Lepton flavor violating processes in the bimaximal texture of neutrino mixings

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We investigate lepton flavor violation in the framework of the minimal supersymmetric standard model with right-handed neutrinos taking the large mixing angle Mikheyev-Smirnov-Wolfenstein solution in the quasidegenerate and inverse-hierarchical neutrino masses. We predict the branching ratio of $\mu \to e+\gamma$ and $\tau \to \mu+\gamma$ processes assuming degenerate right-handed Majorana neutrino masses. We find that the branching ratio in the quasidegenerate neutrino mass spectrum is 100 times smaller than the ones in the inverse-hierarchical and hierarchical neutrino spectra. We emphasize that the magnitude of U_{e3} is one of the important ingredients to predict BR($\mu \to e+\gamma$). The effect of the deviation from the completely degenerate right-handed Majorana neutrino masses is also estimated. Furtheremore, we examine the $S_{3L}\times S_{3R}$ model, which gives the quasidegenerate neutrino masses, and the Shafi-Tavartkiladze model, which gives the inverse-hierarchical neutrino masses. Both predicted branching ratios of $\mu \to e+\gamma$ are smaller than the experimental bound.

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I. INTRODUCTION

The Super-Kamiokande Collaboration has almost confirmed neutrino oscillation in atmospheric neutrinos, which favors the $\nu_\mu \to \nu_\tau$ process [1]. For solar neutrinos [2,3], the recent data of the Super-Kamiokande and Sudbury Neutrino Observatory experiments also suggest the neutrino oscillation $\nu_e \to \nu_x$ with the large mixing angle (LMA) Mikheyev-Smirnov-Wolfenstein (MSW) solution, although other solutions are still allowed [4,5]. If we take the LMA MSW solution, neutrinos are massive and the flavor mixings are almost bimaximal in the lepton sector.

If neutrinos are massive and mixed in the standard model (SM), there exists a source of lepton flavor violation (LFV) through the off-diagonal elements of the neutrino Yukawa coupling matrix. However, because of the smallness of the neutrino masses, the predicted branching ratios for these processes are so tiny that they are completely unobservable [6].

On the other hand, in the supersymmetric framework the situation is quite different. Many authors have already studied LFV in the minimal supersymmetric standard model (MSSM) with right-handed neutrinos assuming the relevant neutrino mass matrix [7–10]. In the MSSM with soft breaking terms, there exist lepton flavor violating terms such as off-diagonal elements of slepton mass matrices $(\mathbf{m}^2_{\tilde{L}})_{ij}$, $(\mathbf{m}^2_{\tilde{e}_R})_{ij}$ and trilinear couplings $\mathbf{A}^e_{ij}$. Strong bounds on these matrix elements come from requiring branching ratios for LFV processes to be below observed ratios. For the present, the most stringent bound comes from the $\mu \to e+\gamma$ decay [$\mathrm{BR}(\mu \to e+\gamma) < 1.2\times 10^{-11}$] [11]. However, if the LFV occurs at the tree level in the soft breaking terms, the branching ratio of this process exceeds the experimental bound considerably. Therefore one assumes that the LFV does not occur at the tree level in the soft parameters. This is realized by making the assumption that soft parameters such as $(\mathbf{m}^2_{\tilde{L}})_{ij}$, $(\mathbf{m}^2_{\tilde{e}_R})_{ij}$, and $\mathbf{A}^e_{ij}$, are universal, i.e., proportional to the unit matrix. This assumption follows from minimal supergravity (MSUGRA). However, even though there is no flavor violation at the tree level, it is generated by the effect of the renormalization group equations (RGE's) via neutrino Yukawa couplings. Suppose that neutrino masses are produced by the seesaw mechanism [12]; there are right-handed neutrinos above a scale M_R. Then neutrinos have the Yukawa coupling matrix $\mathbf{Y}_\nu$ with off-diagonal entries in the basis of the diagonal charged-lepton Yukawa couplings. The off-diagonal elements of $\mathbf{Y}_\nu$ drive off-diagonal ones in the $(\mathbf{m}^2_{\tilde{L}})_{ij}$ and $\mathbf{A}^e_{ij}$ matrices through running of the RGE's [13].

One can construct $\mathbf{Y}_\nu$ from the recent data of neutrino oscillations. Assuming that oscillations need only account for the solar and atmospheric neutrino data, we take the LMA MSW solution for the solar neutrino. Then the lepton mixing matrix, which may be called the Maki-Nakagawa-Sakata (MNS) matrix or the MNS-Pontecorvo (MNSP) matrix [14,15], is given in Ref. [16]. Since the data for neutrino oscillations only indicate the differences of the mass square Δm^2_{ij}, neutrinos have three possible mass spectra: the hierarchical spectrum $m_{\nu 3} \gg m_{\nu 2} \gg m_{\nu 1}$, the quasidegenerate one $m_{\nu 1} \simeq m_{\nu 2} \simeq m_{\nu 3}$, and the inverse-hierarchical one $m_{\nu 1} \simeq m_{\nu 2} \gg m_{\nu 3}$.

We have already analyzed the effect of neutrino Yukawa couplings for the $\mu \to e+\gamma$ process assuming the quasidegenerate and inverse-hierarchical spectra [17]. In this paper, we present the detailed formulas in our calculations of $\mu \to e+\gamma$ and discuss the dependence of the supersymmetry (SUSY) breaking parameters on the branching ratio. In the [17], the right-handed Majorana neutrino masses were assumed to be completely degenerate. We study the effect of deviation from this degeneracy in this work. The correlation between $\mathrm{BR}(\mu \to e+\gamma)$ and $\mathrm{BR}(\tau \to \mu+\gamma)$ is also calculated. Furthermore, two specific models of the neutrino mass matrix are examined in the $\mu \to e+\gamma$ process.

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This paper is organized as follows. In Sec. II, we give the general form of $\mathbf{Y}_\nu$ and $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$, which play a crucial role in generating the LFV through the running RGE's. In Sec. III, we calculate the branching ratios of the processes $\mu \to e + \gamma$ and $\tau \to \mu + \gamma$ in the three neutrino mass spectra. In Sec. IV, we examine the $S_{3L} \times S_{3R}$ model, which gives the quasidegenerate neutrino masses, and the Shafi-Tavartkiladze model, which gives the inverse-hierarchical neutrino masses. In Sec. V, we summarize our results and give a discussion.

II. LFV IN THE MSSM WITH RIGHT-HANDED NEUTRINOS

A. Yukawa couplings

In this section, we introduce the general expression for the neutrino Yukawa coupling $\mathbf{Y}_\nu$, which is useful in the following arguments, and investigate the LFV triggered by the neutrino Yukawa couplings. The superpotential of the lepton sector is described as follows:

$$W_{\text{lepton}} = \mathbf{Y}_e L H_d e_R^c + \mathbf{Y}_\nu L H_u \nu_R^c + \frac{1}{2} \nu_R^{cT} \mathbf{M}_R \nu_R^c, \tag{2.1}$$

where H_u, H_d are chiral superfields for Higgs doublets, L is the left-handed lepton doublet, and e_R and ν_R are the right-handed charged lepton and the neutrino superfields, respectively. $\mathbf{Y}_e$ is the Yukawa coupling matrix for the charged lepton and $\mathbf{M_R}$ is the Majorana mass matrix of the right-handed neutrinos. We take $\mathbf{Y}_e$ and $\mathbf{M_R}$ to be diagonal.

It is well known that the neutrino mass matrix is given by

$$\mathbf{m}_\nu = (\mathbf{Y}_\nu v_u)^{\mathrm{T}} \mathbf{M_R}^{-1} (\mathbf{Y}_\nu v_u), \tag{2.2}$$

via the seesaw mechanism, where v_u is the vacuum expectation value (VEV) of the Higgs particle H_u. In Eq. (2.2), the Majorana mass term for left-handed neutrinos is not included since we consider the minimal extension of the MSSM.

The neutrino mass matrix $\mathbf{m}_\nu$ is diagonalized by a single unitary matrix:

$$\mathbf{m}_\nu^{\text{diag}} \equiv \mathbf{U}_{\text{MNS}}^{\mathrm{T}} \mathbf{m}_\nu \mathbf{U}_{\text{MNS}}, \tag{2.3}$$

where $\mathbf{U}_{\text{MNS}}$ is the lepton mixing matrix. Following the expression in Ref. [10], we write the neutrino Yukawa coupling as

$$\mathbf{Y}_\nu = \frac{1}{v_u} \sqrt{\mathbf{M}_{\mathbf{R}}^{\text{diag}}} \mathbf{R} \sqrt{\mathbf{m}_\nu^{\text{diag}}} \mathbf{U}_{\text{MNS}}^{\mathrm{T}}, \tag{2.4}$$

or explicitly

$$\mathbf{Y}_\nu = \frac{1}{v_u} \begin{pmatrix} \sqrt{M_{R1}} & 0 & 0 \\ 0 & \sqrt{M_{R2}} & 0 \\ 0 & 0 & \sqrt{M_{R3}} \end{pmatrix} \mathbf{R} \times \begin{pmatrix} \sqrt{m_{\nu 1}} & 0 & 0 \\ 0 & \sqrt{m_{\nu 2}} & 0 \\ 0 & 0 & \sqrt{m_{\nu 3}} \end{pmatrix} \mathbf{U}_{\text{MNS}}^{\mathrm{T}}, \tag{2.5}$$

where $\mathbf{R}$ is a 3×3 orthogonal matrix, which depends on the model. Details are given in Appendix A.

First, let us take the degenerate right-handed Majorana masses $M_{R1} = M_{R2} = M_{R3} \equiv M_R$. This assumption is reasonable for the case of quasidegenerate neutrino masses. Otherwise close agreement would be needed between $\mathbf{Y}_\nu$ and $\mathbf{M_R}$. This assumption is also made for the cases of inverse-hierarchical and hierarchical neutrino masses. We also discuss later the effect of the deviation from the degenerate right-handed Majonara neutrino masses.

Then we get

$$\mathbf{Y}_\nu = \frac{\sqrt{M_R}}{v_u} \mathbf{R} \begin{pmatrix} \sqrt{m_{\nu 1}} & 0 & 0 \\ 0 & \sqrt{m_{\nu 2}} & 0 \\ 0 & 0 & \sqrt{m_{\nu 3}} \end{pmatrix} \mathbf{U}_{\text{MNS}}^{\mathrm{T}}, \tag{2.6}$$

and

$$\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu = \frac{M_R}{v_u^2} \mathbf{U}_{\text{MNS}} \begin{pmatrix} m_{\nu 1} & 0 & 0 \\ 0 & m_{\nu 2} & 0 \\ 0 & 0 & m_{\nu 3} \end{pmatrix} \mathbf{U}_{\text{MNS}}^{\mathrm{T}}, \tag{2.7}$$

or equivalently

$$(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{\alpha\beta} = \frac{M_R}{v_u^2} \sum_{i=1}^{3} m_{\nu i} U_{\alpha i} U_{\beta i}^*, \tag{2.8}$$

where the $U_{\alpha\beta}$'s are the elements of $\mathbf{U}_{\text{MNS}}$. It is remarked that $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ is independent of $\mathbf{R}$ in the case of $M_{R1} = M_{R2} = M_{R3} \equiv M_R$. It may be important to consider the deviation from the degenerate right-handed Majonara neutrino masses. A detailed discussion is given in Sec. III B.

