Flavor without flavor symmetry reexamined

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The mixing of the light families of fermions with other, heavy vectorlike families can lead to a nontrivial family structure without any symmetry that distinguishes among the families. When this idea is combined with grand unification, predictive models of quark and lepton masses can be constructed. In this paper, these old ideas are reexamined, and two realistic examples are given.

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I. INTRODUCTION

There are two outstanding features of the quark and lepton spectrum. First, the masses of the fermions of each kind exhibit an interfamily hierarchy: $m_3 \gg m_2 \gg m_1$. Second, these hierarchies appear to be nearly aligned, at least in the case of the quarks, in the sense that u_3 is aligned with d_3, u_2 with d_2, and u_1 with d_1. That is to say, the mixing angles are very small. There is also some evidence that the lepton hierarchies are aligned, namely the fact that the lepton mixing angle U_{e3} is very small. But the large mixing angle $U_{\mu 3}$ seen in atmospheric neutrino oscillations shows that the alignment is not as good for the leptons as for the quarks.

Attempts to explain the quark and lepton masses are usually based on either grand unification or flavor symmetry, or a combination of the two approaches. The way that flavor symmetry would explain the parallel hierarchies of the fermion masses is quite straightforward. If flavor symmetry distinguished the different families from each other, then the masses of the fermions of different families would typically arise at different order in flavor symmetry breaking. Moreover, the mixing of families would also be a flavor-symmetry-breaking effect. Consequently, if family-symmetry-breaking effects were small, one might expect both a hierarchical pattern of masses and small mixing angles, as observed.

Turning to grand unification, one finds that grand unified gauge symmetries are able to explain small mixing angles in a completely different way, which is most readily understood by considering the minimal $SO(10)$ model. In minimal $SO(10)$ all four Dirac mass matrices, those of the neutrinos, up quarks, down quarks, and charged leptons (which matrices we denote henceforth as N,U,D,L) are exactly proportional: $N = U \propto D = L$. Obviously this means that the hierarchies in minimal $SO(10)$ are *exactly* aligned, and the Cabibbo-Kobayashi-Maskawa (CKM) angles vanish. In realistic unified models the relation between the mass matrices is more complicated, but for models based on $SO(10)$ and some related groups the CKM angles do tend to be small.

The question arises whether grand unification can also explain the other main feature of the quark and lepton spectrum—the interfamily mass hierarchies—without flavor symmetry. If so, then it becomes attractive to dispense with flavor symmetry altogether. By flavor symmetry, we mean here any symmetry that distinguishes among the three families.

If there is no flavor symmetry, then how could one explain that some families have much larger mass than others? One possibility is that fermions from different families may mix differently with superheavy fermions that are vectorlike under the standard model group [1–6]. Such a situation tends to arise very naturally in the context of grand unification, as first noted in [1]. In this paper we reexamine this old idea and attempt to improve upon some of the previous model-building attempts that are based on it.

In Sec. II we shall explain the basic idea and review at some length some of the past model-building attempts. In Sec. III, we propose two new models which further develop the idea.

II. REVIEW OF THE IDEA AND SOME EXISTING MODELS

The basic idea that we are reexamining in this paper is that in grand unified models the different families can have different masses by virtue of mixing differently with superheavy fermions that are in real representations of the standard model group. There may be arbitrarily many such real representations, and—assuming the "Georgi Survival Hypothesis" [7], which is a consequence of naturalness—their existence will have no effect on the number of light families.

There are various possibilities for these real representations of fermions. One possibility is that they come from anomaly-free sets of complex representations of the grand unified group as in [1]. Another possibility is that they come from irreducible real representations of the grand unified group, as in [2], where fermions in the **10** and **45** representations of $SO(10)$ were introduced in addition to the usual three spinors. A third possibility is that they come from conjugate pairs of complex representations. A particularly interesting case of this is that they are in $\mathbf{16}+\overline{\mathbf{16}}$ pairs of $SO(10)$, i.e., that they are in one or more conjugate "family-antifamily" pairs. The idea of using such family-antifamily pairs to get interesting textures was first proposed by [3] and independently by [4]. A particularly realistic interfamily hierarchy can result if the mixing is predominantly with just one family-antifamily pair [3,4,5], as will be explained below.

