10^{-3} \leqslant |U_{e2}|^2 \leqslant 1 \times 10^{-2} \quad \text{for SMA MSW}, \tag{19}$$

for the large mixing angle (LMA) and the small mixing angle (SMA) Mikheyev-Smirnov-Wolfenstein (MSW) solutions. From the atmospheric neutrino experiment [1], we also have

$$\Delta m_{23}^2 = m_3^2 - m_2^2 = \Delta m_{\mathrm{atm}}^2 = (1-7) \times 10^{-3} \ \mathrm{eV}^2. \tag{20}$$

Combining these experimental data with Eq. (14), we obtain the allowed regions in the m_2^2-m_1^2 plane, which are shown in

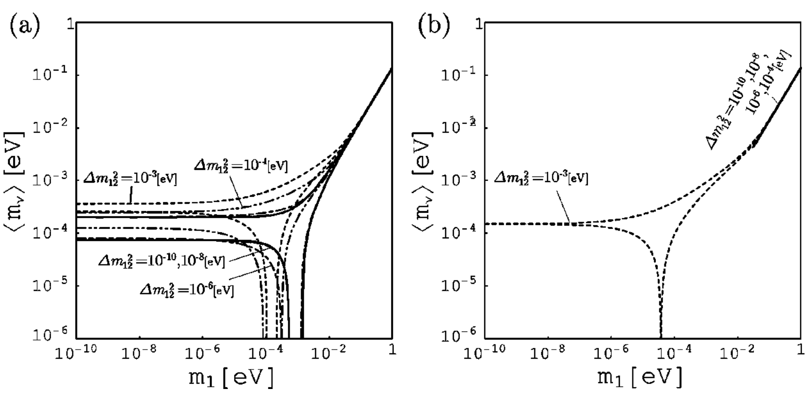

FIG. 3. The allowed regions of $\langle m_\nu \rangle$ and m_1. The allowed regions are inside the boundary curves for each fixed Δm_{12}^2. (a) Normal neutrino mass hierarchy case $\Delta m_{23}^2 = +(1-7) \times 10^{-3}$ eV^2. (b) Inverse neutrino mass hierarchy case $\Delta m_{23}^2 = -(1-7) \times 10^{-3}$ eV^2.

Fig. 1. The allowed regions in the $|U_{e2}|^2$-m_1 plane are shown in Fig. 2. From Eq. (15), we also obtain the allowed regions in the $\langle m_\nu \rangle$-m_1 plane as shown in Fig. 3. It turns out from Fig. 2 and Fig. 3 that large $\langle m_\nu \rangle$ prefers the large mixing angle MSW solution for the solar neutrino problem. From Fig. 3 and the recent experimental upper bound $\langle m_\nu \rangle < 0.2$ eV [18], we obtain the upper bound for m_1 as $m_1 \lesssim 1.5$ eV. If we impose the constraint of neutrino masses from the astrophysical observation [19], $\Sigma_i m_i < 1.8$ eV, in Fig. 3, we obtain

$$\langle m_\nu \rangle \lesssim 0.08 \ \text{eV}. \tag{21}$$

Recently, Klapdor-Kleingrothaus *et al.* [20] have discussed the evidence for neutrinoless double beta decay. If this is true, it is big news. They have reported $\langle m_\nu \rangle = 0.05 - 0.84$ eV (95% C.L.), which is consistent with Eq. (21).

The mass matrices for quarks M_d and M_u in Eq. (2) are the same as in our previous model and the phenomenological results from them were discussed in Ref. [2]. M_d and M_u are assumed to be of type I, similarly to the charged leptons given in Eq. (8). The Cabibbo-Kobayashi-Maskawa [21] quark mixing matrix derived from those M_d and M_u is consistent with the experimental data. (See Ref. [2] for details.)

Finally, let us incorporate the seesaw mechanism in our model. In the seesaw mechanism [22], the neutrino mass matrix M_ν is given by $M_\nu = -M_D^T M_R^{-1} M_D$. Here M_D is the Dirac neutrino mass matrix and M_R is the Majorana mass matrix of the right-handed components. In this paper, we furthermore assume that M_D and M_R have the same zero texture as M_ν of Eq. (5):

$$M_D = \begin{pmatrix} 0 & A_D & 0 \\ A_D & B_D & C_D \\ 0 & C_D & D_D \end{pmatrix}_{2\leftrightarrow 3 sym.part} = \begin{pmatrix} 0 & Y_D & Y_D \\ Y_D & Z_D & W_D \\ Y_D & W_D & Z_D \end{pmatrix},$$
$$M_R = \begin{pmatrix} 0 & A_R & 0 \\ A_R & B_R & C_R \\ 0 & C_R & D_R \end{pmatrix}_{2\leftrightarrow 3 sym.part} = \begin{pmatrix} 0 & Y_R & Y_R \\ Y_R & Z_R & W_R \\ Y_R & W_R & Z_R \end{pmatrix}. \tag{22}$$

This form conserves its form via the seesaw mechanism as discussed in Ref. [10]; namely, the M_ν given by $M_\nu = -M_D^T M_R^{-1} M_D$ takes the same form too and is given by Eq. (5).

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