Note that this representation of the Yukawa coupling is given at the electroweak scale. Since we need the Yukawa coupling at the grand unified theory (GUT) scale, Eq. (2.5) should be modified by taking account of the effect of the RGE's [18–20]. Modified Yukawa couplings at the scale M_R are given by

$$\mathbf{Y}_\nu = \frac{\sqrt{M_R}}{v_u} \mathbf{R} \begin{pmatrix} \sqrt{m_{\nu 1}} & 0 & 0 \\ 0 & \sqrt{m_{\nu 2}} & 0 \\ 0 & 0 & \sqrt{m_{\nu 3}} \end{pmatrix} \times \mathbf{U}_{\text{MNS}}^{\mathrm{T}} \sqrt{I_g I_t} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \sqrt{I_\tau} \end{pmatrix} \tag{2.9}$$

with

$$I_g = \exp\left[\frac{1}{8\pi^2} \int_{t_Z}^{t_R} - c_i g_i^2 dt \right],$$

$$I_t = \exp\left[\frac{1}{8\pi^2} \int_{t_Z}^{t_R} y_t^2 dt \right],$$

$$I_\tau = \exp\left[\frac{1}{8\pi^2}\int_{t_Z}^{t_R} y_\tau^2 dt\right], \tag{2.10}$$

where $t_R = \ln M_R$ and $t_Z = \ln M_Z$. Here, the g_i's ($i=1,2$) are gauge couplings and y_t and y_τ are Yukawa couplings, and the c_i's are constants ($\frac{3}{5}$,3). We shall calculate the LFV numerically by using the modified Yukawa coupling in the following sections.

As mentioned in the previous section, there are three possible neutrino mass spectra. The hierarchical type ($m_{\nu 1} \ll m_{\nu 2} \ll m_{\nu 3}$) gives the neutrino mass spectrum as

$$m_{\nu 1} \sim 0, \quad m_{\nu 2} = \sqrt{\Delta m_\odot^2},$$
$$m_{\nu 3} = \sqrt{\Delta m_{\rm atm}^2}, \tag{2.11}$$

the quasidegenerate type ($m_{\nu 1} \sim m_{\nu 2} \sim m_{\nu 3}$) gives

$$m_{\nu 1} \equiv m_\nu, \quad m_{\nu 2} = m_\nu + \frac{1}{2m_\nu}\Delta m_\odot^2,$$
$$m_{\nu 3} = m_\nu + \frac{1}{2m_\nu}\Delta m_{\rm atm}^2, \tag{2.12}$$

and the inverse-hierarchical type ($m_{\nu 1} \sim m_{\nu 2} \gg m_{\nu 3}$) gives

$$m_{\nu 2} \equiv \sqrt{\Delta m_{\rm atm}^2}, \quad m_{\nu 1} = m_{\nu 2} - \frac{1}{2m_{\nu 2}}\Delta m_\odot^2,$$
$$m_{\nu 3} \simeq 0. \tag{2.13}$$

We take the typical values $\Delta m_{\rm atm}^2 = 3\times 10^{-3}$ eV^2 and $\Delta m_\odot^2 = 7\times 10^{-5}$ eV^2 in our calculation of the LFV.

We take typical mixing angles of the LMA MSW solution such as $s_{23} = 1/\sqrt{2}$ and $s_{12} = 0.6$ [16], in which the lepton mixing matrix is given in terms of the standard parametrization of the mixing matrix [23] as follows:

$$\mathbf{U}_{\rm MNS} = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\phi} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\phi} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\phi} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\phi} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\phi} & c_{23}c_{13} \end{pmatrix}, \tag{2.14}$$

where $s_{ij} \equiv \sin\theta_{ij}$ and $c_{ij} \equiv \cos\theta_{ij}$ are mixings in vacuum, and ϕ is the CP violating phase. The reacter experiment of CHOOZ [21] presented an upper bound on s_{13}. We use the constraint from the two-flavor analysis, which is $s_{13} \leqslant 0.2$ in our calculation. If we take account of the recent result of the three-flavor analysis [22], the upper bound of s_{13} may be smaller than 0.2. Then, if we use the results in [22], our results for $\mu \to e + \gamma$ are reduced at most by a factor of 2. In our calculation, the CP violating phase is neglected for simplicity.

B. LFV in slepton masses

Since SUSY is spontaneously broken at low energy, we consider the MSSM with the soft SUSY breaking terms:

$$\begin{aligned} -\mathcal{L}_{\rm soft} = {} & (\mathbf{m}_{\tilde Q}^2)_{ij}\tilde Q_i^\dagger \tilde Q_j + (\mathbf{m}_{\tilde u}^2)_{ij}\tilde u_{Ri}^* \tilde u_{Rj} + (\mathbf{m}_{\tilde d}^2)_{ij}\tilde d_{Ri}^* \tilde d_{Rj} + (\mathbf{m}_{\tilde L}^2)_{ij}\tilde L_i^\dagger \tilde L_j + (\mathbf{m}_{\tilde e}^2)_{ij}\tilde e_{Ri}^* \tilde e_{Rj} + (\mathbf{m}_{\tilde \nu}^2)_{ij}\tilde \nu_{Ri}^* \tilde \nu_{Rj} + \tilde m_{H_d}^2 H_d^\dagger H_d + \tilde m_{H_u}^2 H_u^\dagger H_u \\ & + (B\mu H_d H_u + \tfrac{1}{2}B_{\nu ij}M_{Rij}\tilde\nu_{Ri}^*\tilde\nu_{Rj}^* + \text{H.c.}) + \Big(\mathbf{A}_{ij}^d H_d \tilde d_{Ri}^* \tilde Q_j + \mathbf{A}_{ij}^u H_u \tilde u_{Ri}^* \tilde Q_j + \mathbf{A}_{ij}^e H_d \tilde e_{Ri}^* \tilde L_j + \mathbf{A}_{ij}^\nu H_u \tilde \nu_{Ri}^* \tilde L_j + \tfrac{1}{2}M_1 \tilde B_L^0 \tilde B_L^0 \\ & + \frac{1}{2}M_2 \tilde W_L^a \tilde W_L^a + \frac{1}{2}M_3 \tilde G^a \tilde G^a + \text{H.c.}\Big), \end{aligned} \tag{2.15}$$

where $\mathbf{m}_{\tilde Q}^2, \mathbf{m}_{\tilde u}^2, \mathbf{m}_{\tilde d}^2, \mathbf{m}_{\tilde L}^2, \mathbf{m}_{\tilde e}^2$ and $\mathbf{m}_{\tilde \nu}^2$ are the mass squares of the left-handed squark, the right-handed up squark, the right-handed down squark, the left-handed charged slepton, the right-handed charged slepton, and the sneutrino, respectively. $\tilde m_{H_d}^2$ and $\tilde m_{H_u}^2$ are the mass squares of the Higgs bosons, $\mathbf{A}_d, \mathbf{A}_u, \mathbf{A}_e$, and $\mathbf{A}_\nu$ are the A parameters for squarks and sleptons, and M_1, M_2, and M_3 are the gaugino masses.

Note that the lepton flavor violating processes come from diagrams including nonzero off-diagonal elements of the soft parameter. In this paper we assume MSUGRA; therefore we put the assumption of universality for soft SUSY breaking terms at the unification scale:

$$(\mathbf{m}_{\tilde L}^2)_{ij} = (\mathbf{m}_{\tilde e}^2)_{ij} = (\mathbf{m}_{\tilde \nu}^2)_{ij} = \cdots = \delta_{ij} m_0^2,$$

$$\tilde m_{H_d}{}^2 = \tilde m_{H_u}{}^2 = m_0^2,$$

$$\mathbf{A}^\nu = \mathbf{Y}_\nu a_0 m_0, \quad \mathbf{A}^e = \mathbf{Y}_e a_0 m_0,$$

$$\mathbf{A}^u = \mathbf{Y}_u a_0 m_0, \quad \mathbf{A}^d = \mathbf{Y}_d a_0 m_0, \tag{2.16}$$

where m_0 and a_0 stand for the universal scalar mass and the universal A parameter, respectively. Because of universality, LFV is not caused at the unification scale.

To estimate the soft parameters at low energy, we need to know the effect of radiative corrections. As a result, lepton flavor conservation is violated at low energy.

The RGE's for the left-handed slepton soft mass are given by

$$\mu \frac{d}{d\mu}(\mathbf{m}_{\tilde{L}}^2)_{ij} = \mu \frac{d}{d\mu}(\mathbf{m}_{\tilde{L}}^2)_{ij}\bigg|_{\mathrm{MSSM}} + \frac{1}{16\pi^2}[(\mathbf{m}_{\tilde{L}}^2 \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu + \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \mathbf{m}_{\tilde{L}}^2)_{ij} + 2(\mathbf{Y}_\nu^\dagger \mathbf{m}_{\tilde{\nu}} \mathbf{Y}_\nu + \tilde{m}_{H_u}^2 \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu + \mathbf{A}_\nu^\dagger \mathbf{A}_\nu)_{ij}], \tag{2.17}$$

while the first term in the right-hand side is the normal MSSM term which has no LFV, and the second one is a source of LFV through the off-diagonal elements of the neutrino Yukawa couplings. The RGE's are summarized in Appendix B.

III. NUMERICAL ANALYSES OF BRANCHING RATIOS

Let us calculate the branching ratio of $e_i \to e_j + \gamma (j < i)$. The amplitude of this process is given by

$$T = e\epsilon^{\alpha *}(q)\overline{u_j}(p) m_{e_i} i\sigma_{\alpha\beta} q^\beta (A^L P_L + A^R P_R) \times u_i(q-p), \tag{3.1}$$

where u_i is the wave function of the ith charged lepton e_i, p and q are the momenta of e_j and the photon, respectively, e is the electric charge, ϵ is the polarization vector of the photon, and $P_{L,R}$ are projection operators: $P_{L,R} = (1 \mp \gamma_5)/2$. $A^{L,R}$ are decay amplitudes and explicit forms are given in Appendix C. It is easy to see that this process changes the chirality of the charged lepton. The decay rate can be calculated using $A^{L,R}$ as

$$\Gamma(e_i \to e_j + \gamma) = \frac{e^2}{16\pi} m_{e_i}^5 (|A^L|^2 + |A^R|^2). \tag{3.2}$$

Since we know the relation $m_{e_i}^2 \gg m_{e_j}^2$, then we can expect $|A^R| \gg |A^L|$. The $A^{L,R}$ contain the contribution of the neutralino loop and the chargino loop as seen in Fig. 1. We calculate the branching ratio using Eq. (3.2) and the formulas in Appendix C. In order to clarify parameter dependence, let us present an approximate estimation. The decay amplitude is approximated as

$$|A^R|^2 \simeq \frac{\alpha_2^2}{16\pi^2} \frac{|(\Delta \mathbf{m}_{\tilde{L}}^2)_{ij}|^2}{m_S^8} \tan^2\beta, \tag{3.3}$$

where α_2 is the gauge coupling constant of $SU(2)_L$ and m_S is a SUSY particle mass. The RGE's develop the off-diagonal elements of the slepton mass matrix and A term. These terms at low energy are approximated by

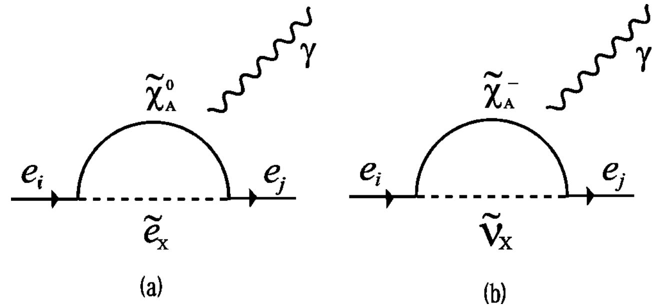

FIG. 1. Feynman diagrams that contribute to the branching ratio of $e_i \to e_j + \gamma$. There are two types of diagram, (a) neutralino-slepton loop and (b) chargino-sneutrino loop.

$$(\Delta \mathbf{m}_{\tilde{L}}^2)_{ij} \simeq -\frac{(6+2a_0^2)m_0^2}{16\pi^2}(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{ij} \ln\frac{M_X}{M_R}, \tag{3.4}$$

where M_X is the GUT scale. Therefore, the off-diagonal elements of $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{ij}$ are the crucial quantities to estimate the branching ratio.

As discussed in Sec. II, $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{ij}$ is given by neutrino masses and mixings at the electroweak scale. Therefore, we can compare the quantity $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{ij}$ among the three neutrino mass spectra: the degenerate, the inverse-hierarchical, and the hierarchical masses. In this section, we present numerical results in these three cases. Here, we use Eq. (3.2) and the vertex functions in Appendix C for the calculation of the branching ratio including the RGE effect.