In order to get fully realistic textures it is useful, as we will see, to posit the existence of superheavy fermions both in family-antifamily pairs as in [3] and in irreducible real representations as in [2]. Examples of such models are those in [4,6,8,9].

The way that mixing with superheavy real fermion representations can lead to different families having different masses can be illustrated very simply by looking at the model of [4]. Suppose that we consider an $SO(10)$ model in which, in addition to the three families $\mathbf{16}_i$, $i=1,2,3$, there is a ''vectorlike'' family-antifamily pair $\mathbf{16}+\overline{\mathbf{16}}$. We can imagine a Z_2 parity under which the ordinary families are odd and the vectorlike ones are even. We do not consider this a flavor symmetry because it does not distinguish among the three families. Suppose further that there are Higgs bosons in the vector and adjoint representations of $SO(10)$, $\mathbf{10}_H$ and $\mathbf{45}_H$. If these are also odd under the Z_2, then only the following types of renormalizable Yukawa coupling are allowed [4]:

$$W_{Yukawa}=M(\overline{\mathbf{16}}\mathbf{16})+a_i(\overline{\mathbf{16}}\mathbf{16}_i)\mathbf{45}_H+b_i(\mathbf{16}\mathbf{16}_i)\mathbf{10}_H. \tag{1}$$

The mass M is of the grand unified theory (GUT) scale. The interesting point to note (which was first noted in [3–5]) is that even though no symmetry has been imposed that distinguishes among the families, two of the light families get mass and one does not. The reason for this is that the two Yukawa coupling vectors, a_i and b_i, span only a two dimensional subspace of the full three dimensional family space. To be more concrete, one can without any loss of generality choose the axes in family space so that $a_i=a(0,0,1)$ and $b_i=b(0,\sin\theta,\cos\theta)$. It is then clear that the first family has no Yukawa couplings at all and remains exactly massless. The other families get mass through mixing, as follows. The first two terms of Eq. (1) lead to a superheavy fermion mass term that can be written as $\overline{\mathbf{16}}(M\mathbf{16}+a\langle\mathbf{45}_H\rangle\mathbf{16}_3)$. One sees that the superheavy spinor is a linear combination of the $\mathbf{16}$ and $\mathbf{16}_3$. The linear combination orthogonal to this is light and in fact is just the third family. In other words, the $\mathbf{16}$ without an index actually contains in part the light fermions of the third family. Consequently, the third term in Eq. (1) generates weak-scale mass terms of the form $b\langle\mathbf{10}_H\rangle\mathbf{16}_3(\cos\theta\mathbf{16}_3+\sin\theta\mathbf{16}_2)$. That is, it generates 23, 32, and 33 elements of the light fermion mass matrices [3–5].

To summarize what is going on in this example, the light fermions are only able to obtain mass through mixing with superheavy fermions. But not all the three families are able to mix in this way, since there are not enough superheavy fermions for them all to mix with. Thus, perforce, an interfamily mass hierarchy results even though all three families have exactly the same quantum numbers.

Let us examine the structure we have just described in more detail. One may write the vacuum expectation value (VEV) of the adjoint Higgs field as $\langle\mathbf{45}_H\rangle=\Omega T$, where T is a generator of $SO(10)$. Then, integrating out the vectorlike fields $\overline{\mathbf{16}}+\mathbf{16}$, as shown in Fig. 1, one obtains the effective operator

$$\begin{aligned}W_{eff}&\cong[a_i\langle\mathbf{45}_H\rangle\mathbf{16}_i][b_j\langle\mathbf{10}_H\rangle\mathbf{16}_j]/M\\&=(a_3T\cdot\mathbf{16}_3)(b_3\mathbf{16}_3+b_2\mathbf{16}_2)\langle\mathbf{10}_H\rangle(\Omega/M)\\&\propto T_{(16_3)}\mathbf{16}_3(\cos\theta\mathbf{16}_3+\sin\theta\mathbf{16}_2).\end{aligned} \tag{2}$$