A. $\mu \to e + \gamma$

We present a qualitative discussion of $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21}$ before predicting the branching ratio $\mathrm{BR}(\mu \to e + \gamma)$. This is given in terms of neutrino masses and mixings at the electroweak scale as follows:

$$(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21} = \frac{M_R}{v_u^2}[U_{\mu 2} U_{e2}^*(m_{\nu 2} - m_{\nu 1}) + U_{\mu 3} U_{e3}^*(m_{\nu 3} - m_{\nu 1})], \tag{3.5}$$

where $v_u \equiv v \sin\beta$ with $v = 174$ GeV is taken as the usual notation and the unitarity condition of the lepton mixing matrix elements is used. Taking the three cases of neutrino mass spectra, the degenerate, the inverse-hierarchical, and the normal hierarchical masses, one obtains the following forms, respectively:

$$(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21} \simeq \frac{M_R}{\sqrt{2} v_u^2} \frac{\Delta m_{\mathrm{atm}}^2}{2 m_\nu} \times \left[\frac{1}{\sqrt{2}} U_{e2}^* \frac{\Delta m_\odot^2}{\Delta m_{\mathrm{atm}}^2} + U_{e3}^*\right] \quad \text{(degenerate)}$$

$$\simeq \frac{M_R}{\sqrt{2} v_u^2} \sqrt{\Delta m_{\rm atm}^2} \times \left[\frac{1}{2\sqrt{2}} U_{e2}^* \frac{\Delta m_\odot^2}{\Delta m_{\rm atm}^2} - U_{e3}^* \right] \quad (\text{inverse})$$

$$\simeq \frac{M_R}{\sqrt{2} v_u^2} \sqrt{\Delta m_{\rm atm}^2} \times \left[\frac{1}{\sqrt{2}} U_{e2}^* \sqrt{\frac{\Delta m_\odot^2}{\Delta m_{\rm atm}^2}} + U_{e3}^* \right] \quad (\text{hierarchy}), \quad (3.6)$$

where we take the maximal mixing for the atmospheric neutrinos. Since $U_{e2} \simeq 1/\sqrt{2}$ for the bimaximal mixing matrix, the first terms in the square brackets on the right-hand sides of Eqs. (3.6) can be estimated by putting in the experimental data. For the case of degenerate neutrino masses, $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21}$ depends on the unknown neutrino mass scale m_ν. As one takes smaller m_ν, one predicts a larger branching ratio. In our calculation, we take $m_\nu = 0.3$ eV,[1] which is close to the upper bound from the neutrinoless double beta decay experiment [25], and also leads to the smallest branching ratio.

We also note that the degenerate case gives the smallest branching ratio BR($\mu \to e + \gamma$) among the three cases as seen in Eqs. (3.6) owing to the scale of m_ν. It is easy to see the fact that the second terms in Eqs. (3.6) are dominant up to $U_{e3} \gtrsim 0.01$ (degenerate), 0.01 (inverse), and 0.07 (hierarchy), respectively. The magnitude and the phase of U_{e3} are important in the comparison between the cases of the inverse-hierarchical and the normal hierarchical masses. In the limit of $U_{e3} = 0$, the predicted branching ratio in the case of the normal hierarchical masses is larger than the other one. However, for $U_{e3} \simeq 0.2$ the predicted branching ratios are almost the same in both cases.

First, we present numerical results in the case of degenerate neutrino masses assuming $\mathbf{M_R} = M_R \mathbf{1}$. The magnitude of M_R is considerably constrained if we impose the b-τ unification of Yukawa couplings [26]. In the case of $\tan\beta \leqslant 30$, the lower bound of M_R is approximately 10^{12} GeV. We also take $M_R \leqslant 10^{14}$ GeV, in order that neutrino Yukawa couplings remain below $\mathcal{O}(1)$. Therefore, we use $M_R = 10^{12}$, 10^{14} GeV in our following calculation.

We take a universal scalar mass (m_0) for all scalars and $a_0 = 0$ as a universal A term at the GUT scale ($M_X = 2 \times 10^{16}$ GeV. The branching ratio of $\mu \to e + \gamma$ is given versus the left-handed selectron mass $m_{\tilde{e}_L}$ for each $\tan\beta = 3,10,30$ and a fixed W-ino mass M_2 at the electroweak scale. In Fig. 2, the branching ratios are shown for $M_2 = 150,300$ GeV in the case of $U_{e3} = 0.2$ with $M_R = 10^{14}$ GeV and $m_\nu = 0.3$ eV; the solid curves correspond to $M_2 = 150$ GeV and the dashed ones to $M_2 = 300$ GeV.

[1]Recently, a positive observation of the neutrinoless double beta decay was reported in [24], where the degenerate neutrino mass of $m_\nu = 0.3$ eV is a typical one.

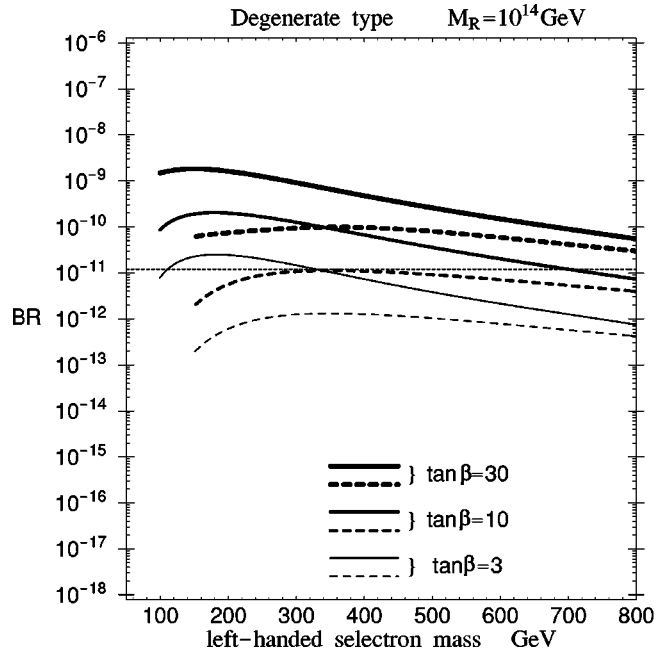

FIG. 2. Predicted branching ratio BR($\mu \to e + \gamma$) versus the left-handed selectron mass for $\tan\beta = 3,10,30$ in the case of degenerate neutrino masses. Here $M_R = 10^{14}$ GeV and $U_{e3} = 0.2$ are taken. The solid curves correspond to $M_2 = 150$ GeV and the dashed ones to $M_2 = 300$ GeV. The horizontal dotted line denotes the experimental upper bound.

The threshold of the selectron mass is determined by the recent CERN e^+e^- collider LEP2 data [27] for $M_2 = 150$ GeV, but for $M_2 = 300$ GeV by the constraint that the left-handed slepton should be heavier than the neutralinos. As $\tan\beta$ increases, the branching ratio increases because the decay amplitude from the SUSY diagrams is approximately proportional to $\tan\beta$ [7]. It is found that the branching ratio is almost larger than the experimental upper bound in the case of $M_2 = 150$ GeV. On the other hand, the predicted values are smaller than the experimental bound except for $\tan\beta = 30$ in the case of $M_2 = 300$ GeV.

Our predictions depend strongly on M_R, because the magnitude of the neutrino Yukawa coupling is determined by M_R as seen in Eq. (2.5). If M_R is reduced to 10^{12} GeV, the branching ratio becomes 10^4 times smaller since it is proportional to M_R^2. The numerical result is shown in Fig. 3. We will examine a model [28,29] that gives the degenerate neutrino masses with $U_{e3} \sim 0.05$ in Sec. IV.

Next we show results in the case of the inverse-hierarchical neutrino masses. As expected from Eq. (3.6), the branching ratio is much larger than the one in the degenerate case. In Fig. 4, the branching ratio is shown for $M_2 = 150,300$ GeV in the case of $U_{e3} = 0.2$ with $M_R = 10^{14}$ GeV. In Fig. 5, the branching ratio is shown for $U_{e3} = 0.05$ with $M_R = 10^{14}$ GeV. The M_R dependence is the same as in the case of quasidegenerate neutrino masses. The predictions almost exceed the experimental bound as long as $U_{e3} \geqslant 0.05$, $\tan\beta \geqslant 10$, and $M_R \simeq 10^{14}$ GeV. This result is based on the assumption $\mathbf{M_R} = M_R \mathbf{1}$; however, it is not guaranteed in the case of the inverse-hierarchical neutrino

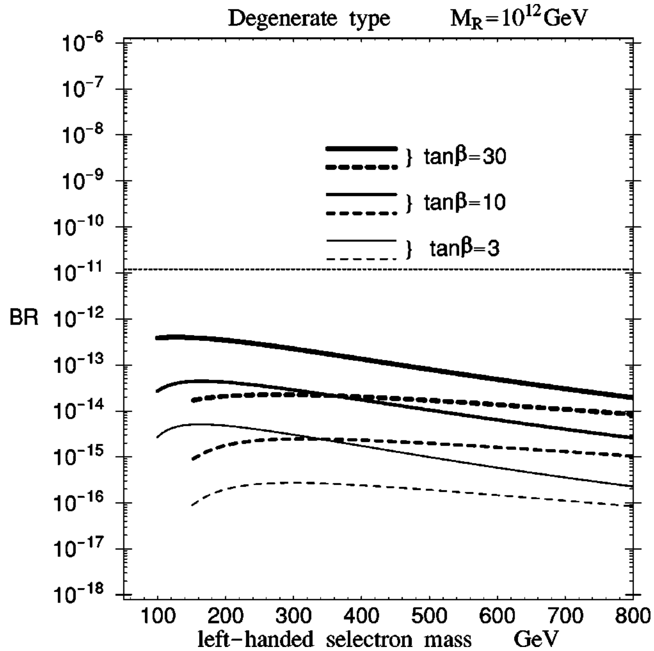

FIG. 3. Predicted branching ratio BR($\mu \to e + \gamma$) versus the left-handed selectron mass for $\tan\beta = 3,10,30$ in the case of degenerate neutrino masses. Here $M_R = 10^{12}$ GeV and $U_{e3} = 0.2$ are taken. The solid curves correspond to $M_2 = 150$ GeV and the dashed ones to $M_2 = 300$ GeV.

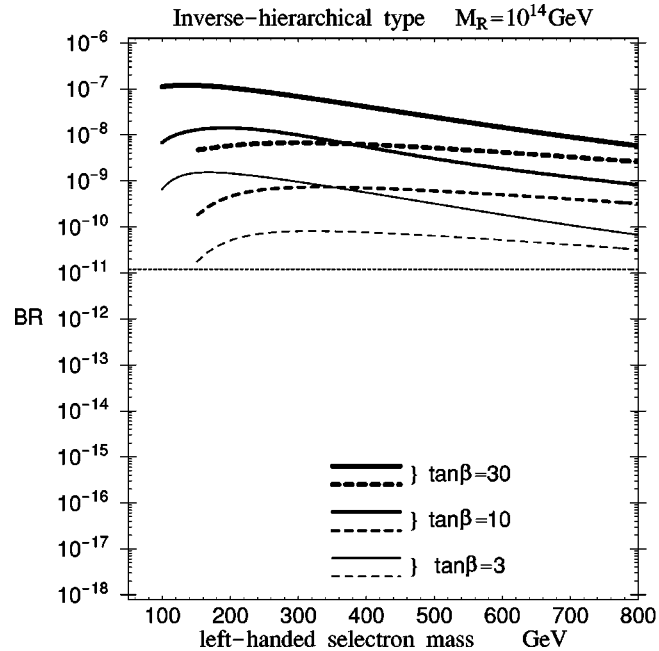

FIG. 4. Predicted branching ratio BR($\mu \to e + \gamma$) versus the left-handed selectron mass for $\tan\beta = 3,10,30$ in the case of inverse-hierarchical neutrino masses. Here $M_R = 10^{14}$ GeV and $U_{e3} = 0.2$ are taken. The solid curves correspond to $M_2 = 150$ GeV and the dashed ones to $M_2 = 300$ GeV.

masses. We will examine a typical model [30] that gives $\mathbf{M_R} \neq M_R \mathbf{1}$ in Sec. IV.