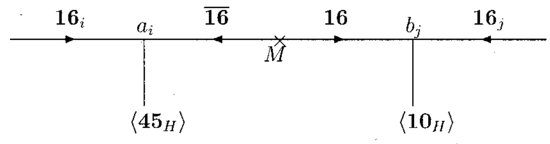

FIG. 1. Diagram that leads to the effective operator given in Eq. (2).

Consider now fermions of type f, where f can be (left-handed) up-type quarks, down-type quarks, charged leptons, or neutrinos. The left-handed antifermions are denoted f^c. There arises straightforwardly from the previous equation the following effective three-by-three mass matrix for the light fermions of type f:

$$f_i^c M_{ij} f_j \cong M_f(f_1^c, f_2^c, f_3^c)\times\begin{pmatrix}0&0&0\\0&0&\sin\theta T_f\\0&\sin\theta T_{f^c}&\cos\theta(T_f+T_{f^c})\end{pmatrix}\times\begin{pmatrix}f_1\\f_2\\f_3\end{pmatrix}. \tag{3}$$

The factor M_f has one value, which we shall call M_U, for up quarks and neutrinos, and another value, which we shall call M_D, for down quarks and charged leptons. The symbols T_f and T_{f^c} stand for the charges of the fields f and f^c under the $SO(10)$ generator T.

There are several interesting features of this structure that have been pointed out in earlier papers. One interesting feature [4,5] is that the structure not only singles out one family as massless (despite there being no flavor symmetry that distinguishes one family from another) but also naturally accommodates a mass hierarchy between the second and third families as well. For example, if we assume that $T_f \sim T_{f^c}$ and that $\sin\theta$ is somewhat small, then $m_{f_2}/m_{f_3}\sim 1/4\tan^2\theta \ll 1$.

Another interesting feature of this structure [4,6] is that it naturally explains why the minimal $SU(5)$ relation $m_b \cong m_\tau$ works well while the corresponding relation for the second family, $m_s \cong m_\mu$, does not. The reason has to do with the way the $SO(10)$ generators appear in Eq. (3). Since the same Higgs doublet H_d in $\mathbf{10}_H$ couples to both $d^c d$ and $l^+ l^-$, it follows that $T_{d^c}+T_d=T_{l^+}+T_{l^-}=-T_{H_d}$. Consequently, the 33 elements of the mass matrices for the down quarks and charged leptons are approximately equal. However, the 23 and 32 elements are different for the two matrices since T_d and T_{d^c} are not equal to T_{l^-} and T_{l^+}.

Finally, the structure incorporates automatically a sort of Fritzschian form [10] for the heavier families, with a ''texture zero'' in the 22 element. This relates the smallness of V_{cb} to the smallness of m_c/m_t and m_s/m_b.

It is remarkable that the one effective Yukawa term given in Eq. (2) goes so far toward providing a satisfactory framework for describing the masses and mixings of the fermions of the second and third families. In several earlier papers attempts were made to construct models of the quark and lepton masses and mixings on the basis of exactly this term. The first attempt was [4], where the second and third families were described using *only* the operator of Eq. (2). However, the resulting fits were not satisfactory. There are four dimensionless quantities (m_μ/m_τ, m_s/m_b, m_c/m_t, and V_{cb}) that had to be fit using two parameters, namely θ and a parameter specifying the $SO(10)$ generator T. [Since T must commute with the standard model group, it must be a linear combination of weak hypercharge and the generator X in $SU(5)\times U(1)_X$, or equivalently a linear combination of $B-L$ and I_{3R}. Thus, only a single parameter is needed to specify T.] It turned out that at least one quantity got very badly fit for all choices of parameters. In the same paper, a better fit was sought by extending the model in the obvious way to the group E_6. One more parameter was thereby introduced, since in E_6 two parameters are needed to specify the generator T. It seems almost prophetic in light of recent results that the best fit obtained in the E_6 model had very small m_s and a large mixing between μ_L^- and τ_L^-, i.e., a large contribution to the leptonic mixing $U_{\mu 3}$. Unfortunately, the values of m_s obtained were *too* small even compared to the recent lattice results, and the value of the $\mu\tau$ mixing angle was not large *enough* to account for the atmospheric neutrino oscillations.