For comparison, we show the branching ratio in the case of hierarchical neutrino masses in Fig. 6. It is similar to the case of the inverse-hierarchical neutrino masses. The branching ratio in the case of degenerate neutrino masses is 10^2 times smaller than that for the inverse-hierarchical and hierarchical neutrino spectra.

In our numerical analyses we assumed $a_0 = 0$ at the GUT scale M_X for simplicity. Let us comment on the A-term dependence, namely, $a_0 \neq 0$ at M_X. We estimate the branching ratio for $a_0 = \pm 1$ at M_X ($\mathbf{A} = \mathbf{Y} a_0 m_0$). In the degenerate type, the predicted branching ratio is 1.02 ($a_0 = 1$) or 1.07 ($a_0 = -1$) times as large as the one in the case of $a_0 = 0$ ($\tan\beta = 30, U_{e3} = 0.2$). In the inverse-hierarchical type, the predicted branching ratios are 1.56 ($a_0 = 1$) and 1.54 ($a_0 = -1$) times as large as the one in the case of $a_0 = 0$ ($\tan\beta = 30, U_{e3} = 0.2$). Therefore the A-term dependence is insignificant in our analyses.

In our calculations, we use the universality condition at M_X. We also examine the no-scale condition $m_0 = 0$ at M_X. It is found that the predicted branching ratio is 10 times smaller than the one in the case of nonzero universal scalar mass.

B. Nondegeneracy effect of $\mathbf{M}_R$

The analyses in the previous section depend on the assumption of $M_{R1} = M_{R2} = M_{R3} \equiv M_R$. In the case of the quasidegenerate neutrino masses in Eq. (2.12) this complete degeneracy of $\mathbf{M}_R$ may deviate with the following magnitude without fine-tuning:

$$\frac{M_{R3}^2}{M_{R1}^2} \simeq 1 \pm \frac{\Delta m_{\rm atm}^2}{m_\nu^2}, \quad \frac{M_{R2}^2}{M_{R1}^2} \simeq 1 \pm \frac{\Delta m_\odot^2}{m_\nu^2}. \tag{3.7}$$

Therefore, we parametrize M_R as

$$\mathbf{M_R} = M_R \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1+\varepsilon_2 & 0 \\ 0 & 0 & 1+\varepsilon_3 \end{pmatrix}, \tag{3.8}$$

where $\varepsilon_2 \simeq \Delta m_\odot^2 / 2 m_\nu^2$ and $\varepsilon_3 \simeq \Delta m_{\rm atm}^2 / 2 m_\nu^2$. By using Eq. (2.6), we obtain

$$\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu = \frac{M_R}{v_u^2} \mathbf{U}_{\rm MNS} \begin{pmatrix} \sqrt{m_{\nu 1}} & 0 & 0 \\ 0 & \sqrt{m_{\nu 2}} & 0 \\ 0 & 0 & \sqrt{m_{\nu 3}} \end{pmatrix} \times \mathbf{K} \begin{pmatrix} \sqrt{m_{\nu 1}} & 0 & 0 \\ 0 & \sqrt{m_{\nu 2}} & 0 \\ 0 & 0 & \sqrt{m_{\nu 3}} \end{pmatrix} \mathbf{U}_{\rm MNS}^{\rm T}, \tag{3.9}$$

where

$$\mathbf{K} \equiv \mathbf{R}^\dagger \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1+\varepsilon_2 & 0 \\ 0 & 0 & 1+\varepsilon_3 \end{pmatrix} \mathbf{R}. \tag{3.10}$$

Thus, we have

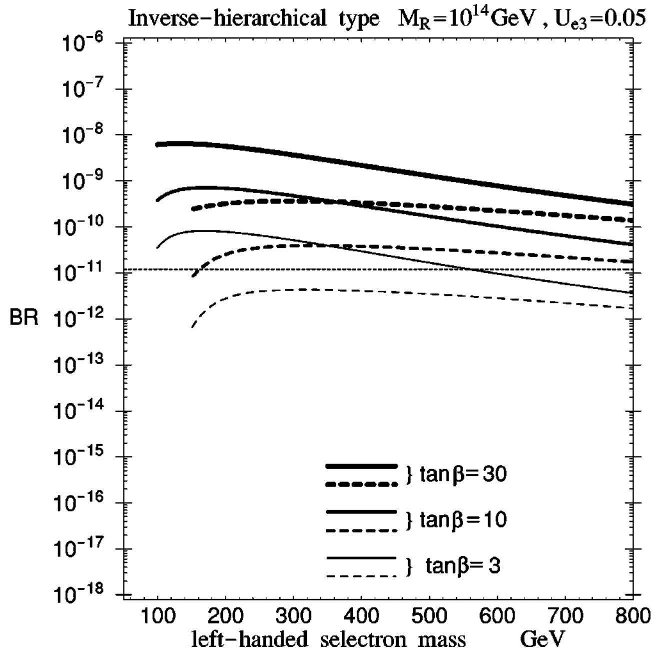

FIG. 5. Predicted branching ratio BR($\mu \to e+\gamma$) versus the left-handed selectron mass for tan $\beta=3,10,30$ in the case of the inverse-hierarchical neutrino masses. Here $M_R=10^{14}$ GeV and $U_{e3}=0.05$ are taken. The solid curves correspond to $M_2=150$ GeV and the dashed ones to $M_2=300$ GeV.

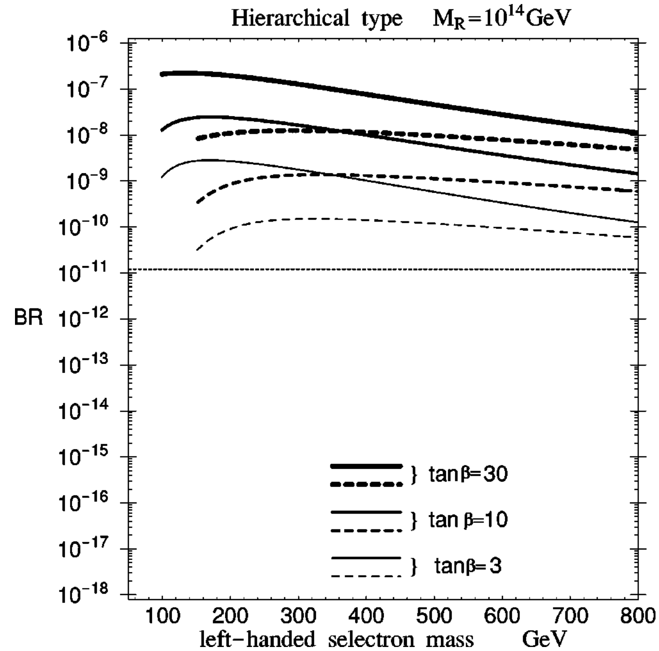

FIG. 6. Predicted branching ratio BR($\mu \to e+\gamma$) versus the left-handed selectron mass for tan $\beta=3,10,30$ in the case of the hierarchical neutrino masses. Here $M_R=10^{14}$ GeV and $U_{e3}=0.2$ are taken. The solid curves correspond to $M_2=150$ GeV and the dashed ones to $M_2=300$ GeV.

$$(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21}=\frac{M_R}{v_u^2}\sum_{i,j}^{3} U_{2i}U_{1j}(K_{ij}\sqrt{m_{\nu i}}\sqrt{m_{\nu j}}), \quad (3.11)$$

with

$$K_{ij}=\delta_{ij}+\varepsilon_2 R_{2i}R_{2j}+\varepsilon_3 R_{3i}R_{3j}, \quad (3.12)$$

where we used $\mathbf{R}^{\mathrm{T}}\mathbf{R}=\mathbf{1}$.[2] So, we get

$$(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21}=(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21}|_{\mathbf{M_R}\propto\mathbf{1}}+\Delta(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21}, \quad (3.13)$$

where the first term is the $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21}$ element in Eq. (3.5), which corresponds to the $\mathbf{M_R}\propto\mathbf{1}$, while the second term stands for the deviation from it as follows:

$$\Delta(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21}=\frac{M_R}{v_u^2}\sum_{i,j}^{3} U_{2i}U_{1j}\sqrt{m_{\nu i}}\sqrt{m_{\nu j}} \times(\varepsilon_2 R_{2i}R_{2j}+\varepsilon_3 R_{3i}R_{3j}). \quad (3.14)$$

In order to estimate the second term, we use $\varepsilon_2=0.0001$ and $\varepsilon_3=0.01$, taking account of $\varepsilon_2 \simeq \Delta m_\odot^2/2m_\nu^2$ and $\varepsilon_3 \simeq \Delta m_{\mathrm{atm}}^2/2m_\nu^2$, where we use $m_\nu=0.3$ eV. Since $m_{\nu i}\simeq m_{\nu j}$ and $R_{ij}\leqslant 1$, we get

$$\Delta(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21}\sim \frac{M_R}{v_u^2}\sum_{i,j}^{3} U_{2i}U_{1j}m_\nu\varepsilon_3 R_{3i}R_{3j}$$

$$\leqslant \frac{M_R}{v_u^2}\frac{1}{2\sqrt{2}}m_\nu\varepsilon_3 \sim 3.5\times 10^{-3}. \quad (3.15)$$

Taking this maximal value, we can estimate the branching ratio as follows:

$$\frac{\mathrm{BR(nondegenerate}\ \ M_R)}{\mathrm{BR(degenerate}\ \ M_R)}\leqslant\left(\frac{2.6+3.5}{2.6}\right)^2\simeq 5.5. \quad (3.16)$$

Therefore, the enhancement due to the second term is at most a factor of 5. This conclusion does not depend on the specific form of $\mathbf{R}$.

Consider the case of the inverse-hierarchical type of neutrino masses. We take $\varepsilon_2\sim 0.01$ with a similar argument to that for the quasidegenerate type neutrino masses, because $m_{\nu 1}$ and $m_{\nu 2}$ are almost degenerate and $\varepsilon_2\simeq\Delta m_\odot^2/2\Delta m_{\mathrm{atm}}^2$ in this case. Then we get

$$\Delta(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21}=\frac{M_R}{v_u^2}\sum_{i,j}^{2} U_{2i}U_{1j}m_{\nu 1}\varepsilon_2 R_{2i}R_{2j}$$

$$\leqslant \frac{M_R}{v_u^2}\frac{1}{2\sqrt{2}}m_{\nu 1}\varepsilon_2\sim 0.063\times 10^{-2}, \quad (3.17)$$

where we assume $\varepsilon_2 \geqslant \varepsilon_3$ and use $m_{\nu 3}\simeq 0$, $m_{\nu 1}\simeq m_{\nu 2}\simeq 0.054$ eV, and $R_{ij}\leqslant 1$. Taking the maximal value, we get

[2]We assume $\mathbf{R}$ to be real for simplicity.

$$\frac{\mathrm{BR}(\text{nondegenerate } M_R)}{\mathrm{BR}(\text{degenerate } M_R)} \leqslant \left(\frac{2.7+0.063}{2.7}\right)^2 \simeq 1.04. \tag{3.18}$$

Thus, the effect of the $\Delta(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21}$ is very small in the case of inverse-hierarchical neutrino masses.

These discussions in this subsection are also qualitatively applicable for the $\tau \to \mu + \gamma$ process.

C. $\tau \to \mu + \gamma$

Let us study the $\tau \to \mu + \gamma$ process. In this case, we should discuss

$$(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{32} = \frac{M_R}{v_u^2}[U_{\tau 2} U_{\mu 2}^*(m_{\nu 2} - m_{\nu 1}) + U_{\tau 3} U_{\mu 3}^*(m_{\nu 3} - m_{\nu 1})]. \tag{3.19}$$

It should be stressed that it is independent of U_{e3}, in contrast to $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21}$. Therefore we can determine the following form of $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{32}$ at the electroweak scale by using the bimaximal mixing matrix:

$$\begin{aligned}
(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{32} &\simeq \frac{M_R}{v_u^2}\left[-\frac{1}{4}\frac{\Delta m_\odot^2}{2m_\nu} + \frac{1}{2}\frac{\Delta m_{\mathrm{atm}}^2}{2m_\nu}\right] \\
&\simeq \frac{M_R}{4v_u^2}\frac{\Delta m_{\mathrm{atm}}^2}{m_\nu} \quad (\text{degenerate}) \\
&\simeq \frac{M_R}{v_u^2}\left[\frac{1}{8}\frac{\Delta m_\odot^2}{\sqrt{\Delta m_{\mathrm{atm}}^2}} - \frac{1}{2}\sqrt{\Delta m_{\mathrm{atm}}^2}\right] \\
&\simeq -\frac{M_R}{2v_u^2}\sqrt{\Delta m_{\mathrm{atm}}^2} \quad (\text{inverse}) \\
&\simeq \frac{M_R}{v_u^2}\left[-\frac{1}{4}\sqrt{\Delta m_\odot^2} + \frac{1}{2}\sqrt{\Delta m_{\mathrm{atm}}^2}\right] \\
&\simeq \frac{M_R}{2v_u^2}\sqrt{\Delta m_{\mathrm{atm}}^2} \quad (\text{hierarchy}).
\end{aligned} \tag{3.20}$$

We see that the cases of the inverse-hierarchical masses and the hierarchical masses are almost the same as seen in Eqs. (3.20).

Let us present the numerical results for $\mathrm{BR}(\tau \to \mu + \gamma)$ [31] versus $\mathrm{BR}(\mu \to e + \gamma)$ [11] in the case of degenerate neutrino mass, where $\tan\beta = 3,10,30$ are taken. In Fig. 7, the branching ratio is plotted for $M_2 = 150,300$ GeV for $U_{e3} = 0.2$ with $M_R = 10^{14}$ GeV. Dotted lines are the experimental upper bounds for $\mathrm{BR}(\tau \to \mu + \gamma)$ and $\mathrm{BR}(\mu \to e + \gamma)$, respectively. The dependence on $\tan\beta$ is the same as in the case of $\mu \to e + \gamma$. It is found that the branching ratio is always smaller than the experimental upper bound in the case of $\tau \to \mu + \gamma$ in contrast with the case of $\mu \to e + \gamma$.

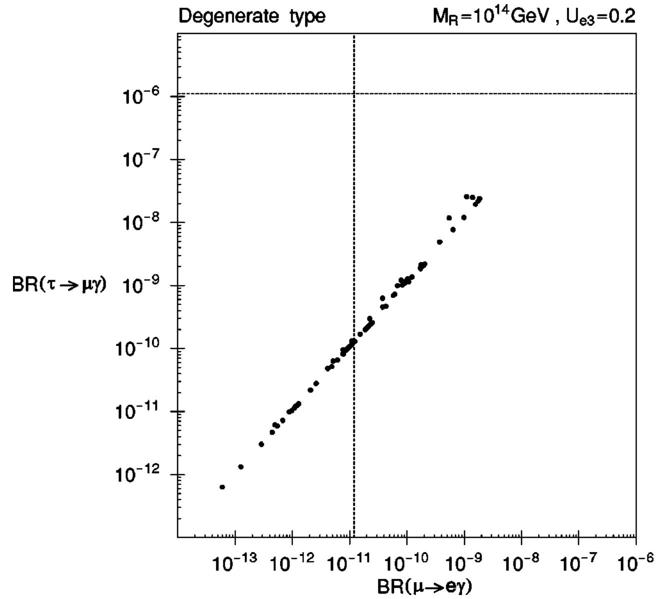

FIG. 7. Predicted branching ratio $\mathrm{BR}(\tau \to \mu + \gamma)$ versus $\mathrm{BR}(\mu \to e + \gamma)$ for $\tan\beta = 3,10,30$ in the case of degenerate neutrino masses. Here $M_R = 10^{14}$ GeV, $m_\nu = 0.3$ eV, and $U_{e3} = 0.2$ are taken and the left-handed selectron mass is taken the same as in Figs. 2–4.

Next we show the results in the case of the inverse-hierarchical neutrino masses. As expected from Eqs. (3.20), the branching ratio is much larger than the one in the degenerate case. In Fig. 8, the branching ratio is shown for $M_2 = 150,300$ GeV in the case of $U_{e3} = 0.2$ with $M_R = 10^{14}$ GeV. In conclusion, the predicted branching ratio is larger than the one in the case of degenerate neutrino mass, and it is smaller than the experimental upper bound for $\tau \to \mu + \gamma$ in contrast with $\mu \to e + \gamma$. The constraint for $\mathrm{BR}(\mu \to e + \gamma)$ is always more severe than the one in the case of $\mathrm{BR}(\tau \to \mu + \gamma)$.

IV. TYPICAL MODELS AND NUMERICAL ANALYSES

A. $S_{3L} \times S_{3R}$ flavor symmetry model—degenerate type

In this section we examine the neutrino model proposed by Fukugita, Tanimoto, and Yanagida [28], which derives the quasidegenerate masses $m_{\nu 1} \sim m_{\nu 2} \sim m_{\nu 3}$. This model is based on the $S_{3L} \times S_{3R}$ flavor symmetry [32]. Taking $\mathbf{M_R} = M_R \mathbf{1}$, the neutrino Yukawa coupling is given as follows:

$$\mathbf{Y}_\nu = Y_{\nu 0}\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon_\nu & 0 \\ 0 & 0 & \delta_\nu \end{pmatrix}\right], \tag{4.1}$$

where we take the diagonal basis for the neutrino sector. The first matrix is the S_{3L} invariant one, and the second one is the symmetry breaking term. The parameters $Y_{\nu 0}, \epsilon_\nu$, and δ_ν are constrained by the experimental values of $\Delta m_{\mathrm{atm}}^2$ and $\Delta m_\odot^2$. Therefore, the flavor mixings come from the charged lepton Yukawa couplings.

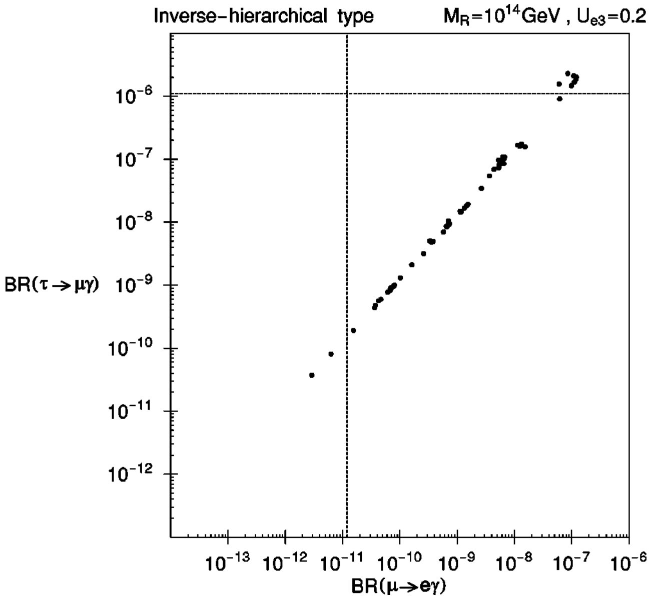

FIG. 8. Predicted branching ratio BR($\tau \to \mu + \gamma$) versus BR($\mu \to e + \gamma$) for tan $\beta = 3,10,30$ in the case of inverse-hierarchical neutrino masses. Here $M_R = 10^{14}$ GeV and $U_{e3} = 0.2$ are taken and the left-handed selectron mass is taken the same as in Figs. 2–4.

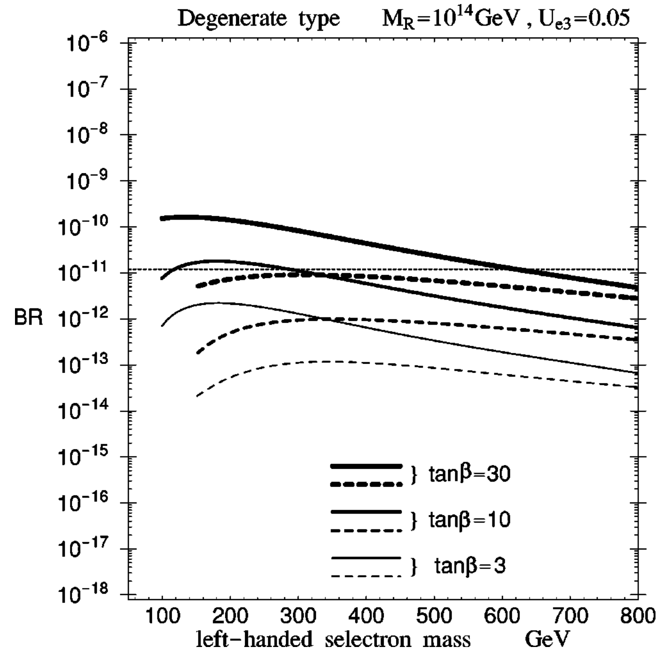

FIG. 9. Predicted branching ratio BR($\mu \to e + \gamma$) versus the left-handed selectron mass for tan $\beta = 3,10,30$ in the case of the $S_{3L} \times S_{3R}$ flavor symmetry model. Here $M_R = 10^{14}$ GeV is taken. The solid curves correspond to $M_2 = 150$ GeV and the dashed ones to $M_2 = 300$ GeV.

The charged lepton Yukawa coupling is given by the symmetry breaking parameters ϵ_l, δ_l as follows:

$$\mathbf{Y}_e = Y_{e0}\left[\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} -\epsilon_l & 0 & 0 \\ 0 & +\epsilon_l & 0 \\ 0 & 0 & +\delta_l \end{pmatrix}\right]. \tag{4.2}$$

Since Y_{e0}, ϵ_l and δ_l are fixed by the charged lepton masses, one gets the lepton mixing matrix elements as follows:

$$\mathbf{U}_{\mathrm{MNS}} \simeq \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & \sqrt{2/3}\sqrt{m_e/m_\mu} \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix}. \tag{4.3}$$

As a result, we see that $U_{e3} = \sqrt{2/3}\sqrt{m_e/m_\mu} \sim 0.05$ from Eq. (4.3).

We estimated the branching ratio of the processes $\mu \to e + \gamma$ and $\tau \to \mu + \gamma$ by using $U_{e3} = 0.05$. We show the branching ratio for $M_2 = 150$ and 300 GeV taking tan $\beta = 3,10,30$ in Fig. 9. Because of the smallness of U_{e3}, we see that BR($\mu \to e + \gamma$) is smaller than the experimental upper bound except for tan $\beta = 30$ and $M_2 = 150$ GeV.

We have also estimated the branching ratio BR($\tau \to \mu + \gamma$) for tan $\beta = 30$, which is much smaller than the experimental bound BR($\tau \to \mu + \gamma$) $< 1.1 \times 10^{-6}$. Thus, the $\mu \to e + \gamma$ process provides a severe constraint compared with $\tau \to \mu + \gamma$ in the present experimental situation.

B. The Shafi-Tavartkiladze model—inverse-hierarchical type

The typical model of the inverse-hierarchical neutrino masses is the Zee model [33], in which the right-handed neutrinos do not exist. However, one can also consider a Yukawa texture which leads to inverse-hierarchical masses through the seesaw mechanism, namely, the Shafi-Tavartkiladze model [30].