In [6] a realistic model was obtained by introducing structures that went beyond Eq. (2) to account for the heavy two families. It was assumed that $T=(I_{3R})+\epsilon(B-L)$, $\epsilon\ll 1$, which gave an appealingly simple explanation of the Georgi-Jarlskog relation; however, the ratio m_c/m_t remained a problem, since with this choice of T, the matrices in Eq. (3) give the minimal $SO(10)$ result $m_c/m_t \cong m_s/m_b$. A way of suppressing m_c/m_t was found in [6] that, although elegant, was somewhat involved.

In [8,9] a very successful model of quark and lepton models was constructed that also included the operator in Eq. (2). (For a similar model see [11].) However, to give a satisfactory account of the heavy two families, two other operators in addition to that in Eq. (2) were needed. Nevertheless, this did not lead to a loss of predictivity for two reasons. First, the generator T was fixed to be exactly $B-L$ in order to solve the doublet-triplet splitting problem via the Dimopoulos-Wilczek mechanism [12], thus reducing the number of parameters by one. Second, with $T=B-L$, the 33 elements in Eq. (3) vanish, since $(B-L)_f+(B-L)_{f^c}=0$, making the parameter θ irrelevant. (This necessitated, of course, that a different operator be introduced to generate the 33 elements.) The model of [8] and [9] was extremely predictive and simple in structure. One of its great successes was that it naturally accounted for the largeness of the atmospheric neutrino mixing angle. However, though very simple, it made an enormous sacrifice from the point of view of the idea we are exploring in the present paper: it was based on a flavor symmetry, i.e., a symmetry that distinguished the three families from each other.

III. SOME NEW MODELS

To summarize the past efforts based on the group $SO(10)$ and the effective operator of Eq. (2), one can say that no completely satisfactory model along these lines exists that succeeds in explaining the flavor structure of the quarks and leptons without a flavor symmetry. In this paper we shall pursue this approach again. We present two models. The first is similar in spirit to the models of [4] and [6]. It is rather simple and has a single prediction, namely the mass of the strange quark, which comes out smaller than the Georgi-Jarlskog prediction and more in line with the recent lattice estimates. The second is very close to the model of [8] and [9], but is obtained without recourse to flavor symmetry.

Model 1. A realistic variant of the models of [8] and [9] can be constructed in a simple fashion. Consider an $SO(10)$ model with the following Yukawa superpotential:

$$\begin{aligned} W_{Yukawa} = {} & M(\overline{\mathbf{16}}\mathbf{16}) + a_i(\overline{\mathbf{16}}\mathbf{16}_i)\mathbf{1}_H \\ & + M'(\overline{\mathbf{16}}'\mathbf{16}') + b_i(\overline{\mathbf{16}}'\mathbf{16}_i)\mathbf{45}_H \\ & + c(\mathbf{16}\mathbf{16})\mathbf{10}_H^{up} + d(\mathbf{16}'\mathbf{16})\mathbf{10}_H^{down}. \end{aligned} \tag{4}$$

The vector Higgs fields $\mathbf{10}_H^{up}$ and $\mathbf{10}_H^{down}$ are supposed, respectively, to obtain VEVs in their $Y/2=+1/2$ and $Y/2=-1/2$ components. Thus the former gives mass only to up quarks and neutrinos, while the latter gives mass to down quarks and charged leptons.