Shafi and Tavartkiladze utilize anomalous U(1) flavor symmetry [34]. In this model, due to the Froggatt-Nielsen mechanism [35], one of the Yukawa interaction terms in the effective theory is given by

$$e^c_{Ri} L_j H_d \left(\frac{S}{M_{\mathrm{pl}}}\right)^{m_{ij}}, \tag{4.4}$$

where e^c_{Ri} and L_j are the right-handed charged lepton and the left-handed lepton doublet, respectively, H_d is the Higgs doublet, and S is a singlet field. The effective Yukawa couplings are given in terms of

$$\lambda \equiv \frac{\langle S \rangle}{M_{\mathrm{pl}}} \simeq 0.2. \tag{4.5}$$

The neutrino mass matrix is given in Appendix D. Fixing the U(1) flavor charges k, n, k' as $k=0,\ n=2,\ k'=2$, which are consistent with neutrino mass data, the Yukawa coupling is given by

$$\mathbf{Y}_\nu = \begin{pmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ 1 & 0 & 0 \end{pmatrix}, \tag{4.6}$$

and the right-handed neutrino Majorana mass matrix is given by

$$\mathbf{M_R} = M_R \begin{pmatrix} \lambda^4 & 1 \\ 1 & 0 \end{pmatrix}. \tag{4.7}$$

It is remarked that right-handed neutrinos contain only two generations in this model. In Eq. (4.6), components 2-2 and 2-3 must be zero for the sake of holomorphy of the superpotential; it is called the SUSY zero. The neutrino mass matrix is given by the seesaw mechanism as

$$\mathbf{m}_\nu = \mathbf{Y}_\nu^{\mathrm{T}} \mathbf{M_R}^{-1} \mathbf{Y}_\nu v_u^2 = \frac{\lambda^2 v_u^2}{M_R} \begin{pmatrix} \lambda^2 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \tag{4.8}$$

where the order 1 coefficient in front of each entry is neglected. This mass matrix gives the inverse-hierarchical neutrino masses. $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ is given as

$$\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu = \begin{pmatrix} 1+\lambda^8 & \lambda^6 & \lambda^6 \\ \lambda^6 & \lambda^4 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^4 \end{pmatrix}. \tag{4.9}$$

It is noticed that the component $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21}$ is suppressed as

$$(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21} \sim \lambda^6 \sim \mathcal{O}(10^{-5}). \tag{4.10}$$

Thus we expect that the branching ratio of $\mu \to e + \gamma$ in this model is much smaller than the one in the case of $(\mathbf{M_R})_{3\times 3} = M_R (\mathbf{1})_{3\times 3}$ in Sec. III.

In Fig. 10, the branching ratio is shown for $M_2 = 150{,}300$ GeV. The predictions are given by taking $\lambda = 0.2$ and all the order 1 coefficients in the Yukawa couplings are fixed to be 1. The predicted value is much smaller than the one in the inverse-hierarchical case discussed in the Sec. III. Because $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21}$ is proportional to λ^6, the smallness of the branching ratio is understandable.

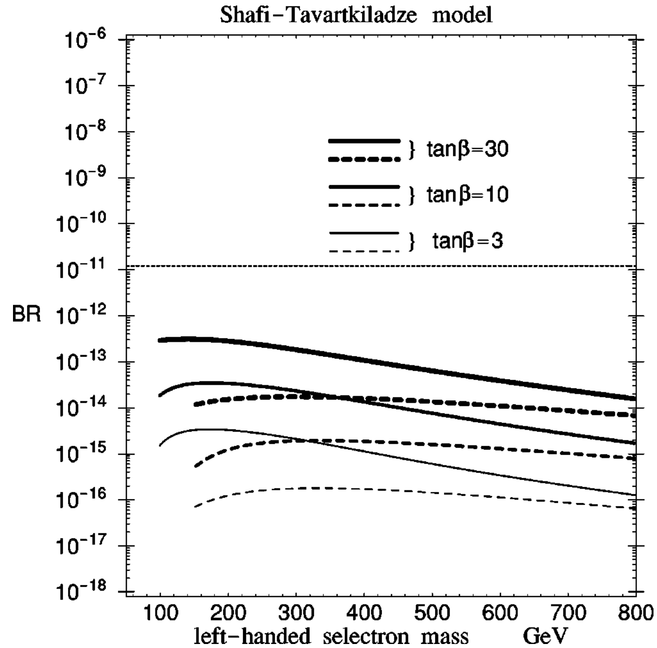

FIG. 10. Predicted branching ratio $\mathrm{BR}(\mu \to e + \gamma)$ versus the left-handed selectron mass for $\tan\beta = 3{,}10{,}30$ in the case of the Shafi-Tavartkiladze model [30]. The solid curves correspond to $M_2 = 150$ GeV and the dashed ones to $M_2 = 300$ GeV.

V. SUMMARY AND DISCUSSION

We have investigated the lepton flavor violating processes $\mu \to e + \gamma$ and $\tau \to \mu + \gamma$, in the framework of the MSSM with right-handed neutrinos. Even if we impose the universality condition for the soft scalar masses and A terms at the GUT scale, off-diagonal elements of the left-handed slepton mass matrix are generated through the RGE's running effects from the GUT scale to the right-handed neutrino mass scale M_R. We have taken the LMA MSW solution for the neutrino masses and mixings.

The branching ratios of $\mu \to e + \gamma$ and $\tau \to \mu + \gamma$ processes are proportional to $|(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{ij}|^2$. Since $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{ij}$ depends on the mass spectrum of the neutrinos, we can compare the branching ratios of three cases of neutrino mass spectra: the degenerate, the inverse-hierarchical, and the hierarchical case.

First, we have studied the three types in the case of $\mu \to e + \gamma$, in which we take $\mathbf{M_R} = M_R \mathbf{1}$. For the case of degenerate neutrino masses, the branching ratio depends on the unknown neutrino mass m_ν. We have taken $m_\nu = 0.3$ eV, which gives us the largest branching ratio. It is emphasized that the magnitude of U_{e3} is one of the important ingredients in predicting $\mathrm{BR}(\mu \to e + \gamma)$. The branching ratio of the inverse-hierarchical case almost exceeds the experimental upper bound and is much larger than the degenerate case for $M_2 = 150$ GeV and $M_2 = 300$ GeV. In general, we expect the relation $\mathrm{BR(degenerate)} \ll \mathrm{BR(inverse\text{-}hierarchical)} < \mathrm{BR(hierarchical)}$. The effect of the deviation from $\mathbf{M_R} = M_R \mathbf{1}$ has been estimated. The enhancement of the branching ratios are at most a factor of five in the case of the quasidegenerate neutrino mass spectrum.

Second, we have studied the three cases in $\tau \to \mu + \gamma$. It is noticed that the branching ratio is independent of U_{e3} in contrast to the case of $\mu \to e + \gamma$. For degenerate neutrino masses, the branching ratio is always smaller than the experimental upper bound. For inverse-hierarchical neutrino masses, the branching ratio is smaller than the experimental bound. The constraint of $\mathrm{BR}(\mu \to e + \gamma)$ is always severer than the one in the case of $\mathrm{BR}(\tau \to \mu + \gamma)$.

Finally, we have investigated the branching ratio of $\mu \to e + \gamma$ in the typical models of the degenerate and inverse-hierarchical cases. Since the $S_{3L} \times S_{3R}$ model, which is a typical one in the degenerate case, predicts $U_{e3} \simeq 0.05$, the branching ratio is much smaller than in the case of $U_{e3} \simeq 0.2$. The Shafi-Tavartkiladze model, which is a typical one of the inverse-hierarchical case, predicts a very small branching ratio. Thus, the models can be tested by the $\mu \to e + \gamma$ process.

The branching ratio of $\mu \to e + \gamma$ and $\tau \to \mu + \gamma$ will be improved to the level 10^{-14} in the PSI and 10^{-7} to 10^{-8} in the B factories at KEK and SLAC, respectively. Therefore, future experiments can probe the framework for the neutrino masses.

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APPENDIX A: YUKAWA MATRIX

The Yukawa matrix is determined in general as follows [10]. The left-handed neutrino mass matrix is given as

$$\mathbf{m}_\nu=(\mathbf{Y}_\nu v_u)^{\mathrm{T}}\mathbf{M}_{\mathbf{R}}^{-1}(\mathbf{Y}_\nu v_u), \tag{A1}$$

via the seesaw mechanism, where v_u is the vacuum expectation value of the Higgs boson H_u. One can always take the diagonal form of the right-handed Majorana neutrino mass matrix $\mathbf{M}_{\mathbf{R}}=\mathbf{M}_{\mathbf{R}}^{\mathrm{diag}}$. The neutrino mass matrix $\mathbf{m}_\nu$ is diagonalized by a single unitary matrix

$$\mathbf{m}_\nu^{\mathrm{diag}}\equiv\mathbf{U}_{\mathrm{MNS}}^{\mathrm{T}}\mathbf{m}_\nu\mathbf{U}_{\mathrm{MNS}}, \tag{A2}$$

where $\mathbf{U}_{\mathrm{MNS}}$ is the MNS matrix. In Eqs. (A1) and (A2), one can divide $\mathbf{M}_{\mathbf{R}}^{\mathrm{diag}}$ into square roots:

$$\begin{aligned}\mathbf{m}_\nu^{\mathrm{diag}}&=\mathbf{U}_{\mathrm{MNS}}^{\mathrm{T}}\mathbf{Y}_\nu^{\mathrm{T}}(\mathbf{M}_{\mathbf{R}}^{\mathrm{diag}})^{-1}\mathbf{Y}_\nu\mathbf{U}_{\mathrm{MNS}}v_u^2\\&=\mathbf{U}_{\mathrm{MNS}}^{\mathrm{T}}\mathbf{Y}_\nu^{\mathrm{T}}\sqrt{(\mathbf{M}_{\mathbf{R}}^{\mathrm{diag}})^{-1}}\sqrt{(\mathbf{M}_{\mathbf{R}}^{\mathrm{diag}})^{-1}}\\&\quad\times\mathbf{Y}_\nu\mathbf{U}_{\mathrm{MNS}}v_u^2.\end{aligned} \tag{A3}$$

Multiplying the inverse square root of the matrix $\mathbf{m}_\nu^{\mathrm{diag}}$ from both right- and left-hand sides of Eq. (A3), one gets the following form:

$$\begin{aligned}\mathbf{1}&=\sqrt{(\mathbf{m}_\nu^{\mathrm{diag}})^{-1}}\mathbf{U}_{\mathrm{MNS}}^{\mathrm{T}}\mathbf{Y}_\nu^{\mathrm{T}}\sqrt{(\mathbf{M}_{\mathbf{R}}^{\mathrm{diag}})^{-1}}\\&\quad\times v_u^2\sqrt{(\mathbf{M}_{\mathbf{R}}^{\mathrm{diag}})^{-1}}\mathbf{Y}_\nu\mathbf{U}_{\mathrm{MNS}}\sqrt{(\mathbf{m}_\nu^{\mathrm{diag}})^{-1}}\\&\equiv\mathbf{R}^{\mathrm{T}}\mathbf{R},\end{aligned} \tag{A4}$$

where one has defined the complex orthogonal 3×3 matrix

$$\mathbf{R}\equiv v_u\sqrt{(\mathbf{M}_{\mathbf{R}}^{\mathrm{diag}})^{-1}}\mathbf{Y}_\nu\mathbf{U}_{\mathrm{MNS}}\sqrt{(\mathbf{m}_\nu^{\mathrm{diag}})^{-1}}, \tag{A5}$$

and $\mathbf{R}$ depends on the model. Therefore, one can write the neutrino Yukawa coupling as

$$\mathbf{Y}_\nu=\frac{1}{v_u}\sqrt{\mathbf{M}_{\mathbf{R}}^{\mathrm{diag}}}\mathbf{R}\sqrt{\mathbf{m}_\nu^{\mathrm{diag}}}\mathbf{U}_{\mathrm{MNS}}^{\mathrm{T}}, \tag{A6}$$

or explicitly

$$\begin{aligned}\mathbf{Y}_\nu&=\frac{1}{v_u}\begin{pmatrix}\sqrt{M_{R1}}&0&0\\0&\sqrt{M_{R2}}&0\\0&0&\sqrt{M_{R3}}\end{pmatrix}\\&\quad\times\mathbf{R}\begin{pmatrix}\sqrt{m_{\nu1}}&0&0\\0&\sqrt{m_{\nu2}}&0\\0&0&\sqrt{m_{\nu3}}\end{pmatrix}\mathbf{U}_{\mathrm{MNS}}^{\mathrm{T}}.\end{aligned} \tag{A7}$$

APPENDIX B: RGE'S

1. From M_X to M_R

$$\mu\frac{d}{d\mu}g_i^2=\frac{1}{8\pi^2}b_ig_i^4,\quad (b_1,b_2,b_3)=\left(\frac{33}{5},1,-3\right),$$

$$\mu\frac{d}{d\mu}M_i=\frac{b_i}{2\pi}\alpha_iM_i,\quad \alpha_i=\frac{g_i^2}{4\pi}\quad (i=1,2,3),$$

$$\mu\frac{d}{d\mu}\mathbf{Y}_e^{ij}=\frac{1}{16\pi^2}\left[\left\{-\frac{9}{5}g_1^2-3g_2^2+3\,\mathrm{Tr}(\mathbf{Y}_d\mathbf{Y}_d^\dagger)+\mathrm{Tr}(\mathbf{Y}_e\mathbf{Y}_e^\dagger)\right\}\mathbf{Y}_e^{ij}+3(\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e)^{ij}+(\mathbf{Y}_e\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^{ij}\right],$$

$$\mu\frac{d}{d\mu}\mathbf{Y}_\nu^{ij}=\frac{1}{16\pi^2}\left[\left\{-\frac{3}{5}g_1^2-3g_2^2+3\,\mathrm{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger)+\mathrm{Tr}(\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger)\right\}\mathbf{Y}_\nu^{ij}+3(\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^{ij}+(\mathbf{Y}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e)^{ij}\right],$$

$$\mu\frac{d}{d\mu}\mathbf{Y}_u^{ij}=\frac{1}{16\pi^2}\left[\left\{-\frac{13}{15}g_1^2-3g_2^2-\frac{16}{3}g_3^2+3\,\mathrm{Tr}(\mathbf{Y}_u\mathbf{Y}_u^\dagger)+\mathrm{Tr}(\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger)\right\}\mathbf{Y}_u^{ij}+3(\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u)^{ij}+(\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d)^{ij}\right],$$

$$\mu\frac{d}{d\mu}\mathbf{Y}_d^{ij}=\frac{1}{16\pi^2}\left[\left\{-\frac{7}{15}g_1^2-3g_2^2-\frac{16}{3}g_3^2+3\,\mathrm{Tr}(\mathbf{Y}_d\mathbf{Y}_d^\dagger)+\mathrm{Tr}(\mathbf{Y}_e\mathbf{Y}_e^\dagger)\right\}\mathbf{Y}_d^{ij}+3(\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d)^{ij}+(\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u)^{ij}\right],$$

$$\mu\frac{d}{d\mu}(\mathbf{m}_{\tilde{L}}^2)_i^j=\frac{1}{16\pi^2}\Bigg[(\mathbf{m}_{\tilde{L}}^2\mathbf{Y}_e^\dagger\mathbf{Y}_e+\mathbf{Y}_e^\dagger\mathbf{Y}_e\mathbf{m}_{\tilde{L}}^2)_i^j+(\mathbf{m}_{\tilde{L}}^2\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu+\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{m}_{\tilde{L}}^2)_i^j+2(\mathbf{Y}_e^\dagger\mathbf{m}_{\tilde{e}}^2\mathbf{Y}_e+m_{H_d}^2\mathbf{Y}_e^\dagger\mathbf{Y}_e+\mathbf{A}_e^\dagger\mathbf{A}_e)_i^j$$
$$+2(\mathbf{Y}_\nu^\dagger m_{\tilde{\nu}}^2\mathbf{Y}_\nu+m_{H_u}^2\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu+\mathbf{A}_\nu^\dagger\mathbf{A}_\nu)_i^j-(\frac{6}{5}g_1^2|M_1|^2+6g_2^2|M_2|^2)\,\delta_i^j\Bigg],$$

$$\mu\frac{d}{d\mu}(\mathbf{m}_{\tilde{e}}^2)_j^i=\frac{1}{16\pi^2}\left[2(\mathbf{m}_{\tilde{e}}^2\mathbf{Y}_e\mathbf{Y}_e^\dagger+\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{m}_{\tilde{e}}^2)_j^i+4(\mathbf{Y}_e\mathbf{m}_{\tilde{L}}^2\mathbf{Y}_e^\dagger+m_{H_d}^2\mathbf{Y}_e\mathbf{Y}_e^\dagger+\mathbf{A}_e\mathbf{A}_e^\dagger)_j^i-\frac{24}{5}g_1^2|M_1|^2\,\delta_j^i\right],$$

$$\mu\frac{d}{d\mu}(\mathbf{m}_{\tilde{\nu}}^2)_j^i=\frac{1}{16\pi^2}[2(\mathbf{m}_{\tilde{\nu}}^2\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger+\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{m}_{\tilde{\nu}}^2)_j^i+4(\mathbf{Y}_\nu\mathbf{m}_{\tilde{L}}^2\mathbf{Y}_\nu^\dagger+m_{Hu}^2\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger+\mathbf{A}_\nu\mathbf{A}_\nu^\dagger)_j^i],$$

$$\mu\frac{d}{d\mu}\mathbf{A}_e^{ij}=\frac{1}{16\pi^2}\Bigg[\left\{-\frac{9}{5}g_1^2-3g_2^2+3\,\mathrm{Tr}(\mathbf{Y}_d^\dagger\mathbf{Y}_d)+\mathrm{Tr}(\mathbf{Y}_e^\dagger\mathbf{Y}_e)\right\}\mathbf{A}_e^{ij}+2\Bigg\{-\frac{9}{5}g_1^2M_1-3g_2^2M_2+3\,\mathrm{Tr}(\mathbf{Y}_d^\dagger\mathbf{A}_d)$$
$$+\mathrm{Tr}(\mathbf{Y}_e^\dagger\mathbf{A}_e)\Bigg\}\mathbf{Y}_e^{ij}+4(\mathbf{Y}_e\mathbf{Y}_e^\dagger\mathbf{A}_e)^{ij}+5(\mathbf{A}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e)^{ij}+2(\mathbf{Y}_e\mathbf{Y}_\nu^\dagger\mathbf{A}_\nu)^{ij}+(\mathbf{A}_e\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^{ij}\Bigg],$$

$$\mu\frac{d}{d\mu}\mathbf{A}_\nu^{ij}=\frac{1}{16\pi^2}\Bigg[\left\{-\frac{3}{5}g_1^2-3g_2^2+3\,\mathrm{Tr}(\mathbf{Y}_u^\dagger\mathbf{Y}_u)+\mathrm{Tr}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)\right\}\mathbf{A}_\nu^{ij}+2\Bigg\{-\frac{3}{5}g_1^2M_1-3g_2^2M_2+3\,\mathrm{Tr}(\mathbf{Y}_u^\dagger\mathbf{A}_u)$$
$$+\mathrm{Tr}(\mathbf{Y}_\nu^\dagger\mathbf{A}_\nu)\Bigg\}\mathbf{Y}_\nu^{ij}+4(\mathbf{Y}_\nu\mathbf{Y}_\nu^\dagger\mathbf{A}_\nu)^{ij}+5(\mathbf{A}_\nu\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^{ij}+2(\mathbf{Y}_\nu\mathbf{Y}_e^\dagger\mathbf{A}_e)^{ij}+(\mathbf{A}_\nu\mathbf{Y}_e^\dagger\mathbf{Y}_e)^{ij}\Bigg],$$

$$\mu\frac{d}{d\mu}\mathbf{A}_u^{ij}=\frac{1}{16\pi^2}\Bigg[\left\{-\frac{13}{15}g_1^2-3g_2^2-\frac{16}{3}g_3^2+3\,\mathrm{Tr}(\mathbf{Y}_u^\dagger\mathbf{Y}_u)+\mathrm{Tr}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)\right\}\mathbf{A}_u^{ij}+2\Bigg\{-\frac{13}{15}g_1^2M_1-3g_2^2M_2$$
$$-\frac{16}{3}g_3^2M_3+3\,\mathrm{Tr}(\mathbf{Y}_u^\dagger\mathbf{A}_u)+\mathrm{Tr}(\mathbf{Y}_\nu^\dagger\mathbf{A}_\nu)\Bigg\}\mathbf{Y}_u^{ij}+4(\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{A}_u)^{ij}+5(\mathbf{A}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u)^{ij}+2(\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{A}_d)^{ij}+(\mathbf{A}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d)^{ij}\Bigg],$$

$$\mu\frac{d}{d\mu}\mathbf{A}_d^{ij}=\frac{1}{16\pi^2}\Bigg[\left\{-\frac{7}{15}g_1^2-3g_2^2-\frac{16}{3}g_3^2+3\,\mathrm{Tr}(\mathbf{Y}_d^\dagger\mathbf{Y}_d)+\mathrm{Tr}(\mathbf{Y}_e^\dagger\mathbf{Y}_e)\right\}\mathbf{A}_d^{ij}+2\Bigg\{-\frac{7}{15}g_1^2M_1-3g_2^2M_2-\frac{16}{3}g_3^2M_3$$
$$+3\,\mathrm{Tr}(\mathbf{Y}_d^\dagger\mathbf{A}_d)+\mathrm{Tr}(\mathbf{Y}_e^\dagger\mathbf{A}_e)\Bigg\}\mathbf{Y}_d^{ij}+4(\mathbf{Y}_d\mathbf{Y}_d^\dagger\mathbf{A}_d)^{ij}+5(\mathbf{A}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d)^{ij}+2(\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{A}_u)^{ij}+(\mathbf{A}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u)^{ij}\Bigg],$$

$$\mu\frac{d}{d\mu}(m_{H_u}^2)=\frac{1}{16\pi^2}\Bigg[6\,\mathrm{Tr}[\mathbf{m}_{\tilde{Q}}^2\mathbf{Y}_u^\dagger\mathbf{Y}_u+\mathbf{Y}_u^\dagger(\mathbf{m}_{\tilde{u}}+m_{H_u}^2)\mathbf{Y}_u+\mathbf{A}_u^\dagger\mathbf{A}_u]+2\,\mathrm{Tr}[\mathbf{m}_{\tilde{L}}^2\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$$
$$+\mathbf{Y}_\nu^\dagger(\mathbf{m}_{\tilde{\nu}}^2+m_{H_u}^2)\mathbf{Y}_\nu+\mathbf{A}_\nu^\dagger\mathbf{A}_\nu]-\left(\frac{6}{5}g_1^2|M_1|^2+6g_2^2|M_2|^2\right)\Bigg],$$

$$\mu\frac{d}{d\mu}(m_{H_d}^2)=\frac{1}{16\pi^2}\Bigg[6\,\mathrm{Tr}[\mathbf{m}_{\tilde{Q}}^2\mathbf{Y}_d^\dagger\mathbf{Y}_d+\mathbf{Y}_d^\dagger(\mathbf{m}_{\tilde{d}}+m_{H_d}^2)\mathbf{Y}_d+\mathbf{A}_u^\dagger\mathbf{A}_d]$$
$$+2\,\mathrm{Tr}[m_{\tilde{L}}^2\mathbf{Y}_e^\dagger\mathbf{Y}_e+\mathbf{Y}_e^\dagger(\mathbf{m}_{\tilde{e}}^2+m_{H_d}^2)\mathbf{Y}_e+\mathbf{A}_e^\dagger\mathbf{A}_e]-\left(\frac{6}{5}g_1^2|M_1|^2+6g_2^2|M_2|^2\right)\Bigg].$$

APPENDIX C: NOTATIONS AND CONVENTIONS IN THE MSSM

1. Mass matrix and mixings

In this appendix, we give our notation for SUSY particle masses and mixings in our calculation.