The structure that emerges from these terms can be understood readily. As before, we can write $\langle \mathbf{45}_H \rangle = \Omega T$, where T is an $SO(10)$ generator, which we choose to parametrize as $T=2I_{3R}+3d(B-L)$. (The parameter called d here is the same up to a normalization as the parameter called ϵ in [6].) Without loss of generality a basis in family space can be chosen so that the Yukawa coupling vectors take the form $a_i=a(0,0,1)$ and $b_i=b(0,\sin\theta,\cos\theta)$. The combination of the two terms involving the $\overline{\mathbf{16}}$, namely the term with M and the term with $\mathbf{1}_H$, cause the fields $\mathbf{16}$ and $\mathbf{16}_3$ to mix with each other. That means that the field $\mathbf{16}$ is not purely superheavy, but also contains an admixture of the fields of the third family. Similarly, the two terms involving the $\overline{\mathbf{16}}'$, namely the term with M' and the term with $\mathbf{45}_H$, cause the $\mathbf{16}'$ and the linear combination $b_i\mathbf{16}_i=b(\cos\theta\mathbf{16}_3+\sin\theta\mathbf{16}_2)$ to mix with each other. Thus the $\mathbf{16}'$ is also not purely superheavy, but contains an admixture of the fields of the second and third families in a proportion that depends on the angle θ.

Given these facts, one sees that the term $(\mathbf{16}\mathbf{16})\mathbf{10}_H^{up}$ contributes only to the 33 element of the up quark mass matrix U. Similarly, the term $(\mathbf{16}'\mathbf{16})\mathbf{10}_H^{down}$ contributes to the 23, 32, and 33 elements of the mass matrix of the down quarks, D, and the mass matrix of the charged leptons, L. At this stage, then, one has that the charm quark is massless. It, like the fermions of the first family, is supposed to get mass from other smaller terms. This is not unreasonable, in light of the fact that m_c/m_t is an order of magnitude smaller than m_s/m_b and m_μ/m_τ.

From the foregoing one can write down the following expressions for the mass matrices D and L:

$$D=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & (1-d)\sin\theta \\ 0 & d\sin\theta & \cos\theta \end{pmatrix} M_D,$$

$$L=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & (1+3d)\sin\theta \\ 0 & -3d\sin\theta & \cos\theta \end{pmatrix} M_D. \tag{5}$$

In this model, two parameters, d and θ are available to predict three dimensionless quantities, V_{cb}, m_μ/m_τ, and m_s/m_b. There is therefore one prediction, which can be taken to be for the strange quark mass. For brevity, let us define $t\equiv\tan\theta$, $v\equiv V_{cb}$, and $3l\equiv m_\mu/m_\tau$. Then, to the leading two orders in small quantities one can write

$$v\equiv V_{cb}=\frac{dt}{1+[(1-d)^2-d^2]t^2}, \tag{6}$$

$$l\equiv\frac{1}{3}\frac{m_\mu}{m_\tau}=\frac{d(1+3d)t^2}{1+[(1+3d)^2+(3d)^2]t^2}, \tag{7}$$

$$m_s/m_b=\frac{d(1-d)t^2}{1+[(1-d)^2+d^2]t^2}. \tag{8}$$

Equation (6) can be inverted to give $d=(v/t)((1+t^2)/(1+2vt))$. Substituting this into Eq. (7), one finds that the expressions simplify to yield a quadratic equation for the parameter $t=\tan\theta$ in terms of the experimentally known v and l:

$$0=t^2\left[5v\left(1-\frac{34}{5}l\right)\right]+t[1-10l]-[l/v+18lv-3v]. \tag{9}$$