The slepton mass $\hat{\mathbf{M}}^2$ term is

$$(\tilde{e}_L^\dagger, \tilde{e}_R^\dagger)\begin{pmatrix} \mathbf{m}_L^2 & \mathbf{m}_{LR}^{2\,\mathrm{T}} \\ \mathbf{m}_{LR}^2 & \mathbf{m}_R^2 \end{pmatrix}\begin{pmatrix} \tilde{e}_L \\ \tilde{e}_R \end{pmatrix}, \tag{C1}$$

with

$$(\mathbf{m}_L^2)_{ij} = (\mathbf{m}_{\tilde{L}}^2)_{ij} + m_{e_i}^2\delta_{ij} + m_Z^2\delta_{ij}\cos 2\beta\left(-\frac{1}{2}+\sin^2\theta_{\mathrm{W}}\right), \tag{C2}$$

$$(\mathbf{m}_R^2)_{ij} = (\mathbf{m}_{\tilde{e}_R})_{ij} + m_{e_i}^2\delta_{ij} - m_Z^2\delta_{ij}\cos 2\beta\,\sin^2\theta_{\mathrm{W}}, \tag{C3}$$

$$(\mathbf{m}_{LR}^2)_{ij} = \frac{\mathbf{A}_{ij}^e v\cos\beta}{\sqrt{2}} - m_{e_i}\mu\tan\beta, \tag{C4}$$

where $(\mathbf{m}_L^2)_{ij}$ and $(\mathbf{m}_R^2)_{ij}$ are 3×3 matrices. The slepton mass matrix can be diagonalized as

$$U^f\hat{\mathbf{M}}^2 U^{f\mathrm{T}} = (\mathrm{diagonal}), \tag{C5}$$

where U^f is a real orthogonal 6×6 matrix.

The chargino mass term is

$$-\mathcal{L} = (\overline{\tilde{W}_R^-}, \overline{\tilde{H}_{2R}^-}) \times \begin{pmatrix} M_2 & \sqrt{2}m_{\mathrm{W}}\cos\beta \\ \sqrt{2}m_{\mathrm{W}}\sin\beta & \mu \end{pmatrix}\begin{pmatrix} \tilde{W}_L^- \\ \tilde{H}_{1L}^- \end{pmatrix} + \mathrm{H.c.} \tag{C6}$$

The chargino mass matrix can be diagonalized as

$$O_R M_C O_L^{\mathrm{T}} = \mathrm{diag}(M_{\tilde{\chi}_1^-}, M_{\tilde{\chi}_2^-}), \tag{C7}$$

where O_L and O_R are real orthogonal 2×2 matrices. The mass eigenstates $\tilde{\chi}_{AL}$ and $\tilde{\chi}_{AR}$ $(A=1,2)$ are

$$\begin{pmatrix} \tilde{\chi}_{1L}^- \\ \tilde{\chi}_{2L}^- \end{pmatrix} = O_L\begin{pmatrix} \tilde{W}_L^- \\ \tilde{H}_{1L}^- \end{pmatrix}, \quad \begin{pmatrix} \tilde{\chi}_{1R}^- \\ \tilde{\chi}_{2R}^- \end{pmatrix} = O_R\begin{pmatrix} \tilde{W}_R^- \\ \tilde{H}_{2R}^- \end{pmatrix}, \tag{C8}$$

and

$$\tilde{\chi}_A^- = \tilde{\chi}_{AL}^- + \tilde{\chi}_{AR}^- \quad (A=1,2) \tag{C9}$$

forms the Dirac fermion with mass $M_{\tilde{\chi}_A^-}$.

The neutralino mass term is

$$-\mathcal{L} = \frac{1}{2}(\tilde{B}_L, \tilde{W}_L^0, \tilde{H}_{1L}^0, \tilde{H}_{2L}^0) M_N \begin{pmatrix} \tilde{B}_L \\ \tilde{W}_L^0 \\ \tilde{H}_{1L}^0 \\ \tilde{H}_{2L}^0 \end{pmatrix} + \mathrm{H.c.}, \tag{C10}$$

where

$$M_N = \begin{pmatrix} M_1 & 0 & -m_Z\sin\theta_{\mathrm{W}}\cos\beta & m_Z\sin\theta_{\mathrm{W}}\sin\beta \\ 0 & M_2 & m_Z\cos\theta_{\mathrm{W}}\cos\beta & -m_Z\cos\theta_{\mathrm{W}}\sin\beta \\ -m_Z\sin\theta_{\mathrm{W}}\cos\beta & m_Z\cos\theta_{\mathrm{W}}\cos\beta & 0 & -\mu \\ m_Z\sin\theta_{\mathrm{W}}\sin\beta & -m_Z\cos\theta_{\mathrm{W}}\sin\beta & -\mu & 0 \end{pmatrix}. \tag{C11}$$

The neutralino mass matrix can be diagonalized as

$$O_N M_N O_N^{\mathrm{T}} = (\mathrm{diagonal}), \tag{C12}$$

where O_N is a real 4×4 orthogonal matrix. The mass eigenstate is

$$\tilde{\chi}_{AL}^0 = (O_N)_{AB}\tilde{\chi}_{BL}^0 (A,B=1,\ldots,4),$$

$$\tilde{\chi}_{BL}^0 = (\tilde{B}_{\mathrm{L}}, \tilde{W}_{\mathrm{L}}^0, \tilde{H}_{1\mathrm{L}}^0, \tilde{H}_{2\mathrm{L}}^0), \tag{C13}$$

and

$$\tilde{\chi}_A^0 = \tilde{\chi}_{AL}^0 + \tilde{\chi}_{AR}^0 \quad (A=1,\ldots,4) \tag{C14}$$

forms a Majorana spinor with mass $M_{\tilde{\chi}_A^0}$.

The chargino vertex functions are

$$C^{R(l)}_{eAX} = -g_2 (O_R)_{A1} U^{\nu}_{X,1}, \tag{C15}$$

$$C^{L(l)}_{eAX} = g_2 \frac{m_e}{\sqrt{2} m_{\mathrm{W}} \cos\beta} (O_L)_{A2} U^{\nu}_{X,1}, \tag{C16}$$

and the neutralino vertex functions are

$$N^{R(l)}_{eAX} = -\frac{g_2}{\sqrt{2}} \left\{ [-(O_N)_{A2} - (O_N)_{A1} \tan\theta_{\mathrm{W}}] U^{l}_{X,1} + \frac{m_e}{m_{\mathrm{W}} \cos\beta} (O_N)_{A3} U_{X,4} \right\}, \tag{C17}$$

$$N^{L(l)}_{eAX} = -\frac{g_2}{\sqrt{2}} \left\{ \frac{m_e}{m_{\mathrm{W}} \cos\beta} (O_N)_{A3} U_{X,1} - 2 (O_N)_{A1} \tan\theta_{\mathrm{W}} U^{l}_{X,4} \right\}. \tag{C18}$$

2. Decay amplitudes $A^{L,R}$

For the amplitudes $A^{L,R}$, there are contributions of the chargino loop and the neutralino loop:

$$A^{L,R} = A^{(c)L,R} + A^{(n)L,R}. \tag{C19}$$

The contributions from the chargino loop are

$$A^{(c)L} = -\frac{1}{32\pi^2} \frac{1}{m^2_{\tilde{\nu}_X}} \left[C^{L(l)}_{jAX} C^{L(l)*}_{iAX} \frac{1}{6(1-x_{AX})^4} \times (2 + 3x_{AX} - 6x^2_{AX} + x^3_{AX} + 6x_{AX} \ln x_{AX}) + C^{L(l)}_{jAX} C^{R(l)*}_{iAX} \frac{M_{\tilde{\chi}^-_A}}{m_j} \frac{1}{(1-x_{AX})^3} \times (-3 + 4x_{AX} - x^2_{AX} - 2\ln x_{AX}) \right], \tag{C20}$$

$$A^{(c)R} = A^{(c)L}|_{L \leftrightarrow R}, \tag{C21}$$

where x_{AX} is defined as

$$x_{AX} = \frac{M^2_{\tilde{\chi}^-_A}}{m^2_{\tilde{\nu}_X}}. \tag{C22}$$

Here $m_{\tilde{\nu}_X}$ is the sneutrino mass and $M_{\tilde{\chi}^-_A}$ is the chargino mass. The contributions from the neutralino loop are

$$A^{(n)L} = \frac{1}{32\pi^2} \frac{1}{m^2_{\tilde{l}_X}} \left[N^{L(l)}_{jAX} N^{L(l)*}_{iAX} \frac{1}{6(1-y_{AX})^4} \times (1 - 6y_{AX} + 3y^2_{AX} + 2y^3_{AX} - 6y^2_{AX} \ln y_{AX}) + N^{L(l)}_{jAX} N^{R(l)*}_{iAX} \frac{M_{\tilde{\chi}^0_A}}{m_i} \frac{1}{(1-y_{AX})^3} \times (1 + y^2_{AX} + 2y_{AX} \ln y_{AX}) \right], \tag{C23}$$

$$A^{(n)R} = A^{(n)L}|_{L \leftrightarrow R}, \tag{C24}$$

where y_{AX} is defined as

$$y_{AX} = \frac{M^2_{\tilde{\chi}^0_A}}{m^2_{\tilde{\ell}_X}}. \tag{C25}$$

Here $m_{\tilde{l}_X}$ is the charged slepton mass and $M_{\tilde{\chi}^0_A}$ is the neutralino mass.

APPENDIX D: ANOMALOUS U(1) FLAVOR SYMMETRY

We review the anomalous U(1) flavor symmetry [34] which is utilized in the Shafi-Tavartkiladze model [30] discussed in Sec. IV. The anomalous U(1) flavor symmetry can arise from string theory. The cancellation of this anomaly is due to the Green-Schwarz mechanism [36]. The associated Fayet-Iliopoulos term is given by [37]

$$\xi \int d^4\theta V_A \quad \text{with} \quad \xi = \frac{g^2_A M^2_{\mathrm{pl}}}{192\pi^2} \mathrm{Tr}\, Q. \tag{D1}$$

The D term is given by

$$\frac{g^2_A}{8} D^2_A = \frac{g^2_A}{8} \left(\sum Q_a |\varphi_a|^2 + \xi \right)^2, \tag{D2}$$

where Q_a is the "anomalous" charge of φ_a. For U(1) breaking, we introduce the singlet field S under the SM gauge group with U(1) charge Q_S. Assuming $\mathrm{Tr}\, Q > 0$, we can ensure the cancellation of D_A in Eq. (D2). Taking $Q_S = -1$, we can ensure the nonzero VEV of S, $\langle S \rangle$, which is given as $\langle S \rangle = \sqrt{\xi}$.

Due to the Froggatt-Nielsen mechanism, the Yukawa interaction term in the effective theory is given by

$$e^c_{Rj} L_i H_d \left(\frac{S}{M_{\mathrm{pl}}} \right)^{m_{ij}}, \tag{D3}$$

where e^c_{Rj} and L_i are the right-handed charged lepton and left-handed lepton doublet, respectively, H_d is the Higgs doublet, and S is a singlet field. The effective Yukawa couplings are given in terms of

$$\lambda \equiv \frac{\langle S \rangle}{M_{\mathrm{pl}}}. \tag{D4}$$

In order to make the interaction term Eq. (D3) neutral, Shafi and Tavartkiladze assigned U(1) flavor charges as follows:

$$Q_{\mathrm{L}_1} = k+n, \quad Q_{\mathrm{L}_2} = Q_{\mathrm{L}_3} = k,$$
$$Q_{\mathrm{N}_1} = -Q_{\mathrm{N}_2} = k+k',$$
$$Q_{H_u} = Q_{H_d} = 0, \quad Q_S = -1, \tag{D5}$$

where $k,n,k' > 0, n \geqslant k'$. Thus they obtained

$$\mathbf{Y}_\nu = \begin{pmatrix} \lambda^{2k+n+k'} & \lambda^{2k+k'} & \lambda^{2k+k'} \\ \lambda^{n-k} & 0 & 0 \end{pmatrix}, \tag{D6}$$

and

$$\mathbf{M_R} = M_R \begin{pmatrix} \lambda^{2k+2k'} & 1 \\ 1 & 0 \end{pmatrix}. \tag{D7}$$

In conclusion, the neutrino mass matrix is given by

$$\mathbf{m}_\nu = \mathbf{Y}_\nu^{\mathrm{T}} \mathbf{M_R}^{-1} \mathbf{Y}_\nu v_u^2 = \frac{\lambda^{2k+n} v_u^2}{M_R} \begin{pmatrix} \lambda^n & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \tag{D8}$$

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