Using $v=0.035$ and $l=1/50.4$ (these are evaluated at the GUT scale), one finds that

$$t\equiv\tan\theta=0.536,\quad d=0.081. \tag{10}$$

Note that the angle θ is not particularly small. This means that the Yukawa vectors a_i and b_i are not "unnaturally" aligned in family space. This is consistent with the philosophy that there is no family symmetry. The small parameter in this model is really d. As is evident from Eqs. (6)–(8), it is the smallness of d that accounts for the mass hierarchy between the second and third families and for the smallness of the mixing between them. That is, remarkably, it is not a flavor symmetry that produces these "flavor" features, but the pattern of $SO(10)$ breaking. Note also that in the limit $d\to 0$, the Georgi-Jarlskog relation $m_s/m_b=\frac{1}{3}m_\mu/m_\tau$ becomes exact, as can be seen from Eqs. (7) and (8). In fact, this is how the Georgi-Jarlskog relation was obtained in the model of [6]. However, since the parameter d is significantly different from zero, there is a significant deviation from the exact Georgi-Jarlskog prediction for m_s. One finds that

$$m_s/m_b\cong 0.866(m_s/m_b)_{GJ}\cong 1/58.2. \tag{11}$$

This is the value at the unification scale. It translates into a strange quark mass of about 137 MeV at 1 GeV, or about 100 MeV at 2 GeV, which is in the range given by recent lattice calculations.

Model 2. The second model is a realization of the model of [8] and [9] constructed without use of flavor symmetries. Again based on $SO(10)$ it has the following Yukawa superpotential terms:

$$\begin{aligned} W_{Yukawa}=&M(\overline{\mathbf{16}}\mathbf{16})+a_i(\overline{\mathbf{16}}\mathbf{16}_i)\mathbf{1}_H \\ &+M'(\mathbf{10}\mathbf{10})+b_i(\mathbf{10}\mathbf{16}_i)\mathbf{16}_H \\ &+c(\mathbf{16}\mathbf{16})\mathbf{10}_H+d(\mathbf{16}\mathbf{10})\mathbf{16}'_H \\ &+g_i(\mathbf{16}\mathbf{16}_i)\mathbf{10}_H\mathbf{45}_H\mathbf{1}_H/M_G^2. \end{aligned} \tag{12}$$

Here, the spinor Higgs field $\mathbf{16}_H$ is supposed to acquire a VEV in the $SU(5)$-singlet direction, i.e., one that commutes with the standard model group, whereas $\mathbf{16}'_H$ is supposed to acquire a VEV in the weak-doublet direction [13]. As in the model of [8] and [9], the adjoint Higgs field $\mathbf{45}_H$ is supposed to acquire a VEV in the $B-L$ direction, as needed to solve the doublet-triplet splitting problem by the Dimopoulos-Wilczek mechanism. This set of terms can be shown to be the most general that is consistent with a Z_3 symmetry under which all the quark and lepton multiplets transform trivially except for the three ordinary families $\mathbf{16}_i$, which all transform nontrivially and in the same way.

The form of the mass matrices that result from these Yukawa terms can be determined by the same kind of reasoning that was used above. As before, we can choose our axes to make $a_i=a(0,0,1)$ and $b_i=b(0,\sin\theta,\cos\theta)$. The coefficient c_i in the higher-dimension operator will then in general have three nonzero components. It is easy to see that integrating out the $\overline{\mathbf{16}}$ leads to a mixing of the $\mathbf{16}$ and $\mathbf{16}_3$, as discussed before. Similarly, integrating out the superheavy fermions in the $\mathbf{10}$ leads to a mixing of the $SU(5)$ $\overline{\mathbf{5}}$'s in $\mathbf{10}$ and in $b_i\mathbf{16}_i=b(\cos\theta\mathbf{16}_3+\sin\theta\mathbf{16}_2)$. The term with coefficient c then gives 33 entries to all the Dirac mass matrices N, U, D, and L. These contributions are denoted "1" in the matrices shown below. The term with coefficient d gives contributions only to the matrices D and L. This can be seen by looking at the $SU(5)$ decomposition of this term: $d[\mathbf{10}(\mathbf{16})\overline{\mathbf{5}}(\mathbf{10})]\overline{\mathbf{5}}(\mathbf{16}'_H)$. This reduces to a term proportional to $\mathbf{10}_3(\cos\theta\overline{\mathbf{5}}_3+\sin\theta\overline{\mathbf{5}}_2)\overline{\mathbf{5}}_H$. These contributions are denoted by "σ" in the matrices below. Note that these contributions are "lopsided," giving only a contribution to D_{23} but not D_{32} and to L_{32} but not L_{23}. Finally, the higher-dimension operator with coefficient g_i gives terms proportional to $\mathbf{16}_3(g_3\mathbf{16}_3+g_2\mathbf{16}_2+g_1\mathbf{16}_1)$, which are denoted by "$\epsilon_i$" in the matrices below. Altogether, then, the matrices have the form

$$U=\begin{pmatrix} 0 & 0 & -\epsilon_1/3 \\ 0 & 0 & -\epsilon_2/3 \\ \epsilon_1/3 & \epsilon_2/3 & 1 \end{pmatrix} M_U, \tag{13}$$

$$D=\begin{pmatrix} 0 & 0 & -\epsilon_1/3 \\ 0 & 0 & \sigma s_\theta - \epsilon_2/3 \\ \epsilon_1/3 & \epsilon_2/3 & 1+\sigma c_\theta \end{pmatrix} M_D,$$

$$L=\begin{pmatrix} 0 & 0 & \epsilon_1 \\ 0 & 0 & \epsilon_2 \\ -\epsilon_1 & \sigma s_\theta - \epsilon_2 & 1+\sigma c_\theta \end{pmatrix} M_D,$$

where $s_\theta \equiv \sin\theta$, $c_\theta \equiv \cos\theta$. By rotations in the 1-2 planes these can be brought to the forms

$$U=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\epsilon/3 \\ 0 & \epsilon/3 & 1 \end{pmatrix} M_U,$$

$$D\cong\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sigma s_\theta - \epsilon_2/3 \\ 0 & \epsilon/3 & 1+\sigma c_\theta \end{pmatrix} M_D, \tag{14}$$

$$L\cong\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon \\ -\epsilon_1 & \sigma s_\theta - \epsilon_2 & 1+\sigma c_\theta \end{pmatrix} M_D,$$

where $\epsilon \equiv \sqrt{\epsilon_1^2+\epsilon_2^2}$. These matrices go over to those of the model of [8] in the limit that $\cos\theta \rightarrow 0$ and $\epsilon_1/\epsilon_2 \rightarrow 0$. As far as the fits to the quark masses and mixings and the charged lepton masses are concerned, the parameter ϵ_1 makes very little difference. (It does, however, make a contribution to the neutrino mixing parameter U_{e3}.) The presence of the $\cos\theta$ and $\sin\theta$ in these matrices is important, on the other hand, since it introduces an additional parameter compared to the model of [8]. There it was found that an excellent fit was obtained with $\sigma \cong 1.7$ and $\epsilon \cong 0.14$. Here, because of the additional parameter θ a slightly better fit is possible. We find the best fit to be $\sigma \cong 1.6$, $\epsilon \cong 0.15$, and $\cos\theta \cong 0.13$.

Essentially, then, the model is the same as that in [8], although slightly less predictive. It has the same important feature of highly ''lopsided'' mass matrices D and L, i.e., $D_{32} \ll D_{23} \sim 1$ and $L_{23} \ll L_{32} \sim 1$. This, as emphasized in [8], gives a natural explanation of why the atmospheric neutrino mixing ($U_{\mu 3}$) is so large. Other important features are the natural explanation of the Georgi-Jarlskog factor and of the fact that $m_c/m_t \ll m_s/m_b$. The reader is referred to [8] for further details.

The interesting thing is that we have succeeded in taking a highly successful model of quark and lepton masses that exists in the literature and that was constructed by means of flavor symmetries which distinguish among the three families and constructing a model that is virtually the same without making any use of flavor symmetries of that kind.

What we have shown is that in the context of grand unification it is possible to construct interesting and predictive models which reproduce the important features of the quark and lepton mass spectrum, while at the same time treating all three families on exactly the same footing, that is, giving them the same quantum numbers. In other words, one can have flavor without flavor symmetry.

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